Online Appendix to "Centralized School Choice with Unequal Outside Options"

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A Additional Theoretical Results

A.1 Proof of Theorem 1

We first prove a lemma that is going to be useful in the proof. This lemma shows that if, for some school, the expected number of assigned students is less than its capacity, then all students are assigned to either that school or more popular ones. This lemma holds for *all* standard mechanisms. Let n_j denote the expected fraction of students assigned to school s_j and let ℓ be the least popular public school that students with the outside option prefer to the outside option school.

LEMMA 1 *In any Bayesian Nash equilibrium of any standard mechanism, if $n_k < q_k$ for some k, then:*

1. $n_j = 0, \forall j > k$,
2. $\sum_{j=1}^{k} n_j = 1$, *if* $k \leq \ell$.

Proof. We first show that for all $j > k$, $n_j = 0$. Suppose not. Then there exists a school s_i (where $i > k$) such that $n_i > 0$. As a result, there exist students who have ex ante positive probabilities of being assigned to s_i. Consider one of these students. This student should at least for a realization of her valuation vectors v with $f(v) > 0$ choose a strategy such that a list, l, in which s_i is ranked higher than s_k, is played with a positive probability; otherwise, she is never assigned to s_i as s_k has empty seats. This cannot be a behavior that emerges in an equilibrium. To see why, notice that this student can do better by playing a list l' instead of l, made from l by shifting down all schools $s_j, j > k$, ranked higher than s_k, including s_i, below s_k. By playing l', she will be assigned to schools $s_j, j > k$ with zero probability as they are ranked below s_k and s_k has empty seats. Moreover, the probability of being assigned to s_k and any school ranked higher than s_k in l' is weakly higher (and strictly higher for at least one of them) under playing l' than l. As such, she is better off.

Next, we show $\sum_{j=1}^{k} n_j = 1$, if $k \leq \ell$. From the first part, for all $j > k$, we know that $n_j = 0$. Assume $\sum_{j=1}^{k} n_j < 1$. Then, the expected number of unassigned students is positive, i.e., $n_\emptyset > 0$. The rest of the proof is similar to the previous part, by showing that students who have positive probability of being unassigned can improve their allocation by changing their strategy.

Lemma 2 *In any Bayesian Nash equilibrium of any standard mechanism, the expected fraction of assigned students n_j is equal to q_j, for all $j \leq \ell$*

Proof. Suppose not. Then there exists $k \leq \ell$ such that $n_k < q_k$. By lemma 1, it must be that $\sum_{j=1}^{k} n_j = 1$. Therefore, $\sum_{j=1}^{k} q_j > \sum_{j=1}^{k} n_j = 1$. Recall that we assumed $\sum_{j=1}^{\ell} q_j \leq 1$, which is a contradiction.

The rest of the proof is similar to the proof of the ACY main theorem. Fix a (potentially manipulable) mechanism such as the Boston mechanism. We will refer to it as the *mechanism* in this proof. Let $\sigma_w^*(v)_{v\in\mathcal{V}}$ and $\sigma_{w/o}^*(v)_{v\in\mathcal{V}}$ be the symmetric equilibrium strategies of the students with and without the outside option, respectively. Let $\pi_j^w(v^i)$ be the probability that a student with the outside option and with valuation v^i goes to school j in the Nash equilibrium. Define $\pi_j^{w/o}(v^i)$ similarly for students without the outside option. Our goal is to show that for a student i with outside option:

$$\sum_{j=1}^{\ell} \pi_j^w(v^i)v_j^i \geq \sum_{j=1}^{\ell} n_j v_j^i. \tag{2}$$

The left-hand side and the right-hand side are the expected utilities of students with outside option under the mechanism and Deferred Acceptance, respectively. Recall that s_l was the last public school that a student i with outside option preferred to his outside option.

Now, using a similar strategy to ACY, suppose students with outside option follow a different strategy and 'mimic' the population: for a student i with access to the outside option and valuation vector v^i, with probability $\eta f(v)$ they play the strategy $\sigma_w^*(v)$ and with probability $(1-\eta)f(v)$ they play the strategy $\sigma_{w/o}^*(v)$. In playing $\sigma_{w/o}^*(v)$, students with outside option drop schools with value less than their outside option from the list. The probability of going to school j under this strategy is at least:

$$\sum_{v\in\mathcal{V}} \Big(\eta\pi_j^w(v) + (1-\eta)\pi_j^{w/o}(v)\Big) f(v) = n_j. \tag{3}$$

The left-hand side is just the definition of n_j in the continuum economy.

The utility of a students with outside option from this new strategy is at least:

$$\sum_{i=1}^{\ell} v_j^i \Big(\sum_{v\in\mathcal{V}} (\eta\pi_j^w(v) + (1-\eta)\pi_j^{w/o}(v)) f(v) \Big) = \sum_{i=1}^{\ell} v_j^i n_j. \tag{4}$$

This is exactly their utility under Deferred Acceptance. Note that this is a lower bound on their utility, since by dropping those schools with value less than the outside option, they potentially increase their chances of going to schools they like.

This shows that utility for a student with access to the outside option under this new strategy is at least equal to his utility under Deferred Acceptance. Clearly, they must be weakly better off under the original equilibrium strategy σ_w^*, or else they could deviate to this new strategy that we just constructed. Hence, students with the outside option are weakly better off under any standard mechanism such as the Boston mechanism than under Deferred Acceptance. To complete the proof, note that if a student does not have the outside option, then our opening example shows that he can be worse off under a symmetric equilibrium produced by the Boston mechanism (compared to Deferred Acceptance equilibrium) so he will not always prefer the Boston mechanism. This completes the proof. ■

A.2 Segregation

One may be curious to see whether there are some plausible conditions under which students without an outside option always prefer the Deferred Acceptance mechanism to the Boston mechanism. To introduce one such condition, we first introduce the notion of a 'single-minded' student; in words, a student is single-minded if he only wants to attend the most popular school inside the centralized system or else prefers the school outside the centralized system.

Definition **2** *A student i is* ***single-minded*** *iff* $v_1^i \geq v_o^i \geq v_2^i \geq \cdots \geq v_M^i$.

The following theorem identifies one condition under which students without an outside option always prefer Deferred Acceptance.

Proposition **1** *Suppose students with the outside option are single-minded, and all students without the outside option have the same valuation vectors. Then, students without the outside option always prefer the Deferred Acceptance mechanism to the Boston Mechanism.*

Proof. When students with the outside option are single-minded, they will always report truthfully. Therefore, their ex ante probability of going to s_1 is at least q_1.

