THE DEVOLUTION OF DECLINING INDUSTRIES*

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In declining industries capacity must be reduced in order to restore profitability. Who bears this burden? Where production is all or nothing, there is a unique subgame-perfect equilibrium: the largest firms exit first [Ghemawat and Nalebuff, 1985]. In this paper firms continuously adjust capacity. Again, there is a unique subgame-perfect equilibrium. All else equal, large firms reduce capacity first, and continue to do so until they shrink to the size of their formerly smaller rivals. Intuitively, bigger firms have lower marginal revenue and correspondingly greater incentives to reduce capacity. This prediction is supported by empirical findings.

I. INTRODUCTION

Models of dynamic competition generally take a rosy view of time: markets expand; better technologies become available; information improves. In this preoccupation with time as an engine of progress, environments in which time is an agent of regress have been shunted aside. Yet, declining industries form an important part of developed economies: more than 10 percent of the United States' 1977 manufacturing output was accounted for by industries whose real output had shrunk over the 1967–1977 period.[1]

In declining industries the important competitive moves pertain to disinvestment rather than investment. An industry facing decline must reduce its capacity in order to remain profitable. Capacity reduction, however, is a public good that must be provided privately.[2] Each firm would like its competitors to shoulder the reduction: a firm may even maintain excess capacity—and sustain losses—in order to force competitors to withdraw sooner. The question arises: who gives in first?

The timing game in a declining industry is therefore a war of attrition rather than a race to preempt. In the original model of the war of attrition [Maynard Smith, 1974], each competitor chooses between continuing to "fight" at a prespecified level of intensity or conceding; the competitor that hangs in the longest wins the prize. Ghemawat and Nalebuff [1985] applied this model, with its

*We are very grateful for comments provided by John Londregan, Mike Whinston, and two anonymous referees.

1. This estimate is based on Bureau of Labor Statistics Data for ninety-five "Economic Growth" industries. The estimate would be even higher at a lower level of aggregation.

2. For a discussion on motivating private provision of public goods, see Bliss and Nalebuff [1984].

The Quarterly Journal of Economics, February 1990

dichotomous choice, to declining industries by restricting production to be an all-or-nothing decision for each firm. This paper, in sharp contrast, allows firms greater strategic flexibility by letting them continuously adjust their capacities as demand declines.

We show that with continuous adjustment, there is a unique subgame-perfect outcome to the battle over declining markets. The Davids cut the Goliaths down to their own size: large firms are the first to reduce capacity, and they continue to do so until they have shrunk to the size of their formerly smaller rivals. Ceteris paribus, survivability is inversely related to size. This prediction appears to fit with recent empirical findings.

The paper's outline is as follows. Section II discusses theoretical work on declining industries and introduces our results. Section III offers corroborative empirical evidence. Section IV presents the formal model and its equilibrium. Section V provides a brief conclusion. The Appendix contains the proofs of all lemmata.

II. Theoretical Literature

Our previous paper examined market decline in a highly stylized setting [Ghemawat and Nalebuff, 1985]. Firms were perfectly informed about their competitors' costs and capacities. Reentry was not allowed after exit. Demand declined continuously and deterministically. Exit was an all-or-nothing decision. Under these four assumptions, there was a unique subgame-perfect equilibrium: the smaller of two equally efficient duopolists forced its larger rival to exit as soon as duopoly profits turn negative.

Does the smallest firm continue to enjoy a competitive advantage in a more general setting? It is useful to recapitulate what we know about the effects of relaxing our previous assumptions.

Incomplete Information. In a paper concurrent with our first model, Fudenberg and Tirole [1986] examine the exit decision in an environment of incomplete information where each firm is uncertain about its rival's costs. They provide conditions for the existence and uniqueness of sequential equilibrium in duopoly. When expectations are symmetric, if exit occurs, it is the less efficient firm that leaves.

Reentry. Londregan [1986], in a model of the industry lifecycle, allows for the possibility of reentry after entry and exit. He shows that if reentry costs are positive, there is a unique subgame-perfect equilibrium in the all-or-nothing exit game with complete information: smallness continues to be an advantage during decline and, by backward induction, also during the growth phase. (See also Fishman [1989] for the effect of decline and exit on entry deterrence.)

Probabilistic Decline. Fine and Li [1986] consider a market that is declining probabilistically. In each period the probability distribution of demand is stochastically worse than before. If the intervals between decisions are sufficiently short, there is a unique subgame-perfect equilibrium in which the smaller firm outlasts its larger competitor.[3] Huang and Li [1986] study exit decisions in a model with random drifts in demand. Because demand may not decline, there is no endgame to work backwards from. Even so, if the state space is continuous so that demand changes smoothly, there is again a unique subgame-perfect equilibrium in which the smaller firm will never be forced out by its larger rival.

Capacity Adjustment. Whinston [1987, 1988] examines an oligopoly in which capacity is adjustable in lumps equal to plant size. In this framework he shows that it is difficult to reach any general conclusions about the pattern of plant closures. When each firm controls several differently sized plants, there is no theoretical prediction about the order of exit. There are several complications. A firm that withdraws a small plant now may be at a strategic disadvantage later if its remaining plants are large. Or a firm with many small plants may find this flexibility disadvantageous against a larger firm with one big plant. Thus, it is hard to separate out the effect of flexibility versus size. To focus on size alone, Whinston considers a special case when all plants equally are sized. There is still a complication; who moves first to break a tie between the two largest firms? The structured pattern of exit returns when the equilibrium play is independent of the tiebreaking rule (a quasi-markov equilibrium)—only the largest firms reduce capacity. Following the proof of Theorem 1, we discuss the relationship between our results in greater detail.

