

# Exit

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*In a declining industry, shrinking demand creates pressure for capacity to be reduced. Who exits first? There is a unique perfect equilibrium for firms with asymmetric market shares and identical unit costs in which survivability is inversely related to size: the largest firm is the first to leave and the smallest firm the last. The intuitive reason is that a small firm can profitably "hang on" longer than a large firm. Sufficiently strong scale economies can, by conferring cost advantages on larger firms, reverse this outcome. Numerical examples, however, suggest that the required cost advantage for large firms to outlast smaller ones may be surprisingly substantial.*

## 1. Introduction

■ Economists have lavished considerable efforts on understanding oligopolistic interactions in growing markets. Contracting ones, by contrast, have commanded only limited attention (Friedman, 1979; Fudenberg and Tirole, 1983). Shrinkage and exit may be dismal phenomena, but the dismal state of smokestack sectors such as chemicals, fibers, and steel suggests that they are important ones. A better understanding of the impact of decline on market structure is a task of some urgency.

Our article aims to enhance that understanding by examining the exit decisions of oligopolists with asymmetric market shares. We focus on how relative size affects the order of exit. The key proposition advanced is that survivability is inversely related to size: smaller firms have, *ceteris paribus*, lower incentives to reduce capacity in declining industries than do their larger competitors.

We prove this proposition in the context of a model that is stylized to highlight the exit decision. The model explores Cournot competition in a declining market.<sup>1</sup> Since competitors in such markets are typically well acquainted, their capacities and production costs are assumed to be common knowledge. As demand shrinks, capacity must be reduced or eliminated to maintain profitability. At each instant, active firms make a single, dichotomous

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<sup>1</sup> Our results also carry over to Bertrand competition. We assume that capacity constraints are binding, and thus the Bertrand and Cournot equilibria coincide; see Kreps and Scheinkman (1983).

decision about whether to continue to operate at their initial capacity levels or to exit the market *in toto*.

The Cournot-Nash equilibrium concept does not generate a unique outcome to this exit game because it effectively allows firms to precommit to production. Since each firm takes the others' strategies as given, large firms can threaten to remain active and thereby force small firms to leave first; the reverse is equally possible.<sup>2</sup> But the outcome in which larger firms leave before smaller ones emerges as the sole equilibrium if we impose the requirement of subgame perfection (Selten, 1975; Kreps and Wilson, 1982). This condition dictates that each player's strategy be an optimal response to other players' strategies and that this optimality be continuously reevaluated as the game proceeds. Using this requirement, we work backwards in time to show that a small firm, because of its longer tenure as a profitable monopolist, can credibly promise to produce over a longer period than its larger rivals and thus can induce earlier exit by them.

While this outcome may appear counterintuitive, it is supported by case studies. In the declining U.K. steel castings industry, for instance, the foundries closed between 1975 and 1981 included some of the industry's largest and most efficient ones (Fuller and Hill, 1984). The machine tool sector provides another example; faced with declining demand, Cleveland Twist Drill's management recognized that its ability to survive would have been improved had it operated a small plant rather than the industry's largest (Hamermesh, 1983). Indeed, the literature on business policy stresses the advantage of small plants over large ones in declining environments (Hall, 1980).

Fudenberg and Tirole (1983) apply the same equilibrium concept that we use to a related but slightly different economic problem. They formulate the exit game as one of incomplete information in which each firm is uncertain about its rival's costs. By assuming a positive probability that no exit will be required, they arrive at a unique equilibrium in which the most efficient firm outlasts its rival. This seems to be a good representation of an immature industry with low sunk costs. By contrast, our model may better depict a declining industry in which substantial resource commitments have already been made and information about competitors' costs has been disseminated.

The exit decision is more closely related to recent models of patent races: continuing to produce is like continuing to do research; exit is comparable to abandoning research. The simplifying assumptions are also similar: production has only two speeds—on or off. Fudenberg *et al.* (1983) show that the firm that is first able to commit credibly to continued research even under the pessimistic assumption of continued competition can force its rival to exit. The analogy for a declining industry is that eventually, only a small firm will be profitable. This eventual profitability provides the credible commitment mechanism that enables the small firm to force the large firm to exit first.

Our results are simplified by, but do not depend on, the assumption of all-or-nothing exit decisions. When firms can reduce capacity (and the associated cost flow) almost continuously, the larger firm reduces capacity alone until its size equals that of its (formerly) smaller competitor. Once parity is reached, the firms shrink at the same pace. The formal model and proof for the "almost continuous" case are provided in Appendix B of Ghemawat and Nalebuff (1985).