Next, note that all students without the outside option will play the same strategy, because we are studying the symmetric NE and they all have the same valuation vector. Now, if all students without the outside option report s_1 as their top choice with probability 1, they all have a q_1 chance of going to s_1, and by symmetry, the outcome is the same as under Deferred Acceptance. Suppose, on the contrary, that in the symmetric NE students without the outside option assign a non-zero chance to a rank-order list that does *not* put s_1 at the top, as in our illustrative example. Then, the probability that students without the outside option go s_1 would be strictly less than q_1 and they are all strictly worse off. ■

The above theorem states that if we assume that students without the outside option have homogeneous intensity of preferences, then single-minded students with the outside option are all better off under the Boston mechanism, while students without the outside option are all worse off.

Assuming identical valuations for students without the outside option essentially shuts down the channel by which the Boston mechanism enhances efficiency. However, this is not a knife-edge result. One can in principle use the above theorem and a continuity argument to show that for "small enough" variations in preferences, the same insight goes through. In other words, the Boston mechanism hurts students without the outside option by forcing them to compete more both within type and with students of the other type. As long as the preference-signaling gains from the mechanism are smaller than the cost, students without the outside option are worse off.

This result suggests that students with better outside options are more likely to attend the most popular schools in the centralized system when the mechanism is not strategy-proof. Hence, a direct prediction of our model is that manipulability can segregate students according to the constraints they face outside the centralized school system; that is, under a manipulable system, the most popular school in the centralized system will have more students with the outside option and fewer students without the outside option.

PROPOSITION **2 (Segregation)** *In any symmetric Nash equilibrium produced by the Boston mechanism, the fraction of students with the outside option who attend the most popular school in the centralized system is weakly higher than their population share* η*, and the fraction of students without the outside option who attend the most popular school in the centralized system is weakly lower than their population share* $1 - \eta$*. In addition, these weak inequalities hold strictly for some parameters.*

B Data construction

B.1 Data sources

Data for this project come from the New Haven Public Schools. We use two main data sources. The first are records from the choice process for assignment to school starting in the Fall of academic years 2015 through 2020. The second are student enrollment records from the fall of each year from 2014 through 2019.

We construct the dataset used in our main difference-in-differences analysis by merging the choice records on to the enrollment dataset. The sample universe is defined as students enrolled in NHPS pre-Ks in their age-4 year. These are students eligible for advancing to Kindergarten. Student covariates in this analysis come from school enrollment files. We match enrollment records to choice records using unique district student identifiers.

In our supplemental school-lottery based analysis (reported in Online Appendix D) the sample universe is defined by participation in a Pre-Kindergarten lottery. We then merge on both enrollment records (to obtain first stage estimates of enrollment effects) and Kindergarten choice behavior (to obtain IV estimates of the effects of enrollment on choice behavior). Student covariates in this analysis come from the school lottery files. District student IDs are not always available for students in the lottery dataset who have not previously enrolled in the NHPS system. Students for whom district identifiers are not available are merged by name and birthdate to enrollment and choice outcomes.

B.2 School classification

We identify and classify schools using NHPS records and state accountability data. Table B1 lists all the schools available to students participating in Kindergarten choice. The "Available years" column lists the years in which the school accepted Kindergarten applications. All schools are available in all years except Highville Charter, which accepted students for the first time in 2020.

Table B2 lists students' enrollment options for Pre-K, the year those options were available (between 2014 and 2019), and whether the school offered a continuation option into Kindergarten.

Figure B 1 reproduces the results in Figure 1 for each year in our data. Application patterns are broadly similar across years.

Table B1: Available lottery options in Kindergarten

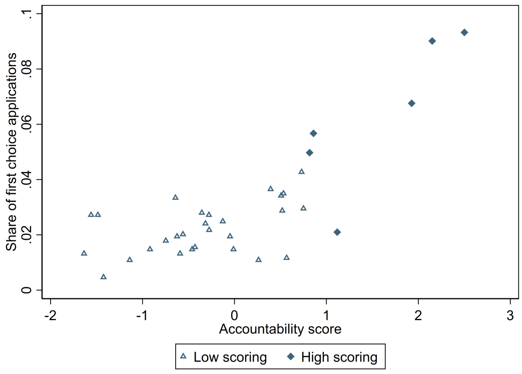
	Available in years
Amistad Academy Elementary & Middle Charter	2016-2020
Augusta Lewis Troup School	2016-2020
Barnard Environmental Studies Interdistrict Magnet	2016-2020
Benjamin Jepson Multi-Age Interdistrict Magnet	2016-2020
Bishop Woods Executive Academy	2016-2020
Booker T. Washington Academy Charter	2016-2020
Brennan-Rogers: The Art of Communication & Media Magnet	2016-2020
Celentano Biotech Health & Medical Magnet	2016-2020
Christopher Columbus Family Academy	2016-2020
Clinton Avenue School	2016-2020
Davis Street Arts & Academics Interdistrict Magnet	2016-2020
East Rock Community Magnet	2016-2020
Edgewood Magnet	2016-2020
Elm City College Preparatory Charter	2016-2020
Elm City Montessori Magnet	2016-2020
Fair Haven School	2016-2020
Harry A. Conte-West Hills Magnet: A School of Exploration & Innovation	2016-2020
Highville Charter School and Change Academy	2020
Hill Central School	2016-2020
John C. Daniels School of International Communication Interdistrict Magnet	2016-2020
John S. Martinez Magnet School	2016-2020
King/Robinson Interdistrict Magnet: An International Baccalaureate World School	2016-2020
L.W. Beecher Museum School of Arts & Sciences Interdistrict Magnet	2016-2020
Lincoln-Bassett Community School	2016-2020
Mauro-Sheridan Science, Technology & Communications Interdistrict Magnet	2016-2020
Nathan Hale School	2016-2020
Quinnipiac Real World Math STEM Magnet	2016-2020
Roberto Clemente Leadership Academy	2016-2020
Ross Woodward Classical Studies Interdistrict Magnet	2016-2020
Strong/Obama Magnet	2016-2020
Truman School	2016-2020
West Rock Author's Academy Interdistrict Magnet	2016-2020
Wexler-Grant Community School	2016-2020
Wintergreen Interdistrict Magnet	2016-2020
Worthington Hooker School	2016-2020

