Institution vs. Behavior: Robustness of Continuous Double Auction Market Outcomes to Noisy Implementation of Four Key Rules[1]

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First Draft June 10, 2023

Abstract

To what extent do the outcomes of markets and other social institutions emerge from complex micro-level interactions within the constraints of their structures, as opposed to being constructed from behavior of individual agents populating them? We present results of a computational experiment in which four important rules of laboratory continuous double auctions (CDAs) are relaxed by introducing white noise in their implementation: (1) no-loss constraint on bids and offers, (2) price priority among bids and asks, (3) bid-ask matching, and (4) trade order of multiple endowed units. Market level outcomes --allocative efficiency, trading volume, and prices--in CDAs are relatively robust to the presence of noise. Erosion of these outcomes (relative to Walrasian equilibrium predictions) is gradual with the rise in noise levels and becomes significant only at relatively high levels of noise. This robustness of market outcomes may help explain the tendency of certain markets to yield outcomes close to theoretical equilibria derived from traders' profit-maximization under a broad range of circumstances.

JEL Classification: C92, D44, D50, D70, D82, G14
Keywords: Social institutions, Double auction market, rules, robustness to errors, computational experiment

[1] We are grateful to Gabriel Faulhaber for research and programming assistance, Karim Jamal for valuable discussions during early phase of this project, with usual disclaimer about responsibility.

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"Rules alone can unite an extended order. … Neither all ends pursued, nor all means used, or need to be known to anybody, in order for them to be taken account of within a spontaneous order."
Hayek (1988, 19-20)

1. Introduction

Auction markets are an important class of institutions in most economies. Of various auction forms, double auctions are especially dominant in the financial sector. Laboratory and computational experiments have revealed that in a variety of settings outcomes of these markets approximate the competitive equilibrium even when they are populated by a mere handful of profit motived traders (Smith 1962), or by zero-intelligence (algorithmic) traders (who bid and ask randomly subject to a no-loss constraint (Gode and Sunder 1993). Smith (1982) laid out a list of precepts he thought to be necessary conditions for an experimental market to function effectively. These include salience (that is use of performance-based rewards), no-loss rule (budget constraint) on bids and asks, and binding contracts (which occur when a buyer accepts the offer of a seller or vice versa (bids and ask match or cross). Gode and Sunder (1997) specify a hierarchy of double auction rules that contribute to allocative efficiency of this market institution.

Institutional scholarship points to the critical role of formal rules and informal social norms and expectations as determinants of their outcomes (North 1991, Smith 2007). Experimental economics literature has already documented the properties of double auction market outcomes when all or some of their rules are enforced. In software-controlled environments of laboratory experiments it is possible to write software to render most violations of rules difficult, even impossible. Experienced researchers can step in to stop rule violations in manual laboratory experiments also. Since outside the controlled environments of laboratory where most markets of substantive interest operate, the effectiveness of rule enforcement is often unknown. The generalizability of laboratory results to markets outside is also limited. Knowledge of the robustness of properties of market outcomes to various rule violations is therefore valuable for design and regulation of markets. The present computational experiment reports on properties of market outcomes when enforcement of four rules is noisy (i.e., prone to random errors) in varying degrees.

In summary, the results suggest that the outcomes—as assessed by allocative efficiency, prices, trade volume and root mean squared deviation of transaction from equilibrium prices--are relatively robust to introduction of small amounts of noise in enforcement of the four important rules of double auction markets (see the opening paragraph above for the list of rules). Significant deviations from theoretical model predictions occur only when the noise in enforcement of rules reaches high levels. This robustness of DA properties to noisy rule enforcement may help us understand why and how the outcomes of double auctions correspond so well to theoretical prediction in a variety of circumstances such as (1) absence of atomistic competition in Smith 1962, (2) markets populated with zero intelligence traders in Gode and Sunder (1993), (3) fixed instead of salient payments to experimental subjects in Jamal and Sunder (1991), and (4) when rules of markets are not perfectly enforced, or traders make unsystematic errors. This robustness may also have helped this auction form to evolve and survive in society over the ages. The concluding section explores some implications for market design and regulation.

2.1 The market mechanism

The experimental market is organized as a computational continuous double auction with $I + K$ traders ($I = 20$ buyers and $K = 20$ sellers in the present implementation). Each market is run for a single period with 10,000 bid/ask steps (chosen to allow for adequate opportunities for trading). The trading mechanism is described in more detail later.

Each buyer (seller) bids (asks) randomly from its feasible range using a uniform distribution (see Gode and Sunder 1993). The default range of feasible bids for buyer i is $r_{bi,}$ = [0, v_i], and the range of feasible asks for seller k is r_{ak} = [c_k, 1000]. This base range is modified by introducing noise in the no-loss constraint in selected treatments as described in Section 2.3 (see Table 1).

Each bid, ask and transaction is valid for a single token. A transaction erases any unaccepted bids and asks from the book. A trader can only engage in either buy or sell transactions up to the maximum number of endowed tokens. A trading session terminates at the earlier of (1) when all traders run out of tokens, or (2) when the number of steps reaches the allowed maximum of 10,000.

In each step of a trading period a buyer or a seller is randomly selected from among all traders who have any tokens available for trading. If a buyer (seller) is selected, a bid (ask) is randomly generated from the buyer's (seller's) range of feasible bids (asks). In Treatments M01, M04, M05, M07, M09, M12, M13, and M15 a randomly generated white noise (i.i.d.) term is included in this bid(ask) as described in Section 2.3 below (Table 1).

[INSERT TABLE 1 HERE]

The newly generated bid (ask) replaces the existing high bid (low ask) in the market if the new bid (ask) satisfies the price improvement rule applicable to the respective treatment. Noise is introduced in this price-priority rule in Treatments M02, M04, M06, M07, M10, M12, M14, and M15 as described in Section 2.3 (see Table 1).

When the current high bid equals or exceeds the current low ask a transaction takes place between the two traders. However, in Treatments M03, M05, M06, M07, M11, M13, M14, and M15 this bid-ask matching is also subject to noise as described in Section 2.3 (Table 1).

Finally, in standard CDA, when endowed with multiple units, buyers buy their highest value units before buying the others (and sellers sell their lowest cost units before selling the others). In Treatments M08-M15 this trade order is also subject to noise as described in Section 2.3.

2.2 Demand and supply functions

At the beginning of each trading period, each buyer $i = 1, 2, \ldots I$ is endowed with the right to buy n tokens, and each seller $k = 1, 2, \ldots K$ is endowed with the right to sell n tokens. The j^{th} unit held by buyer i has a redemption value of v_{ij} where $j = 1, 2, \ldots n$ (ordered highest to lowest). The profit earned from buying a unit at price p_{ij} is $(v_{ij} - p_{ij})$. The redemption values assigned to all buyers constitute the demand function for the market.

The cost of the tokens held by seller k is c_{kj} , where $j = 1, 2, \ldots n$ (ordered lowest to highest). The profit from selling one unit at price p_{kj} is $(p_{kj} - c_{kj})$. The costs assigned to all sellers constitute the supply function for the market.

In all 17 treatments in the present experiment, the number of buyers and sellers is set at 20 each ($I = K = 20$). In one set of 8 single-unit treatments, endowment of each trader is set to 1 (n = 1). Buyer values are set 30 apart in range 800-230, and seller costs are set 30 apart in range 200-770. Two panels of Figure 1 show the market demand and supply functions of 20 steps each with each step belonging to a different trader.

[INSERT Figure 1 HERE]

In a second set of 9 multi-unit treatments (not yet completed) this endowment is set to 5 (n =5). Buyer values are set 6 points apart in range 800-206, and seller costs are set 6 points apart in range 200-794. Individual demand functions are created by randomly assigning one of the twenty steps from each quintile of the market demand function to one of the twenty buyers, so each buyer has exactly five units. Analogously, individual supply functions are created by randomly assigning one of the twenty steps from each quintile of the market supply function to one of the twenty sellers, so each seller has exactly five units. This procedure ensures that all twenty buyers are statistically identical, and same is true of the twenty sellers. The two panels of Figure 2 show the market demand and supply function of 100 steps each, along with 20 individual demand and supply functions (each consecutive individual function shifted to the right to avoid overlap).

[INSERT Figure 2 HERE]

In treatments with single-token endowments market demand and supply functions ($P = 800 - 30Q$ and $P = 200 + 30Q$, respectively) yield an equilibrium price of 500 per token, an equilibrium quantity of 10 tokens, and only 10 buyers and 10 sellers out of the 20 would trade in equilibrium.

2.3 Noisy Implementation of Trading Rules

In standard continuous double auctions (CDA), four constraints play important roles: (1) *No-Loss Constraint*, i.e., traders do not propose bids/offers whose acceptance may inflict a loss on them; (2) *Price Priority*, i.e. higher bids and lower asks receive priority over existing bids and asks; (3) *Bid-Ask Matching Rule*, i.e., bids and asks that match or cross trigger completion of a transaction at a price within the range between these two numbers;[2] and (4) *Trade Order*, i.e., when

[2] The mean of high bid and low ask is often used as the price (and is also reported here), but any convex combination of these two numbers may be chosen as the price.

buyers (sellers) are endowed with multiple units of tokens, they trade their higher value (lower cost) units before trading the other units.

