Chapter 16

Using Experimental Data to Model Bargaining Behavior in Ultimatum Games

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1. INTRODUCTION

In ultimatum games two players bargain anonymously to divide a fixed amount between them, using a computer or human intermediary for communication. One player (proposer) proposes a division of the "pie" and the other player (responder) decides whether to accept the proposal. If accepted, the proposal is implemented so both players receive their agreed upon shares; if rejected, players receive nothing.

Harsanyi and Selten (1972) generalized the Nash equilibrium solution to provide a unique axiomatic equilibrium solution to this two-person bargaining game with incomplete information. Rubinstein (1985) showed that when the players try to maximize their own profits, there exists a unique subgame perfect equilibrium solution to this ultimatum bargaining problem: the proposer demands all but the smallest possible portion of the pie for himself, and the responder accepts any positive offer. However, the data gathered from most laboratory experiments on ultimatum or other noncooperative bargaining games exhibit significant discrepancies from these theoretical equilibrium solutions. The stylized facts are: proposers offer more than the minimal amount; responders reject not only the minimal but often even larger amounts.[1]

Literature on attempts to close this gap between the theory and data is large, and expanding fast. We do not attempt a review here.[2] These attempts can be approximately classified into four broad groups.[3] One approach is to consider the boundaries of the game the laboratory subjects are thought to be playing beyond its formal specification, and even beyond the walls of the laboratory (e.g., Hoffman et al. 1994, and Bolton and Zwick 1995). In a second approach subjects' beliefs and expectations about their environment, and possible dynamic modifications of these beliefs as the agents act and observe the results, are considered (e.g., Binmore et al. 1985, Binmore et al. 1995, Harrison and McCabe 1992, Slonim and Roth 1998, and Ochs and Roth 1989). A third approach is to add social (other regarding) arguments to agent preferences. These include fairness, envy, reciprocity, trust, intent, etc., e.g., Kahneman, Knetsch, and Thaler 1986, Bolton 1991, Bolton and Ockenfels 2000, Rabin 1993, Fehr and Schmidt 1999, Falk and Fischbacher 1998, and Berg et al. 1995. A fourth approach focuses on understanding and modeling the individual decision processes (e.g., Camerer et al. 1993, and Samuelson 2001). All these efforts are directed at developing theories to economically and simultaneously explain the agent behavior in ultimatum, and possibly other games.

We model the expectations of proposers on the basis of data gathered by others in several previous experiments.[4] In this framework, both the equal split (fairness) solution and the usual subgame perfect solution arise as special cases from different beliefs that might be held by the proposer. We ask: how well can a simple and controllable specification of preferences organize the data and theory in relation to each other by selecting the proposer's belief which are consistent with responder's rejections?

In Section 2, we examine the data from prior experiments, and use it in Sections 3 and 4 to develop and estimate two static models of the responder's behavior, and assess their effectiveness in organizing the responder data from previously published experiments. In Section 5, we develop a static model of optimal response of the proposer to the responder's behavior, and examine the ability of this model to organize the data from previously published experiments. Section 6 presents some concluding remarks.

2. MODELING BEHAVIOR OF THE RESPONDER

The proposer of the division of the "pie" must assess the decision rule of the responder. It seems reasonable, *a priori*, for the proposer to assume that whether the responder accepts or rejects the proposal depends on what that proposal is. What do we know about the proposer's beliefs about this relationship? Subgame perfect predictions are often derived by assuming that the responder will accept any positive amount and reject only those proposals that offer her nothing. The rationale for this assumption is that in a single play game, if payoff from the game is the only argument of the responder's preference function, and preference is increasing in this argument, the responder is better off accepting any non-zero offer from the proposer. It is possible that the responder's preferences include arguments in addition to the absolute amount offered by the proposer. It is also possible that the responders may reject such offers to "educate" the population of proposers at large about the toughness of the responders in general. The issue can be resolved only by reference to data. We return to this issue in the concluding section after analyzing data from previously reported laboratory studies.