Table B2: Available enrollment options in PreK

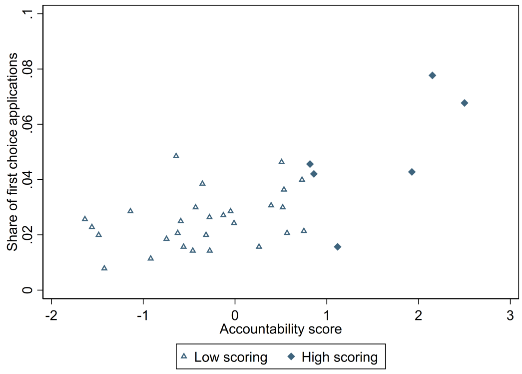
	Open in years	OO
Augusta Lewis Troup School	2014-2019	No
Barnard Environmental Studies Interdistrict Magnet	2014-2019	Yes
Benjamin Jepson Head Start	2014-2019	No
Benjamin Jepson Multi-Age Interdistrict Magnet	2014-2019	Yes
Bishop Woods Executive Academy	2014-2016	No
Brennan-Rogers: The Art of Communication & Media Magnet	2014-2019	Yes
Celentano Biotech Health & Medical Magnet	2014-2019	Yes
Christopher Columbus Family Academy	2014-2019	No
Davis Street Arts & Academics Interdistrict Magnet	2014-2019	Yes
Dr. Reginald Mayo Early Learning Center	2015-2019	No
East Rock Community Magnet	2014-2019	No
Fair Haven School	2014-2019	No
Harry A. Conte-West Hills Magnet: A School of Exploration & Innovation	2014-2019	Yes
Hill Central Music Academy	2014-2019	No
John C. Daniels School of International Communication Interdistrict Magnet	2014-2019	Yes
John S. Martinez Magnet School	2014-2019	No
King/Robinson Interdistrict Magnet School: An International Baccalaureate World School	2014-2019	Yes
L.W. Beecher Museum School of Arts & Sciences Interdistrict Magnet	2014-2019	Yes
Lincoln-Bassett School	2014-2019	No
Lulac	2014-2019	No
Mauro-Sheridan Science, Technology & Communictions Interdistrict Magnet	2014-2019	Yes
Nathan Hale School	2014-2019	No
Ross Woodward Classical Studies Interdistrict Magnet	2014-2019	Yes
Truman School	2014-2019	No
West Rock Author's Academy Interdistrict Magnet	2014-2019	Yes
Wexler-Grant School	2014-2019	No
Zigler PreK Center	2014-2018	No

OO refers to the option of continuing in the same school in kindergarten without another application. Bishop Woods was a PreK school until 2016 but stopped admitting PreK students thereafter. Lulac is not considered an NHPS school for purposes of PreK enrollment reporting in 2019. See Online Appendix C.

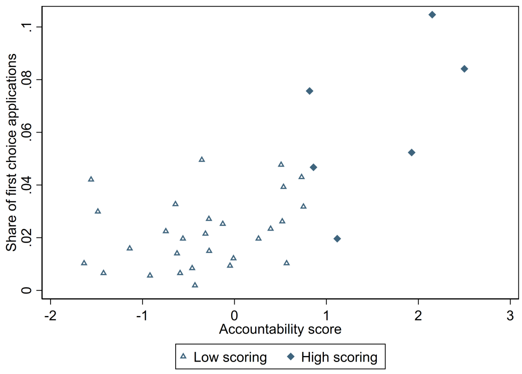
Figure B 1: Share of first choice applications by accountability score by year



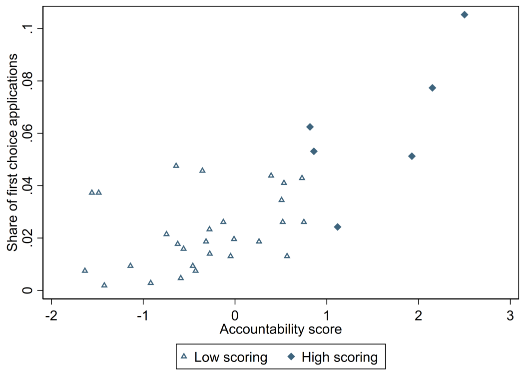
(a) 2016



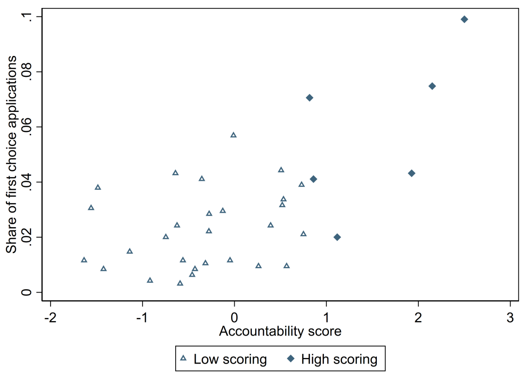
(b) 2017



(c) 2018



(d) 2019



(e) 2020

Figure displays the share of first-choice applications (vertical axis) by school-level accountability scores (horizontal axis). Panels refer to application statistics for the year listed in the title. See Section 3.1 for details.

C Additional analyses and results

C.1 Alternate approaches to inference and sample selection

In our main difference-in-differences analysis, our statistical tests are based on heteroscedasticity-robust standard errors that treat each individual as an independent observation. While we think the assumption that individual school choice applications are statistically independent is reasonable in our context, one common approach in difference-in-difference specifications is to cluster standard errors by the unit at which treatment is assigned (Bertrand, Duflo and Mullainathan, 2004). In our setting, that corresponds to the Pre-K school, because it is the schools where students are enrolled that determine outside option availability. Table C1 repeats the exercise in Table 2 with standard errors clustered at the level of the Pre-K school. In addition to clustered standard errors, we report p-values from a test of the null hypothesis of zero effect obtained using a clustered wild bootstrap-t procedure (Roodman et al., 2019). This procedure improves inference when the number of clusters is relatively small or clusters are of different sizes (Cameron, Gelbach and Miller, 2008; MacKinnon and Webb, 2017). Compared to our main estimates, these changes tighten our standard errors and yield p-values closer to zero.