In the present analysis we prefer to maintain the assumption of complete information: in the typical declining industry competitors are well acquainted, and the production technologies embodied in

3. In their paper they argue that probabilistic decline allows the possibility of multiple equilibria. The multiplicity, however, is an artifact of the long time periods between decision making. Imagine that the time interval between decisions is sufficiently long that only one decision is made for all time. Then, we are in the traditional one-shot or static Nash equilibrium model where we know multiple solutions are possible. It is possible to extend the Fine and Li model to prove that if the decision periods are sufficiently short, there is a unique sequential equilibrium in which the smaller firm always outlasts its larger rival.

extant investments are common knowledge. The payoffs to allowing rèentry or stochastic demand trajectories are probably limited: the papers cited above suggest that smallness continues to be a competitive advantage with these generalizations.

We believe that there is a large payoff in extending the models of exit beyond the all-or-nothing production technology. Although such technologies characterize some industries with large, inflexible plants such as alumina refining (see Ghemawat and Nalebuff [1985]), firms usually shrink continuously as demand declines (soda ash, rayon, baby foods, vacuum tubes, cigars, and electric coffee percolators are some of the many examples: see Harrigan [1980]). We study competition under the opposite of all-or-nothing adjustment; we focus on production technologies where capacity is continuously adjustable. This allows us to model the effect of size differences without the complication of differential flexibility.

Allowing variable capacity is an important extension for an additional reason: it represents a significant generalization of the standard war of attrition. Wars of attrition, as usually formulated, allow only two actions: fight or concede. The possibility of variable capacity corresponds to allowing variable levels of concession, which considerably complicates calculating equilibrium. To our knowledge, this paper and one by Whinston [1987, 1988] are the first to characterize the equilibria in a war of attrition with variable response possibilities. The cost of this extension is that we must confine our attention to a highly stylized oligopoly with uniform and constant marginal costs.

We demonstrate that given continuous capacity adjustment, there is a unique subgame-perfect equilibrium in declining industries. In this equilibrium the largest of several equally efficient firms will reduce capacity alone until its market share is equal to that of its next smallest rival. Once parity is reached, the two largest firms reduce capacity together while all others maintain capacity. Then, when they reach the size of the third biggest firm, all three start to shrink at an equal rate, and so on.

The motivation behind this result is that marginal revenue is inversely proportional to firm size. Firm i's marginal revenue equals $Q_iP'(Q) + P(Q)$; this is a declining function of Q_i since a bigger firm suffers more from a decline in price. In the absence of economies of scale, bigger firms have, therefore, a greater incentive to reduce capacity. The force of this marginal revenue argument is quite distinct from the level that operates when capacity decisions are all-or-nothing. In the all-or-nothing environment the survivor of competition among equally efficient rivals is the firm that has the longest profitable tenure as a monopolist.

To help illustrate the novel results of this paper, consider a duopoly where firm 1 operates four machines and firm 2 operates two machines.[4] If the firms must make a dichotomous choice between operating at full capacity or exiting in toto, Ghemawat and Nalebuff [1985] demonstrate that the capacity vector evolves from (4,2) to (0,2) to (0,0) as the industry declines. In contrast, if firms may withdraw machines continuously, the only subgame-perfect equilibrium for the industry capacity vector is (4,2) to (X,2) to (2,2) to (Y,Y) to (0,0), where X falls smoothly from 4 to 2 and Y falls smoothly from 2 to 0. In words, the larger firm reduces capacity until its size equals that of its smaller competitor. Once parity is reached, the firms shrink together.

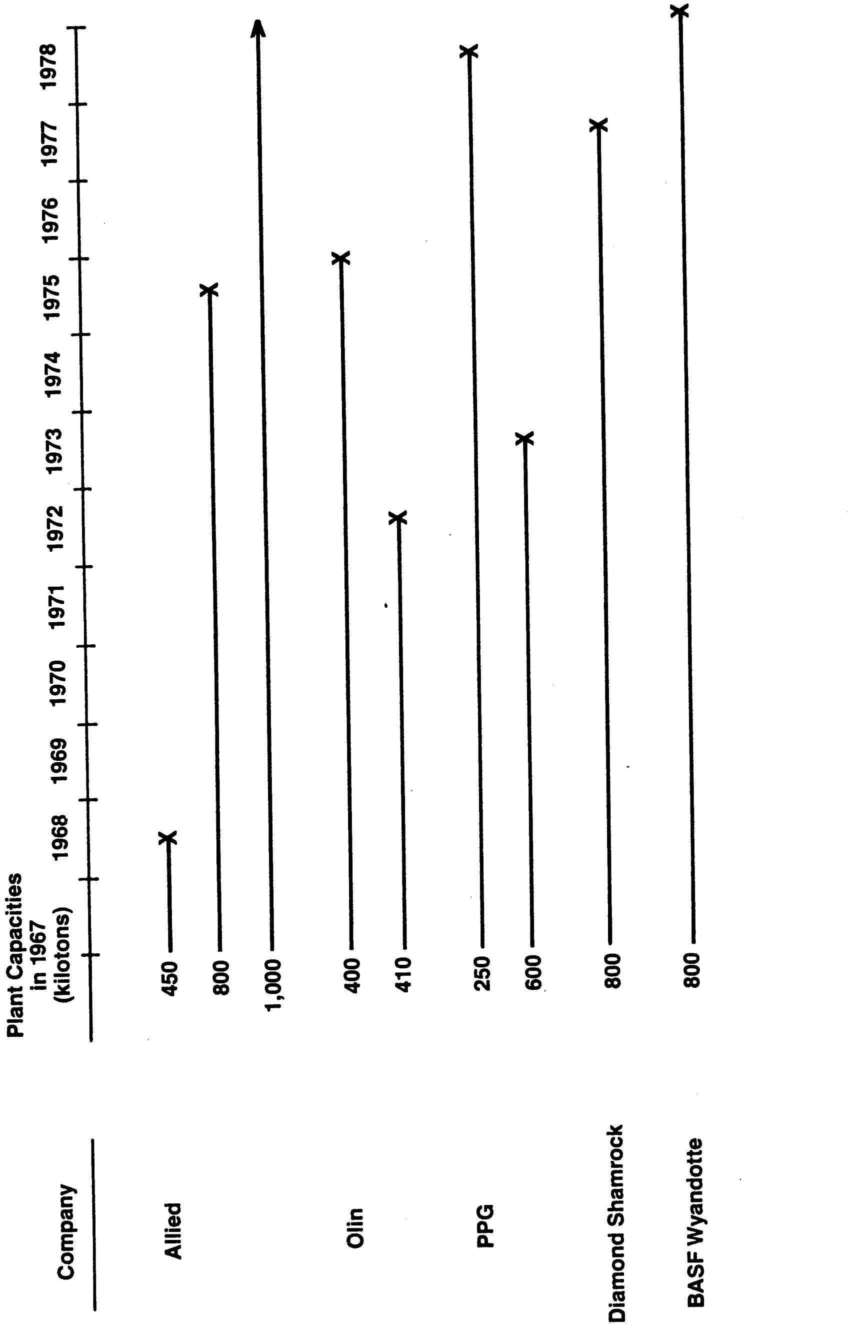
The move toward equalization of market shares provides the testable hypothesis that large firms undertake a disproportionate share of the capacity reductions during decline. Empirical support for this hypothesis is marshaled in the following section. The formal propositions are then proved in the context of the stylized model presented in Section IV.