Slembeck (1999) reports the most extensive data relevant to our inquiry. In his experiment, Slembeck had 19 pairs of players play 20 consecutive rounds of a single-play ultimatum game anonymously. Identity and role of each member of the pairs remained fixed and anonymous through the twenty rounds.[5] Figure 1 shows the relative frequencies of responders' acceptance as a function of the fraction of the total pie demanded by the proposers for the entire pooled data set of 380 observations. The range of the proposer's demand between 0 and 1 is divided into ten equal intervals, and each interval is labeled by its mean value. The number of observations in each interval (from which relative frequencies are calculated) is shown above each bar. The relative frequency of acceptance is close to 100 percent when the proposer demands 30 percent or less of the total. Beyond this level, the relative frequency of acceptance progressively declines to about 10 percent when the proposer demands more than 90 percent of the pie. We repeated our analysis by partitioning the data into the early and late (first and last ten) rounds of the experiment to detect any effects of learning. The general result, that the chances of acceptance of the offer by the responder decline as the proposer demands more, remains unaltered through the early and later rounds.

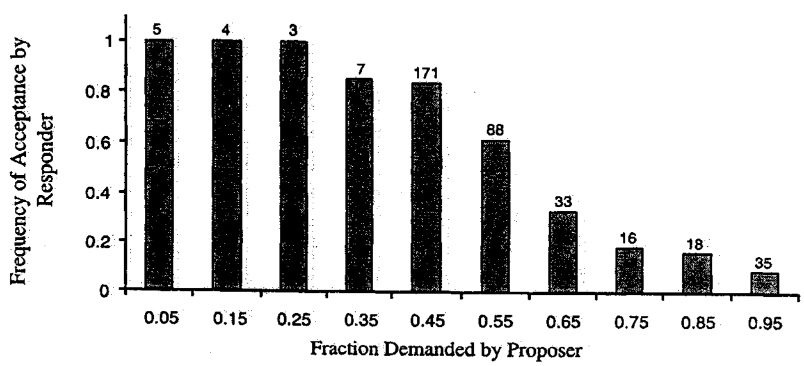


Figure 1. Frequency of Acceptance in Slembeck (1999) Data (No. of Observations at the Top of Each Bar)

The data reported in the published versions of other studies are not as detailed as in Slembeck. However, we analyzed the data from several other studies (Guth et al. (1982), Guth/Tietz (1988), Neelin et al. (1988), Ochs/Roth (1989)) of ultimatum games (either single play experiments, or from the last round of the multiple-play experiments). Results of this analysis are shown in Figure 2 in a format similar to the format of Figure 1. Panels A-E show the analysis of data from individual studies; Panel F shows the same analysis for data pooled from all five panels. Again, similar to Slembeck's data, we see a general pattern of declining relative frequency of acceptance by the responder as the proportion of the pie demanded by the proposer increases. Statistically, it is improbable for such a pattern to appear in data by random chance if the actual probability function matched the standard assumption that the probability of acceptance by the responder is constant at 1 over $0 \leq d_1 < 1$ and 0 at $d_1 = 1$.

The experimental data from the laboratory suggest that the probability of the responder accepting a proposal declines with the increase in the fraction of the pie demanded by the proposer. Is it reasonable to generalize this characteristic from laboratory to the field, and conclude that agents in economically significant work situations also have such a declining probability of acceptance? Whether fraction of the pie or the absolute magnitude of the offer is the relevant argument is not clear. Croson (1996) reports significant differences in the behavior of both players when proposals are presented in absolute versus fraction-of-the-pie amounts; and when the responders are or are not informed. However, as the size of the pie increases, so does the absolute dollar cost of rejection. Telser (1995) uses data for contracts of professional baseball players to show that their salaries and net marginal revenue products are consistent with the Law of Demand.

A second key question is: what should we assume the proposer believes about the responder's acceptance probability function.[6] There are many candidate assumptions to choose from. For example, predictions of the subgame perfect equilibrium are usually derived from the assumption that the proposer believes the responder will accept any offer greater than zero. This assumption is illustrated by the solid thick rectangular line marked a = 0 in Figure 3. Under this specific assessment of the responder's behavior, the proposer will demand almost the whole pie $(1 - \varepsilon)$, and the responder will accept the offer ε, where ε represents the smallest possible positive amount.[7]

Alternatively, we could assume that the proposer's assessment of the probability of acceptance by the responder declines, as the proposer demands a larger share. The class of functions that satisfy this condition is large and we need further restrictions to define a narrower class. One reasonable restriction would be that the function be non-increasing. An example of such a function, valued 1 at $d = 0$, and decreasing linearly to 0 as the demand of the proposer increases from 0 to 1, is shown by the thick dashed diagonal line in Figure 3 marked $a = 100$.[8]

