The Commitment to Seniority in Self-Governing Groups

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No sane man would for one moment think of making a graduate from West Point a field general, or one from Annapolis an admiral, or one from any university or college chief of a great newspaper, magazine, or business house. A priest or preacher who has just taken orders is not immediately made a bishop, archbishop, or cardinal. In every walk of life, "men must tarry at Jericho till their beards are grown."

Champ Clark, Speaker of the U.S. House of Representatives

For a period from about World War I until the mid 1970s, the so-called "textbook Congress" period (Shepsle), committees in the U.S. House of Representatives were structured according to a strict seniority arrangement, an arrangement that survives in weaker form to this day. According to this structuring principle, members of a committee's party delegation were assigned places in a committee queue depending only on their length of continuous committee service. Prizes, in the form of access to committee resources (office space, staff, budgetary discretion) and authority over the committee's agenda (committee and subcommittee chairmanships), were a function of queue position (and whether the party was in the majority or

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minority). Queue position, moreover, was a more-or-less enforceable property right inasmuch as no member's position was threatened from below. A member moved up the queue as those above him left the committee; members joining the committee after him held strictly inferior queue positions.

Seniority is not restricted to congressional committees. It is a structuring principle of wide generality and applicability. We observe its practice in an extensive range of groups and situations. Examples include employment contracts, civil-service grades and steps, academic tenure tracks, the stratification of generations (as in social-security systems), even sociobiological pecking orders. Seniority, in fact, is a special form of hierarchy or status differentiation. While the latter may be constituted in accord with any number of different factors, ranging from age and other indices to more meritocratic or performance-based criteria, seniority shall be taken here to be a measure of "organizational age" (although sometimes it is indistinguishable from "calendar age").

A question arises as to why seniority is practiced at all. Seniority is observed in a wide variety of contexts and the explanation for its practice is to some extent context-dependent. Thus, it is necessary first to characterize the context: Is the group self-governing, in which case seniority is a matter of choice, or is its organization at least partially subject to the will of others, in which case seniority is a matter of imposition or negotiation? Is the group endowed with productive capabilities, in which case attention to production incentives is required, or is it simply endowed with material wealth, in which case distribution is the only thing that matters? If the former, is its product a public good, restricted to the group but consumed in common by all its members, or can its product be partitioned into shares? What is the mechanism by which the group (if it chooses its own arrangements) or a hierarchical superior (if such arrangements are imposed or negotiated) sustains the arrangements—that is, what is the technology for punishing violators of group practices? Responses to questions of this sort determine the viability and advantages of seniority. With these specific contexts in mind, we will attempt to answer the question, "Why seniority?"

The results presented here are more a prolegomenon than a definitive treatment of seniority. We employ a simple overlapping-generations model of group interaction to examine the effects of organizing group life by a seniority system. Does such a system enhance contemporaneous group welfare? Does it produce a flow of benefits that maximizes (discounted) lifetime net welfare of each group number? Is it sustainable in the sense that it is vulnerable neither to opportunistic defection by individual group members, which would cause the activities of the group to unravel, nor to subgroup jealousies, which would lead to expropriation of individual benefits? In short, we are interested in both the advantages of seniority and its sustainability or fitness.

We begin with the case where the product of the group activity is a public good equally available for all group members to consume. We then examine circumstances where the group's product is divisible and appropriable, with individual members enjoying shares. In each instance we want to determine whether an optimal organization of group production displays some version of a seniority principle.

Before addressing this question, we discuss a related role for seniority—it may serve as a facilitator of intergenerational risk-sharing. Our research agenda begins in Section 2. We adapt the model of Cremer to study cooperation in self-governing groups, with special emphasis on the role of seniority in legislatures. In the following sections, the results are extended to include varying productivities, private goods, and expropriation. The article concludes with a discussion of future directions for research.

1. SENIORITY AND INTERGENERATIONAL TRANSFERS

One motivation for seniority is the benefit of intertemporal redistribution in the absence of complete markets. Imagine a species whose members live for two periods, youth and old age. At the end of the first period, a member parthenogenetically reproduces an offspring. The species may be thought of as consisting of a string of overlapping generations: a member is, in the first period, a daughter interacting with her mother and, in the second period, a mother interacting with her daughter. At birth a member is endowed with two food units. The food units last only one period after which they spoil. Were it not for spoilage a member would be self-sufficient, consuming one food unit per period. Spoilage prevents this form of self-sufficiency.

This setting, which Binmore calls the Mother–Daughter Game and which Hammond earlier described as the Poverty Game, is one in which the intertemporal smoothing of consumption is the only issue. There is no capital market or other venue in which to save for one's old age. How can the participants avoid a life consisting of a prosperous youth (two food units) and an impoverished old age (no food units)? The solution requires intergenerational sharing. A daughter contributes some amount of food to her mother and receives, in turn, that same amount of food from her daughter a period later.

Although the transfers go only in one direction (from young to old), Binmore establishes that a sharing outcome is sustainable as part of a subgame-perfect equilibrium. The reason is that, in equilibrium, the daughter donates to the mother if and only if the mother donated to her donating mother. Thus, a daughter who fails to support her mother cannot expect to be taken care of by her own daughter.[1] This variation of the biblical "an eye

1. Out of equilibrium, a daughter must punish (not donate to) her mother if her mother failed to take care of the grandmother; otherwise, the daughter's daughter will not take care of her.

for an eye" allows smooth intertemporal consumption as the norm. An occasional (off-the-equilibrium-path) defection by a selfish daughter is punished, but the punisher is not punished in turn, so that a return to the sharing regime occurs.

Essentially, this same logic has been employed to study social-security systems (Sjoblom), intertemporal institutions (Engineer and Bernhardt), and social contracts more generally (Kotlikoff et al.). In all these instances, there is some form of a seniority effect. A "privileged" senior agent (or generation) receives special treatment. Although the seniors have neither production capability nor savings, the youth support them in the style to which the latter desire to be supported when they are seniors (Binmore; Hammond). Young workers transfer financial resources to the older generation via social security (Sjoblom). In other applications, younger generations may pay the older generation a fee for the asset rights to the latter's political institutions (Kotlikoff et al.; Engineer and Bernhardt). In all these cases, there is a redistribution from young to old, maintained as an equilibrium to effect the smoothing of intertemporal consumption.[2]

In any general theory of seniority effects, we would want to be able to accommodate intertemporal smoothing as in the examples just given. But notice that there is no productive activity involved (and, thus, no need to maintain incentives to produce). The problem is only that of distributing an exogenously provided endowment. For many groups, however, the central challenge is to maintain production incentives. One stumbling block in these circumstances is finite-lived agents. The oldest group member, just about to "retire," has no future in the group; the group, therefore, has no hold on this oldest member and, in particular, may not have any method for inducing him to work in the group's behalf. This, in turn, affects the incentives of the next oldest group member, and the next oldest after that, etc. To consider the problem of maintaining incentives, we employ an overlapping-generations model. The specific application we have in mind is the use of seniority in politics as a discipline device for the self-governing members of a legislative party.

2. SENIORITY AS A DISCIPLINE DEVICE

To consider seniority in a particular context, such as a political party, we must begin the task of trying to analyze the group's objective. In the traditional economic model, the organization is already in existence and coherent in its intentions. For example, a firm's goal is to maximize profits—and the question before it is to determine what structuring principles will best help it accomplish this objective. However, what about a group whose purposes are

2. These studies pay homage to Samuelson's classic statement of overlapping-generations models.

less coherent or for whom the content of its productive activity is itself something to be chosen? A legislative party, after all, may produce any of a number of different policies, and its members may not be in agreement on what exactly it should do. This suggests a different form of seniority: senior group members have greater influence over what the group chooses to do.