We examine the consequences of introducing noise in this standard trading protocol on performance of CDA as assessed by allocative efficiency, and bias and volatility of prices and trading volume (using predictions of Walrasian equilibrium predictions as benchmark). We conduct two sets of computational treatments. In the first set, each trader is endowed with one token (and the trade-order is not relevant). Noise is introduced to various combinations of the first three constraints yielding a total of seven treatments: one at a time, two at a time, and all three simultaneously (plus the no-noise treatment). The second set of seven treatments each trader is endowed with multiple tokens, and the trade order is also subjected to noise. Table 1 summarizes all 15 treatments (seven in the first and eight in the second set). In addition, M00 and M16 are the control treatments without any noise for the single and multiple token endowments, respectively.

2.3.1 Noisy No-Loss Constraint

(In Treatments M01, M04, M05, M07, M09, M12, M13, M15; see Table 1).

In markets with noisy no-loss constraint, range of bids from buyers and asks from sellers is specified as follows:

BUYERS: The default (noise-free) range of feasible bids for the buyer i is $r_{bi,} = [0, v_i]$, where v_i is the redemption value of the token. Noise is introduced to v_i by replacing it with $v_i\ (1 + \tilde{\boldsymbol{e}}_{\boldsymbol{1}})$ where $\tilde{e}_1$ is an i.i.d. random variable $\tilde{e}_1 \sim$ U $(-x_1, x_1)$, and subject to $\max[x_1] = 1$. For example, if input $x_1 = 0.5$, noise term $\tilde{e}_1 \sim$ U (-0.5, 0.5). If -0.1 is drawn for $\tilde{e}_1$, then range of bids is modified to $r_{bi,} = [0, 0.9v_i]$; if 0 is drawn, then is unchanged at $r_{bi,} = [0, v_i]$; and if 0.1 is drawn, then $r_{bi,} = [0, 1.1v_i]$.

SELLERS: The default (noise-free) range of feasible asks for the seller k is $r_{ak} = [c_k, y]$ where c_k is the cost of the token to the seller, and y is an exogenously chosen upper bound for all market bids, asks, and transaction prices in the entire experiment. Noise is introduced to c_k by replacing it with c_k $(1 + \tilde{e}_2)$ where $\tilde{e}_2$ is another i.i.d. random variable $\tilde{e}_2 \sim U(-x_2, x_2)$, subject to $\max[x_2] = 1$. For example, assume that input $x_2 = 0.1$, noise term $\tilde{e}_2 \sim U(-0.1, 0.1)$. If -0.05 is drawn, then, range changes to $r_{ak} = [0.95c_k, y]$; if 0 is drawn, then range is unchanged at $r_{ak} = [c_k, y]$; and if 0.05 is drawn, then $r_{ak} = [1.05c_k, y]$.

2.3.2 Noisy Price Priority

(In Treatments M02, M04, M06, M07, M10, M12, M14, M15; see Table 1)

In markets with noisy price priority, the following applies to buyers and sellers.

BUYERS: A new bid *b* replaces the current bid (*cb*) if and only if ($b > cb\ (1+ \tilde{e}_3)$) where $\tilde{e}_3$ is an i.i.d. random variable $\tilde{e}_3 \sim U[-x_3, x_3]$ subject to $\max(x_3) = 1$. For example, if b = \$30 and cb= \$28, *b* replaces *cb* to be stored as the new *cb* in a standard CDA. Now assume that input $x_3 = 0.5$, so $\tilde{e}_3 \sim U[-0.5, 0.5]$ and 0.2 is drawn. The noisy current bid = 28 × (1+0.2) = \$33.60. In this case, *b* will not replace the current bid, even though $b > cb$. Similarly, when the value of $\tilde{e}_3$ drawn is sufficiently negative, *b* may replace *cb* even if the former is lower. For example, if b = \$28, cb= \$30, *b* will not replace *cb* in a standard CDA. However, if $\tilde{e}_3 = -0.2$ is drawn from $\tilde{e}_3 \sim U[-0.5, 0.5]$, the noisy current bid = 30 × (1-0.2) = \$24.00 will be replaced by *b* even though $b < cb$.

SELLERS: In a market with noisy price priority, a new ask *a* replaces the current ask *ca* if and only if ($a < ca\ (1- \tilde{e}_4)$) where random variable $\tilde{e}_4 \sim U[-x_4, x_4]$ subject to $\max(x_4) = 1$. For example, if a = \$25, ca = \$28, *a* replaces *ca* in a standard CDA to be stored as the new *ca*. Now assume that $\tilde{e}_4 \sim U[-0.5, 0.5]$ and 0.2 is the random draw for $\tilde{e}_4$. The noisy current ask = \$28 × (1-0.2) = \$22.40. In this case, *a* will not replace the current ask, even though $a < ca$. Analogous argument applies to the possibility of a higher *a* replacing a noisy *ca* when the value of $\tilde{e}_4$ drawn is sufficiently negative.

2.3.3 Noisy Bid-Ask Match

(In Treatments M03, M05, M06, M07, M11, M13, M14, M15; see Table 1)

In markets with noisy bid-ask match, a transaction occurs between a current bid, *cb* and a current ask, *ca*, if the following condition is met:

$$ca \leq cb \times (1+ \tilde{e}_5),$$

where an i.i.d. random variable $\tilde{e}_5 \sim U\,[-x_5, x_5]$ subject to $\max(x_5) = 1$. The transaction price is calculated as the arithmetic mean of the crossing current bid *cb* and current ask *ca*.

When zero is drawn for $\tilde{e}_5$, we have the standard CDA in which the transaction occurs when the current ask is equal to or lower than the current bid. For $\tilde{e}_5 > 0$, a transaction can occur even when *cb* is less than *ca*. For example, if *ca* is \$55 and *cb* is \$50. No transaction occurs under the standard CDA. Assume that $\tilde{e}_5 \sim U\,[-0.5, 0.5]$ and 0.2 is drawn from the distribution, the noisy *cb* becomes (\$50 x (1+0.2) = \$60). A transaction is executed in this case because *ca* = \$55 < noisy *cb* = \$60, at the (mid-point) transaction price (\$50+\$55)/2 = \$52.50. The buyer pays \$2.50 above its bid and the seller gets \$2.50 less than its ask. Analogous logic applies when $\tilde{e}_5 < 0$, and a transaction may fail to be executed even when the *cb* is above *ca*.

2.3.4 Noisy Trade Order for Multiple Unit Endowments

(In Treatments M08-M15 and M16 with no trade order noise; see Table 1)

When each trader is endowed with more than one token having different reservation values or costs, in a standard CDA it is assumed that every trader transacts its high value (low cost) unit before taking up the next highest value (next lowest cost) unit for trade. Relaxing this constraint on the order in which endowment of multiple units is traded is the fourth source of noise we consider in implementation of CDA in this experiment (in Treatments M08-M15 and M16 with no trade order noise; see Table 1). Assume that all units in the endowment of each buyer *i* have been ordered in a non-increasing vector ($v_{i1}, v_{i2}, \ldots$) and all units of each seller *k* have been ordered in a non-decreasing vector (c_{j1}, c_{j2}, ...). The degree of noise in trade order is parameterized by $0 \leq \theta < 1$, where the probability of n^{th} of the units remaining in the hands of a trader being picked for the next transaction is θ times the probability of the $(n - 1)^{\text{th}}$ unit being picked. This implies that if a trader has q units in hand at any time, the probability of the first unit (of highest value or lowest cost) of these being picked is $\frac{(1-\theta)}{(1-\theta^q)}$, and the probability of the second unit being picked is this expression multiplied by θ, and so on, such that the total probability of one of the q units being picked adds up to one, that is, $\sum_{n=1}^{k} \theta^{n-1} \left(\frac{1-\theta}{1-\theta^k}\right) = 1$; $\theta = 0$ implies no noise, and as θ approaches 1, the probability of any unit on hand being picked for trade approaches equal probability. As an illustration, Table 2 provides the individual probabilities of the q units at each stage of the trading, for $\theta = 0.5$.

[INSERT TABLE 2 HERE]

The distributions for the six noise parameters ($\tilde{e}_1$ to $\tilde{e}_5$ and θ) are specified at the start of each market session. Depending on the experimental treatments, some of the noise parameters are active, while others are muted by setting the chosen noise parameter ($\tilde{e}_1$ ... $\tilde{e}_5$) to zero and θ to one (see Table 1). In the first set of treatments (Treatments M01-M07), each buyer (seller) is endowed with one token each. In the second set of treatments (Treatments M08 to M15) each buyer (seller) is endowed with multiple (five) tokens. We added two noise free control treatments, M00 (for single token) and M16 (for multiple tokens).

Each step of simulation consists of the following elements in the given order: (1) either the pool of buyers (B) or the pool of sellers (S) is randomly picked with a 50/50 probability, (2) if B (S) is picked, one buyer (seller) is randomly picked from the pool of all buyers (sellers) who have non-zero token balances and asked to submit a bid (ask). The range of values (costs) available to buyers (sellers) is subject to specified levels of no-loss constraint noise ($\tilde{e}_1$ for buyer and $\tilde{e}_2$ for seller) in treatments M01, M04, M05 and M07. In these treatments, a new random draw for the noise is made before the buyer (seller) is asked to submit its bid (ask). Similarly, a new random draw for the noise for price priority, $\tilde{e}_3$ for buyer and ($\tilde{e}_4$) for seller, is made in treatments M02, M04, M06 and M07. Finally, the noise for the bid-ask match, $\tilde{e}_5$, is drawn in treatments M03, M05, M06 and M07.