In Section 2 we present our model of duopolists who face dichotomous exit decisions. These results are extended in Section 3 to an oligopoly. Section 4 generalizes the duopoly model to allow differences in costs. The conclusions are summarized in Section 5.

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<sup>2</sup> The war of attrition literature provides important insights into the solution of our model (Hammerstein and Parker, 1983). Typically, there is a problem due to the multiplicity of equilibria (Nalebuff and Riley, 1985). Wilson (1983) demonstrates that most (and sometimes all) equilibria disappear when the game is truncated at some finite time; in a declining industry, all profitability is eventually exhausted, and this is what leads to a unique outcome.

## 2. The duopoly model

■ Consider an undifferentiated market that is shrinking for exogenous reasons. Let  $p$  denote price,  $q$  stand for industry output, and  $t$  index time, and assume that for all dates  $t \in (0, \infty)$ ,

$$\frac{\partial p(q, t)}{\partial t} < 0, \quad (1a)$$

$$\lim_{t \rightarrow \infty} p(q, t) = 0, \quad (1b)$$

$$\frac{\partial p(q, t)}{\partial q} < 0, \quad (1c)$$

$$\frac{\partial p(q, t)q}{\partial q} > 0. \quad (1d)$$

Condition (1a) requires that the price decline smoothly over time. Condition (1b) dictates that demand at any price asymptotically approach 0; this assumption is stronger than needed, but is made for simplicity. Finally, condition (1d) implies that marginal revenue is always positive; again, with the assumed structure of the exit decision, this is not necessary, but it does help us avoid getting bogged down in the mechanics of short-run output adjustment. These requirements are satisfied, for example, by suitably parameterized isoelastic demand functions of the form used in Section 4.

At time 0, two firms, indexed by  $i = 1, 2$ , serve this market. Firm  $i$ 's capacity is  $K_i$ :  $K_1$  is greater than  $K_2$ .<sup>3</sup> Both firms' cost flows per unit of capacity are identical and equal to  $c$ , and this is common knowledge. To keep the firm operating imposes a cost flow proportional to capacity. Neither firm incurs any other operating costs, so that, given our assumption of positive marginal revenue, available capacity is always fully used. The capacity cost,  $cK_i$ , can be avoided only by exit. Note that  $cK_i$  is *not* a sunk cost. Firms can either pay at the rate of  $cK_i$  and operate or not pay and shut down.

This cost structure is stylized to highlight the fact that in many declining industries, the marginal cost of producing up to historic capacity levels is low; fixed costs dominate and thus result in tremendous pressures to fill up capacity. The usual reason is that the scale of operation tends to be inflexible. Continuous production processes illustrate this particularly well. In alumina refining, for instance, operating below full capacity utilization is technically inefficient in that it alters product characteristics. Operating below 70% utilization levels is infeasible because a minimal amount of chemicals has to be fed through the machinery to keep it running. A strategy of frequently shutting down and starting up is not a viable option either. After each shut-down, corrosive chemicals accumulate in the machinery (e.g., caustic soda in the digestors). Start-up requires cleaning out the machinery and readjusting and "debottlenecking" it—a process that sometimes takes over a year (Wells, 1984). In effect, then, alumina refineries face a choice between operating at close to capacity and shutting down permanently.

At the beginning of the planning horizon ( $t = 0$ ), both firms are just breaking even:<sup>4</sup>

$$p(K_1 + K_2, 0) = c. \quad (2)$$

<sup>3</sup> In the special case where both firms have identical capacities and costs, their exit times as monopolists are also equal ( $t^* = t_1^* = t_2^*$ ). In addition to the two pure strategy solutions, this special case also yields a continuum of mixed strategy equilibria. The existence of a multiplicity of mixed strategy equilibria indexed by the toughness of the two firms is a knife-edge result; any perturbation of the problem that results in a difference between  $t_1^*$  and  $t_2^*$  leads to a unique perfect equilibrium in pure strategies. For further discussion of this point, see Ghemawat and Nalebuff (1985) and Nalebuff and Riley (1985).

<sup>4</sup> Since exit *never* occurs before the date at which price first hits cost, we restrict our analysis to this domain without affecting its generality.

Over time, as demand shrinks, pressures will build up for this capacity to be eliminated. At each instant, a firm that is still active in the market chooses between continuing to operate at its initial capacity level and exiting costlessly and completely from the market.<sup>5</sup> Entry by other firms is precluded by set-up costs. Reentry is not allowed. Even if reentry were possible, it would not occur in equilibrium except as noted in footnote 9 of Section 4.