Table C2 again repeats the exercise in Table 2, this time excluding the 2020 choice process from the sample. As noted above, applicants prior to 2020 could list four schools on their application, while in applicants in 2020 could list six. Changes in application length could alter strategic play separately from the change in assignment mechanism (Haeringer and Klijn, 2009). In addition, NHPS conducted an informational intervention in 2020 designed to encourage applicants to submit longer applications, with higher placement chances (Arteaga et al., Forthcoming). This intervention did not affect first choice selections, but does represent a break from past choice procedures that could in principle shape application outcomes. It turns out, however, that results that exclude the 2020 process are very similar to our main findings. The 2020 changes in application length and choice outreach are not compelling explanations for the changes in choice behavior we observe.

The exercise reported in Table C2 also helps assess robustness to a change in the analysis sample that took place in 2020. In that year, one of the larger Headstart programs in New Haven was reclassified as no longer being part of NHPS for the purposes of enrollment reporting. It is therefore excluded from our sample in that year. That excluding 2020 data does not affect our findings indicates that the exit of this program from the sample did not drive our results. This is consistent with the observation from Table 2 that the relationship between access to the outside option and observable predictors of choice behavior did not change in the post period relative to the pre period. The students who exit the sample in 2020 have similar backgrounds and behave similarly to other students without an outside option.

C.2 Event study

Event study specifications that compute year-by-year differences across groups before and after the change in the policy treatment are a common component of difference-in-difference analyses; in particular, they can provide a useful visual check for confounding "pre-trends" (Roth, 2018; Kahn-Lang and Lang, 2020). We observe three years of data under Boston and two years of data under DA, so this exercise is technically possible in our setting. Unfortunately, the small sample of individuals who participate in choice despite having an outside option available (204 students across all five years of data, as reported in Table 1) renders the results imprecise and uninformative. For transparency, we report our findings from this exercise in Online Appendix Table C3. For school quality measures, we cannot reject the null hypothesis that the coefficient on OO_i in year t is equal to the coefficient on OO_i in the year preceding the policy change (2018) for any year $t \neq 2018$. For the demand-share measure, the only difference that is statistically significant at conventional levels is the decline in the outside option vs. no outside option gap in 2019 relative to 2018.

C.3 Additional comments on comparing empirical evidence to theoretical predictions

In subsection 3.5, we discuss how to compare the empirical results to the predictions from the model when moving from a Boston mechanism that greatly rewards strategic behavior, to a truncated Deferred Acceptance mechanism. In the empirical results we observe full or nearly-full convergence in choice behavior between students with and without an outside option. This suggests that the distinction between strategy-proof Deferred Acceptance with unlimited list length and the non-strategy-proof truncated Deferred Acceptance mechanism that we observe in our setting (and that is common in school choice practice) did not seem to have first-order implications for choice behavior, at least as it interacts with access to outside options.

One feature of our results worth noting is that the gap in rates of listing high-achieving schools first between students with and without an outside option closes after the switch to Deferred Acceptance because the rate for the former group *falls*, not because the rate for the latter group *rises*.[10] This contrasts with the example presented in Section 2.1, in which convergence takes place as students without an outside option become more likely to list desirable schools first on their application. We note that the unreliability of causal inferences drawn from single-difference comparisons (such as the before-after comparison of rates of listing high-achieving schools for students without an outside option) is precisely why researchers employ difference-in-differences designs in the first place. For example, a reduction in

[10] Online Appendix Figure C 2 illustrates this point.

the relative preferences for high-achieving schools for all students over time would produce this type of result for reasons unrelated to the test we are trying to conduct.

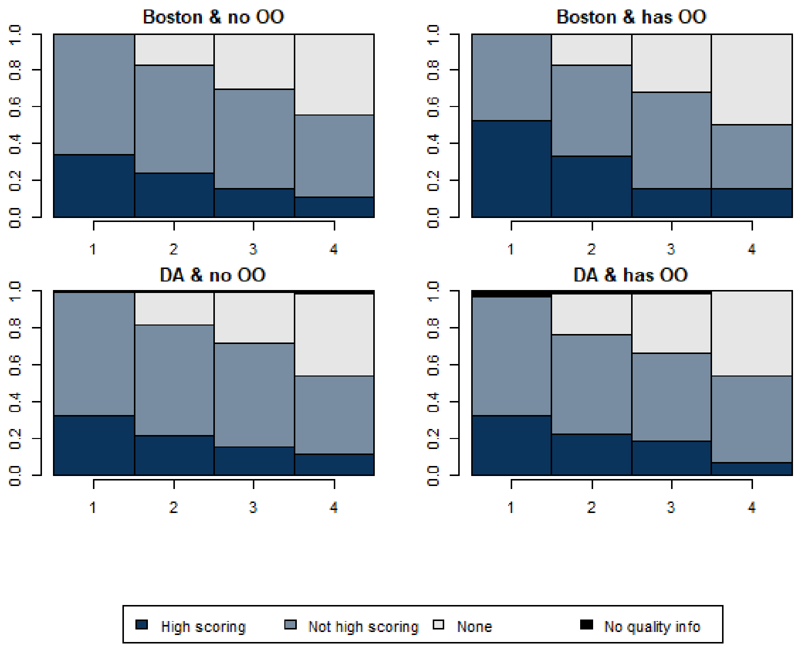
A second feature our results is that, although the share of students with and without an outside option who choose schools in the high scoring category fully converges, average quality at the first listed school only partially converges. Our model predicts full convergence of preference reports. Mechanically, what happens here is that the average quality of first-listed schools in the high scoring category for students with an outside option remains somewhat higher than for other students in the post period. One explanation for this behavior is a violation of the model's assumption that all students have the same preference rankings. If preferences over schools are differentiated horizontally (for example, due to location), then differences in rank lists may persist even if strategic reporting is eliminated.

C.4 Random assignment of outside options

Our main difference-in-differences approach relies on the assumption that, in the absence of the mechanism change, changes in choice behavior would have been the same for students with and without an outside option. The observable characteristics of the two groups are stable over the period in question, which suggests the assumption may be reasonable, but the pre-K that students attend is not randomly assigned and it is possible that group composition changes differentially in ways we cannot observe but that affect choice behavior.