III. Empirical Evidence

The available empirical evidence on decline suggests that firms with larger market shares experience greater pressures to shrink as an industry devolves. Herein, we provide a review of three case studies and relevant cross-sectional evidence.

The first case describes the decline of the U. S. synthetic soda ash industry over the 1967–1978 period (see Harrigan [1980, Ch. 5]). In 1967 almost three quarters of the soda ash consumed in the United States was synthesized from limestone and salt. Five firms (listed in Figure I) accounted for 99 percent of the domestic synthetic soda ash capacity. All five employed the mature and very capital-intensive Solvay process. The remainder of the U. S. soda ash market was supplied by natural reserves that had begun to be mined in Wyoming in the 1950s. Natural soda ash cost significantly less to "produce" than did synthetic soda ash; the costs of transporting it, however, were higher because most soda ash customers were located east of the Mississippi, closer to the synthetic soda ash capacity.

4. The machines are all of equal size and equal efficiency.



SOURCE: Harrigan (1980), Ch. 5.

FIGURE I
Plant Closures in the Synthetic Soda Ash Industry

Under pressure from natural soda ash, synthetic soda ash producers operated at 89 percent of their capacities in 1967—an amount probably just below their break-even level. Over the next decade the tide continued to turn against synthetic producers. Wyoming relaxed its regulations governing the mining of natural soda ash. Higher energy costs and stricter environmental regulations undercut the Solvay process because it was more energy- and emission-intensive. Since the Solvay capacity required substantial reinvestment to be kept operational, the stage was set for plant closures.

Over the next decade the output of synthetic soda ash fell by nearly two thirds. In 1978 only one Solvay plant, out of the original nine, remained operational.[5] Figure I depicts the devolution of that industry. The pattern of closures supports our predictions in two respects. First, each of the three multiplant firms closed a plant before either of the two single-plant operators. Second, none of the five firms dropped out of the market until all firms were down to one plant apiece.[6]

Our second example is based on Baden Fuller and Hill's [1984] case study of the U. K. steel castings industry. The demand for U. K. steel castings declined by 42 percent over the 1975–1981 period, and the industry's return to sales dropped from 11 percent to 1 percent. Competitors adjusted their capital stocks by closing foundries. Executives of the two largest firms, F. H. Lloyd and the Weir Group, "felt that they had borne the brunt of the costs of rationalization" [Baden Fuller and Hill, 1974, p. 23]. They accounted for 41 percent of industry output in 1975, but for 63 percent of the capacity that was withdrawn over the 1975–1981 period; this reduced their combined market share to 24 percent. Some of the Lloyd and Weir foundries that were closed had been more efficient than foundries that their competitors continued to operate.

The U. S. integrated steel-making industry provides a third example of devolution. During the 1960s and 1970s, imports and

5. The sole survivor, Allied Chemical, operated the largest plant in the industry. Harrigan [1980, pp. 136–37] notes that it benefited from the most advanced Solvay technology, access to a saline lake (which facilitated waste disposal), and the easternmost location of any synthetic soda ash plant (which gave it the biggest advantage in terms of transportation cost over producers of natural soda ash).

6. Note that some of this evidence goes against the predictions of Ghemawat and Nalebuff [1985] that the order of exit should follow plant size. Of course, the smaller firms may be producing below efficient scale. Or, they may be producing for a market niche so that their fate will be less affected by the great winds blowing against the industry.

minimills (which recycle steel scrap) intensified the pressure on integrated steelmakers. Deily [1985] studied the contraction of the eight largest integrated steelmakers which together accounted for 90 percent of U. S. capacity in 1960. Because they faced large barriers to exit, Deily focused on their replacement investments over the 1962–1979 period. After controlling for plant efficiency and reinvestment requirements, she concluded that [p. 125] "firm size seemed to have a negative impact on *all* major investments during these years. It is possible that some of the laggardness reflected the strategic behavior described by Ghemawat and Nalebuff [1985]." It is worth adding that the eight-firm concentration ratio in steelmaking fell from 90 percent in 1960 to 76 percent by 1979.