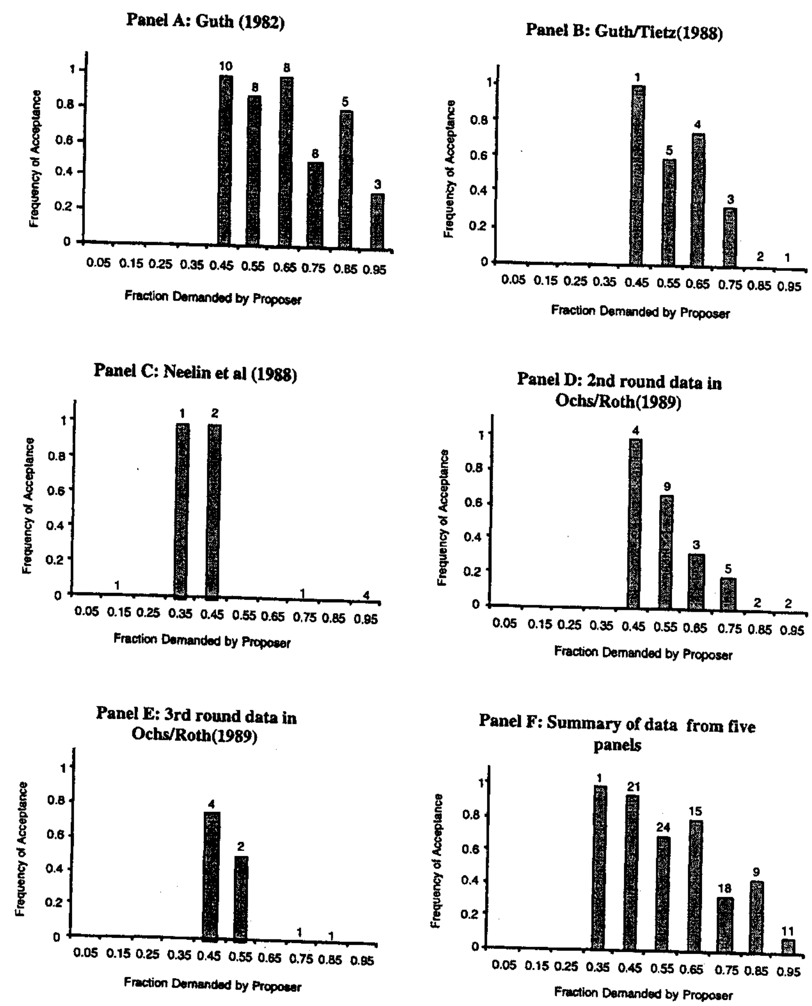


Figure 2. Frequency of Acceptance in Data from Other Prior Studies (No. of the Observations at the Top of Each Bar)

Given the problem of observability, how do we choose a candidate for the proposer's beliefs about the responder's probability of acceptance from a countless set of candidate beliefs? We start with Muth's (1961) rational expectation assumption: if economic agents have the ability to learn from experience, their beliefs about the probability distribution of their environment

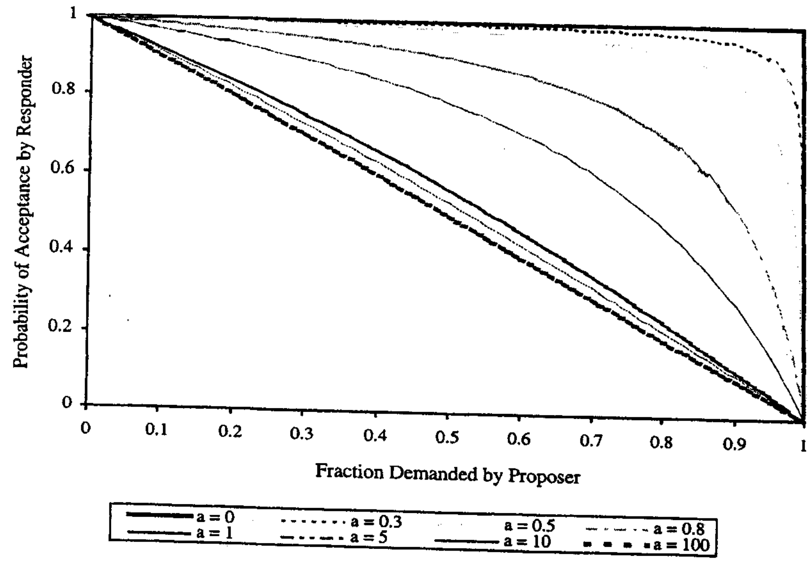


Figure 3. Hyperbolic Family of Probability of Acceptance Functions (with Parameter *a*)

should come arbitrarily close to the probability distribution of their environment itself.[9]