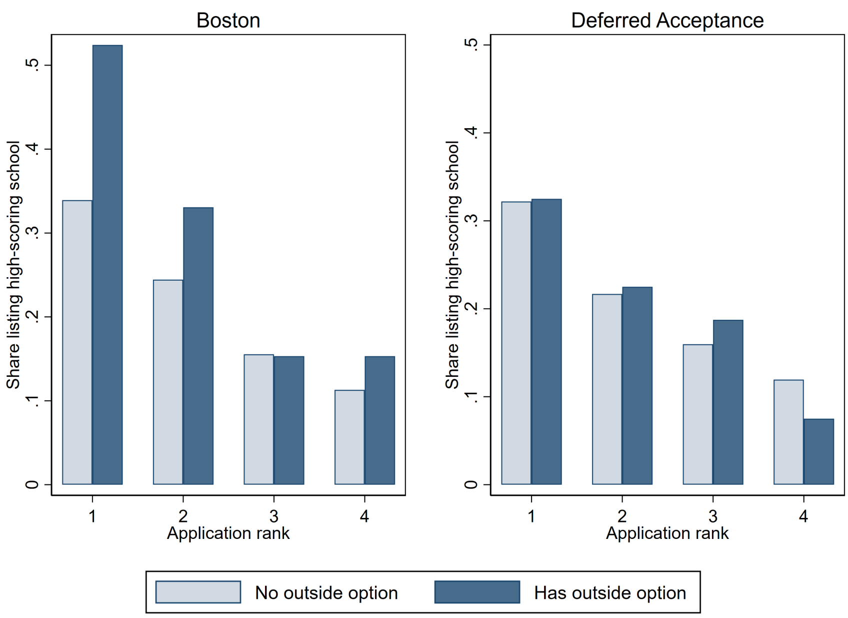
To address this possibility, we considered additional analyses that exploit the random assignment of students to *pre-Ks* that arises from tiebreaking lotteries in the *pre-K* choice process. This approach compares Kindergarten choice behavior for students who were assigned to pre-Ks that give students the option to continue Kindergarten to choice behavior for students who were assigned to pre-Ks without that option, using the econometric strategy from Abdulkadiroğlu et al. (2017). This analysis is challenging because participating in Kindergarten choice is an endogenous response to pre-K placement, so we cannot condition on participation while also effectively leveraging the randomness of lottery assignment. Including non-participants in the analysis requires defining new outcome variables that do not condition on choice participation, and for which the relationship to theory is less clear. This approach also reduces statistical precision. Online Appendix D describes the exercise in detail. Our results are too imprecise to provide compelling evidence one way or the other; we report them for completeness and transparency.

Figure C 1: School accountability scores by application rank, mechanism, and outside option availability



Horizontal axis in each graph is application position. Vertical axis is the share of applications. Upper two graphs show results under the Boston mechanism, lower two graphs show results under the DA mechanism. Sample: Pre-K students applying to Kindergarten.

Figure C 2: Share of high scoring choices by application rank



Sample: Students enrolled in PreK4 at a New Haven Public School and participating in the NHPS Kindergarten lottery in a year in which either the Boston or Deferred Acceptance mechanism was in place. Under the Boston (Deferred Acceptance) mechanism 124 (80) students have the option to continue Kindergarten without application at their PreK school and 1698 (770) students don't have this option.

Table C1: Differences in the outside-option effect before and after the mechanism change to Deferred Acceptance – *Standard errors clustered at PreK school level*

	List high scoring school 1^{st}		Quality of 1^{st}	Demand share of 1^{st}
Controls	Prediction	Actual	-listed school	-listed school
No controls				
Coeff.	0.010	-0.182	-0.463	-0.014
Std. err.	(0.019)	(0.056)	(0.129)	(0.004)
p-value	[0.668]	[0.003]	[0.002]	[0.002]
Demographics				
Coeff.		-0.190	-0.468	-0.014
Std. err.		(0.060)	(0.127)	(0.004)
p-value		[0.006]	[0.003]	[0.003]
+ *school zone*				
Coeff.		-0.195	-0.500	-0.015
Std. err.		(0.049)	(0.105)	(0.003)
p-value		[0.000]	[0.000]	[0.001]
N	2672	2672	2667	2643

Results from difference-in-difference estimates of equation 1 for the outcome listed in each column. The coefficients reported are from the $OO_i \times DA_t$ interaction term. Sample: students who are enrolled in a NHPS Pre-K and participated in a kindergarten lottery between 2016 and 2020. Sample counts differ slightly between columns 2 and 3 because one school does not have an accountability score; it is included in the left columns as a non-high-scoring school. Column 4 excludes 2020 applicants whose first choice school was new in 2020 and therefore does not have a 2019 demand share. Standard errors in parentheses are clustered at the level of the PreK enrollment school. p-values are in brackets and are obtained using a wild clustered bootstrap-t procedure with 1999 resamplings (Roodman et al., 2019).

Table C2: Differences in the outside-option effect before and after the mechanism change to Deferred Acceptance – *K lottery participants (excl. 2020 lottery)*

	List high scoring school 1^{st}		Quality of 1^{st}	Demand share of 1^{st}
Controls	Prediction	Actual	-listed school	-listed school
No controls				
Coeff.	0.032	-0.180	-0.618	-0.015
Std. err.	(0.028)	(0.089)	(0.236)	(0.005)
Demographics				
Coeff.		-0.189	-0.626	-0.016
Std. err.		(0.091)	(0.237)	(0.005)
+ *school zone*				
Coeff.		-0.214	-0.701	-0.018
Std. err.		(0.089)	(0.227)	(0.005)
N	2313	2313	2313	2313

Results from difference-in-difference estimates of equation 1 for the outcome listed in each column. The coefficients reported are from the $OO_i \times DA_t$ interaction term. Sample: students who are enrolled in a NHPS Pre-K and participated in a kindergarten lottery between 2016 and 2019. Robust standard errors in parentheses.

Table C3: Event Study estimates of the differences in the outside-option effect by year − *K lottery participants*

	Observational sample		
Controls	List high scoring school 1^{st}	Quality of 1^{st}-listed school	Demand share of 1^{st}-listed school
Demographics + school zone			
OO ×2016	0.117	0.521	0.006
Std. Err.	(0.117)	(0.295)	(0.007)
OO ×2017	0.144	0.369	0.009
Std. Err.	(0.109)	(0.294)	(0.006)
OO ×2019	-0.123	-0.406	-0.013
Std. Err.	(0.110)	(0.292)	(0.005)
OO ×2020	-0.078	0.054	-0.006
Std. Err.	(0.113)	(0.305)	(0.006)
N	2672	2667	2643

Event study estimates of equation 1 for the outcome listed in each column. The coefficients reported are from interactions between OO_i and year, with the 2018 interaction term normalized to zero. Sample: students who are enrolled in a NHPS Pre-K and participated in a kindergarten lottery between 2016 and 2020. Sample counts differ slightly between columns 1 and 2 because one school does not have an accountability score; it is included in the left columns as a non-high-scoring school. Column 3 excludes 2020 applicants whose first choice school was new in 2020 and therefore does not have a 2019 demand share. Robust standard errors in parentheses.