In order to ascertain whether the market-share effects identified in these case studies hold up more broadly, Ghemawat [1985] used Harvard University's PICA database to analyze the determinants of changes in four-firm concentration ratios over the 1967–1977 period in 294 four-digit U. S. manufacturing industries. Since it is difficult to control for intraindustry differences in efficiency based on publicly available data, the basic specification did not attempt to do so. This omission is likely to bias the results toward the null hypothesis—that declining demand does not decrease concentration—in proportion to one's prior belief that size is positively correlated with efficiency.

Ghemawat found that even in the absence of controls for cost differences, a dummy variable indicating declines in real industry output over the sampled period was positively associated, as the 5 to 10 percent level of statistical significance, with decreases in industry concentration. When the rate of decline was introduced as a continuous independent variable, again he found a significant correlation: the higher the rate of decline, the greater the observed rate of decrease of concentration.

Lieberman [1989] offers the most careful and comprehensive cross-sectional tests of the predictions implied by the theoretical models of exit and devolution. His results are based on a sample of thirty chemical products, each of which experienced chronic declines in output lasting five years or longer. "In general in declining industries, it appears that small producers suffer disproportionately high mortality rates whereas large-share firms make more frequent incremental reductions of capacity. When analysis is limited to survivor firms in steeply declining industries, there is significant evidence that firm size convergence is due to more rapid divestment by the largest producers" [p. 17]. Specifically, of the 15 products in Lieberman's sample that experienced declines in capacity of 40 percent or more, 12 exhibited convergence in the sizes of the survivors. This evidence, together with the cases discussed above, is suggestive of a tendency for market shares to converge during decline.

IV. Model Specification

The formal structure of our model consists of the following notation and assumptions:

A1. There are m competitors in the market. Initially, firm i has a capacity level of $k_i(0)$. The industry capacity vector at time t is represented by $K(t) \equiv [k_1(t), \ldots, k_m(t)]$.

A2. For each firm i the flow cost of maintaining its capacity is C per unit. There are no other operating costs. Production is constrained by capacity; setup costs preclude the addition of new capacity or the reintroduction of previously withdrawn capacity.

A3. Capacity can be adjusted continuously. Reductions are irreversible.

A4. Time is quantized into periods of duration Δ. Period n begins at time t^n. An approximation of the continuous time solution is the equilibrium in the limit where Δ approaches zero.

A5. A withdrawal of capacity is effected at the beginning of the relevant time interval and is reflected immediately in cost reduction. Withdrawal decisions are made simultaneously.

A6. Firm i's cost of withdrawing capacity is S per unit. This cost will be positive if exit costs (e.g., severance payments) predominate or negative if the retired capacity has a sufficiently large salvage value. If the output price is zero, exit costs are not sufficiently large to prevent firms from withdrawing capacity: $rS < C$, where r is the discount rate.

A7. Each firm's output is a perfect substitute for every other firm's output. Define total output at t by $Y(t) \equiv k_1(t) + k_2(t) + \ldots + k_m(t)$. The price to firm i is $P(Y(t),t)$.[7]

7. We are implicitly assuming that firms produce at full capacity. Although this simplifies the exposition, the results generalize directly to the case where firms may produce at less than full capacity. The reason is that production costs depend only on capacity. Thus, in a one-period problem, no firm would maintain excess capacity. We then show (Theorem 1) that the unique subgame perfect equilibrium follows the sequence of period-by-period maximizations. This remains true whether or not there is the option of maintaining idle capacity.

A8. The inverse demand function $P(Y,t)$ is well defined and continuous for all $Y \geq 0$. $P(0,t)$ is bounded, and $P(Y,t)$ equals zero for some finite Y. Furthermore, $P(Y,t)$ has continuous second derivatives and is downward sloping: $P'(Y,t) \equiv dP(Y,t)/dY < 0$, $P''(Y,t) \equiv d^2P(Y,t)/dY^2$ is continuous.

A9. Firm i's marginal revenue at any Cournot-Nash equilibrium output level (k_i^*) is a nonincreasing function of total industry output: $P'(Y,t) + k_i^* P''(Y,t) \leq 0$ for all possible Y. Hahn [1962] demonstrates that this condition ensures the stability of the Nash equilibrium.[8]

A10. Demand is declining toward zero for exogenous reasons. For any $\epsilon > 0$ there exists a T such that $P(0,t) < \epsilon$ for all $t \geq T$.

A11. Marginal revenue at full capacity, $P(Y,t) + k_i P'(Y,t)$, is a strictly decreasing function of t for all Y. Assumptions A10 and A11 are both satisfied, for example, if $P(Y,t) = P(Y)e^{-gt}$.

These assumptions lead to a unique subgame-perfect equilibrium. The firms with the largest capacity are the first to shrink. When the formerly larger firms shrink to the size of their smaller rivals, the latter join in subsequent reductions of capacity.

Although the arguments are technical, the intuition is simple. Imagine that all firms act "myopically"; i.e., each firm seeks to maximize only its current period profits. This behavior leads to a sequence of period-by-period Cournot-Nash equilibria. Since marginal costs are equal, the firms with the smallest outputs have the greatest marginal revenue. This implies that the smallest firms never act first to reduce capacity.