Applying the rational expectations assumption to the proposer's beliefs about the probability of acceptance of a proposal by the responder takes us back to the data presented in Figures 1 and 2. If these data are representative of the environment in which the ultimatum game is played, it is reasonable for us to assume that the proposer's beliefs about the responder's probability of acceptance should correspond to these data. In the following section, we identify a plausible class of non-increasing functions, and then estimate the parameters of the function from Slembeck's data.

3. A CLASS OF HYPERBOLIC FUNCTIONS FOR RESPONDER'S PROBABILITY OF ACCEPTANCE

We would like to identify a class of concave downward sloping functions which are plausible candidates to have generated the data in Figures 1 and 2 and include the rectangular ($a = 0$ in Figure 3) and linear ($a \Rightarrow \infty$ in Figure 3) functions as special cases. In this section we consider the hyperbolic functions followed by piecewise linear functions in the following section.

We picked equilateral hyperbola as a function that might have these desired characteristics. Let $(f-b)(d-c)=\frac{a^2}{2}$, define the functional form where f is the probability of acceptance by the responder, d is the proposer's demand, and b and c should be functions of parameter a.

Since all those functions should pass through two points: (0,1) and (1,0), we can get two equations with two unknowns:

$$(1-b)(-c)=\frac{a^2}{2}, \tag{1}$$

$$(-b)(1-c)=\frac{a^2}{2}, \tag{2}$$

We solve these equations for b and c in terms of a, and derive two solutions for equilateral hyperbola f:

$$f=\frac{a^2}{2\left(d-\frac{1+\sqrt{1+2a^2}}{2}\right)}+\frac{1+\sqrt{1+2a^2}}{2}, \tag{3}$$

and

$$f=\frac{a^2}{2\left(d-\frac{1-\sqrt{1+2a^2}}{2}\right)}+\frac{1-\sqrt{1+2a^2}}{2}. \tag{4}$$

Equations (3) and (4) define a single parameter family of hyperbolas. Equation (3) shows a class of concave functions and equation (4) a class of convex functions.

Figures 1 and 2 suggest that there is little drop in the probability of acceptance until the demand of the proposer exceeds 0.40 of the pie. The responder reacts to further increases in the proposer's demand by reducing the probability of acceptance. In Figure 3 as parameter a approaches zero, we get the rectangular function; when it increases without limit, the hyperbola becomes linear in the limit. We therefore choose the class of concave function (3) to represent the probability of acceptance as a function of the proposer's demand. Figure 3 also shows the hyperbolic functions as parameter a takes value 0, 0.3, 0.5, 0.8, 1, 5, 10, and 100. In the next section, we estimate (3) from Slembeck's experimental data.

(1) Estimating the Probability of Acceptance Function from Experimental Data

For statistical estimation of parameter a, we classify randomly chosen one half of Slembeck's raw data of 380 observations (19 fixed pairs of players played 20 rounds). We selected 190 observations from the odd-numbered rounds for odd-numbered pairs and the even-numbered rounds for even-numbered pairs (henceforth, called the first sub-sample) to estimate model (3). We then use the other half of the sample (henceforth called the second sub-sample) to cross validate the ability of the model to predict the behavior of responders.[10]

The observations are indexed $i = 1, 2, \ldots n$. Each observation consists of a pair (d_i, D_i), where d_i is the fraction of the pie demanded by the proposer, and D_i is the observed decision of the responder to accept ($D_i = 1$) or reject ($D_i = 0$) the proposer's proposal. We can rewrite equation (3) as $f_i = \phi(d_i, a)$, where f_i is the model value of the acceptance probability calculated from equation (3). We can write the sum of squared deviations between this model value of probability of acceptance and observed decisions of the responder as

$$S = \sum_{i=1}^{n} (D_i - \phi(d_i, a))^2. \tag{5}$$

We can solve this equation for the value of a that minimizes the sum of squared deviations S. Solver function in computer worksheet Excel is a convenient tool for doing this. This will give us the least squared error (LSE) estimate of a, denoted by $\hat{a}_{\mathrm{LSE}}$.