In the second set (multi-unit treatments M08-M16) with trade order noise), parameter θ is active throughout except in M16. The process for setting the other noise parameters ($\tilde{e}_1$ to $\tilde{e}_5$) is the unchanged from the first set of treatments described in the preceding paragraph.

A schematic representation of the trading with noise parameter is presented in Figure 3, and an illustration of how transactions can take place is provided in Appendix 1.

INSERT Figure 3 HERE

3. Computational Results

The parameters of the computational market described above are summarized in the Appendix. These include the number of periods in each market (1), the number of steps in each period (10,000), the number of independent runs of each treatment (100), market and individual demand and supply functions, and the noise levels introduced in implementation of CDA rules. Since no information, learning or balances are carried over from one period to the next, single-period market tells us all we can learn. One hundred repetitions of the single period market yield the statistical sampling distribution of market performance statistics. We expect 10,000 steps to be sufficient to complete virtually all transactions in a period.

3.1 Prices and trading volume

Figure 4 Panels A and B present cloud plots of all transaction prices observed in 100 independent runs of each of the eight single-unit endowment Treatments M00-M07 (against the transaction sequence numbers on the x-axis). Treatment M00 (the top row of charts in Figure 4A) has no noise. In the other seven rows of Figure 4, charts in the left and right columns show results for noisy error ranges 0-10 and 0-20 percent respectively (i.e., for $x_i = 0.1$ and 0.2 respectively, as explained in Section 2.3 above, and in the Appendix). Horizontal red line in each chart is the equilibrium price 500 and the black curved line is the mean price for respective transaction sequence numbers. The +/- one standard deviation range of prices is also down in a broken horizonal curve. The mean allocative efficiency calculated for the 100 replications of each treatment is given as a number in each chart.

(INSERT FIGURE 4A & B HERE)

In all eight panels of Figure 4, mean transaction prices are close to the equilibrium level (500)[3]. This is true not only in absence of errors in implementation of rules (see the top row charts in Fig. 4), but also in charts in all other seven rows; errors of implementation being white noise, they do not introduce any up or down bias in transaction prices.

Eight panels of Figure 5 show the behavior of root mean squared deviation (RMSD) of transaction prices by transaction sequence numbers for eight single-unit treatments for the two levels of noise (continuous line for 10 and broken line for 20 percent noise). Note that:

- In all panels, the RMSD tends to decline with higher transaction sequence numbers (later in the period) because market demand and supply units closer to the equilibrium prices are

[3] Note that the mean of final one or two transactions (sequence numbers 12 or above) tends to deviate (up or down) noticeably from the equilibrium level. Since the equilibrium transaction volume in these markets is 10, only a small number of 100 replications reach volume of 12 or more, in which case the mean transaction prices is calculated from a small number of observations and has a larger standard error of estimation.

more likely be traded later, at prices closer to equilibrium. RMSD is near 100 for early transactions and 40-60 at the end of the period.

- There is no noise in Treatment M00. In the other seven treatments, when the noise level increases from 10 to 20 percent, RMSD rises (as indicated by broken lines being almost entirely above the continuous lines in Figure 5) by 3.14-17.13 on average across all transactions in the seven single-unit Treatments M01-M07. Of these, the the largest increase of 17.13 in mean of RMSD occurs in Treatment M07 (all three kinds of noise present simultaneously).

(INSERT FIGURE 5 HERE)

Although the average trading volume under Treatments M00-M07 can be discerned from panels of Figure 4, we also present Panels A and B of Figure 6 to explicitly show the mean, standard deviation and distributions (histograms) of trading volume (across 100 replications of each treatment) under the single-unit endowment Treatments M00-M07. The red vertical line in each chart indicates the equilibrium trading volume of 10 as a standard of comparison for the results of the computational markets. Introduction of noise in implementation of rules tends to increase the chances of an extra-marginal in the demand function being matched with an intra-marginal unit in the supply function (and vice-versa), thus allowing the trading volumes to exceed the equilibrium level. In Figure 6, histograms, this tendency can be seen in the form of higher mean trading volume and a right-tail skewness of histograms under the noise Treatments M01-M07. Furthermore, this effect increases when the magnitude of noise is increased from 10 percent in the left column of charts to 20 percent in the right column of charts.

INSERT Figure 6 HERE

3.2 Allocative Efficiency

Allocative efficiency is the fraction of maximum possible total (buyer + seller) surplus extracted by the traders in a market. Mean allocative efficiency (across 100 runs of each treatment) is inserted as a number in the panels of Figures 4A (for M00-M03) and 4B (for M04-M07). For sake of easy comparisons of the effects of introducing noise in implementation of three rules of CDA, these numbers are also charted in three panels of Figure 7. The upper (blue) line is the efficiency under 10 percent noise treatments and the lower (broken red) line is the efficiency of the same treatments with 20 percent noise. Each of the three panels includes five overlapping treatments. All three panels have the no-noise (M00) at the left end and three kinds of noise (M07) at the right end. In the first panel, the three intermediate points are for noise in No-Loss (M01), No-Loss plus Price Priority (M04), and No-Loss plus Bid/Ask (M05) treatments. In the second panel, the three intermediate points are for noise in Price Priority (M02), Price Priority plus No-Loss (M04), and Price Priority plus Bid Ask Match (M06) treatments. In the third panel, the three intermediate points are for noise in Bid Ask Match (M03), Bid-Ask Match plus No-Loss (M05), and Bid-ask Match plus Price Priority (M06) treatments.

INSERT FIGURE 7 HERE

In absence of any noise (Treatment M00), mean efficiency of these markets is the highest among all treatments at 96.8 percent. The 3.2 percent drop in efficiency arises when an extramarginal trader displaces an intramarginal trader which depends on the shape of extramarginal segments of demand and supply functions (see Gode and Sunder 2004). As seen in the left panel, introduction of noise in implementing the No-Loss rule, whether alone or in combination with noise in the Price Priority and/or Bid-Ask Match rules causes major drops in efficiency to (94.2%, 94.9%, 89.9%, 90.9%) with 10 percent noise and (85.4%, 86.7%, 71.3%, 73.5%) with 20 percent noise. In contrast, whenever noise is introduced to only the Price Priority and/or Bid-Ask Match rules, the drop in efficiency is smaller at (96.7%, 93.8%, 93.9%) for 10 percent noise and (96.3%, 91.0%, 91.0%) for 20 percent noise. These efficiency comparisons suggest that the effect of introducing noise to the No-Loss rule is the largest of the three, followed by the effect of noise in the Bid-Ask Match. Introduction of noise to the Price Priority rule appears to cause only a small drop in allocative efficiency of CDAs.

Table 3 shows the results of regression analysis of the effect of introducing noise in various combinations of the three CDA rules on their allocative efficiency. Results for 10 percent noise are given in the middle column and for 20 percent noise in the right column. The regression analysis also supports the conclusion that the introduction of noise in the price priority rule makes little difference to allocative efficiency, even when the noise level is raised to 20 percent. The adverse effects of noise in No-Loss and Bid-Ask March rules are of comparable magnitude. Further, these effects are approximately additive in presence of multiple kinds of noise.

(INSERT Table 3 HERE)

4. Discussion and concluding remarks

Designs of social institutions have significant consequences for properties of their outcomes, and consequently for our welfare. Rules and constraints imposed on individual behavior, as well as rules by which interaction among individual actions occurs to produce outcomes, are important aspects of institutional design. To the extent rules and constraints may prevent some or all participants from taking their preferred actions, or achieve their desired individual goals, their implementation may be resisted, and present a challenge for social organization. Desirability of allocating social resources to implementation should be related in some appropriate manner to the extent of harm the violations of specific rules and constraints may bring to society.

With this perspective, we report the results of a computational experiment with a social institution called double auction which is especially important in financial sector of economy. We examine the allocative efficiency (and some other) consequences of introducing two alternative levels of noise in four rules often used in CDAs. We find that imperfect (noisy) implementation of two of these rules (No loss constraint and bid-ask match) has significant negative social consequences in form of reduced allocative efficiency, which tends to be relatively insensitive to noise in implementation of a third rule (price priority). At the time of this writing, consequences of noise in the fourth rules are still being determined.

The present report is an initial attempt in a program to examine consequences of imperfect implementation of specific rules, which may help us design better social institutions, implementation, and regulatory strategies.