This sequence of period-by-period Nash equilibrium strategies is the unique subgame-perfect equilibrium. When each firm acts myopically, any deviation in output must lower current period profits. The only reason to deviate, therefore, would be to raise future profits. In Lemma 2 we show that maintaining capacity in excess of the myopic equilibrium has no effect on future outcomes. Since demand is declining, the firm will already have some excess capacity; extra units of excess capacity do not convince competitors to further reduce their output. In Lemma 4, we show that a reduction of capacity below the myopic solution also fails to

8. Note that Hahn [1962] makes an a priori assumption that there is a unique Nash equilibrium. Since we are also interested in proving uniqueness, his condition must be modified so that the stated inequality holds at any Nash equilibrium. The general condition will be satisfied if $YP'(Y,t)$ is a declining function of Y.

increase future profits; it may even decrease profits by encouraging competitors to increase their future outputs (i.e., temper their reductions). This intuition is now made rigorous.

LEMMA 1. There exists a *unique* Cournot-Nash equilibrium to the single-period problem where each firm maximizes its current flow of profits subject to the constraint that no firm produces above its capacity: $k_i(t^n) \leq k_i(t^{n-1})$.

Proof of Lemma 1. See the Appendix.

The output at this Cournot-Nash equilibrium is denoted by $k_i^*(K(t^{n-1}),t^n)$. We refer to the strategy of maximizing current profits as *myopic* behavior.

DEFINITION 1. $K^*(K(t^{n-1}),t^n)$ is the vector of *myopic* Nash strategies:[9]

$$K^*(K(t^{n-1}),t^n) \equiv [k_1^*(K(t^{n-1}),t^n), \ldots, k_m^*(K(t^{n-1}),t^n)].$$

The *sequence* of myopic Nash equilibrium strategies is then defined by calculating the myopic Nash equilibrium level in period t^n, where the initial capacity vector is given by the myopic Nash equilibrium from period t^{n-1}. Proposition 1 states that in this sequence of myopic Nash equilibrium strategies, only the largest firms reduce capacity.

PROPOSITION 1. Order the firms so that $k_1(0) \leq k_2(0) \leq \ldots \leq k_m(0)$. In the sequence of myopic Nash equilibria, each period there exists some $k^*(t^n)$ such that all firms with initial capacity below $k^*(t^n)$ remain at their initial capacity and all firms with initial capacity above $k^*(t^n)$ have reduced their capacity to $k^*(t^n)$:

$$K^*(K(t^{n-1}),t^n) = [k_1(0),k_2(0), \ldots, k_j(0),k^*(t^n), \ldots, k^*(t^n)],$$

where $k_j(0)$ is the maximum initial capacity less than $k^*(t^n)$.

Proof of Proposition 1. Define $k^*(t^n)$ by

$$\int_0^{\Delta} \{k^*(t^n)P'(K^*,t^n + \tau) + P(K^*,t^n + \tau) - (C - rS)\}e^{-r\tau}\, d\tau = 0,$$

where K^* is shorthand for the industry's unique, myopic Cournot-Nash equilibrium level of output in period t^n. All firms with capacity $k^*(t^n)$ are maximizing current period profits. Because P' is

9. For notational consistency, let $K^*(K(t^{-1}),t^0)$ equal $K(0)$, the initial capacity vector.

negative, all firms with $k_i(0) < k^*(t^n)$ have positive marginal profit at full capacity utilization. In the myopic solution all such firms are capacity constrained. Hence, $K^*(K(t^{n-1}),t^n)$ is a Cournot-Nash equilibrium for period t^n, and by Lemma 1 this equilibrium is unique.

Q.E.D.

To calculate a subgame-perfect equilibrium for this model poses no theoretical difficulties. We proceed using the logic of dynamic programming and backward induction. The primary conceptual issue is to characterize the nature of the solution. We show that for equally efficient oligopolists, only the largest firms reduce capacity in response to declining demand. This characterization of the subgame-perfect Nash equilibrium coincides with the description of the sequence of myopic Nash equilibrium strategies proved in Proposition 1, as we now show that the two solutions are the same.

THEOREM 1. Under Assumptions A1–A11, $K^*(K(t^{n-1}),t^n)$ is the unique subgame-perfect equilibrium capacity vector in period t^n for all n.

Proof of Theorem 1. The proof is based on backward induction. We first show that the result is true past some time T. Then by assuming the result from period t^{n+1} onwards, this implies that from time t^n onwards there exists a unique subgame-perfect equilibrium which coincides with the myopic Nash equilibrium in each period. Intermediate steps in the argument are provided by Lemmata 2–5, which are stated and proved in the Appendix.

Since demand is declining toward zero, eventually all firms will cease to produce, even as a monopolist. Lemma 5 shows that there exists a time T such that once $t^x \geq T$, the unique myopic solution coincides with any subgame-perfect equilibria: all firms have zero capacity.

The inductive hypothesis tells us that over $[t^{n+1},\infty)$ there is a unique subgame perfect equilibrium which coincides with the sequence of period-by-period Cournot-Nash solutions. We need to extend this back one period to t^n. By Lemma 1 the myopic Nash equilibrium in period t^n is unique. To demonstrate that this is also the unique subgame-perfect equilibrium, it is sufficient to show that producing either more or less than $k_i^*(K(t^{n-1}),t^n)$ results in lower aggregate profits over $[t^n,T]$. Any deviation away from k_i^* lowers profits over $[t^n,t^{n+1}]$, since k_i^* is the one-period profit-maximizing strategy. The only reason to deviate, therefore, is that it might produce higher profits over $[t^{n+1},T]$. In this interval the inductive hypothesis allows us to consider the sequence of period-by-period Cournot-Nash solutions as the unique subgame-perfect equilibrium. We show for this sequence of myopic Nash equilibrium strategies, any deviation away from $k_i^*(K(t^{n-1}),t^n)$ cannot raise profits over $[t^{n+1},T]$.