D Random assignment to outside options

D.1 Empirical framework

This section uses variation in assignment to schools with and without an outside option generated by the pre-K application process to conduct additional tests of model predictions. The intuition is as follows: to gain admission to pre-K schools with Kindergarten continuation options, students must apply through the centralized assignment process in either their age-3 or age-4 year. We can exploit random variation in assignment outcome from the pre-K lottery to estimate the effect of having an outside option on Kindergarten choice behavior.

To conduct this analysis we use data on the universe of Pre-K applications to the centralized choice system in the years 2015 through 2019. Following Abdulkadiroğlu et al. (2020), we estimate instrumental variables specifications of the form

$$\begin{aligned} Y_i &= \beta D_i + \Gamma_1 \mathbf{P}_i + e_i \\ D_i &= \tau Z_i + \Gamma_0 \mathbf{P}_i + v_i \end{aligned} \tag{5}$$

where Y_i is a kindergarten choice behavior of interest, D_i is an indicator variable for enrollment in a school with an outside option in a student's age-4 pre-K year, Z_i is an indicator for assignment to a school with an outside option in the choice lottery, and $\mathbf{P}_i$ is a vector of dummies for each value of a student's propensity to be placed in a school with an outside option, given their application and the rules of the choice lottery, rounded to the nearest 0.001. The intuition is that assignment to an outside option school is random within groups defined by assignment propensity, as in Rosenbaum and Rubin (1983).

We compute propensity scores P_i using a resampling process that follows Agarwal and Somaini (2018) and Kapor, Neilson and Zimmerman (2020), and builds on insights from Azevedo and Leshno (2016). This procedure relies on the observation that the Boston and Deferred Acceptance mechanisms are Report Specific Priority + Cutoff (RSP+C) mechanisms in the Agarwal and Somaini (2018) sense. In RSP+C mechanisms, the admissions chances for a given application to a given school depend on that application's report specific priority (i.e., a student's admissions priority group at the school) and the cutoff value for a school, which is chosen to reflect the school's capacity constraint.

The resampling process works as follows. Within each market (defined here by year and grade) we draw a large number ($N = 201$) of resampled markets from the population iid with replacement. Each resampled market is a list of individuals with a participation decision, a report if they participated in the lottery, and a priority at each school. In each resampled market, we solve for market-clearing cutoffs by running the assignment mechanism. Using j to index schools, the cutoffs $\left\{\pi_j^{(k)}\right\}_{k=1,\ldots,N}$

allow us to calculate admissions chances for each individual i at each school j in each resampling k. See Kapor, Neilson and Zimmerman (2020) for details of this procedure. We obtain our estimates of the propensity to be placed in a school with an outside option (the $\mathbf{P}_i$) by averaging school-specific placement probabilities over the resampled market-clearing cutoffs and then adding the placement probabilities for all outside-option schools.

With propensity scores $\mathbf{P}_i$ in hand, we compute estimates of Equation 5 separately in two samples, defined by the assignment mechanism used when applicants are scheduled to enter the Kindergarten choice process if they follow the normal grade progression. Students applying to age-4 pre-K programs in 2018 and earlier are in the Boston mechanism sample, as are students applying to age-3 pre-K programs in 2017 and earlier. Other students are in the Deferred Acceptance sample. Students who have not reached the age of scheduled kindergarten application by the time our panel ends in 2020 are excluded from the sample. This exclusion eliminates, for example, students applying to age-3 pre-K in 2019, because they are not yet eligible for Kindergarten choice in 2020, the last year of our data. We split the sample on the basis of the choice mechanism students experience in their Kindergarten choice process because that is age at which difference in continuation options across schools emerge.

D.2 Data description

Table D1 describes the data used for the lottery-based analysis. 3,363 individuals participate in a pre-K lottery during our sample period. Students occasionally participate in a pre-K lottery more than once if they want to switch schools between their age-3 and age-4 years, so the count of lottery participants at the student-year level is larger, equal to 3,941. 51% of participants are applying for placement in their age 3 year, denoted PK3 in the table. In what follows, all observations are at the level of the pre-K *application*. In our regression analysis we account for the multi-time applicants in inference by clustering standard errors at the person level.

Compared to statistics for our enrollment sample reported in Table 1, pre-K choice applicants are somewhat likely to be white, though Black and Hispanic students still make up more than 80% of the sample.

All pre-Ks to which students apply through the choice process are magnet schools that offer outside options. Students may enroll in the remaining stand-alone pre-Ks outside the choice process.[11] 26.7% of students who participate in choice receive a placement, and 22.1% enroll in a magnet (outside option) school the following year. 47.4% of participants enroll in some NHPS pre-K program; the additional students beyond those enrolled in magnets are attending stand-alone pre-Ks without

[11]Non-magnet pre-K enrollment is decentralized; this contrasts with magnet pre-K choice and with all Kindergarten choice, as described in the main text.

continuation options. Remaining students attend non-NHPS pre-Ks or receive care outside of the formal schooling system.

22.9% of students enroll in a pre-K with an outside option in their age-4 year. These are students we define as having outside options in the Kindergarten choice process. The number differs from the 22.1% year-after enrollment share because some students applying to pre-K for the age-3 year may apply again in their age 4 year.

48.5% of applicants have interior placement probabilities. Our lottery-based analysis focuses on these students. Consistent with descriptive statistics for aggregate placement rates, the mean estimated placement propensity is 0.256; the mean for students with interior placement probabilities is 0.257.

Panel III of Table D1 describes how students in the sample participate in Kindergarten choice. 54% of Pre-K lottery participants go on to participate in Kindergarten choice. This figure rises to 64% for those without an outside option.

The remaining rows of Panel III describe the outcome variables for Equation 5. A challenge here is how to define Kindergarten choice outcomes in a sensible way for the nearly half of pre-K choice participants who do not go on to participate in Kindergarten choice. This is critical for our analysis because, as we discuss in the main text, access to outside options dramatically changes rates of choice participation. This contrasts with our difference-in-differences tests in Section 3 of the main text, where we *condition* on participation in Kindergarten choice. We cannot do that here because Kindergarten participation is an outcome of pre-K lottery assignment, and conditioning on it would undo the benefits we get from randomization.