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Table 1: Experimental Treatments

Treatment	***Noise in No-loss Constraint*** $\tilde{e}_1, \tilde{e}_2$	***Noisy Price Priority*** $\tilde{e}_3, \tilde{e}_4$	***Noisy Bid-Ask Match*** $\tilde{e}_5$	***Noisy Trade Order*** θ	***Active Noise Parameters***
Panel A: Set of treatments where each trader is endowed with a single unit					
M00					*No noise*
M01	Present				$\tilde{e}_1, \tilde{e}_2$
M02		Present			$\tilde{e}_3, \tilde{e}_4$
M03			Present		$\tilde{e}_5$
M04	Present	Present			$\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \tilde{e}_4$
M05	Present		Present		$\tilde{e}_1, \tilde{e}_2, \tilde{e}_5$
M06		Present	Present		$\tilde{e}_3, \tilde{e}_4, \tilde{e}_5$
M07	Present	Present	Present		$\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \tilde{e}_4, \tilde{e}_5$
Panel B: Set of treatments where each trader is endowed with multiple (5) units					
M08				Present	θ
M09	Present			Present	$\tilde{e}_1, \tilde{e}_2, \theta$
M10		Present		Present	$\tilde{e}_3, \tilde{e}_4, \theta$
M11			Present	Present	$\tilde{e}_5, \theta$
M12	Present	Present		Present	$\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \tilde{e}_4, \theta$
M13	Present		Present	Present	$\tilde{e}_1, \tilde{e}_2, \tilde{e}_5, \theta$
M14		Present	Present	Present	$\tilde{e}_3, \tilde{e}_4, \tilde{e}_5, \theta$
M15	Present	Present	Present	Present	$\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \tilde{e}_4, \tilde{e}_5, \theta$
M16					*No noise*

Table 2: Probability of an Ordered Unit being Picked with k Units Remaining (for θ = 0.5)

Units remaining (k) for buyer	Probability of nth unit is given by $\theta^{n-1}\left(\frac{1-\theta}{1-\theta^k}\right)$					Total probability
	1st (highest)#	2nd	3rd	4th	5th (lowest)	
5	0.516	0.258	0.129	0.065	0.032	1.00
4	0.533	0.267	0.133	0.067		1.00
3	0.571	0.286	0.143			1.00
2	0.667	0.333				1.00
1	1.000					1.00

#In the case of sellers, in the absence of trade order noise, unit with the lowest cost is traded first, and the unit with the highest cost is traded last.

Table 3: Results of Allocative Efficiency Regression on Single-Unit Treatment Dummies for (M00-M07)

	Noise Level = 10 percent	**Noise Level = 20 percent**
	Coefficient (t Value)	**Coefficient (t Value)**
Intercept	0.97 (241.7)	0.97 (180.9)
No-Loss Noise	-0.03 (-4.6)	-0.11 (-15.1)
Price Priority Noise	-0.001 (-0.2)	-0.01 (-0.7)
Bid-Ask Noise	-0.03 (-5.4)	-0.06 (-7.8)
NL + PP Noise	-0.02 (-3.6)	-0.10 (-13.4)
NL + BA Noise	-0.07 (-12.2)	-0.26 (-33.7)
PP + BA Noise	-0.03 (-5.2)	-0.06 (-7.7)
NL+PP + BA Noise	-0.06 (-10.52)	-0.23 (-30.9)
DF (Model/Error)	7/ 792	7/ 792
F Value	37.7 (Pr. < 0.0001)	321.7 (Pr. < 0.0001)
R^2 (Adj. R^2)	0.2498 (0.2431)	0.7398 (0.7375)

Figure 1: Market Demand and Supply Functions for Single-Unit per Trader Treatments (M00-M07): Equilibrium Price = 500; Equilibrium Quantity = 10

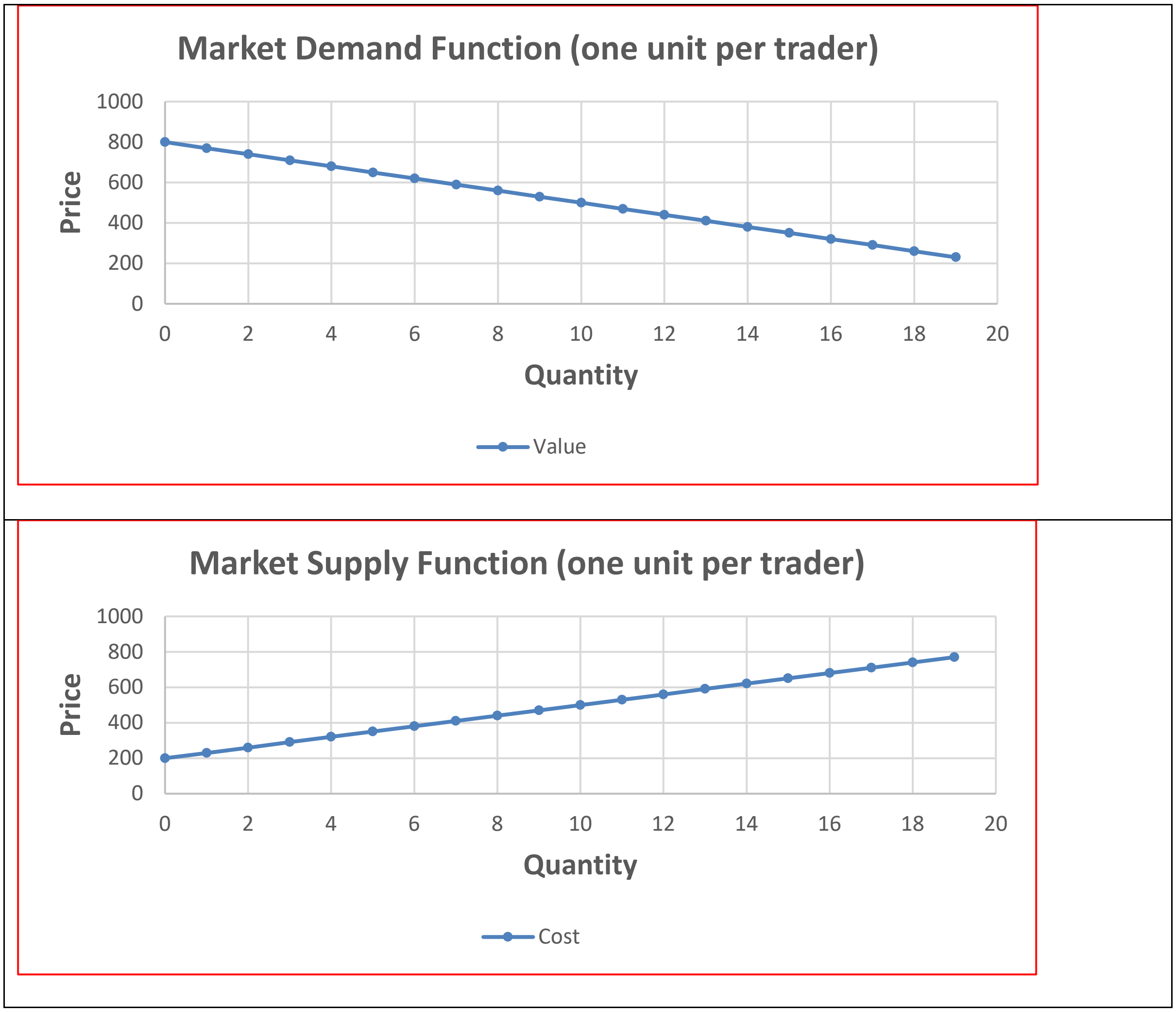
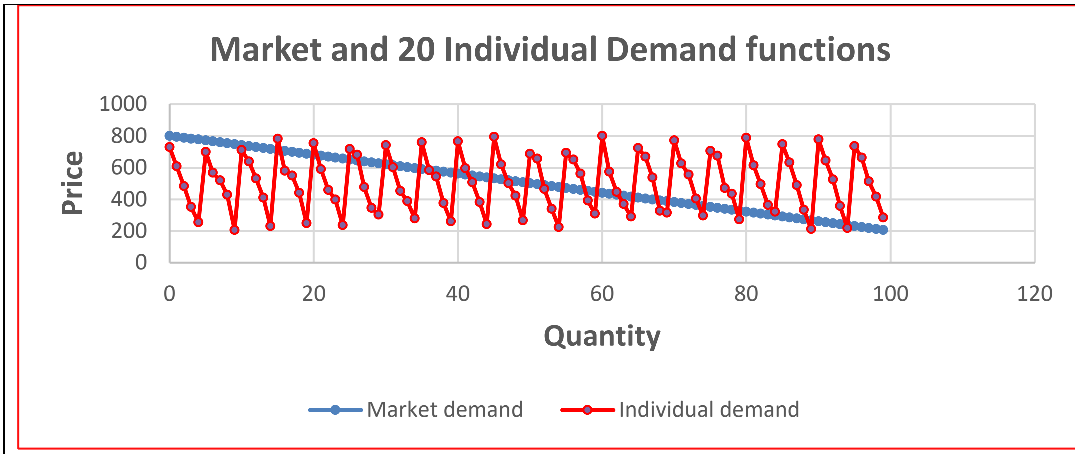


Figure 2: Market and Individual Demand and Supply Functions for Multiple-Unit per Trader Treatments (M08-M16): Equilibrium Price = 500; Equilibrium Quantity = 50

(Note: For clarity, each consecutive individual function shifted right by 5 units)