The idea (at least as proposed by the senior members) is that the junior representatives "pay their dues." Over time, the junior members become more senior and the party's position begins to reflect their views to a greater extent. As a reward for loyalty, the senior-most members of the party can exert their leadership position to push some of their own agenda.

One advantage of this type of seniority is that it helps the incumbent members get reelected. Constituents prefer to reelect legislators who have more influence. Hence, incumbent legislators are inclined to institute a seniority system that gives them an advantage over a challenger. This view of seniority is studied in a recent article by McKelvey and Riezman.[3]

The point we wish to focus on is the way in which a seniority system allows a party to gain greater cohesiveness and become more productive. The effectiveness of a political party depends on each of its members sacrificing some of her individual goals and preferences. Even recognizing the need for compromise, there remains the issue of who makes the compromises, how much, and when. As a result of internal negotiations, the direction of the party is endogenously determined by its membership. But all members may not get an equal say. We illustrate this use of seniority as an incentive device in an overlapping-generations model of Congress.

A congressman gets elected at age 1 and plans on retiring at age n. At any point in time, a party is made up of legislators of different ages. We take the case where there is one of each age. The age distribution, thus, stays constant over time. Each period can be thought of as an election. The senior-most representative retires at age n. He is replaced by a "freshman" representative, age 1. The other $n - 1$ incumbent congressmen are reelected and each becomes one period more senior.[4] How much cooperation can such a party achieve?

3. One way that senior members may have more influence is if the legislature gives them differential control over the agenda. McKelvey and Riezman consider an elected legislature, some of whose members have had previous legislative experience. The legislature's first task is to decide whether to institute a seniority system. Next, proposals are entertained on how to divide a dollar. A proposal comes to the floor when it is made by a *recognized* legislator. If there is no seniority system, then recognition is completely random. If, on the other hand, legislators have elected to institute seniority, then those with seniority are recognized with probability p_s, and those without seniority with probability $p_j < p_s$. If a proposal is made and approved by a majority, the game ends; if rejected, then the recognition phase is repeated. In effect, the authors have added a seniority stage to the Baron and Ferejohn bargaining-in-legislatures game. In their game-theoretic treatment, McKelvey and Riezman extend the analysis to elections, deriving an incumbency effect in which constituents prefer to reelect experienced legislators who, in turn, are favorable toward instituting a seniority system.

4. This description of a congressional life cycle makes the presumption that a congressman

We answer this question by adapting Cremer's model of cooperation in ongoing organizations. The story that goes with the model has a member of age i choosing an effort level x_i, which has an associated disutility $a(x_i)$. But, in the case of elected officials, effort does not seem to capture the moral hazard problem. While it is certainly the case that some elected officials shirk on the job, there is also the problem of motivating congressmen to cooperate with each other. Congressmen are more effective when they band together as part of a cohesive political party. Achieving cohesion requires individual sacrifice, perhaps giving up special perks for one's district or voting against one's personal preferences for the sake of the party. Thus, we think of each representative choosing some level of cooperative effort, x_i. A member who exerts great influence in setting the party position has a low level of x_i—the party cooperates with him rather than vice versa. A member who has to compromise some of her personal or district's preferences in order to support the party position is said to exert a greater level of cooperative behavior: x_i is large.

Since cooperation has an implied personal sacrifice, there is a disutility $a(x_i)$ associated with cooperation level x_i. The power of the party depends on the sum of its parts. Given cooperation levels $\{x_1, \ldots, x_n\}$, the party's power is represented by the function $f[\Sigma x_i]$. The output of the party may be thought of as a public good that is consumed by all members.[5] Putting costs and benefits together, a vector of cooperation levels $\{x_1, \ldots, x_n\}$ leads to utility for member i:

$$U_i(x_1, \ldots, x_n) = f\left[\sum x_j\right] - a(x_i).$$

The cost and benefit functions are common to all individuals with $f' > 0$, $f'' < 0$, $a' > 0$, and $a'' > 0$, so that U_i is concave. Furthermore, an anarchistic party $\{x_i = 0, \forall i\}$ produces no benefits for any of its members, $f(0) = a(0) = 0$.

As with the standard public-goods problem, each of the party members has incentive to free-ride. Here, free-riding means pursuing one's own ob-

can plan on reelection. Although reelection is neither effortless nor guaranteed, in recent times the probability has been extraordinarily high—roughly 98 percent. Of course, this rate is not exogenous, but may result from constituents' satisfaction with their representative's effort.

5. This stylization misses some of the important elements of the group production. The benefits of forming a party depend on the degree of heterogeneity of the members' preferences. If all the members have identical preferences, then the party has a unified voice even without cooperative behavior. Conversely, it does not help to increase the size of the party by adding members with opposing views—the resulting compromise position may leave everyone unsatisfied. In our model, the party formation is exogenous. The members of the political party have similar but not identical preferences. Working out these differences makes the party stronger, which is to all members' benefit, but requires compromises that may fall unequally on the members.

jectives while taking advantage of the party's power to achieve this goal. It is assumed that zero sacrifice, $x_i = 0$, is a dominant strategy for each individual in any single play of the game: for $x_i > 0$, $f(y + x_i) - a(x_i) < f(y)$.

The fact that zero cooperation is each player's dominant strategy implies that the seniority game is a multiperson "prisoners' dilemma." But the game is repeated over time, so there is the possibility of supporting positive levels of cooperation over time. As a benchmark, we begin with the consideration of how a party member would maximize her lifetime utility in a cooperative solution (or, equivalently, how the members would maximize "contemporaneous group welfare"). The first-best level of production for this party is obtained by solving

$$\max_{x_1,\ldots,x_n} \; nf\left[\sum_{i=1}^{n} x_i\right] - \sum_{i=1}^{n} a(x_i).$$

Given the convexity of the cost of cooperative effort, the first-best solution involves a common cooperation level for agents of all ages—call it y_n. The cooperation level y_n solves $\max_{y>0}[nf(ny) - na(y)]$. This solution requires all n agents to cooperate. If, on the other hand, only m agents will cooperate (and $n - m$ members free-ride), then the second-best solution that maximizes lifetime utility is obtained by solving $\max_{y>0}[nf(my) - ma(y)]$. Following Cremer, we call this m-cooperation level, y_m. Here an agent enjoys party benefits for all n periods, but bears costs in only m periods (or, equivalently, all n members enjoy the benefits from the party equally, but only m of them contribute to the cooperative effort and bear the associated costs).

Cremer demonstrates conditions under which the $(n - 1)$-cooperation level is sustainable as a stationary equilibrium.[6] There is no way to induce the oldest agent to work. (It is assumed that someone cannot be punished once they have retired.) The best that the group can do is achieve full cooperation from everyone else: for $i < n$, $x_i = y_{n-1}$. Is this $(n - 1)$-cooperation level supportable as an equilibrium of the game? For the cooperating members, incentives are provided by the desire to keep the game going. If any member fails to provide the equilibrium level of cooperation, this will be noticed immediately as the output will be lower than expected. Alerted to the presence of a cheater, the group disbands and all its members get zero thereafter.[7]

6. Stationarity implies that cooperation levels, y_i, are constant over time. Without stationarity, one must consider the possibility of intergenerational transfers that increase without bound.

7. We restrict attention to trigger strategies. Each member follows the equilibrium prescription provided that group output exactly equals $f[(n - 1)y_{n-1}]$. If there is *any* reduction in group output, then all members enter into a punishment phase.