To identify equilibria let us be more explicit about the strategy space for the two firms. The strategies of firms are simply a probability distribution of exit times. These strategies are specified as a function of both time and the other firm's participation. Let  $D_i(t, i + j)$  be firm  $i$ 's action at time  $t$  conditional on the presence of both firm  $i$  and firm  $j$ :  $D_i(t, i + j) = 0$  denotes continuing to produce  $K_i$ ;  $D_i(t, i + j) = 1$  denotes the decision to exit at time  $t$ .

Given this framework, what are the Nash equilibrium exit dates for each firm? Define the dates  $t_1^*$  and  $t_2^*$  such that

$$p(K_i, t_i^*) = c, \quad i = 1, 2. \quad (3)$$

From the first-order conditions for profit maximization, firm  $i$ , if alone in the market, would choose to continue to operate until  $t_i^*$ . Left alone, the smaller firm operates longer:  $t_2^*$  is greater than  $t_1^*$ .

Let  $z$  be the last date such that *both* firms are serving the market.  $C_i(z, t_0)$  denotes firm  $i$ 's nonpositive profits (i.e., losses), discounted back to an arbitrary date,  $t_0$ , from operating over the period  $[t_0, z]$ .  $V_i(z, t_0)$  represents firm  $i$ 's discounted profits from operation after its rival exits at  $z$ . If  $z$  is less than  $t_i^*$  and the discount rate is constant at  $r$ , we have

$$C_i(z, t_0) = \int_{t_0}^z [p(K_1 + K_2, t) - c]K_i e^{-r(t-t_0)} dt, \quad (4a)$$

$$V_i(z, t_0) = \int_z^{t_i^*} [p(K_i, t) - c]K_i e^{-r(t-t_0)} dt. \quad (4b)$$

When  $z$  is greater than or equal to  $t_i^*$ ,  $V_i(z, t_0)$  equals zero.

If firm  $i$  outlasts firm  $j$ , then their respective profits over the time horizon are

$$P_i(z, 0) \equiv V_i(z, 0) + C_i(z, 0), \quad (5a)$$

$$P_j(z, 0) \equiv C_j(z, 0) \leq 0. \quad (5b)$$

The fact that only one firm can make money after time 0 combined with firm  $i$ 's desire to exit, at the very latest, by time  $t_i^*$ , leads to only two possible Nash equilibria. They result from the following pure strategies:

$$D_i(t, 1 + 2) = \begin{cases} 0 & \forall t < t_i^* \\ 1 & \forall t \geq t_i^* \end{cases}, \quad i = 1, 2;$$

$$D_j(t, 1 + 2) = 1 \quad \forall t \geq 0, \quad j \neq i.$$

In words, either firm 1 exits at time 0 and firm 2 at time  $t_2^*$  or firm 2 exits at time 0 and firm 1 at time  $t_1^*$ . The second solution is a Nash equilibrium only if  $P_2(t_1^*, 0)$  is less than or equal to 0. Otherwise, firm 2 finds it profitable to suffer losses up to time  $t_1^*$  to garner monopoly profits from  $t_1^*$  to  $t_2^*$ . This qualification does not apply to the first solution because the larger firm, firm 1, can only expect to lose money if it outlasts firm 2 past time  $t_2^*$ .

No other Nash equilibria exist for firms using either pure or mixed strategies. Consider the options for firm 2 when firm 1 uses a pure strategy. Firm 1 chooses a persistence time,

<sup>5</sup> For a treatment of the case where exit costs are positive, see Ghemawat (1984). If they are not so high as to make staying in the market forever the dominant strategy, they cannot affect the equilibrium order of exit unless one firm suffers disproportionately greater exit costs.

$z_1$ , such that  $D_i(t, 1 + 2) = 0 \Leftrightarrow t < z_i$ . Firm 2 will then either concede or hold. If waiting until  $z_1$  is not worthwhile, firm 2 minimizes its losses by dropping out immediately. Alternatively, if waiting past  $z_1$  can be justified ( $P_2(z_1, 0) > 0$ ), firm 2 anticipates eventually being alone in the market; its optimal exit time is then  $t_2^*$ .<sup>6</sup> Given that firm 2's only Nash strategies are exiting at either 0 or  $t_2^*$ , firm 1's optimal exit time will correspondingly be either  $t_1^*$  or 0. These are the two proposed Nash equilibria in pure strategies. The possibility that both firms could adopt mixed strategies remains open. The impossibility of mixed strategy solutions is shown later in the proof of Proposition 1.