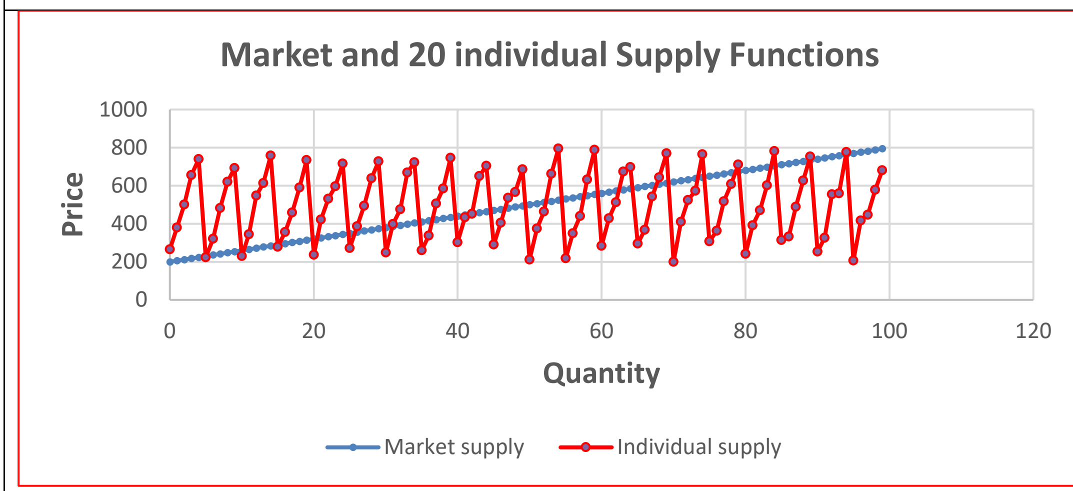


FIG. 3. — Schematic representation of Transaction

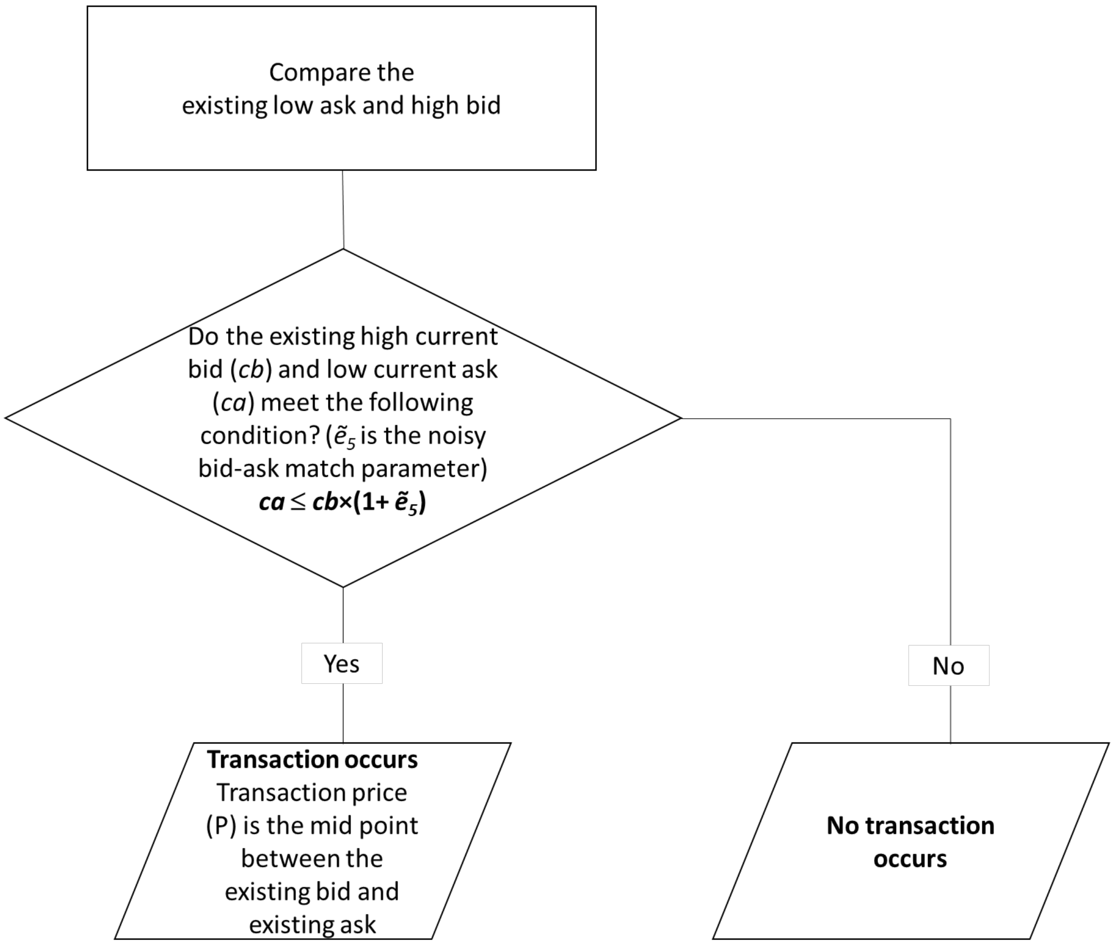


Figure 4A: Transaction Prices and Allocative efficiency in Single-Unit Treatments (M00-M03)