An alternative way to estimate the value of parameter a is using the maximum likelihood method. Since (3) defines the probability of acceptance $\phi(d_i, a)$ for each demand, d_i, submitted by the proposer, the likelihood function is given by

$$L = \prod_{i=1}^{n} \phi(d_i, a)^{D_i} (1 - \phi(d_i, a))^{1-D_i} \tag{6}$$

Again, we can solve this equation for the value of a that maximizes the likelihood L. By using Solver function in computer worksheet Excel, we could get the Maximum Likelihood Estimate (MLE) of parameter a, denoted by $\hat{a}_{\mathrm{MLE}}$.

Figure 4-Panel A shows the relative frequency chart prepared from the first sub-sample.[11] Acceptance probability functions, corresponding to LSE and MLE estimates of a ($\hat{a}_{\mathrm{LSE}} = 1.57$ and $\hat{a}_{\mathrm{MLE}} = 1.46$ respectively) given by equation (3) have been superimposed on the relative frequency chart. Panel B shows the estimation results for the second sub-sample which yields $\hat{a}_{\mathrm{LSE}} = 1.42$ and $\hat{a}_{\mathrm{MLE}} = 1.34$.[12]

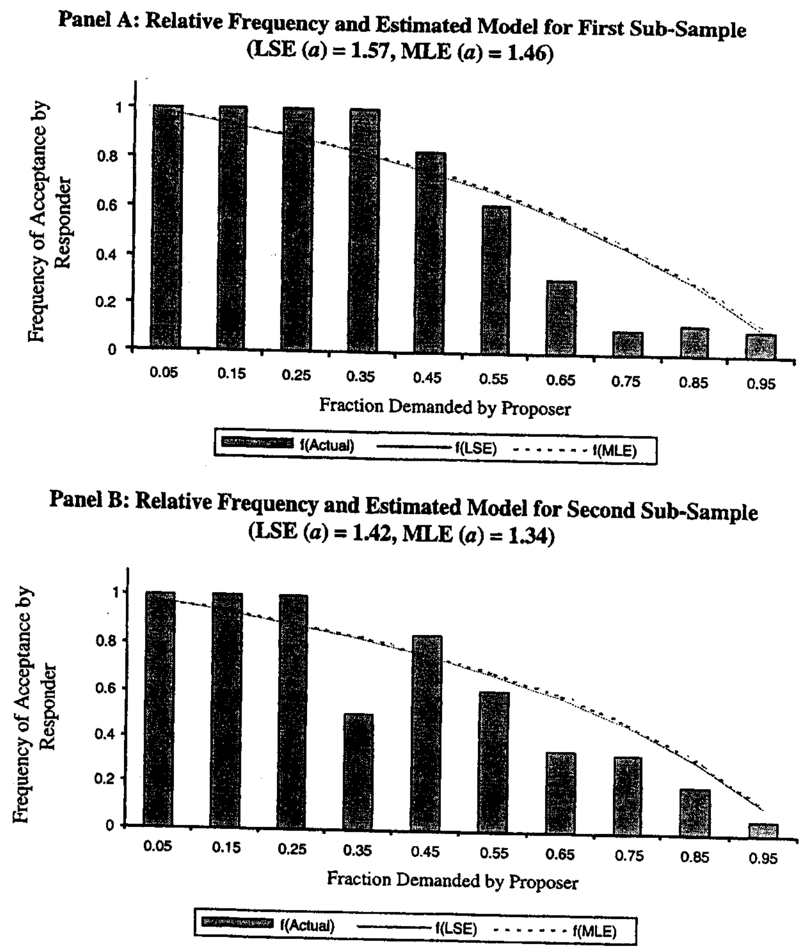
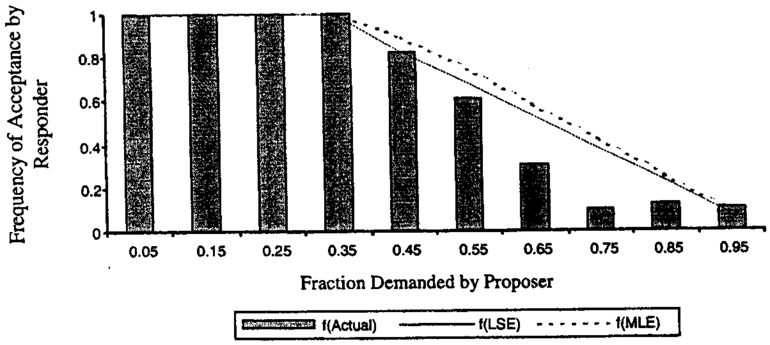