The problem with this punishment strategy is that *everyone* pays for any one's transgression. It is hard to tell the story that a political party would disband (or at least act anarchistically and thus forfeit all its power) upon discovering that one of its members has "cheated" by selfishly pursuing her own objectives at the party's expense. Fortunately, a political party has a more credible threat—expulsion. Since political acts are almost public by definition, a party knows not only that someone has cheated, but who has cheated. An individual that fails to live up to her equilibrium expectation can be thrown out of the group. It is presumed that such an excommunicated representative is either powerless on her own or even that she fails to get reelected without the party support. In either case, a selfish representative gets to take advantage of the party once, but should expect to be punished in the next period (and possibly thereafter).[8]

Our use of expulsion as the punishment is meant as a stylization of a party's internal censure. In the United States, the national political party does not control the selection of candidates running at the local level. But it can certainly hamper the campaign of a "party dissident" by withholding its endorsement and financial support. In contrast, with most European parliamentary systems, the national party has a much greater influence in selecting the local representatives, so that a representative who violates the seniority contract could be denied reelection. Expulsion is also a punishment that may be inappropriately large compared to the crime. A junior member can always be replaced by another junior member. But, it may make more sense to strip a senior member of certain influential and prestigious committee assignments. This would be like a step back in the seniority ladder. Since it would hurt the party to expel one of its senior members, the party will try other methods of censure first. Our first pass at the problem treats all members of the party as equally valuable from the party's perspective. The creation of a seniority system makes some members more influential, but that is an endogenous artifact of the seniority system. In Section 4, we recognize the fact that a party's power may depend on the presence of senior members, and this will affect how the party deals with defections.

Although the interpretation of actions and scope of punishment strategies are somewhat different, the sustainability of $(n - 1)$-cooperation as an equilibrium follows directly from Cremer. His assumption provides a sufficient condition to support cooperation at level $(n - 1)y_{n-1}$:

8. For completeness, we note that this story has to be slightly recast when $n = 2$. In this case, if the junior member cheats, then next period the new junior member is supposed to "expel" the senior member as a punishment. It is more realistic to say that the junior member fails to join the senior member's party. The next generation then rewards this junior member for carrying out the punishment strategy. Of course, for political parties, the two-person version is a rather uninteresting special case and we do not pursue it further.

$$f[(n-1)y_{n-1}] - a(y_{n-1}) \geq 0. \tag{1}$$

That is, all $n - 1$ "cooperating" agents in the second-best solution have nonnegative surplus.

Theorem (Cremer). If (1) holds, then the outcome in which the $n - 1$ youngest agents provide effort level y_{n-1} is sustainable as a stationary equilibrium.

The equilibrium exhibits what we call a "truncated seniority effect" in cooperation levels, viz., $x_i = y_{n-1}$, for $i < n$ and $x_n = 0$. We continue with several observations about truncated seniority.

First, it is worth reemphasizing that with (1), the truncated seniority system maximizes lifetime utility in the second-best world. Since the cost of compromising is convex ($a'' \geq 0$), for any level of group cooperation, the cost is minimized when the compromises are spread equally among the members. However, the incentive constraint implies $x_n = 0$, so the best one can do is to spread the cooperative effort equally among the $n - 1$ other members. Under (1), the second-best amount of group cooperation, $(n - 1)y_{n-1}$, is incentive-compatible, so there is no need to distribute compromises unequally in order to achieve a higher level of group cooperation.

Why is it that $(n - 1)y_{n-1}$ is incentive-compatible? Each member has exactly one chance to act selfishly. Normally, this occurs in the last period but there is the option to free-ride at an earlier stage. A member who free-rides at an earlier stage is expelled from the group and, thus, uses up her one opportunity. Since the oldest member will always act selfishly ($x_n = 0$), if another player decides not to cooperate prior to her retirement, there will only be $n - 2$ others cooperating and her return will be $f[(n - 2)y_{n-1}]$. If, on the other hand, the representative waits until her ultimate period to act selfishly, her return will be $f[(n - 1)y_{n-1}]$. Since there is no discounting, she will want to take her one selfish opportunity when it has the biggest return, namely, when she is the oldest.

By developing this argument further, we show that the assumption (1) is stronger than necessary. Equation (1) can be replaced by (1′), which is the necessary and sufficient condition for the $(n - 1)$-cooperative solution to be a sustainable equilibrium:

$$f[(n-1)y_{n-1}] - a(y_{n-1}) \geq \frac{f[(n-2)y_{n-1}] - f[(n-1)y_{n-1}]}{n-1}. \tag{1'}$$

Corollary 1. Equation (1′) is necessary and sufficient for the truncated seniority system to be sustainable as a stationary equilibrium.

Proof. By rearranging terms, we see that (1′) is equivalent to

$$nf[(n-1)y_{n-1}] - (n-1)a(y_{n-1}) \geq f[(n-2)y_{n-1}]. \qquad (1')$$

The youngest member's lifetime utility in the second-best solution must be at least as high as her payoff from free-riding in the first period. This makes it clear why the condition is necessary. To see that (1′) is sufficient, we consider two cases: (i) $f[(n-1)y_{n-1}] - a(y_{n-1}) \geq 0$, and (ii) $f[(n-1)y_{n-1}] - a(y_{n-1}) < 0$.

In case (i), the payoffs in each of the first $n-1$ periods are positive, so that (1) holds and incentive-compatibility follows the argument of Cremer. In case (ii), a member who considers deviating from the equilibrium would find it is best to do so immediately. Unless the member is planning to cooperate all the way through period $n-1$, there is no advantage to incurring a negative payoff while postponing deviation. Thus, the greatest incentive to free-ride occurs in the first period. Since (1′) is sufficient to ensure cooperation in period 1, it is sufficient to sustain the truncated seniority system as a stationary equilibrium. Q.E.D.

This argument relies on the absence of discounting. There are several reasons to weigh future utility payments less than current ones: certainly, impatience and uncertain prospects of reelection are two factors. Discounting diminishes the sustainability of a seniority system, since the gain from shirking is immediate while the reward for cooperation is delayed until the final period. With discount parameter δ, the necessary and sufficient condition for the $(n-1)$ cooperative solution to be a sustainable equilibrium is

$$\sum_{i=1}^{n-1} \delta^{i-1}[f[(n-1)y_{n-1}] - a(y_{n-1})] + \delta^{n-1} f[(n-1)y_{n-1}]$$

$$\geq f[(n-2)y_{n-1}]. \qquad (1'')$$

Corollary 1′. With discount rate δ, (1″) is necessary and sufficient for the truncated seniority system to be sustainable as a stationary equilibrium.

The proof is contained in the Appendix. In the text, we continue with the no-discounting case.

Returning to the proof of Corollary 1, note that the necessary and sufficient condition is a statement about the youngest member's lifetime utility. Although the truncated seniority system may confront an incentive problem for the youngest member, there is never any incentive problem in the penultimate period. This holds even without (1) or (1′).[9]

9. Alternatively, this says that (1′) is always satisfied in a two-period model.

Corollary 2. There is no advantage to deviating from the truncated seniority system when the free-riding opportunity is only one period away.

Once again, the proof is contained in the Appendix.

The best case for the truncated seniority system arises when the disutility of compromise is linear in x_i—that is, $a(x_i) = ax_i$.[10] The extra burden on the young is just offset by the extra surplus returned to members with seniority. Since disutility is linear, there is no need to smooth cooperation over the lifetime.

Proposition 1. Under (1′) and $a(x_i) = ax_i$, the truncated seniority system achieves the first-best outcome.