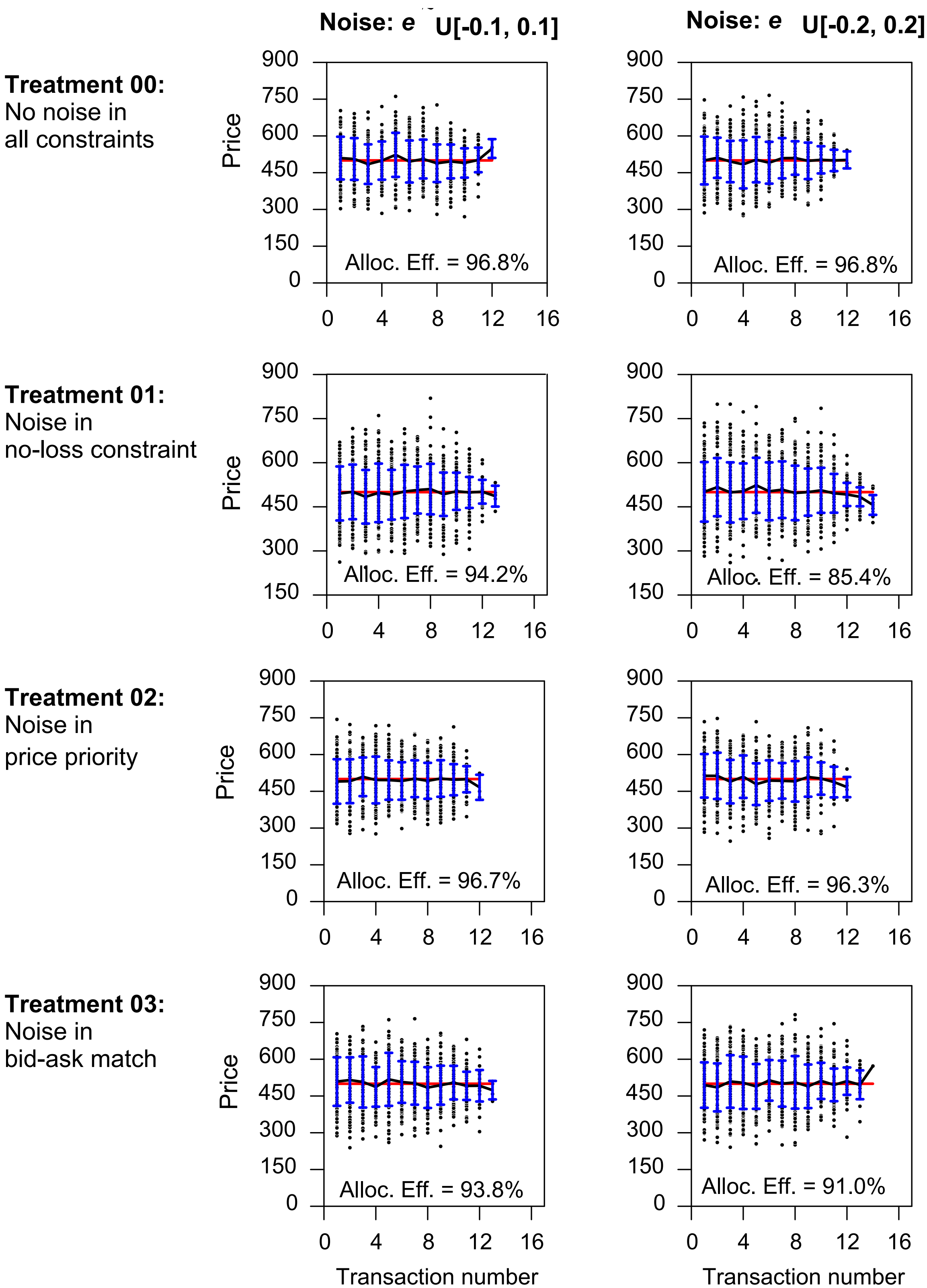
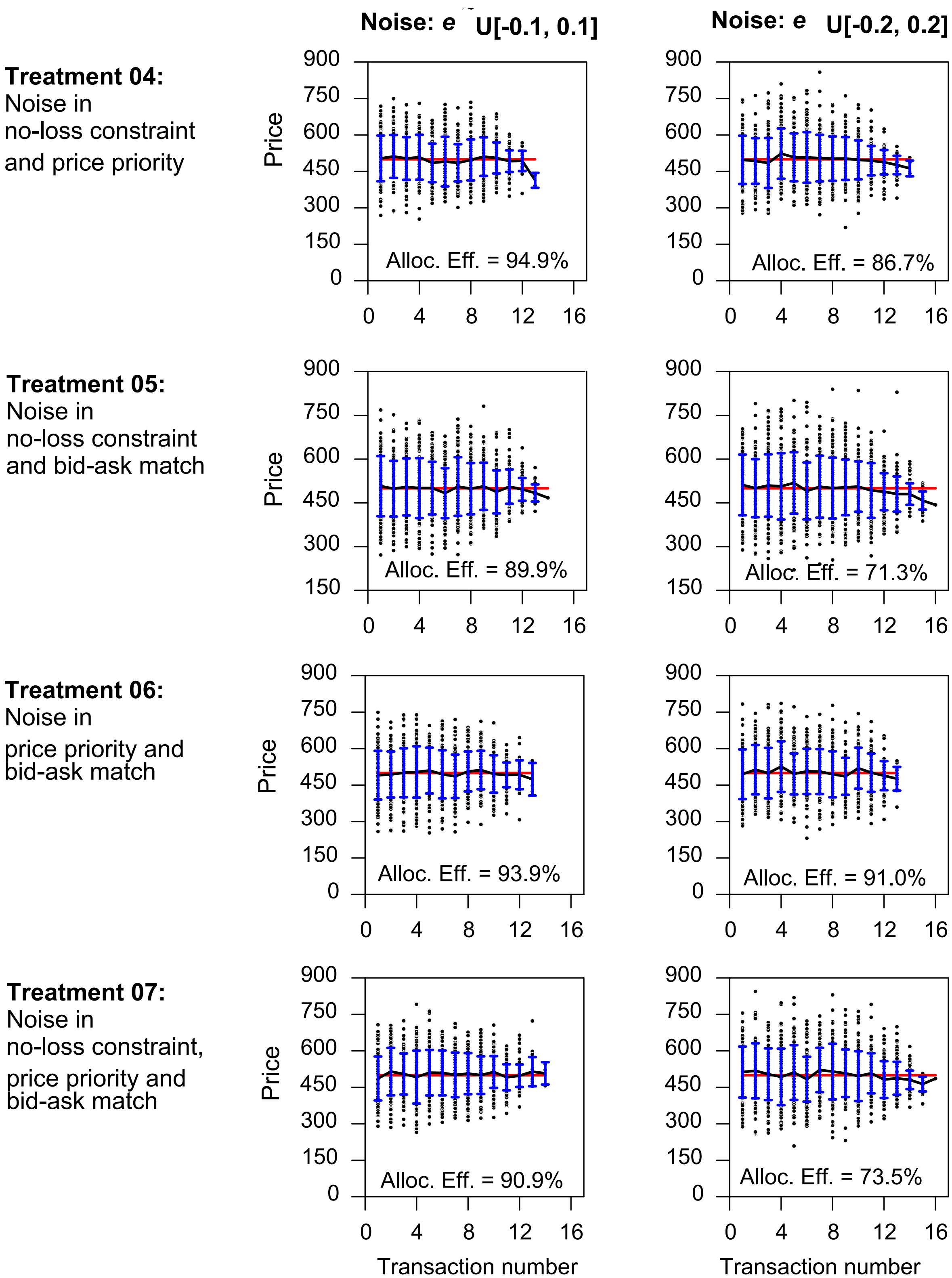
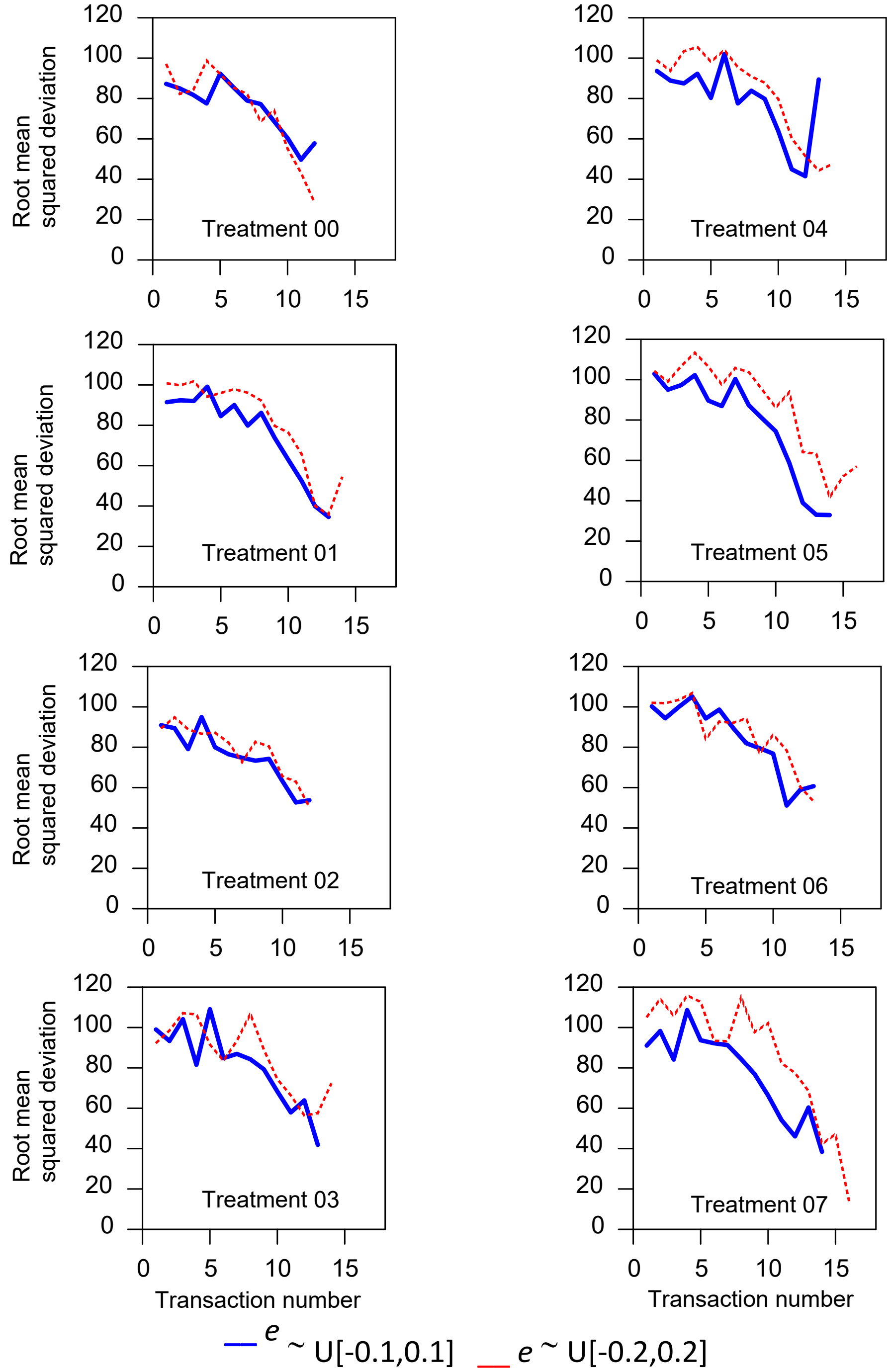


Figure 4B: Transaction Prices and Allocative efficiency in Single-Unit Treatments (M04-M07)



Noise: e U[-0.1, 0.1]
Noise: e U[-0.2, 0.2]
Treatment 04:
Noise in
no-loss constraint
and price priority
Alloc. Eff. = 94.9%
Alloc. Eff. = 86.7%
Treatment 05:
Noise in
no-loss constraint
and bid-ask match
Alloc. Eff. = 89.9%
Alloc. Eff. = 71.3%
Treatment 06:
Noise in
price priority and
bid-ask match
Alloc. Eff. = 93.9%
Alloc. Eff. = 91.0%
Treatment 07:
Noise in
no-loss constraint,
price priority and
bid-ask match
Alloc. Eff. = 90.9%
Alloc. Eff. = 73.5%
Price
Transaction number

Figure 5: Root Mean Squared Deviation of Prices from Equilibrium for Single-Unit Treatments (M00-M07)



Treatment 00: No noise
Treatment 01: Noise in no-loss constraint
Treatment 02: Noise in price priority
Treatment 03: Noise in bid-ask match
Treatment 04: Noise in no-loss constraint and price priority
Treatment 05: Noise in no-loss constraint and bid-ask match
Treatment 06: Noise in price priority and bid-ask match
Treatment 07: Noise in no-loss constraint, price priority and bid-ask match

Figure 6A: Transaction Volume in Single-Unit Treatments (M00-M03)