Consider the payoff to a firm that deviates and chooses to produce more than the myopic level, $k_i' > k_i^*(K(t^{n-1}),t^n)$. Because firm i is able to produce more that k_i^*, it is obviously not constrained by capacity in the period t^n myopic Nash equilibrium. In the sequence of myopic Nash equilibria that follow, Lemma 2 shows that all unconstrained firms produce a strictly smaller output each period. As a result, from period t^{n+1} onwards, the subgame-perfect equilibrium in each period is unchanged: $K^*(K'(t^n),t^{n+1}) = K^*(K(t^n),t^{n+1})$. The additional capacity of firm i held at t^n is eliminated immediately in period t^{n+1} and does not affect profits over $[t^{n+1},T]$.

Consider, alternatively, the payoff to a firm that deviates and chooses to produce $k_i' < k_i^*(K(t^{n-1}),t^n)$. In this case the subgame-perfect equilibrium from periods t^{n+1} onwards may change. Lemma 3 implies that the new subgame-perfect equilibrium involves firm i producing no more and the other firms producing no less than at the no-deviation benchmark. This outcome leads to (weakly) lower profits for firm i by Lemma 4. Intuitively, the reaction curves are negatively sloped so that excess capacity reduction by firm i encourages its rivals' production and thus reduces its own profits.

We have shown that any output other than $k_i^*(K(t^{n-1}),t^n)$ in period t^n results in strictly lower profits in period t^n without any possibility of future gain in the continuation equilibrium. Thus, $k_i^*(K(t^{n-1}),t^n)$ is the unique subgame perfect equilibrium output in period t^n. By induction this is true for all n. The sequence of myopic Nash equilibrium strategies is the unique subgame-perfect equilibrium.

Q.E.D.

Remark 1. One might be tempted to argue for uniqueness more directly. Subgame-perfection is a stronger condition than Nash equilibrium. Since every subgame-perfect equilibrium is a fortiori Nash, if there were multiple subgame-perfect equilibria there would have to be multiple Nash equilibria. It might therefore be argued that since Lemma 1 establishes uniqueness of the Nash equilibrium, there can be at most one subgame-perfect equilibrium. This argument is flawed. Lemma 1 only establishes the uniqueness of the *myopic* Nash equilibrium in each period. If firm i lowers its capacity below $k_i^*(K(t^{n-1}),t^n)$, this can change the sequence of subsequent myopic Nash equilibria. The full Nash equilibrium of the multiperiod problem is *not* unique.

Remark 2. If capacity reductions are reversible, the result remains true, and the argument is even simpler. In each period there is a unique "myopic" Cournot-Nash equilibrium, and this equilibrium is not affected by the previous play (since reductions are reversible). The uniqueness of the single period-by-period Nash play together with a finite horizon immediately implies that there is a unique subgame-perfect equilibrium.

Remark 3. Weaker conditions that also lead to Lemmata 1–5 will provide a generalization of the Theorem 1. For example, the market can decline stochastically, as in Fine and Li [1986].

There is no direct generalization of Theorem 1 when firms have heterogeneous costs. The reason is that without equal marginal costs, we can no longer order marginal revenue by output. As a result, the decline in demand affects firms differentially depending on both size and marginal cost. Even if there remains a unique subgame-perfect Nash equilibrium, there is no longer a simple characterization result (such as Proposition 1) describing an orderly pattern to the capacity reductions.

A comparison with Whinston [1988] suggests that the assumption of continuous capacity adjustment is essential to our result. If capacity adjustment is subject to an integer constraint, the proof of Theorem 1 breaks down. The problem arises in Lemma 1; because of integer constraints, there is no guarantee of a unique solution even in the one-period myopic problem.

It is worth pinpointing the source of difficulty. If capacity lumps come in different sizes, multiple equilibria are a generic problem. But, if capacity lumps are all equally sized, multiple equilibria arise in the one-period problem only to break ties. When there are two equally large firms, which one reduces capacity first? A tiebreaking rule is needed. Because the rule may depend on the earlier play of the game, it can contaminate the entire characterization of the previous play. Whinston solves this problem by restricting firms to "super-markov" strategies; the resolution of ties cannot depend on earlier play. With this additional restriction, he shows that with equally sized lumps of capacity, only the largest firms reduce capacity.

Our result is a limiting case of the discrete model. In the passage to continuous time we do not require super-markov strategies. For intuition, consider the sequence of period-by-period myopic Nash equilibrium strategies. With lumps of any size one of two equally-sized firms must move before the other. With continuous capacity adjustment there is no tension between equally sized firms; they reduce capacity together. There is a unique myopic equilibrium in every period which then coincides with the subgame-perfect solution.

V. Conclusions

We have demonstrated that in situations where equally efficient oligopolists start out with asymmetric market shares, larger firms will bear the brunt of capacity reductions until their market shares equal those of their smaller competitors. Once shares do equalize, all competitors with identical capacities reduce together.

Although some empirical corroboration of these conclusions is available, we still face obvious gaps in our understanding of how industries actually decline. One important item on the research agenda concerns the impact of cost differences (arising, possibly, from economies of scale) on shrinkage patterns. An additional agenda item is to expand firms' strategy spaces to permit cost-reducing investment. The considerable amount of theoretical and empirical work that remains to be done in this area suggests that the study of exit will continue to be a growth industry.

Appendix

This Appendix provides the proofs for Lemmata 1–5.