Proof. First, we show that the total cooperation level is the same as the first-best. The two first-order conditions are

$$n^2 f'[ny_n] - na = 0 \qquad \text{(first best)},$$
$$n(n-1)f'[(n-1)y_{n-1}] - (n-1)a = 0 \qquad \text{(second best)}.$$

Thus total cooperation levels are the same:

$$a = nf'(ny_n) = nf'((n-1)y_{n-1}) \Rightarrow ny_n = (n-1)y_{n-1}. \tag{2}$$

Lifetime utility equals the first-best level,

$$nf[(n-1)y_{n-1}] - (n-1)a(y_{n-1}) = nf[ny_n] - na(y_n), \tag{3}$$

since disutility is linear. Finally, we note that under (1′), y_{n-1} is incentive-compatible (Corollary 1), so that the truncated seniority system is an equilibrium.[11] Q.E.D.

3. SENIORITY AS EFFORT REDUCTION

To begin a more general consideration of seniority, consider the same setup as in the previous section, except that we require $x_1 \geq x_2 \geq \cdots \geq x_{n-1} \geq x_n = 0$. Here, seniority is treated as a "compromise-sharing" of a more general sort. As a member gains seniority, the party increasingly reflects the member's (or her constituents') preferences and secures more special treatment

10. Here we relax the assumption that $a'' > 0$ to allow $a'' = 0$. Although (1′) is still a necessary condition to support the truncated seniority system as an equilibrium, the $(n-1)$-cooperative solution is not the only way to achieve the second-best outcome. All m-cooperation levels lead to identical lifetime utility and even the first best is attainable without (1′); see Propositions 1 and 2.

11. We note that the proof of Corollary 1 does not require the strict convexity of $a(x)$.

for her district. The cooperative burden is distributed more heavily on young members and, consequently, the optimal amount of group cooperation may change.

Let x^* be the value of x that solves $f[\Sigma_i x_i] = a(x^*)$; x^* is the cooperation level for which an agent just breaks even in any period. Then we have a third corollary to Cremer's result.

Corollary 3. If $x_i \leq x^*$, $i = [1,(n-1)]$, and $x_n = 0$, then $\{x_1, \ldots, x_n\}$ is a sustainable equilibrium.

Proof. Since $x_i \leq x^*$, each party member has a weakly positive surplus from participating in the seniority game in periods 1 through $(n-1)$. This surplus is lost if a member fails to provide the equilibrium level of cooperation prior to the preretirement period n; the player is expelled from the party and receives zero thereafter. Therefore, the only reason to sacrifice this flow of gains would be if there were some larger gain available immediately. No such opportunity exists. A player's highest period of surplus occurs in the final period; since everyone else is cooperating, the retiring member gets the maximal benefit, $f[\Sigma_{i=1}^{n-1} x_i]$. Defecting earlier is less valuable as the party's power is diminished by the effect of two noncooperating members, the defector (labeled as j) and the retiring player—output is only $f[(\Sigma_{i=1}^{n-1} x_i) - x_j]$. In short, the maximal payoff opportunity occurs in period n and each of the prior periods has a nonnegative payoff so that waiting is at least costless.

Q.E.D.

The assumption that $x_i \leq x^*$ is the direct analogue of (1). Once again the condition is stronger than necessary. It requires an agent to do no worse than to break even in every period. In the example below, members do not break even in every period, and still have the incentive to conform to the seniority system. But the condition $x_i \leq x^*$ *cannot* be replaced by the analogue of (1′):

$$nf\left[\sum_{i=1}^{n-1} x_i\right] - \sum_i a(x_i) \geq f\left[\sum_{i=2}^{n-1} x_i\right].$$

The incentive to defect from the equilibrium is not monotonic in seniority.

Example 1. An example, adapted from Cremer, illustrates the gain of a truncated seniority system over other alternatives. Consider a group with five members ($n = 5$) that produces a public good with technology $f(y) = y$ and cost-of-cooperation function $a(x) = x^2/2 + x$. The oldest member free-rides, so that the second-best optimum effort level is found by solving

$$F = \max_{x_1, x_2, x_3, x_4} 5f\left[\sum_{i=1}^{4} x_i\right] - \sum_{i=1}^{4} a(x_i).$$

That is, the second-best optimum maximizes lifetime utility, where the latter consists of enjoying the public good for all five periods, but contributing to its production for only four periods. Substituting for f and a,

$$F(x_1, x_2, x_3, x_4) = 4[x_1 + x_2 + x_3 + x_4] - \tfrac{1}{2}[x_1^2 + x_2^2 + x_3^2 + x_4^2].$$

Differentiating, setting to zero, and solving the first-order condition gives $x_1 = x_2 = x_3 = x_4 = 4$. Lifetime utility is greatest when there is a truncated seniority system.

However, other seniority systems, although suboptimal, are nevertheless sustainable as an equilibrium. All that is necessary is that the incentive constraints be satisfied. Table 1 gives five different seniority systems that are sustainable as equilibria. The first seniority system in this table has the largest variance among the cooperating members. In successive rows the variance in cooperation levels declines, with the final row portraying the truncated seniority system. Note that in the first three seniority systems, early periods are associated with negative utility [contrary to (1) but not (1′)].

Returning to the larger theme of this article, Corollary 3 has established circumstances in which a general seniority system, $(x_1, \ldots, x_n)$, richer than the "truncated" system, $(y_{n-1}, \ldots, y_{n-1}, 0)$, is sustainable as an equilibrium. But there is no advantage to this generality if the input level $(n - 1)y_{n-1}$ is incentive-compatible. The advantage to a more general division of cooperation arises only when the truncated seniority system violates the incentive-compatibility constraints. In these cases, the truncated seniority system is not a viable option since it is not a sustainable equilibrium.

With linear disutility, $a(x_i) = ax_i$, it is possible to achieve the first-best outcome even without (1′). Proposition 2 below shows that the first-best outcome is always possible under the allocation $(y_1, 0, 0, \ldots, 0)$. The incentive-compatibility constraint never binds if all the cooperation is

Table 1. Sustainable Seniority Systems

x_1	x_2	x_3	x_4	x_5	Lifetime Utility
8	6	3/2	1/2	0	12.75
7	5	3	1	0	22.00
13/2	9/2	7/2	3/2	0	25.50
4	4	4	4	0	32.00

pushed into the first period. The $m = 1$ solution requires extreme sacrifice by junior members, $x_1 = y_1$ ($= ny_n$). Thereafter, for the next $n - 1$ periods the member can act selfishly and enjoy the benefits from the cooperation of the new junior member. The freshman member has no incentive to shirk even though she is doing all the cooperation; without her there is no output and, thus, no opportunity to free-ride.

The first-best would also be attainable under the truncated seniority system $(y_{n-1}, y_{n-1}, \ldots, y_{n-1}, 0)$ *if this were incentive-compatible.* But when (1′) is violated, the truncated seniority system is not incentive-compatible. Thus, the allocation $(y_1, 0, 0, \ldots, 0)$ can support a higher level of cooperation and consequently a higher level of lifetime utility.

Proposition 2. With $a(x_i) = ax_i$, the seniority system $(y_1, 0, 0, \ldots, 0)$ achieves the first-best outcome as a sustainable equilibrium.

Proof. By an argument parallel to Equation (2) of Proposition 1, the solution to the first-order condition has a constant level of total cooperation for all values of m—simply replace $(n - 1)$ by m:

$$a = nf'[ny_n] = nf'[my_m] \Rightarrow ny_n = my_m.$$

The only issue is whether or not y_m is incentive-compatible. The $m = 1$ allocation of cooperation is always incentive-compatible. If the junior-most member refuses to cooperate, then nobody cooperates and the party produces no benefits. Furthermore, the party punishes this shirking and the junior-most member gets zero thereafter. Thus, shirking gets the junior-most member nothing, while cooperation leads to the first-best outcome. Q.E.D.