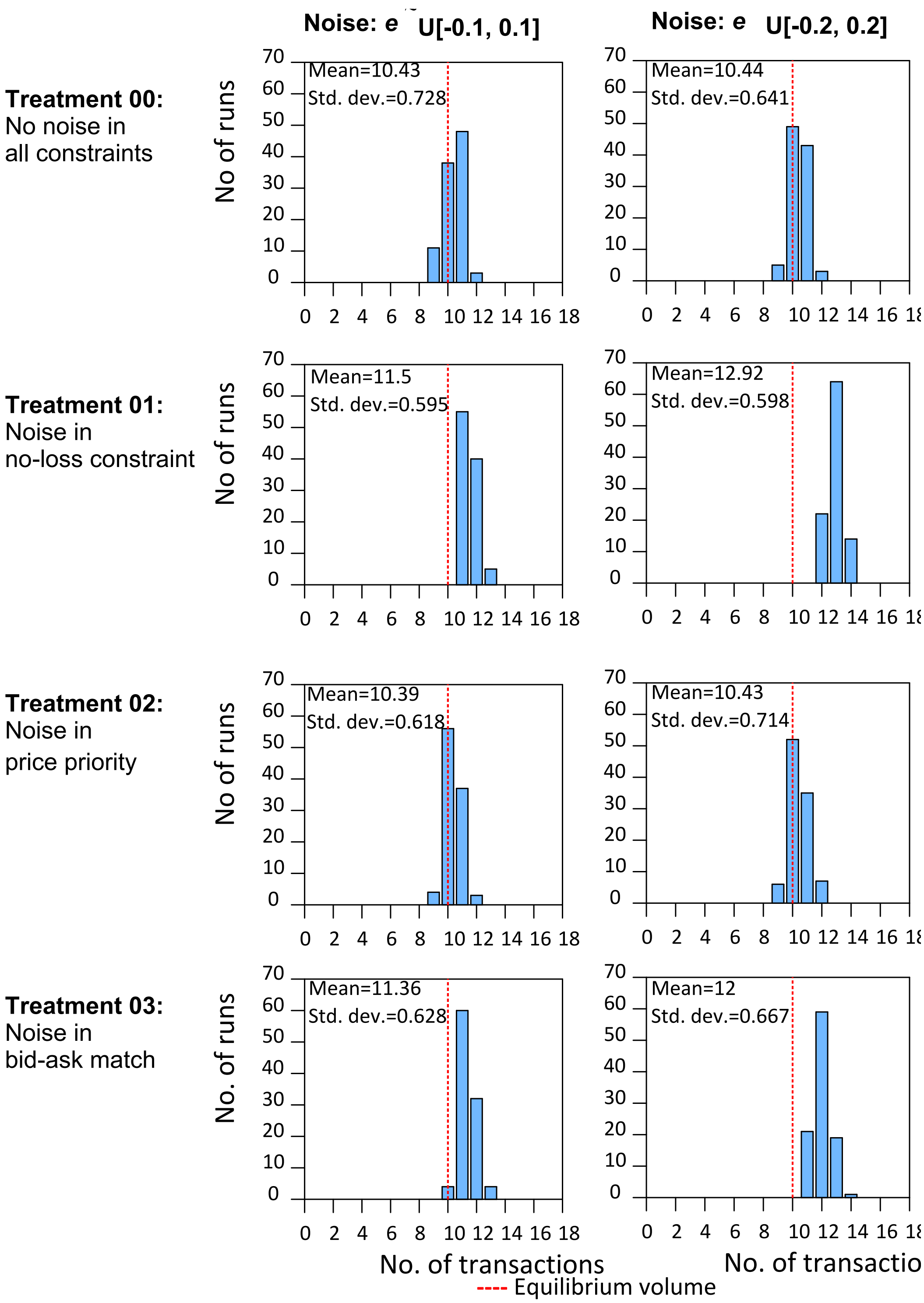


Figure 6B: Transaction Volume in Single-Unit Treatments (M04-M07)

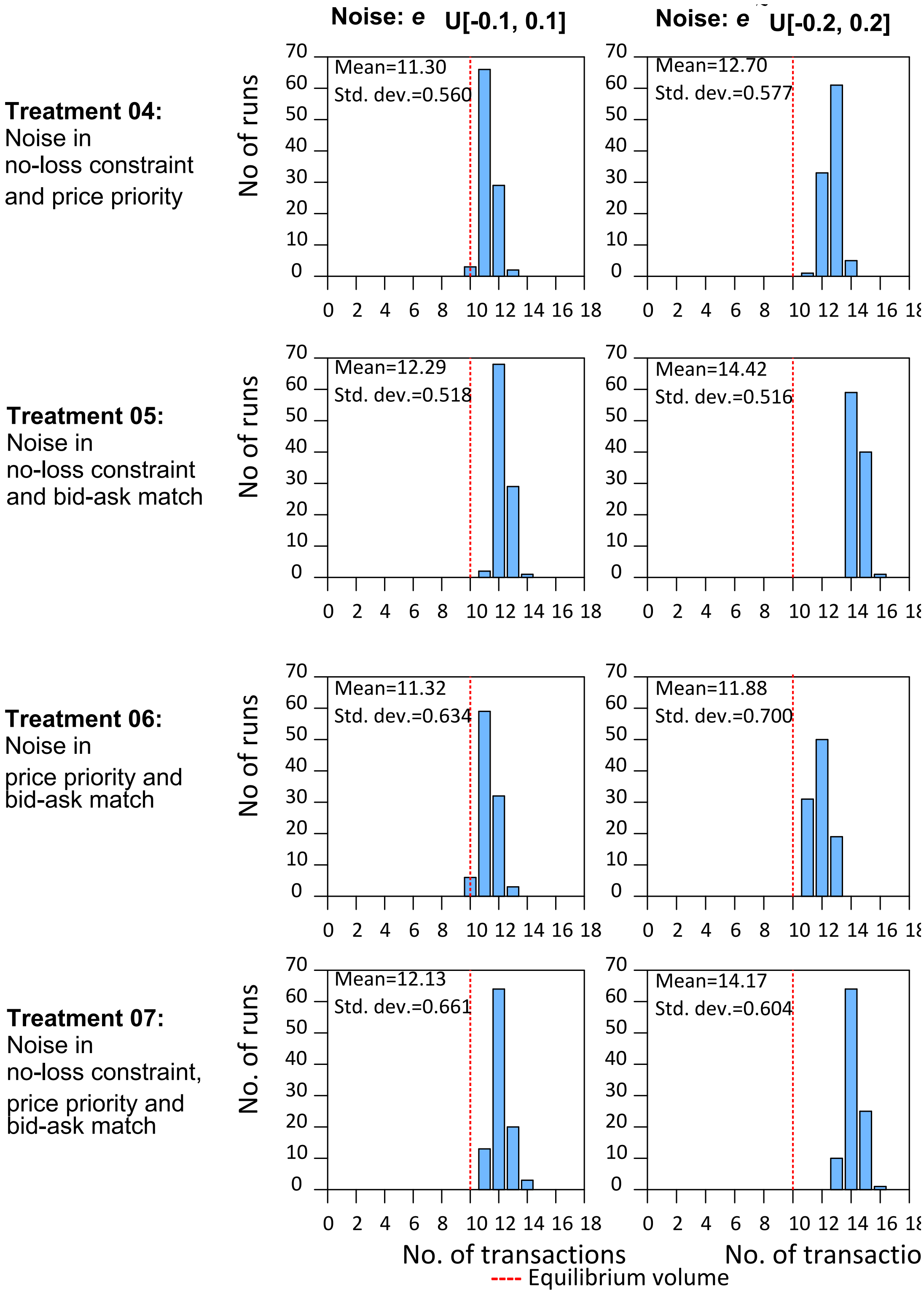
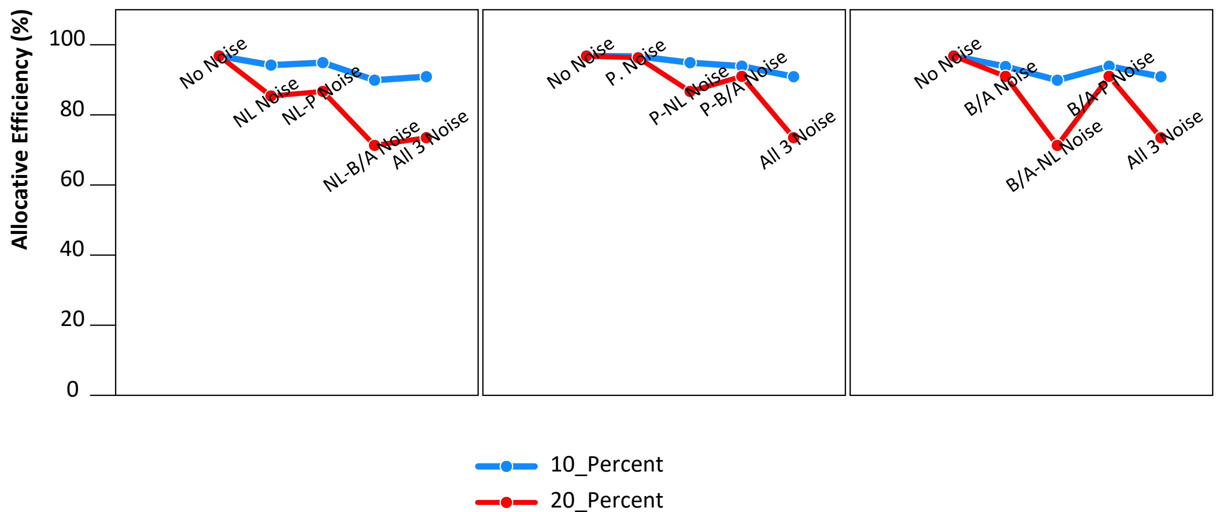