We define three outcomes of interest. The first is an indicator for listing a high scoring school first. This is equal to one for students who participate in Kindergarten choice and rank a high-scoring school first on their application. It is zero for other students, including non-participants. The second is an indicator for either applying and listing a high-scoring school first, or not applying and having a high-scoring school as your outside option. The third describes an outcome of the choice process–either placing at a high-scoring school through the process, or not placing and being defaulted in to a high-scoring outside option. We display descriptive statistics for these variables in the bottom three rows of Table D1.

D.3 Results

Table D2 reports findings estimates from Equation 5. The left three columns report results for students who reach Kindergarten age during the Boston mechanism period, the middle three columns report results for students who reach the Kindergarten age during the Deferred Acceptance period, and the third panel reports the difference between effect estimates in the two periods.

Panel I reports balance tests in which we estimate the first-stage equation with placement as the independent variable and predetermined student covariates as outcomes. We observe some imbalance in student observables in the Boston mechanism period, with Black students more likely to be placed at a school with an outside option and Hispanic students less likely to be placed.

Our concerns about how any imbalance may impact our findings are limited, for several reasons. First, lottery placements are balanced on observables in the Deferred Acceptance period, and we cannot reject the joint null of no difference in estimated effects of placement on observables across periods. Second, balance concerns arise only in lottery-reported race indicators, which have some missing data. We do not observe imbalance in census measures of tract-level demographics or poverty, where no data is missing. Third, the lack of balance we observe appears to arise solely from pre-K lotteries in two years, 2016 and 2017.[12] Supplementary analyses that exclude these years return results similar to what we report here.

Panel II reports first stage estimates of the effect of placement Z_i on enrollment in a (magnet) school with an outside option in students' age-4 year. In both the Boston mechanism period and the Deferred Acceptance period, placement raises the probability of age-4 enrollment by between 36 and 37 percentage points, on a base of 0.229 in the full sample.

In the Boston mechanism period, students placed in schools with an outside option are less likely to enroll in other NHPS schools. We do not see this tradeoff in the Deferred Acceptance period. From the perspective of students' NHPS continuation options in the Kindergarten choice process, this distinction is not an important one. Both students who are enrolled in an NHPS school without a continuation option and students who are not enrolled in an NHPS school at all need to find a placement through the centralized process if they want to avoid an administrative placement.

Panel III reports instrumental variables estimates of the effects of enrollment in a school that offers an outside option during the age-4 year on students' Kindergarten choice behaviors. As expected, the option to continue in one's current school without having to go through the centralized choice process dramatically reduces the likelihood a student participates in the Kindergarten choice process. Magnet enrollment reduces participation in the centralized system by 71 percentage points in the Boston mechanism period and 61 percentage in the Deferred Acceptance period. These effects are each highly statistically significant. We cannot reject that the difference between them is zero at conventional levels.

Enrollment in a school with an outside option reduces the rate at which students *both* apply *and* list a high-achieving school first under both Boston mechanism and Deferred Acceptance. This is unsurprising given how much enrollment reduces application rates. We cannot reject a null of no difference across the Boston mechanism

[12]We do not see any evidence of imbalance in the Kindergarten choice lotteries.

and Deferred Acceptance periods. Turning to the outcomes of primary interest, we cannot reject the null hypothesis of no effect across any specification. Standard errors are in the 0.08 to 0.09 range in all cases, so we cannot rule out effects that are large relative to sample means. Consistent with theory, Deferred Acceptance effects are smaller (more negative) than Boston mechanism effects across each outcome, but we cannot rule out a null of no difference.

Table D3 presents an identical table that adds controls for demographic covariates to equation 5. There is no evidence that this increases precision. As in our main specifications we cannot reject the nulls of no effect and no difference between Boston mechanism and Deferred Acceptance effects at conventional levels.

Table D1: Sample descriptives of PreK Lottery participants

	Any PreK			PreK3			PreK4		
	All	Boston	DA	All	Boston	DA	All	Boston	DA
I. Demographics									
Tract poverty rate	0.228	0.228	0.227	0.226	0.227	0.225	0.230	0.230	0.230
Female	0.510	0.508	0.514	0.508	0.511	0.505	0.512	0.505	0.526
Black	0.433	0.439	0.424	0.437	0.444	0.428	0.429	0.434	0.419
White	0.173	0.163	0.188	0.181	0.171	0.193	0.164	0.155	0.181
Hispanic	0.392	0.396	0.387	0.379	0.381	0.377	0.405	0.408	0.399
Special education	0.047	0.051	0.040	0.044	0.046	0.041	0.050	0.055	0.039
Share PreK3 applicants	0.509	0.461	0.582						
II. Schooling & Choice									
Any magnet placement	0.267	0.229	0.327	0.338	0.298	0.388	0.194	0.170	0.243
Enroll magnet	0.221	0.202	0.252	0.264	0.248	0.285	0.202	0.191	0.223
Enroll any	0.474	0.481	0.463	0.452	0.448	0.457	0.494	0.506	0.468
Enroll magnet in PreK4	0.229	0.213	0.254	0.255	0.239	0.276	0.202	0.191	0.223
Interior placement chance	0.485	0.425	0.579	0.543	0.529	0.560	0.426	0.336	0.606
Magnet placement probability	0.256	0.220	0.311	0.327	0.290	0.373	0.182	0.159	0.226
Magnet placement probability – interior	0.257	0.247	0.269	0.289	0.249	0.335	0.216	0.244	0.184
III. Choices in K *Lottery*									
Participate	0.540	0.557	0.514	0.463	0.481	0.440	0.620	0.622	0.615
Participate | no OO	0.639	0.652	0.619	0.556	0.567	0.542	0.719	0.720	0.718
List high scoring school 1^{st}	0.171	0.179	0.160	0.154	0.161	0.146	0.189	0.194	0.179
High scoring 1^{st} choice or OO	0.195	0.203	0.182	0.175	0.182	0.167	0.215	0.221	0.204
Placed or OO at high scoring school	0.166	0.161	0.174	0.152	0.144	0.162	0.180	0.175	0.190
No. of obs.	3941	2395	1546	2004	1105	899	1937	1290	647
No. of students	3363	2067	1296	2004	1105	899	1937	1290	647