The two Nash equilibria are not equally appealing. Firm 2 is the "stronger" firm in that left alone, it remains profitable longer than firm 1. As a result, the Nash equilibrium which has firm 2 exiting immediately is not a perfect equilibrium. If firm 2 "trembles" and somehow fails to leave immediately, firm 1's plan to remain until  $t_1^*$  is no longer optimal.

*Proposition 1.* The only perfect equilibrium is the one in which firm 1 exits immediately and firm 2 remains until  $t_2^*$ , i.e.,

$$\begin{aligned} D_1(t, 1 + 2) &= 1 & \forall t > 0 \\ D_2(t, 1 + 2) &= 0 & \forall t \in [0, t_2^*]. \end{aligned}$$

*Proof.* The argument is illustrated in Figure 1, where the ordinate measures instantaneous profits and the pattern of shading and overlay follows the first two steps in the proof. Define the time  $x_A$  to be the first date such that firm 2 is willing to sustain losses over the period  $[x_A, t_1^*)$  to reap monopoly profits over  $[t_1^*, t_2^*)$ :

$$P_2(t_1^*, x_A) = V_2(t_1^*, x_A) + C_2(t_1^*, x_A) \equiv 0. \tag{6}$$

Firm 1 must exit by time  $t_1^*$ : that is the date after which its operating profits are negative, even under the optimistic assumption that it manages to monopolize the market. Therefore, the most pessimistic assumption that firm 2 can make is that firm 1 remains active until time  $t_1^*$ . Even with this pessimistic belief, firm 2, upon surviving until time  $x_A$ , will then choose to remain active up to time  $t_2^*$ . Hence, if firm 2 somehow manages to stay in the market until time  $x_A$ , firm 1 can make no credible threat to prevent firm 2 from "surviving" and staying on until  $t_2^*$ . Since the continued operation of both firms implies losses for firm 1 over  $[x_A, t_1^*)$ , firm 1's optimal decision on reaching time  $x_A$  is to exit immediately:

$$D_1(t, 1 + 2) = 1 \quad \forall t \in [x_A, \infty). \tag{7}$$

The argument proceeds by recursion. Firm 2 is assured of monopoly profits after time  $x_A$ . Calculate the earliest date  $x_B$  such that firm 2 is willing to sustain losses over the period  $[x_B, x_A)$  to garner monopoly profits over  $[x_A, t_2^*)$ :

$$P_2(x_A, x_B) = V_2(x_A, x_B) + C_2(x_A, x_B) \equiv 0. \tag{8}$$

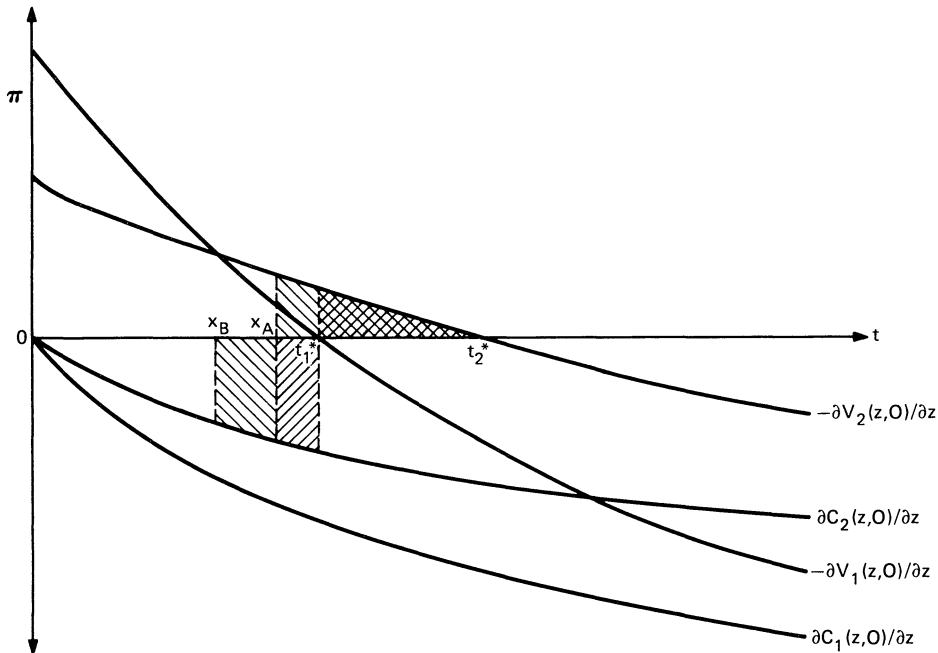
Since firm 2 finds it optimal to remain in the market until  $t_2^*$  upon surviving until time  $x_B$ , firm 1 will find it optimal to exit immediately upon reaching time  $x_B$ . Now, calculate  $x_C$  using the fact that at the very latest, firm 1 must exit by time  $x_B$ . Note that  $x_C$  is a *larger* step back from  $x_B$  than  $x_B$  was from  $x_A$ ; the steps are moving towards the origin at an increasing rate because with each successive step backwards, the total remaining monopoly profits are greater, and because the offsetting duopoly losses are occurring earlier and thus are smaller.<sup>7</sup> Hence, this process of backward induction continues until  $P_2(x_n, 0)$  is greater