Lemma 1. There exists a *unique* Cournot-Nash equilibrium to the single-period problem where each firm maximizes its current flow of profits subject to the constraint that no firm produces above its capacity, $k_i(t^n) \le k^i(t^{n-1})$. The output at this Cournot-Nash equilibrium is denoted by $k_i^*(K(t^{n-1}),t^n)$,

$$k_i^*(K(t^{n-1}),t^n) \equiv \underset{k_i}{\operatorname{argmax}}$$

$$\int_0^{\Delta} k_i\left[P\left(\sum_{j\neq i} k_j^*\,(K(t^{n-1}),t^n) + k_i,t^n + \tau\right) - (C - rS)\right]e^{-r\tau}\,d\tau$$

subject to

$$k_i \le k_i(t^{n-1}).$$

Proof of Lemma 1. Assumptions A8 and A9 provide sufficient continuity to prove the existence of at least one myopic Nash equilibrium (see Friedman [1977]). Here we concentrate on proving uniqueness.[10] At any myopic Nash equilibruim marginal profit for firm i equals[11]

$$(1.A)\qquad M\Pi_i = \int_0^{\Delta} \{k_i P'(K^*,t^n+\tau) + P(K^*,t^n+\tau) - (C - rS)\} e^{-r\tau}\, d\tau \geq 0.$$

Let there be two distinct myopic Nash equilibria, with total capacity levels K_A^* and K_B^*. Without loss of generality, $K_A^* \geq K_B^*$. Since K_A^* and K_B^* are distinct, there must exist some firm i such that $k_{iA}^* > k_{iB}^*$. The fact that $k_{iA}^* > k_{iB}^*$ implies that firm i is not capacity constrained in B, and hence $M\Pi_{iB} = 0$. For firm i in the Nash equilibrium K_A^*, $M\Pi_{iA} \geq 0$. Therefore, $M\Pi_{iA} - M\Pi_{iB} \geq 0$. This leads to a contradiction because

$$(2.A)\qquad \mathrm{M}\,\Pi_{iA} - M\,\Pi_{iB}$$

$$(3.A)\qquad = \int_0^{\Delta} \{k_{iA}^* P'(K_A^*,t^n+\tau) + P(K_A^*,t^n+\tau) - [k_{iB}^* P'(K_B^*,t^n+\tau) + P(K_B^*,t^n+\tau)]\} e^{-r\tau}\, d\tau$$

$$(4.A)\qquad \leq \int_0^{\Delta} \{k_{iA}^* P'(K_B^*,t^n+\tau) - k_{iB}^* P'(K_B^*,t^n+\tau)\} e^{-r\tau}\, d\tau$$

$$(5.A)\qquad \leq 0.$$

Equality (3.A) follows by canceling the cost terms $(C - rS)$. Inequality (4.A) is based on A9, which states that each firm's marginal revenue $k_iP'(K,t) + P(K,t)$ is a nonincreasing function of industry output, K; this implies that $k_{iA}^*P'(K_A^*,t) + P(K_A^*,t) < k_{iA}^*P'(K_B^*,t) + P(K_B^*,t)$ over the interval $t \in [t^n,t^{n+1}]$. The final inequality follows as $P'(K_A^*,t)$ is negative (by A8) and $k_{iA}^* > k_{iB}^*$.

Q.E.D.

DEFINITION 2. $\Pi_i^*(K(t^{n-1}),t^n)$ is the present value of firm i's profits over the period t^n to t^{n+1} in the sequence of myopic Nash equilibria.

10. At any Nash equilibrium the second-order conditions for a maximum will be met as

$$\frac{d^2\Pi_i}{dk_i^2} = \int \{2P'(K,t^n+\tau) + k_i^*P''(K,t^n+\tau)\} e^{-r\tau}\, d\tau < \int \{P'(K,t^n+\tau) + k_i^*P''(K,t^n+\tau)\} e^{-r\tau}\, d\tau \leq 0 \text{ by A9.}$$

11. Firm i's marginal profit may be positive if it is capacity constrained.

The myopic Nash equilibrium outputs in period t^n, $K^*(K(t^{n-1}),t^n)$, are a function only of the previous period's capacity vector; thus, $K(t^{n-1})$ determines the profits in period t^n.

DEFINITION 3. Firm i is *capacity constrained* in the myopic Nash equilibrium $K^*(K(t^{n-1}),t^n)$ if $k_i(0) < k^*(t^n)$, where $k^*(t^n)$ is as defined in the proof of Proposition 1. For all capacity-constrained firms, $k_i^*(K(t^{n-1}),t^n) = k_i(0)$.

LEMMA 2. In the iterated sequence of myopic Nash equilibria, each firm that is not capacity constrained in the period beginning at t^n reduces its output in the next period:

$$k_i^*(K(t^{n-1}),t^n) = k^*(t^n) \Longrightarrow k_i^*((K^*(K(t^{n-1}),t^n)),t^{n+1}) < k^*(t^n).$$

Proof of Lemma 2. Imagine to the contrary that in the unique equilibrium $K^*(K(t^n),t^{n+1})$ there exists some unconstrained firm i with $k_i^*((K^*(K(t^{n-1}),t^n)),t^{n+1}) = k_i^*(K(t^{n-1}),t^n) = k^*(t^n)$. Since in any myopic Nash equilibrium $MR_i \geq 0$, it must be true at t^{n+1} that

$$\text{(6.A)} \quad \int_0^{\Delta} \{k^*(t^n)P'(K^*(K(t^n)t^{n+1}),t^{n+1} + \tau) + P(K^*(K(t^n),t^{n+1}),t^{n+1} + \tau) - (C - rS)e^{-r\tau}\, d\tau \geq 0.$$

This implies that no firm would want to reduce capacity below $k^*(t^n)$ in period t^{n+1}; any firm with a lower capacity would have strictly positive marginal revenue and could only be in equilibrium if it is capacity constrained. By Proposition 1 all firms have capacity less than or equal to $k^*(t^n)$ and hence maintain their capacity. But if no firm reduces capacity between period t^n and t^{n+1}, then $K^*(t^{n+1}) = K^*(t^n)$. The inequality in (6.A) now contradicts A11 which states that if K is held constant, $k_iP'(K,t^n) + P(K,t^n)$ is a strictly declining function of t; this is zero averaged over t^n to t^{n+1} and hence cannot be greater than or equal to zero when averaged over t^{n+1} *to* t^{n+2}.