One of the insights from Proposition 2 is that the incentive constraints are less binding on the junior-most members. If the junior-most member is designated to make all the sacrifices, then selfish behavior is less valuable. The reason is as follows: whatever the pattern of x_i, a member that behaves selfishly in the first period is expelled thereafter and never has another opportunity to cooperate. The summation of $a(x_i)$ is irrelevant as none of it is ever paid—not the first period because of selfish behavior and not later because of expulsion. Thus, the gain to selfishness depends on the cooperation of others, $\Sigma_{i=2}^{n-1} x_i = (\Sigma_{i=1}^{n-1} x_i) - x_1$. Holding the total level of cooperation constant, as the value of x_1 increases, the freshman member gains less from breaking the equilibrium. (Note that by the time only one member is cooperating, the equating of effort with cooperation is dubious.)

Cremer's theorem and Proposition 2 taken together suggest a trade-off. Because of the convex cost of compromising, members would prefer to spread out the cooperative input equally up to their final period. Provided this is incentive-compatible, it is the second-best outcome. But if (1′) fails and the truncated seniority program is not incentive-compatible, then a marginal increase in seniority is costless in terms of disutility of effort but improves the party's ability to increase total cooperation.

The tension between spreading out cooperative effort and front-loading cooperation to ease incentive compatibility becomes even more pronounced when we abandon the simplification that all members of the party are equally productive. Senior members have more experience and have developed more organization-specific human capital. This suggests that for efficiency purposes, senior members should work harder. With this complication, the first best cannot be achieved by front-loading cooperative effort even with a linear disutility function. The freshmen members are simply too green; the party is much more effective when the experienced members contribute their cooperative effort. With these issues in mind, we examine the consequences of rising productivity on seniority systems.

4. SENIORITY WITH RISING PRODUCTIVITY

In this section, we replace the assumption that cooperative effort is equally valuable from all members. This may be done is several ways. Cremer considered the case in which there are several different tasks, and productivity depends on the assigned task. He showed that younger workers should be assigned to the more difficult (and more productive) tasks. Unfortunately, this may not be feasible in the internal organization of a political party. An inexperienced junior member cannot be made more productive simply by being assigned to chair a committee. A junior member in over her head might even have lower productivity. The fact that senior members are more productive, then, exacerbates the incentive-compatibility problem.

In our approach to the incentive problem, we assume that productivity varies with experience, not with the task. In the political arena, productivity is likely to increase with seniority. This is represented by a rising productivity vector, $\pi = (\pi_1, \ldots, \pi_n)$, with $\pi_1 < \cdots < \pi_n$. We maintain the assumption that the senior-most agent's productivity does not induce her participation, so that we are in a second-best situation.[12]

12. Formally, we assume that

$$f\left[\sum_{j=1}^{n} \pi_j x_j\right] - a(x_n) < f\left[\sum_{j=1}^{n-1} \pi_j x_j\right], \qquad x_n > 0.$$

This implies that any incentive-compatible allocation must have $x_n = 0$. Note that this is a stronger assumption than before since the most senior agent is now the most productive.

With agent i cooperating at level x_i, the group produces $f[\Sigma\pi_i x_i]$. Lifetime utility, given the productivity vector π, is

$$F^* = nf\left[\sum_{i=1}^{n-1} \pi_i x_i\right] - \sum_{i=1}^{n-1} a(x_i). \tag{4}$$

The second-best level of production is determined by differentiating F^* with respect to x_i and solving the first-order conditions:

$$nf'\left(\sum \pi_i x_i\right) = \frac{a'(x_i)}{\pi_i}.$$

Since the left-hand side is constant across i, marginal disutility should be proportional to productivity. Dividing respective first-order conditions gives

$$\frac{a'(x_i)}{a'(x_j)} = \frac{\pi_i}{\pi_j}. \tag{5}$$

This implies that $x_{i+1} > x_i$ since $a'' > 0$ and $\pi_{i+1} > \pi_i$. Thus, the second-best requires an increasing cooperation vector $(y_1^*, \ldots, y_{n-1}^*, 0)$ with $y_{i+1}^* > y_i^*$.

Can this second-best outcome, in which cooperation increases with seniority, be sustained as an equilibrium? The answer depends on the punishment given to a member who defects from the equilibrium. The party still has the option of expelling a defector, even when the member is highly productive and, thus, not immediately replaceable. Although this punishment hurts the party in the short run, it remains a credible threat. If the party failed to throw out the defector, then none of the other members would have an incentive to cooperate. On the other hand, less severe punishments may also be sufficient to ensure efficient cooperation. For example, a temporary loss of seniority privileges may be enough to motivate cooperation. If so, then the party should try these punishments first in the unlikely event that a member fails to follow her equilibrium strategy. For simplicity, we continue our analysis of seniority using expulsion as the punishment option.

A sufficient condition to support the second-best outcome is

$$f\left[\sum_{i=1}^{n-1} \pi_i y_i^*\right] - a(y_{n-1}^*) \geq 0. \tag{6}$$

This assumption states that an agent in her penultimate period of group membership will not be a net loser if she continues to contribute at the optimal level. Although (6) is an assumption only about agent $n - 1$, cooperative effort is increasing up to y^*_{n-1}. Thus, it is immediate that (6) implies that all junior members also have a weakly positive surplus in each period. This environment with rising productivity suggests a fourth corollary to Cremer's theorem.

Corollary 4. Under (6), $(y^*_1, \ldots, y^*_{n-1}, 0)$ is a sustainable equilibrium.

Proof. The proof follows the argument of Corollary 1. Under (6), each party member has a weakly positive surplus from participating in the seniority game *in all periods*. The future surplus is lost if a member fails to provide the equilibrium level of cooperation; the player is expelled from the party and receives zero thereafter. Therefore, the only reason to sacrifice this flow of gains would be if there were some larger gain available immediately. No such opportunity exists. A player's highest period of surplus occurs at her final period; since everyone else is cooperating, the retiring member gets the maximal benefit, $f[\Sigma_i \pi_i y^*_i]$. Defecting earlier is less valuable as the party's power is diminished by the effect of two noncooperating members, the defector (labeled as j) and the retiring player—output is only $f[(\Sigma_i \pi_i y^*_i) - \pi_j y^*_j]$. In short, the maximal payoff opportunity occurs in period n, and each of the prior periods has a nonnegative payoff so that waiting is at least costless. Q.E.D.

This result demonstrates that (6) is sufficient for a second-best optimum to be sustainable. Note, however, that in this arrangement more senior agents work harder. That is, there are no rewards for seniority.

As in previous results, (6) is stronger than necessary. For example, to support cooperation at the second-best level, the incentive-compatibility condition for the penultimate period is

$$f\left[\sum_{i=1}^{n-1} \pi_i y^*_i\right] + \left[f\left[\sum_{i=1}^{n-1} \pi_i y^*_i\right] - f\left[\sum_{i=1}^{n-2} \pi_i y^*_i\right]\right] - a(y^*_{n-1}) \geq 0. \quad (7)$$

That is, it is necessary that the cooperation cost borne by member with seniority $n - 1$ must be less than her gain in period n plus the marginal value of her cooperation in period $n - 1$. This necessary condition is clearly weaker than (6). When this condition fails, the second best is not an equilibrium, as seen in the example below.