<sup>6</sup> There is also the possibility of exact indifference,  $P_2(z, 0) = 0$ . In this case firm 2 can randomize between leaving at time 0 and time  $t_2^*$ . Firm 1's best response to this strategy is to exit at an arbitrarily small time,  $\epsilon$ . This contradicts the assumption of firm 2's indifference as  $P_2(\epsilon, 0) > 0$ .

<sup>7</sup> This dominance argument is independent of the discount rate. If any step backwards ( $\Delta$ ) were to be smaller than the previous one, then a comparison of the intervals shows that

$$C_2'(x - \Delta + \delta, x - \Delta) > C_2'(x + \delta, x) \quad \forall \delta \in (0, \Delta),$$

FIGURE 1  
DUOPOLISTS' PROFITS



than or equal to zero. When this inequality is satisfied, firm 2 can credibly commit to staying on over  $[0, t_2^*]$ , and firm 1 finds it optimal to exit at time 0.

Note that a similar argument eliminates all mixed-strategy Nash equilibria. Over  $[x_A, t_1^*)$  firm 1 will not randomize its exit decision; firm 2 stays in with probability 1. Thus, firm 1 will choose to exit with probability 1 by the time  $x_A$ . One can use this argument to work backwards to time  $x_B$  and eventually to time 0. *Q.E.D.*

The order of exit in the presence of capital constraints can be deduced from a similar argument.<sup>8</sup> If firms are restricted in their ability to borrow against future profits, then the magnitude of potential losses becomes important. A capital constraint limits each firm's ability to sustain losses. We have already argued that exit must occur before the minimum of  $(t_1^*, t_2^*)$ : it will occur even sooner if one of the capital constraints becomes binding first. In this case the backwards induction starts at the date when duopoly losses result in the first binding capital constraint. As in Proposition 1, the perfection criterion shows that whichever firm can credibly sustain losses longest is able to force its rival to exit immediately.

The possibility of mergers or takeovers introduces additional considerations into firms' exit decisions. Imagine that total profits are larger if the small firm exits first; yet, when acting separately, the small firm forces the larger one to exit first. This provides an incentive for the larger firm to buy out and then to retire its smaller rival. The radio tube industry provides an example. In 1975 GTE Sylvania purchased the plant and equipment of a smaller competitor, Union Electric, loaded the assets on a barge, and sank them in the Pacific Ocean (Harrigan and Porter, 1979).

Similar analytical approaches can be applied to other extensions of the exit model. Interesting possibilities include inventorying and, on the demand side, cyclicalities, uncertainty, and stabilization of output at positive levels.

where  $C_2$  indicates the instantaneous duopoly profits for firm 2. Integration then implies that the present value of duopoly losses is smaller in the earlier interval. But we already know that earlier profits are larger. Therefore,  $\Delta$  should be larger, not smaller than the previous step backwards.

<sup>8</sup> We are grateful to Douglas Gale for suggesting this point. Note that any explanation for the presence of capital constraints is outside the scope of this article; yet, they have been reported to exist.

### 3. Oligopoly

■ The arguments used to prove Proposition 1 can also be extended inductively to solve for the optimal exit decisions of  $N$  oligopolists. The last firm to exit is the one with the longest profitable tenure as a monopolist; the final two firms are the duopoly pair with the longest profitability, and so on.

Let firm  $i$  have capacity  $K_i$  and order the firms so that  $K_1 > K_2 > \dots > K_N$ . Take unit capacity costs to be equal to  $c$  for all firms. Define  $\hat{t}_i$  to be the time when an oligopoly consisting of firms  $(i, i + 1, \dots, N)$  begins to lose money:

$$p\left(\sum_{j=i}^N K_j, \hat{t}_i\right) - c \equiv 0. \tag{9}$$

Given conditions (1a)–(1d), we can easily determine that  $\hat{t}_1 < \hat{t}_2 < \dots < \hat{t}_N < \infty$ .