Q.E.D.

Next, we show that if a firm reduces its output, this encourages its competitors to raise their production in the future. As a result, such a firm earns (weakly) smaller profits in each subsequent iteration of the myopic Nash equilibrium.

LEMMA 3. Let $K(t^n)$ and $K'(t^n)$ differ only in that firm i has less capacity in K' than in K: $k_i'(t^n) < k_i(t^n)$ and $k_j'(t^n) = k_j(t^n)$ for all $j \neq i$. This implies that

$$k_i^*(K',t^{n+1}) \leq k_i^*(K,t^{n+1}) \text{ and } k_j^*(K',t^{n+1}) \geq k_j^*(K,t^{n+1}).$$

Proof of Lemma 3. If $k_i^*(K',t^{n+1}) = k_i^*(K',t^{n+1})$, then the constrained optimization problem that defines the Nash equilibrium is unaffected by the fact that $k_i^*(K',t^n) < k_i^*(K',t^n)$: the equilibrium following K' and K are identical. By the same token, it would be impossible for the equilibrium following K' to have $k_i^*(K',t^{n+1}) > k_i^*(K',t^{n+1})$; any Nash equilibrium following K' is also feasible following K, and this would contradict the uniqueness result of Lemma 1. It remains to show that if $k_i^*(K',t^{n+1}) < k_i^*(K,t^{n+1})$, then $k_j^*(K',t^{n+1}) < k_j^*(K,t^{n+1})$ leads to a contradiction. An argument parallel to the derivation of equation (6.A) shows that firm j cannot be capacity constrained in $k_j^*(K',t^{n+1})$; its marginal revenue must be zero in the K' solution. This leads to the inequality,

$$\text{(7.A)} \quad \int_0^{\Delta} \{k_j^*(K,t^n)P'(K,t^{n+1} + \tau) + P(K,t^{n+1} + \tau)\}e^{-r\tau}$$
$$\geq \int_0^{\Delta} \{k_j^*(K',t^n)P'(K',t^{n+1} + \tau) + P(K',t^{n+1} + \tau)e^{-r\tau}\, d\tau.$$

This implies for all firms other than i, that $k_j^*(K',t^{n+1}) \leq k_j^*(K',t^{n+1})$. And for firm i, this is true by assumption. Aggregating across firms reveals that $K^{*\prime}(t^{n+1}) < K^*(t^{n+1})$. But the function $k_jP'(K,t^{n+1}) + P(K,t^{n+1})$ is decreasing in both K by A9 and k_j (holding K constant) by A8: this contradicts the inequality in (7.A).

On a more intuitive level, A9 insures that the reaction curves are negatively sloped:

$$dMR_i(K(t^{n-1}),t^n)/dk_j$$
$$= \int_0^{\Delta} \{P'(K^*,t^n + \tau) + k_j P''(K^*,t^n + \tau)\}\, e^{-r\tau}\, d\tau \geq 0.$$

As a result, a reduction by firm i moderates the future reductions by all other firms.

Q.E.D.

LEMMA 4. Let $K(t^n)$ and $K'(t^n)$ differ only in that firm i has less capacity in K' than in K. Then for all $t^x \geq t^n$,

$$\Pi_i^*(K'(t^x),t^{x+1}) \leq \Pi_i^*(K(t^x),t^{x+1}),$$

where $K'(t^x)$ and $K(t^x)$ are the two myopic Nash equilibria that evolve in period t^x given capacity levels $K'(t^n)$ and $K(t^n)$, respectively, in period t^n.

Proof of Lemma 4. To consider the effect on firm i of moving from $K(t^n)$ to $K'(t^n)$, we break up the change into two parts. First consider the effect of firm i moving from $k_i^*(K(t^n))$ to k_i' when all other firms remain at $K(t^n)$. Because $K(t^n)$ is a myopic Nash equilibrium, firm i strictly lowers its current profit by moving to k_i'. Next, consider the effect on firm i of the other firms' movement from $k_j^*(K(t^n),t^{n+1})$ to $k_j^*(K'(t^n),t^{n+1})$. By Lemma 3, all the other firms either maintain or increase their output. Because goods are perfect substitutes, this can only reduce the price and thus further hurt firm i. The same argument applies to all periods past t^{n+1}.

Q.E.D.

Our final lemma establishes the trivial uniqueness of the subgame-perfect equilibrium once the market price is sufficiently low that all firms have $k_i = 0$ as a dominant strategy.

LEMMA 5. There exists some time T such that in any subgame-perfect equilibrium all firms have zero capacity for $t^n \geq T$. This coincides with the myopic Nash equilibrium, $K^*(K(t^{n-1}),t^n)$.

Proof of Lemma 5. Downward sloping demand (A8) implies that a firm could never earn more than the price at zero aggregate supply:

$$\Pi_i(K(t^n),t^n) \leq k_i(t^n)[P(0,t^n) - (C - rS)]/r.$$

Let $\epsilon = [C - rS]$. By A6, $\epsilon > 0$. Then by A10 there exists some T such that $P(0,t) < \epsilon$ for $t \geq T$. Hence, for $t^x \geq T$, $\Pi_i(K(t^x),t^x) < 0$ for all $K(t^x)$ and for all firms. Maintaining any positive capacity is dominated by exiting in toto.

Q.E.D.

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