Figure 7: Allocative Efficiency Comparisons Across Levels of Noise and Treatments

(NL: No-Loss Constraint; PP: Price Priority; BAM: Bid-Ask Match)



Appendix 1: An Overview of Parameters and Treatments of Computational Market Experiment

1. Market treatments (17): M00-M16 (see Table 1).
2. No. of trading periods in each treatment: 1
3. No. of steps (bids and asks generated) in each period: 10,000
4. No. of replications reported for each treatment: 100
5. No. of buyers and sellers: 20 buyers and 20 sellers
6. No. of tokens endowed to each buyer/ seller: 1 (in M00-M07), 5 (in M08-M16)
7. Market demand, supply and equilibrium quantity and price:

   For M00-M07: Market demand: P = 800 -30Q; Market supply: P = 200 +30Q

   Equilibrium quantity: 10; Equilibrium Price: 500

   For M08-M16: Market demand: P = 800 – 6Q; Market supply: P = 200 + 6Q

   Equilibrium quantity: 50; Equilibrium price: 500.
8. Individual demand and supply: Constructed from market demand and supply (see p. ??)
9. Both market and individual supply and demand remain unchanged over all eight in the first set and all nine in the second set of treatments in this experiment.
10. Level of noise is defined by seven parameters: ($\tilde{e}_1 \ldots \tilde{e}_5$, θ, and x_i). $\tilde{e}_i \sim$ U ($-x_1$, x_1).
11. The entire computational experiment is repeated for two fixed values of $x_i = 0.1$ and $x_1 = 0.2$ for all x_i. In combination with no noise treatments M00 and M16, $x_1 = 0.1$ introduces a lower level of noise by constraining noise terms e_i $(i = 1, 2, 3, 4, 5)$ within +/- ten percent, and $x_1 = 0.2$ doubles the range of noise terms e_i *to* +/- twenty percent. Recall that percentage is the appropriate interpretation because e_i are multiplicative error terms.
12. Trade order noise parameter θ has a range of $0 \leq \theta < 1$, and a new value of θ is picked randomly from distribution $\sim U(0,1)$ for each of 100 replications of each of the multi-unit treatments M08-M16. Multiple unit endowments of a buyer are ordered highest to lowest values, and for a seller ordered lowest to highest for costs. The probability of a unit being picked from this ordered vector is based on the realized value of θ and is calculated as given in Section 2.3.4 of the main text and summarized again in the following illustration:

Illustration of the transactions in markets with noisy parameters

1. At the beginning of each of the 100 runs (of Treatments M08-M15), a value of trade order noise parameter θ is drawn from uniform distribution ~*U(0,1*, and it remains in force throughout the run.*)*

$0 \leq \theta < 1$

The degree of this noise in trading order is parameterized by $0 \leq \theta < 1$, where the probability of n^{th} of the remaining units in the hands of a trader being picked for the next transaction is θ times the probability of the $(n-1)^{\text{th}}$ unit being picked. This implies that if a trader has k units in hand at any time, the probability of the first unit (of highest value or lowest cost) being picked is $\frac{(1-\theta)}{(1-\theta^k)}$, and the probability of the second unit being picked is this expression multiplied by θ, and so on, such that the total probability of one of the k units being picked adds up to one, that is, $\sum_{n=1}^{k} \theta^{n-1}\left(\frac{1-\theta}{1-\theta^k}\right) = 1$; $\theta = 0$ implies no noise, and as θ approaches 1, the probability of any unit on hand being picked for trade approaches equality.

Probability of an Ordered Unit being Picked with *k* Units Remaining (for θ = 0.5)

Units remaining (k) for buyer	**Probability of nth unit is given by $\theta^{n-1}\left(\frac{1-\theta}{1-\theta^k}\right)$**					**Total proba bility**
	1st (highest)#	**2nd**	**3rd**	**4th**	**5th (lowest)**	
5	0.516	0.258	0.129	0.065	0.032	1.00
4	0.533	0.267	0.133	0.067		1.00
3	0.571	0.286	0.143			1.00
2	0.667	0.333				1.00
1	1.000					1.00

#In the case of sellers, in the absence of trade order noise, the lower cost units are traded before the higher cost units.

13. At the beginning of each step after the first, identities of the buyer who submitted the highest bid (current bid = c*b*) and the seller who submitted the lowest ask (current ask = *ca*) for the current transaction are already in the memory. identified (on the basis of the current bids and asks). In step 1, *cb =0* and the first bidder becomes the current bidder; *ca* =1,000 and the first asker becomes the current asker.

At each step, a new draw from the distribution of each of the active noise parameters ($\tilde{e}_1$ to $\tilde{e}_5$) is made. Depending on the experimental treatments, some of the noise parameters will be active, while others will be muted by setting the parameters to zero (see **Table 1**).

14. At each step, (1) the pool of buyers (B) or the pool of sellers (S) is picked with a 50/50 probability, (2) if B (S) is picked, one buyer (seller) is randomly picked from the pool of all buyers (sellers) with non-zero inventory balance, and asked to submit a bid (ask). If the trader runs out of tokens (zero inventory), it is not picked again.
15. At each step when a buyer (or a seller) is picked, the buyer (or seller) submits its bid (ask) as follows.

Npoisy No-loss constraint	Description
$\tilde{e}_1 \sim U(-x_1, x_1)$	**BUYERS**: The default range of feasible bids for the buyer *i* is $r_{bi,} = [0, v_i]$, where v_i is the redemption value of the token. Noise is introduced to v_i by replacing it with $v_i (1 + \boldsymbol{\tilde{e}_1})$. For example, assume that $x_1 = 0.1$, $\tilde{e}_1 \sim U(-0.1, 0.1)$. If -0.1 is drawn for $\tilde{e}_1$, then range $r_{bi,} = [0, 0.9v_i]$; if 0 is drawn, then $r_{bi,} = [0, v_i]$; and if 0.1 is drawn, then $r_{bi,} = [0, 1.1v_i]$.
$\tilde{e}_2 \sim U(-x_2, x_2)$	**SELLERS**: The default range of feasible asks for the seller *k* is $r_{ak} = [c_k, y]$ where c_k is the cost of the token to the seller, and *y* is an exogenous upper bound for market bids, asks, and transaction prices. Noise is introduced to c_k by replacing it with $c_k (1 + \boldsymbol{\tilde{e}_2})$. For example, assume that $x_2 = 0.1$, $\tilde{e}_2 \sim U(-0.1, 0.1)$. If -0.05 is drawn, then, range $r_{ak} = [0.95c_k, y]$; if 0 is drawn, then $r_{ak} = [c_k, y]$; and if 0.05 is drawn, then $r_{ak} = [1.05c_k, y]$.

16. The bid (or ask) replaces the current bid (or ask) as follows. The first current bid (ask) in the simulation run will be the bid (ask) from the first buyer (seller) picked.

Noisy Price-priority	Description
$\tilde{e}_3 \sim U[-x_3, x_3]$	**BUYERS:** A new bid *b* replaces the current bid (*cb*) if and only if ($b > cb(1+ \tilde{e}_3)$). For example, if *b* = \$30 and *cb*= \$28, *b* will replace *cb* in a standard CDA. In the case of a noisy price-priority, whether this replacement is made depends on $\tilde{e}_3$. Assume that $\tilde{e}_3 \sim U[-1, 1]$ and 0.2 is drawn. The noisy current bid becomes 28×(1+0.2) = \$33.60. In this case, *b* will not replace the current bid, even though $b > cb$. Similarly, when the value of $\tilde{e}_3$ drawn is sufficiently negative, *b* may replace *cb* even if the former is lower. For example, if *b* = \$28, *cb*= \$30, *b* will not replace *cb* in a standard CDA. However, if $\tilde{e}_3$ = -

	0.2 is drawn from $\tilde{e}_3 \sim U\,[-1, 1]$, the noisy current bid = 30×(1-0.2) = \$24.00 will be replaced by *b* even though $b < cb$.
$\tilde{e}_4 \sim U\,[-x_4, x_4]$	**SELLERS**: A new ask *a* replaces the current ask *ca* if and only if ($a < ca\,(1- \tilde{e}_4)$). For example, if a = \$25, ca = \$28, *a* will replace *ca* in a standard CDA. Now assume that $\tilde{e}_4 \sim U\,[-1, 1]$ and 0.2 is the random draw for $\tilde{e}_4$. The noisy current ask is given by \$28×(1-0.2) = \$22.40. In this case, *a* will not replace the current ask, even though $a < ca$. Analogous argument applies to the possibility of a higher *a* replacing a noisy *ca* when the value of $\tilde{e}_4$ drawn is sufficiently negative.

17. A trade takes place under the following conditions.

Noisy bid-ask match	Description
$\tilde{e}_5 \sim U\,[-x_5, x_5]$,	A transaction occurs between a current bid (*cb*) and a current ask (*ca*), if the following conditions are satisfied: $ca \leq cb \times (1+ \tilde{e}_5)$, The transaction price is calculated as the arithmetic mean of the current bid *cb* and current ask *ca*.