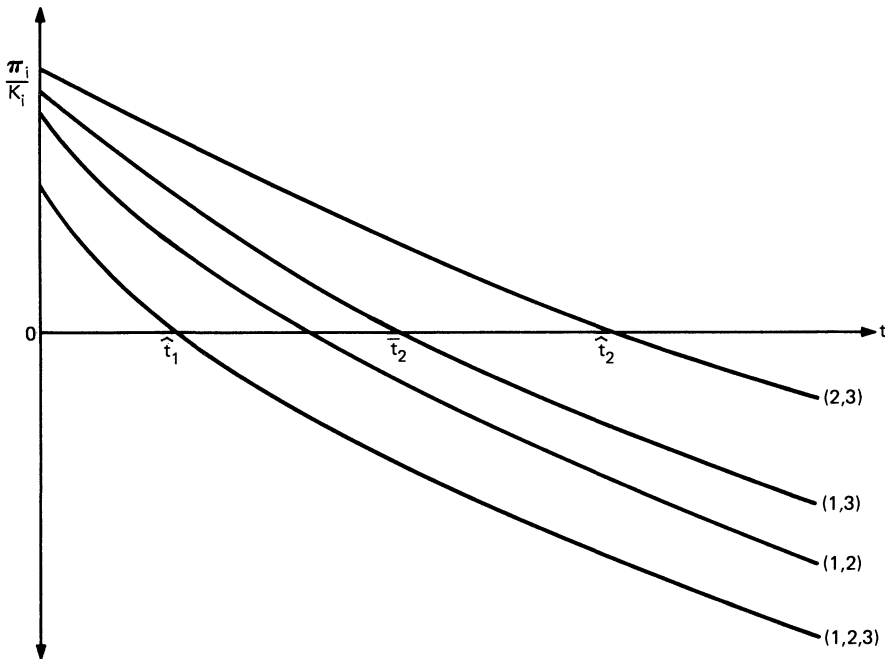
*Proposition 2.* Under the assumptions made above, there exists a unique perfect equilibrium in which the exit sequence corresponds exactly to firm size: firm  $i$  exits at time  $\hat{t}_i$ .

*Proof.* We sketch the inductive argument used in proving the proposition when  $N = 3$ . At the outset define  $\bar{t}_2$  to be the date at which firms 1 and 3 begin to incur losses even as a duopoly (see Figure 2). Note that  $\bar{t}_2 < \hat{t}_2$  because firms 1 and 3 have a greater total capacity than firms 2 and 3.

No exit will occur before time  $\hat{t}_1$  because all firms are profitable. Firm 1 can earn a profit after time  $\hat{t}_1$  if and only if one of its competitors leaves before time  $\bar{t}_2$ . Past  $\bar{t}_2$  firm 1 will be the larger firm in a duopoly incurring losses; Proposition 1 implies that firm 1 will then exit immediately.

Hence, if all three firms are present at time  $\bar{t}_2$ , firm 1 cannot expect to earn profits and will choose to leave. Since firms 2 and 3 can thereby count on firm 1's withdrawal by time  $\bar{t}_2$ , there is some prior time,  $x_A$ , after which firms 2 and 3, attracted by duopoly profits over

FIGURE 2  
OLIGOPOLISTS' PROFITS



$(\bar{t}_2, \hat{t}_2)$ , would be willing to continue producing, even if firm 1 is still active. Sequential rationality then dictates that firm 1 exit immediately on reaching time  $x_A$ , for it can never hope to be profitable again.

Using  $x_A$  as a fulcrum, calculate a time  $x_B$  such that firms 2 and 3 are willing to shoulder losses over  $[x_B, x_A)$  to earn profits over  $[x_A, \hat{t}_2)$ . Firm 1 now finds it optimal to exit as soon as it reaches time  $x_B < x_A$ . Repetition of this argument leads to firm 1's leaving earlier and earlier until time  $t_1$  is reached. At the first moment it incurs losses ( $\hat{t}_1$ ), firm 1 exits. Note that once this happens, firms 2 and 3 enjoy positive profits until time  $t_2$  at which date, by Proposition 1, firm 2 exits. *Q.E.D.*

#### 4. Cost differences

■ Firms rarely have exactly identical costs. If the smaller firm's unit cost ( $c_2$ ) is lower than the larger firm's ( $c_1$ ), then the arguments used to prove Proposition 1 hold *a fortiori*. But if production is subject to economies of scale,  $c_1$  will be less than  $c_2$ , and Proposition 1 may no longer hold.

It is not surprising that if a large firm has a great enough cost advantage, it will be able to outlast a smaller, less efficient competitor. As seen in the example below, however, the cost advantage required by the larger firm to overturn Proposition 1 can be surprisingly large.