This table describes all applicants to PreK3 and PreK4 grades who are of age to enter the Kindergarten lottery between 2016 and 2020. "Any PreK," "PreK3," and "PreK4" panels show results for students appyling to different pre-K grades. Panel I describes student demographics. Panel II describes Pre-K choice behavior. "Any magnet placement" is an indicator for placement in a magnet Pre-K (a pre-K with a continuation option). "Enroll magnet" and "enroll any" describe enrollment outcomes in a magnet pre-K and any pre-K in the year following the lottery. "Enroll magnet in PreK4" is an indicator for magnet enrollment in PreK4. "Interior placement chance" is an indicator for having a magnet placement probability strictly between 0 and 1. "Magnet placement probability — interior" is the probability of magnet placement for applicants with interior placement probabilities. Panel III describes students' participation in Kindergarten choice. "Participate" is a dummy for submitting an application. "Participate — no OO" is a dummy for submitting an application in the sample of students without continuation options. "List high scoring school 1st" is equal to one if a student participates and lists a high scoring school first. "High scoring 1st choice or OO" is an indicator for having a high scoring school as a listed first choice or as an OO. "Placed or OO at high scoring school" is an indicator for either placing at a high scoring school or not placing and having such a school as a continuation option.

Table D2: RDMD Estimates on PK4 enrollment

	Sample: Participates in any PreK lottery								
	Boston			DA			DiD		
	β	SE	N	β	SE	N	β	SE	N
I. Demographics									
Tract poverty rate	-0.003	(0.009)	1011	-0.014	(0.010)	869	-0.010	(0.014)	1880
Female	-0.011	(0.042)	1012	0.049	(0.045)	872	0.061	(0.061)	1884
Has Race	0.022	(0.023)	1012	0.018	(0.017)	872	-0.004	(0.028)	1884
Black	0.109	(0.042)	930	0.032	(0.045)	840	-0.077	(0.061)	1770
White	-0.006	(0.036)	930	0.011	(0.036)	840	0.018	(0.050)	1770
Hispanic	-0.106	(0.038)	935	-0.041	(0.041)	843	0.065	(0.057)	1778
Black – census tract share	-0.010	(0.016)	1011	0.001	(0.019)	869	0.010	(0.024)	1880
White – census tract share	0.021	(0.016)	1011	0.020	(0.020)	869	-0.001	(0.026)	1880
Hispanic – census tract share	-0.010	(0.014)	1011	-0.021	(0.016)	869	-0.010	(0.021)	1880
Joint Test			0.020			0.628			0.440
II. First Stage									
Enroll Magnet	0.360	(0.035)	1012	0.370	(0.035)	872	0.010	(0.049)	1884
Enroll any	0.178	(0.038)	1012	0.314	(0.038)	872	0.136	(0.054)	1884
Enroll other	-0.182	(0.024)	1012	-0.056	(0.032)	872	0.126	(0.040)	1884
III. Choices K - IV									
Participate	-0.715	(0.113)	1012	-0.611	(0.121)	872	0.103	(0.165)	1884
List high scoring school 1^{st}	-0.123	(0.083)	1012	-0.169	(0.091)	872	-0.045	(0.123)	1884
High scoring 1^{st} choice or OO	0.010	(0.087)	1012	-0.062	(0.094)	872	-0.072	(0.128)	1884
Placed or OO at high scoring school	0.015	(0.080)	1012	-0.058	(0.091)	872	-0.074	(0.121)	1884

Coefficients from reduced form and IV estimates of equation 5. Sample: PreK applicants at NHPS schools old enough to participate in the Kindergarten lottery under Boston mechanism (first set of columns) and DA mechanism (second set of columns). Panel I: effects of lottery placement on predetermined covariates. Race data missing for some observations. Panel II: First stage effects of placement on magnet enrollment (schools with outside options), any enrollment, and non-magnet enrollment in students Pre-K4 year. Panel III: IV estimates of the effect of magnet enrollment (and the option to continue in current school through elementary grades) on listed outcome. "Participate" is a dummy for submitting an application. "Participate — no OO" is a dummy for submitting an application in the sample of students without continuation options. "List high scoring school 1st" is equal to one if a student participates and lists a high scoring school first. "High scoring 1st choice or OO" is an indicator for having a high scoring school as a listed first choice or as an OO. "Placed or OO at high scoring school" is an indicator for either placing at a high scoring school or not placing and having such a school as a continuation option.

Table D3: RDMD Estimates on PK4 enrollment with Gender and Race covariates

	Sample: Participates in any PreK lottery								
	Boston			DA			DiD		
	β	SE	N	β	SE	N	β	SE	N
I. First Stage									
Enroll Magnet	0.350	(0.034)	1012	0.368	(0.036)	872	0.018	(0.049)	1884
Enroll any	0.156	(0.037)	1012	0.310	(0.038)	872	0.154	(0.053)	1884
Enroll other	-0.194	(0.024)	1012	-0.057	(0.032)	872	0.137	(0.040)	1884
II. Choices K - IV									
Participate	-0.774	(0.114)	1012	-0.624	(0.121)	872	0.150	(0.166)	1884
List high scoring school 1^{st}	-0.159	(0.085)	1012	-0.175	(0.092)	872	-0.016	(0.125)	1884
High scoring 1^{st} choice or OO	-0.029	(0.090)	1012	-0.066	(0.095)	872	-0.037	(0.130)	1884
Placed or OO at high scoring school	-0.017	(0.083)	1012	-0.065	(0.090)	872	-0.048	(0.123)	1884

Coefficients from reduced form and IV estimates of equation 5. Sample: PreK applicants at NHPS schools old enough to participate in the Kindergarten lottery under Boston mechanism (first set of columns) and DA mechanism (second set of columns). Panel I: effects of lottery placement on predetermined covariates. Race data missing for some observations. Panel II: First stage effects of placement on magnet enrollment (schools with outside options), any enrollment, and non-magnet enrollment in students Pre-K4 year. Panel III: IV estimates of the effect of magnet enrollment (and the option to continue in current school through elementary grades) on listed outcome. "Participate" is a dummy for submitting an application. "Participate — no OO" is dummy for submitting an application in the sample of students without continuation options. "List high scoring school 1st" is equal to one if a student participates and lists a high scoring school first. "High scoring 1st choice or OO" is an indicator for having a high scoring school as a listed first choice or as an OO. "Placed or OO at high scoring school" is an indicator for either placing at a high scoring school or not placing and having such a school as a continuation option. All specifications include controls for race and gender.