Example 2. The structure of example 1 is maintained: $f(y) = y$ and $a(x) = x^2/2 + x$. However, now we have productivity effects. Specifically, suppose $\pi = [1/4, 3/4, 5/4, 7/4,]$.[13] A second-term legislator is three times as productive as a freshman, a third-term legislator is five times as productive, and a fourth-term legislator is seven times as productive. Optimal effort levels are determined by maximizing lifetime utility:

$$F = \max_{x_1,\ldots,x_4} 5f\left[\sum \pi_i x_i\right] - \sum a(x_i).$$

Solving the first-order conditions yields

$$y^* = \left[\frac{1}{4}, \frac{11}{4}, \frac{21}{4}, \frac{31}{4}\right].$$

This produces a lifetime utility of 1524/32. But is this effort level substainable? The answer is no, since the necessary condition given in (7) is violated. Member 4 has an incentive to defect since her net gain is positive:

$$2f\left[\sum_{i=1}^{n-1} \pi_i y_i^*\right] - f\left[\sum_{i=1}^{n-2} \pi_i y_i^*\right] - a(y_4^*) = \frac{1}{32}[1424 - 278 - 1209] = -\frac{63}{32} < 0.$$

Given second-best cooperation levels from the youngest three members, incentive-compatibility requires $x_4 \leq 29.44/4$. But then $x_3 = 21/4$ is not incentive-compatible. The necessary reduction in x_3 further reduces the incentive to cooperate at age 4. The two most senior members will supply an inefficiently small amount of cooperation. In contrast, raising x_1 causes no loss (to the first order) in efficiency, while improving incentives in the later periods. Thus, the youngest member will supply too much cooperation. The optimal seniority profile will be steeper than y^* in this "third-best" outcome.

5. SENIORITY AS A HIERARCHY OF SHARES

To this point, the resultant of cooperative effort is a public good enjoyed by all members of the party. Seniority was used to organize the cooperative input, rather than divide the output. There are other instances where the output of a political party is a private good: for example, the fashioning of an omnibus pork-barrel bill, shares of which are appropriable by members of the party.

13. Since the oldest agent does not work in equilibrium, her productivity is not reported.

For simplicity, we imagine that the party output is a pie that may be divided into slices. Let λ_i be member i's share, where $\Sigma\lambda_i = 1$. We restrict attention to the class of share distributions that display a seniority effect: $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$.

As before, cooperative effort is needed to produce output.[14] The party production technology, $f(\cdot)$, depends on individual cooperative efforts and productivities. But now the output is a private good. The members' utilities are a function of the size of the pie, their respective shares, and their respective inputs:

$$U_i(x_1, \ldots, x_n) = \lambda_i f\left[\sum_j \pi_j x_j\right] - a(x_i).$$

Lifetime utility (or contemporaneous group welfare) is

$$F(x_1, \ldots, x_n) = f\left[\sum_j \pi_j x_j\right] - \sum_j a(x_j).$$

We make two additional departures from the earlier public-good model. First, we assume that punishment is instantaneous. If a member defects, she is denied her contemporaneous share as well as her share of all future group production. Second, we assess the security of individual property rights. Does anyone have an incentive to expropriate another member's share? If one member gets too large a share, others will find it better to reorganize; they will prefer to dissolve the party in order to expropriate a member's "excess" return.

5.1. Instantaneous Punishment

Since punishment is instantaneous, it is no longer the case that the absence of a future necessarily impedes the group's ability to induce effort from the most senior member. Consider the first-best effort level, $(z_1^*, \ldots, z_n^*)$, determined by maximizing F. From the first-order conditions, we see that more productive members (those who are more senior) contribute greater effort: $\pi_i > \pi_j$ implies that $z_i^* > z_j^*$ as

$$a'(z_i^*)/a'(z_j^*) = \pi_i/\pi_j.$$

14. Here, we may think of cooperative effort as legislators taking the heat for pork. For example, in 1988, Congress failed to give itself a pay raise when a majority of the members were unwilling to go on the record as supporting this action.

Can this first-best level of cooperative effort be sustained? We must first determine whether all members have a sufficient incentive to cooperate. If lifetime utility (or contemporaneous group welfare) is positive,

$$f\Big[\sum_j \pi_j z_j^*\Big] - \sum_j a(z_j^*) \geq 0, \tag{8}$$

then there exists a sharing system that satisfies the incentive constraints.

Assumption (8) does not in itself imply that the output is divided according to a seniority system. But in the case of rising productivity, (8) guarantees that there is a seniority sharing rule that sustains the first-best cooperation levels. We begin by showing the existence of a seniority rule that gives positive surplus in each period.

Lemma 1. With (8), if $\pi_1 < \cdots < \pi_n$, then there exists a sharing rule with $\lambda_1 < \cdots < \lambda_n$ such that $\lambda_i f[\Sigma \pi_j z_j^*] - a(z_i^*) \geq 0$.

Proof. Set

$$\lambda_i = \frac{a(z_i^*)}{\Sigma_j a(z_j^*)} \,. \tag{9}$$

With this division of output, the surplus in each period is positive by (8):

$$\lambda_i f\Big[\sum_j \pi_j z_j^*\Big] - a(z_i^*) = \frac{a(z_i^*)}{\Sigma_j a(z_j^*)}\bigg[f\Big[\sum_j \pi_j z_j^*\Big] - \sum_j a(z_j^*)\bigg] \geq 0.$$

The λ_i form an increasing series as

$$\pi_{i+1} > \pi_i \Rightarrow z_{i+1}^* > z_i^* \Rightarrow a(z_{i+1}^*) > a(z_i^*). \qquad \text{Q.E.D.}$$

This variation leads to our fifth corollary of Cremer's theorem.

Corollary 5. With (8) and rising productivity, there exists a seniority sharing rule for which the first best $(z_1^*, \ldots, z_n^*)$ is a sustainable equilibrium.

Proof. Consider the λ-division according to (9). A member who defects from the equilibrium gets zero during the period of defection and thereafter. Since participation in the equilibrium results in positive surplus in each period, there is no incentive to defect. Q.E.D.

The proposed sharing rule is only one of many ways to satisfy the incentive constraints. We turn to a variation on our earlier example to illustrate several other incentive-compatible sharing rules.

Example 3. Let $f(y) = y$, $a(x) = (x^2/2 + x)/5$, and $\pi = (1/5, 3/5, 5/5, 7/5, 9/5)$. The first-best effort levels are $z^* = (0, 2, 4, 6, 8)$. Solving the incentive-compatibility constraints leads to the following four conditions on the λ_i:

$$\lambda_5 \geq 10/35,$$

$$\lambda_4 + \lambda_5 \geq 16/35,$$

$$\lambda_3 + \lambda_4 + \lambda_5 \geq 19/35,$$

$$\lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 \geq 20/35.$$

The "steepest" sustainable share system is (0, 0, 0, 0, 1), in which a group member slaves away until the last period when she gets the entire pie. The "flattest" is (5/28, 5/28, 5/28, 5/28, 8/28), in which a group member is held to a fixed share until the last period when she receives a retirement bonus. Each of these displays a seniority effect in that $\lambda_i \leq \lambda_{i+1}$. Notice, moreover, that an egalitarian system is not sustainable. However, there are share systems that do not display a seniority effect which are sustainable—for example, the "juniority" system (15/35, 1/35, 3/35, 6/35, 10/35). It should be noted that lifetime utility is the same for all sharing systems sustaining first-best effort; however, lifetime utility is lower for those sharing rules that cannot sustain first-best effort levels.