Begin the planning horizon so that at  $t = 0$  both firms are profitable:

$$p(K_1 + K_2, 0) > c_i, \quad i = 1, 2. \tag{10}$$

Define  $t'_i$  to be the date after which firm  $i$  incurs losses as a duopolist:

$$p(K_1 + K_2, t'_i) \equiv c_i, \quad i = 1, 2. \tag{11}$$

Given our assumption that  $c_1 < c_2$ , it follows that  $t'_1 > t'_2$ ; the smaller, less efficient firm begins to lose money as a duopolist before its larger rival. Thus, even if the smaller firm would last longer as a monopolist ( $t_2^* > t_1^*$ ), the initial duopoly losses may induce it to withdraw first.

If firm 1 has a cost advantage so great that  $t_1^* > t_2^*$ , then firm 1 will outlast firm 2. The reasoning exactly parallels that used in the proof of Proposition 1 with the roles of firm 1 and 2 reversed. One cannot make an equally unequivocal prediction about the outcome in the case where firm 2 can last longer as a monopolist but is the first to lose money as a duopolist (see Figure 3). Recursion shows that once firm 2 reaches  $t'_1$ , it will stay in until  $t_2^*$ . But is it worthwhile for firm 2 to shoulder losses over  $(t'_2, t'_1)$  to earn profits later over  $[t'_1, t_2^*)$ ?<sup>9</sup> The general answer depends on the specific parameterization. To gain a sense of the likely outcomes, we examine a particular class of demand functions.

*Example.* Consider isoelastic demand of the form,  $q = bp^{-\epsilon}e^{-gt}$ , where  $q$  is quantity demanded,  $b$  is a constant,  $\epsilon$  is the price elasticity of demand, and  $g$  is the rate at which demand shrinks. The constant  $b$  is normalized so that both firms are profitable at  $t = 0$ , and  $\epsilon$  is taken to be greater than one to guarantee full capacity use. For simplicity, the discount rate,  $r$ , is set to zero; with positive discounting, the results that follow would be less dramatic. Finally, let  $s$  stand for the larger firm's initial share of total industry capacity,  $K$ .

Given these assumptions, the optimal exit times are:

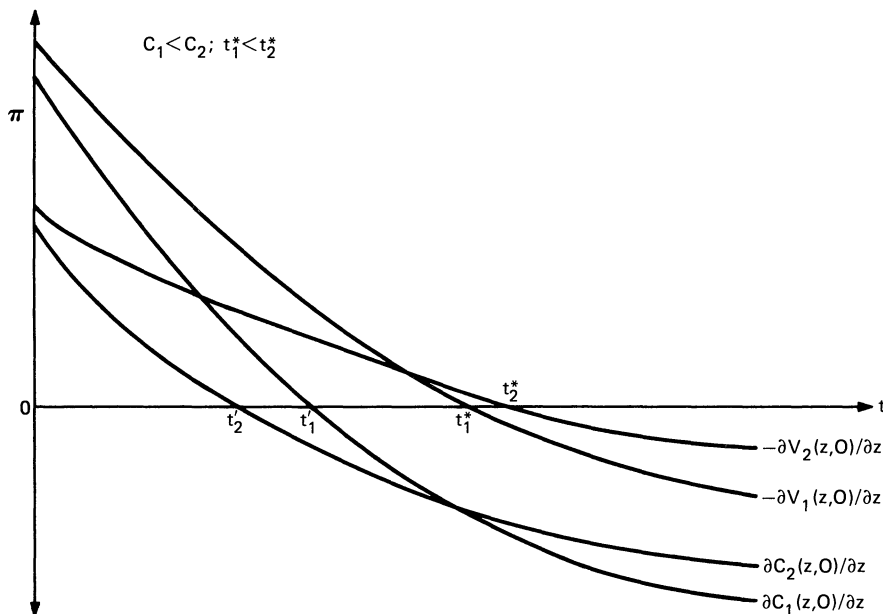
$$t'_i = \frac{1}{g} \ln \left[ \frac{b}{K} c_i^{-\epsilon} \right], \quad i = 1, 2, \tag{12}$$

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<sup>9</sup> Here, the assumption of no reentry is important. If reentry were allowed, firm 2 would temporarily exit at  $t'_2$  and reenter at  $t'_1$  to avoid all duopoly losses. Firm 2's longer tenure as a monopolist ( $t_2^* > t_1^*$ ) then forces firm 1 to exit permanently at  $t'_1$ .