Figure 4. Static Model Estimated from Slembeck Data (1999)

(2) Statistical Test of the Model

The LSE and MLE are close to each other. We have arbitrarily chosen MLE to test whether we can reject the null hypothesis that the two sub-samples of the Slembeck data are drawn from the same distribution. Substituting MLE estimate 1.46 from the first sub-sample into (3), the estimated model from the first half of Slembeck's experimental data is:

Panel C: Relative Frequency and Estimated Linear Model for First Sub-Sample (LSE (a^r) = 0.335, MLE (a^r) = 0.40)



Panel D: Relative Frequency and Estimated Linear Model for Second Sub-Sample (LSE (a^r) = 0.347, MLE (a^r) = 0.45)

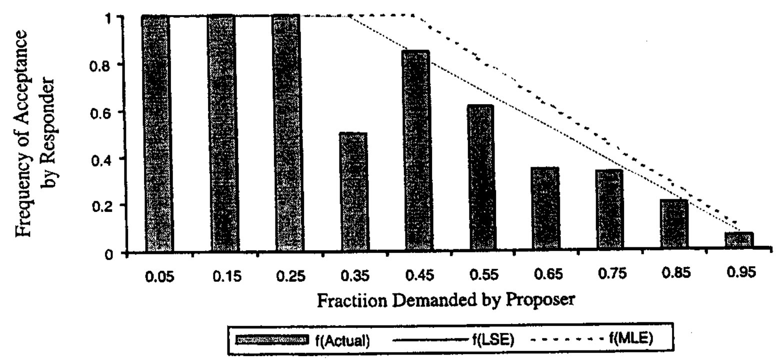


Figure 4. Continued

$$\phi(d_i,\hat{a}) = \frac{1.46^2}{2\left(d_i - \frac{1+\sqrt{1+2*1.46^2}}{2}\right)} + \frac{1+\sqrt{1+2\times1.46^2}}{2}$$

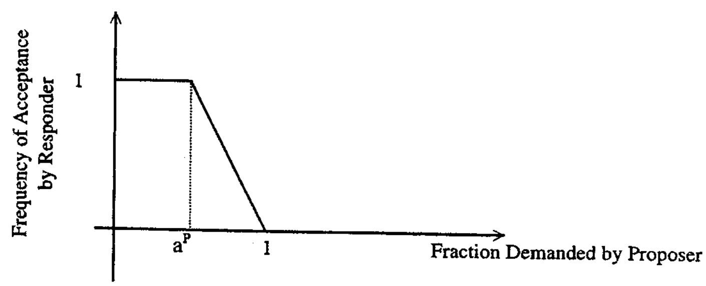
$$= \frac{1.0658}{d_i - 1.647} + 1.647 \qquad (7)$$

We can assess the explanatory power of this model by applying it to the second sub-sample. In other words, if our model could correctly describe

Slembeck's experimental result, the model should explain the variation in D_i of the second sub-sample. The null hypothesis is that the estimated model should apply to both outside and within the sample from which the estimates are derived. For this purpose, we use Chow's forecast test.[13] We know $1.26 < F_{0.01} < 1.29$. Since $1.01 < F_{0.01}$, we could not reject the null hypothesis at 1 percent level of significance, which means the estimated model is applicable to both the first and the second sub-sample. We also calculate the p-value relating to F-statistic of 1.01 in Appendix 1. The approximate p-value 0.5 provides no support for rejecting the null hypothesis, giving us some confidence in the predictive power of the estimated model.[14]

4. A PIECEWISE LINEAR CLASS OF FUNCTIONS FOR THE PROBABILITY OF ACCEPTANCE BY RESPONDERS

A second plausible candidate to describe the observed data can be identified as a piecewise linear model. Define parameter a^P as the critical value of the relative demand by the proposer. We hypothesize that when the proposer demands less than a^P, the responder accepts the offer with probability 1. On the other hand, when the proposer demands more than a^P, the responder's probability of acceptance is a linear decreasing function of the demand. See the following chart.[15]