5.2. Expropriation

Now we want to add another consideration. Is an individual's share so large that other individuals would want to expropriate it? The answer depends on the opportunities for expropriation. We start with the idea that any single member can effect a "coup." If the party is reorganized so as to deny member j her share, the immediate output is divided in a manner preserving the original share structure—that is, $\lambda_i' = \lambda_i/[1 - \lambda_j]$. As a result of the expropriation, the party disbands and the members get zero in all following periods.[15]

With this specification of a reorganization, a "coup" would always be led by the senior-most member. This increases her final payoff and there is no

15. More generally, it may take several members acting in unison to effect an expropriation. These revolutionary members need not get zero thereafter. Following the reorganization, they might form their own group in a way that maximizes their remaining lifetime utility. Of course, they would want to do this in a way that is expropriation-proof. This suggests an interesting topic for future research.

cost in the future since this is her final period of participation. We rule this type of reorganization out-of-court. Our concept is that the senior-most person is the target rather than the instigator of an expropriation.[16] In the discussion that follows, we restrict attention to an expropriation led by one of the $n - 1$ junior members. In many cases, if a "coup" happens, it will be led by the party's second-in-command.

Lemma 2. With any seniority-sharing rule satisfying $\lambda_i f[\Sigma_j \pi_j x_j] - a(x_i) \geq 0$, the member who gains the most from expropriation is the second-most senior member.

Proof. Since a member's immediate gain is proportional to her preexisting share, the second-most senior member has the biggest benefit from the expropriation. Conversely, she has the least to lose. Since the party disbands, each of the members forfeits the sum of future period surpluses (which are positive by assumption). Although everyone loses their last period of surplus, this is the only loss suffered by the second-most senior member. Q.E.D.

Under the conditions of Lemma 2, if the second-most senior member does not seek a reorganization, then the equilibrium is "expropriation-proof." The formal condition for this is provided in (10):

$$(\lambda_{n-1} + \lambda_n) f\left[\sum_j \pi_j z_j^*\right] - a(z_n^*) \geq \frac{\lambda_{n-1}}{1 - \lambda_n} f\left[\sum_j \pi_j z_j^*\right]. \tag{10}$$

Recognizing that $a(z_{n-1}^*)$ is a sunk cost, the left-hand side is the second-most senior member's utility without expropriation. This must exceed the benefit from expropriation as given on the right-hand side.

Putting Lemmas 1 and 2 together underscores the fact that there are two forces that need to be balanced in order to sustain first-best cooperation level with a sharing rule. Shares need to be large enough to induce cooperative effort, on the one hand, yet must not be so large as to encourage a preference for expropriation. Provided that the total surplus is sufficiently large, it is possible to design a seniority-sharing rule that provides incentives to cooperate at the first-best level without creating the incentive to expropriate.

Proposition 3. With $n \geq 3$ and rising productivity vector π, there exists a seniority system, λ, for which the first-best cooperation level, z^*, is sustainable as an expropriation-proof equilibrium if

16. Of course, there is the saying: "The best defense is a good offense."

$$f\left[\sum_{j=1}^{n} \pi_j z_j^*\right] - \sum_{j=1}^{n} a(z_j^*) \geq \frac{a(z_{n-1}^*)[\Sigma_{j=1}^{n} a(z_j^*)]}{\Sigma_{j=1}^{n-2} a(z_j^*)}. \tag{11}$$

Proof. In Lemma 1, it was shown that the seniority-sharing system according to (9) leads to positive surplus in each period. Thus, by Lemma 2, if this seniority system satisfies (10), it is expropriation-proof.

To demonstrate this, we first rearrange (10):

$$\frac{\lambda_n}{1-\lambda_n}[1 - \lambda_{n-1} - \lambda_n] f\left[\sum_{j=1}^{n} \pi_j z_j^*\right] \geq a(z_n^*) \tag{10'}$$

$$\Leftrightarrow$$

$$f\left[\sum \pi_j z_j^*\right] - \sum_{j=1}^{n} a(z_j^*) \geq \frac{a(z_n^*)}{\lambda_n}\left[\frac{1-\lambda_n}{1-\lambda_{n-1}-\lambda_n}\right] - \sum_{j=1}^{n} a(z_j^*). \tag{10''}$$

Substituting the values of λ_{n-1} and λ_n from (9) into (10'') yields the sufficient condition

$$f\left[\sum \pi_j z_j^*\right] - \sum_{i=1}^{n} a(z_i^*) \geq \frac{a(z_{n-1}^*)[\Sigma_{j=1}^{n} a(z_j^*)]}{\Sigma_{j=1}^{n-2} a(z_j^*)}. \qquad \text{Q.E.D.}$$

5.3. Extensions

There is the matter of discounting. Let δ_i be a discount parameter for a member of age i.[17] Individual stage-game utility is unaffected:

$$U_i(x_1, \ldots, x_n) = \lambda_i f\left[\sum_{j=1}^{n} \pi_j x_j\right] - a(x_i).$$

Thus Corollary 5 still holds in the presence of discounting. With instantaneous punishment, there is never any gain from defection, and so the discount rate is irrelevant.

But lifetime utility depends on the discount parameter:

$$F(x_1, \ldots, x_n) = \sum_{i=1}^{n}\left[\prod_{j=0}^{i-1} \delta_j\right] U_i(x_1, \ldots, x_n),$$

17. In the next section, we discuss the case where the discount parameter is endogenous.

where $\delta_0 = 1$. On the other hand, *contemporaneous* group welfare looks identical to the case with $\delta_i = 1$, $\forall i$; a planner maximizing contemporaneous group welfare would find optimal cooperation levels as in the no-discounting case, z_i^*.

We return to our previous example to illustrate an optimal sharing rule. Consider a case where the party's objective is to maximize contemporaneous group welfare.[18] At any given level of group cooperation, contemporaneous welfare is independent of the sharing rule, except as it impinges on the incentive constraints. Thus, the party will try to sustain the first-best level of cooperation and then, among the sharing rules that do so, choose the one that maximizes the members' lifetime utilities.

Example 4. As before, $f(y) = y$ and $a(x) = (x^2/2 + x)/5$. Assume the productivity configuration: $\pi = (3/5, 1, 1, 1, 7/5)$. Maximizing F (with $\delta = 1$) yields a first-best cooperation vector $z^* = (2, 4, 4, 4, 6)$. To simplify things even further, assume that output shares depend only on productivity, so that $\lambda_2 = \lambda_3 = \lambda_4$. For $\delta_i = 4/5$, $i = [1, 5]$, the incentive compatibility constraints at z^* are

$$\begin{aligned}
\lambda_5 &\geq 2/9,\\
5[1 - \lambda_1 - \lambda_5]/3 + 4\lambda_5 &\geq 13/9,\\
45[1 - \lambda_1 - \lambda_5]/3 + 16\lambda_5 &\geq 77/9,\\
305[1 - \lambda_1 - \lambda_5]/3 + 64\lambda_5 &\geq 433/9,\\
625\lambda_1 + 1220[1 - \lambda_1 - \lambda_5]/3 + 256\lambda_5 &\geq 1940.\bar{3}/9.
\end{aligned}$$

In the no-discounting case, any sharing rule-sustaining first-best effort levels generates the same lifetime utility. This is no longer true in a world with discounting. With discounting, members will, ex ante, prefer a sharing arrangement that increases the payoff in early periods. In this example,