FIGURE 3  
COST DIFFERENCES



$$t_i^* = \frac{1}{g} \ln \left[ \frac{b}{K_i} c_i^{-\epsilon} \right], \quad i = 1, 2. \tag{13}$$

For firm 2 to outlast firm 1 despite its cost disadvantage, it must have a longer profitable tenure as a monopoly,

$$t_2^* > t_1^*, \tag{14a}$$

and its initial losses must be less than its monopoly profits,

$$P_2(t_1^*, t_2^*) > 0. \tag{14b}$$

In this example, conditions (14a) and (14b) lead respectively to the requirements that

$$\frac{c_2}{c_1} < \left( \frac{K_1}{K_2} \right)^{1/\epsilon} = \left( \frac{s}{1-s} \right)^{1/\epsilon} \tag{15a}$$

and

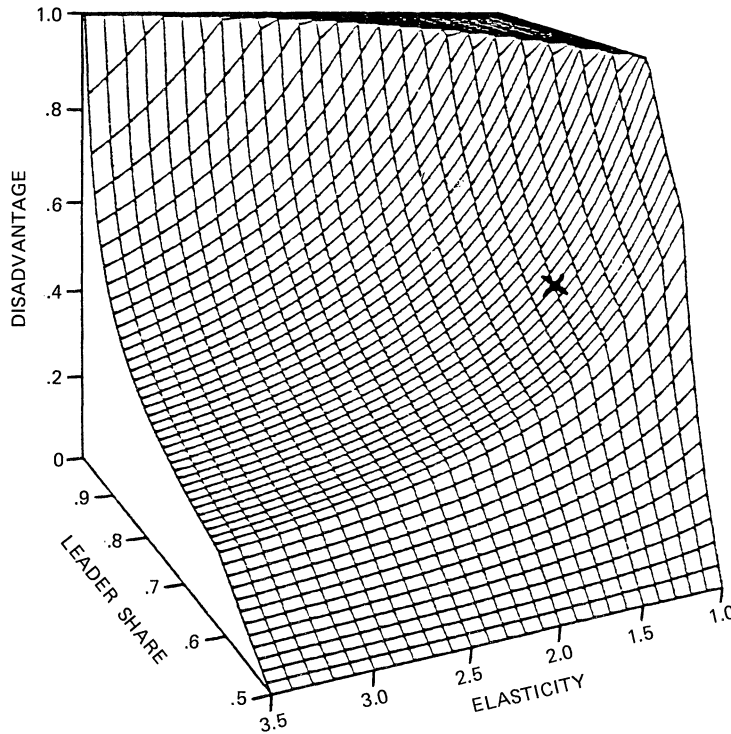
$$\frac{c_2}{c_1} < \frac{\epsilon[(1-s)^{-1/\epsilon} - 1]}{-\ln(1-s)}. \tag{15b}$$

These derivations follow from (5a) and the values of  $t_i'$  and  $t_i^*$ .

Firm 2 will be able to outlast a larger, more efficient rival if and only if both (15a) and (15b) hold. These two constraints on  $c_2/c_1$  simultaneously determine a critical ratio,  $\mathcal{L}^*$ , which depends only on the leader share ( $s$ ) and the demand elasticity ( $\epsilon$ ). Figure 4 plots the implied cost disadvantage,  $\mathcal{L}^* - 1$ .<sup>10</sup> An example marked with an  $x$  can be read from the figure: with  $s = .7$  and  $\epsilon = 1.5$ ,  $c_2$  has to be over 53% greater than  $c_1$  ( $\mathcal{L}^* = 1.53$ ) for firm 2 to drop out before firm 1. Note that unless the price elasticity is very high or the market shares are almost equal ( $s \approx .5$ ), firm 2 must suffer from a *very* severe cost disadvantage to be outlasted by its larger competitor.

<sup>10</sup> The critical ratio  $\mathcal{L}^*$  is graphed between 1 and 2 in Figure 4.

FIGURE 4  
COST DISADVANTAGE REQUIRED OF FIRM 2



## 5. Conclusion

■ Although the exit game with perfect information has a multiplicity of Cournot-Nash equilibria, the subgame perfection criterion can be used to select among them. In the case where firms have asymmetric market shares but identical costs, the subgame perfect equilibrium has firms withdrawing in order of their sizes: the largest firm is the first to shrink and the smallest firm the last. Sufficiently strong scale economies can, by conferring cost advantages on larger firms, overturn this outcome. Numerical examples, however, suggest that the required cost advantages for larger firms may have to be surprisingly substantial.

For the sake of simplicity, we have derived these results for settings in which production has only two speeds: on and off. It is important, however, to recognize that qualitatively similar results can and have been derived for settings in which equally efficient firms can withdraw capacity almost continuously (Ghemawat and Nalebuff, 1985).

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