18. Specifying the group objective function is not a trivial matter. If commitment were possible and decisions were made in a veil of ignorance perspective, then the population would agree that the party's objective should be to maximize lifetime utility, $\Sigma_{i=1}^{n} \delta^{i-1} U_i$. This would also be the objective function for a party that must compete for new members by offering the highest utility possible. When commitment is not possible, then the group has an incentive to change the allocation so as to increase their present welfare, $\Sigma_{i=1}^{n} U_i$. Next period, the group faces the same problem and chooses the same allocation, so that their solution is time consistent. The absence of discounting, of course, favors the more senior members. Note that in this second case, the members are choosing an allocation just for the present period; their decision today does not commit them to use the same division tomorrow. A third quite reasonable approach would be for the group members to permit a change in the sharing rule, provided that change is maintained over time. The group adopts a new rule which stays in place until a later group implements a different rule. In this case, the group objective function would be $U_1 + (1 + \delta)U_2 + (1 + \delta + \delta^2)U_3 + \cdots + (1 + \delta + \cdots + \delta^{n-1})U_n$. Everyone present in the group today will at some point receive the U_n allocation. Only the youngest member cares about U_1. This approach places an even heavier tilt toward the more senior members. This objective function is time consistent as any subsequent group considering a new allocation rule would also have the same composition, one member of each age. Our use of contemporaneous group welfare as the objective function is only an example of an optimal sharing rule.

among incentive schemes that support the first-best level of contemporaneous group welfare, lifetime utility is maximized at $\lambda_1 = 4/9$, $\lambda_2 = \lambda_3 = \lambda_4 = 1/9$, $\lambda_5 = 2/9$.

6. FURTHER DEVELOPMENTS

There are several promising directions for future research.

Overlapping structure. The overlapping-generations model employed by Cremer and maintained here is extremely simple—there is exactly one member in each "cohort" and the cohorts each live precisely n periods. It would be of great interest to generalize seniority effects to situations in which there is a variable number of members of each "type" and a more complicated overlapping structure. Work is already underway in extending the model in this direction (see Cooper and Daughety; Kandori; Salant; Smith).

Discounting. We have only superficially allowed for individual discounting. Of special interest is making the discount rate endogenous. The probability that a legislator is reelected (and hence her discount rate) depends upon how well her constituents have prospered in the past (during her tenure) and how well they expect to do with her representing them in the future. Increased seniority gives constituents a greater incentive to reelect experienced legislators, especially with forward-looking voters. But should the full incumbent advantage commence at the start of a second term? There is a limited amount of seniority that can be distributed. What is the seniority profile that maximizes a freshman legislator's expected lifespan?[19]

Productivity. We have considered an organization in which experience increases productivity. But our specification of productivity effects is exogenous and leaves out proactive investments in human capital. That is, productivity is not only attached to positions and cohorts but also to individuals as a result of their (costly) human capital investments. Incentives to make those investments, just like incentives to cooperate, yield positive externalities to the larger group. Future work on seniority in the context of productivity effects will need to take this endogenous source of productivity into account.[20]

As we believe is evident, seniority is an empirically pervasive practice. There are literatures too large and disparate to cite here on seniority in

19. In intergenerational-transfer settings like Binmore's Mother–Daughter game, there is a similar issue; agents trade off intertemporal smoothing of consumption with front-loading consumption so as to increase the probability of survival into the future.

20. There is a very close relationship between the interests of group members in encouraging one another to make optimal human capital investments and the interests of legislators in encouraging their committees to make optimal investments in "specialization." On the latter, see Gilligan and Krehbiel (1987, 1988a,b, 1989).

legislatures, labor markets, civil-service regimes, social-security systems, and sociobiological communities. The supply of explanations and rationales, however, is surprisingly small. Thus, we need to work toward a better understanding, since seniority is an institutional practice, much like the division and specialization of labor, that vitally affects what groups can accomplish and how they go about their business.

APPENDIX

Corollary 1′. With discount rate δ, (1″) is necessary and sufficient for the truncated seniority system to be sustainable as a stationary equilibrium:

$$\sum_{i=1}^{n-1} \delta^{i-1}[f[(n-1)y_{n-1}] - a(y_{n-1})] + \delta^{n-1} f[(n-1)y_{n-1}] \geq f[(n-2)y_{n-1}]. \tag{1''}$$

Proof. The condition (1″) is the statement that the youngest member's lifetime utility in the second-best solution must be at least as high as her payoff from free-riding in the first period. This makes it clear why the condition is necessary.

To see that it is also sufficient, we consider two cases: (i) $\delta f[(n-1)y_{n-1}] - a(y_{n-1}) \geq 0$, and (ii) $\delta f[(n-1)y_{n-1}] - a(y_{n-1}) < 0$.

In case (i), we show that each member's continuation payoff is decreasing with seniority (up to age $n-1$). For a member of age $t < (n-1)$, the change in the continuation value between age t and $t+1$ is

$$\begin{aligned}\delta^{n-t-1}&\Big[(1-\delta)f[(n-1)y_{n-1}] - [f[(n-1)y_{n-1}] - a(y_{n-1})]\Big]\\ &= \delta^{n-t-1}\Big[-\delta f[(n-1)y_{n-1}] + a(y_{n-1})\Big]\\ &\leq 0.\end{aligned} \tag{A1}$$

The member is one period closer to the reward in period n, but has one less period of surplus along the way. In case (i), this change leads to a loss in the continuation value. Since the payoff to defection is constant at $f[(n-2)y_{n-1}]$, the greatest incentive to defect occurs when the continuation value is lowest, which is the penultimate period. But the member with seniority $n-1$ prefers to cooperate one last time:

$$\begin{aligned} f[(n-1)y_{n-1}] &- a(y_{n-1}) + \delta f[(n-1)y_{n-1}] - f[(n-2)y_{n-1}] \\ &= \Big[\delta f[(n-1)y_{n-1}] - a(y_{n-1})\Big] \\ &\quad + \Big[f[(n-1)y_{n-1}] - f[(n-2)y_{n-1}]\Big] \\ &\geq 0. \end{aligned}$$

The first term is positive by assumption of case (i) and the second term is positive from the monotonicity of f.

Because the inequality in (A1) is reversed in case (ii), each member's continuation payoff is increasing with seniority. The payoff to defection is still constant at $f[(n-2)y_{n-1}]$. Thus, the most advantageous time to defect is the initial period. But (1″) ensures that the lifetime utility from cooperation is superior to defection, so that the youngest members choose not to defect.

Q.E.D.

Corollary 2. There is no advantage to deviating from the truncated seniority system when the free-riding opportunity is only one period away.

Proof. Consider the payoff from shirking in period $n-1$; the member gets $f[(n-2)y_{n-1}]$ but sacrifices the equilibrium payoff of $2f[(n-1)y_{n-1}] - a(y_{n-1})$. We must show that

$$2f[(n-1)y_{n-1}] - a(y_{n-1}) \geq f[(n-2)y_{n-1}]. \tag{A2}$$

We start with the first-order condition that defines y_{n-1}:

$$\begin{aligned} nf'((n-1)y_{n-1}) &= a'(y_{n-1}) \\ \Rightarrow f[(n-1)y_{n-1}] - f[(n-2)y_{n-1}] &\geq y_{n-1} f'((n-1)y_{n-1}) \\ &= y_{n-1}\, a'(y_{n-1})/n \\ &\geq a(y_{n-1})/n, \end{aligned} \tag{A3}$$

where the inequalities use the concavity of f and the convexity of a. Substituting the inequality (A3) into (A2), we have

$$f[(n-1)y_{n-1}] - \frac{n-1}{n}\, a(y_{n-1}) \geq 0. \tag{A4}$$

But this is simply the statement that average utility is (weakly) positive in the second-best solution. Since $y = 0$ leads to zero average utility, the second-best solution can do no worse than zero, and the result follows. Q.E.D.

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