This functional form can be written as follows:

$$\begin{cases} f = 1, & if\, d \in [0, a^P) \\ f = \dfrac{1}{1-a^P} - \dfrac{d}{1-a^P}, & if\, d \in [a^P, 1] \end{cases} \qquad (8)$$

Now in order to combine these two linear models into one regression model, we define two dummy variable b_1 and b_2 as follows:

$$\left\{ b_1 = \begin{matrix} 1, \ if\, d \in [0, a^P) \\ 0, \ otherwise \end{matrix} \text{ and } b_2 = \begin{matrix} 1, \ if\, d \in [a^P, 1] \\ 0, \ otherwise \end{matrix} \right\}. \tag{9}$$

The final piecewise linear model will be: $f = b_1 + b_2\left(\frac{1}{1-a^P} - \frac{d}{1-a^P}\right)$.

This linear model has only one parameter a^P since both b_1 and b_2 are determined by d and a^P. By using the first sub-sample, we could get the least squared error and maximum likelihood estimates for a^P. Define $f_i = \phi(d_i, a^P)$ as the estimated probability of acceptance by the responder conditional on each value of d_i. The sum of squared deviation between the model value $\phi(d_i, a^P)$ and the observed value D_i of the probability of acceptance is written as:

$$S = \sum_{i=1}^{n} (D_i - \phi(d_i, a^P))^2 \tag{5A}$$

The likelihood function is written as:

$$L = \prod_{i=1}^{n} \phi(d_i, a^P)^{D_i} ((1 - \phi(d_i, a^P))^{1-D} \tag{6A}$$

The Solver function in Excel helps us find the 0.335 as the LSE estimate and 0.40 as the MLE estimate. The explanation of the estimates is that if the proposer demands less than 0.335 (or 0.40 for MLE), the responder would accept the demand with probability 1; if the proposer demands 0.335 (or 0.40 for MLE) or more, the responder's probability of acceptance decreases proportionally with increase in the proposer's demand according to (8).

Figure 4-Panel C shows the relative frequency chart prepared from the first sub-sample. The probabilities of acceptance corresponding to the LSE and MLE estimates of parameter a^P in the piecewise linear model are superimposed to the bar chart. Panel D shows the relative frequency chart prepared from the second sub-sample which yields 0.347 and 0.45 as the LSE and MLE estimates of a^P respectively.[16]

Similar to the hyperbolic model testing, we randomly chose MLE estimate to test whether we could reject the null hypothesis that both two sub-samples are drawn from the following distribution:

$$\phi(d_i, 0.40) = b_1 + b_2\left(\frac{1}{1-0.40} - \frac{d_i}{1-0.40}\right) \tag{10}$$

The Chow's forecast test can be applied here. F-statistics can be calculated as follows:

$$|F| = \left| \frac{(RSS_R - RSS_1)/(380 - n_1)}{RSS_1/(n_1 - k)} \right|$$
$$= \left| \frac{(246.0332 - 123.91)/(380 - 190)}{123.91/(190 - 1)} \right| \approx 0.9804$$

Since $0.9804 < F_{0.01}$, we could not reject the null hypothesis at 1 percent level of significance. The p-value for F-statistics of 0.9804 is 0.5542 (see Appendix 1),[17] which reinforces our confidence in the predictive power of the model.

5. OPTIMAL DEMAND DECISION OF THE PROPOSER

If the proposer knows that the probability of acceptance of the proposal by the responder is given by $\phi(d, a)$, and therefore depends on parameter a and their own decision d_i, they can choose their decision to maximize their own expected reward. Assuming that the proposer has a static estimate of parameter a from hyperbolic model (3), what is the optimal decision of the proposer?

Let $\pi(d, \phi)$ be the expected profit of the proposer from demanding d based on his expectation of the responder's decision rule:

$$\pi(d,\phi) = d\phi(d,a). \tag{11}$$

The proposer will choose the optimal demand d^* which meets the first and second order conditions (12) to maximize the expected profit $\pi(d,\phi)$:

$$\frac{\partial \pi(d,\phi)}{\partial d} = 0, \text{ and } \frac{\partial^2 \pi(d,\phi)}{\partial d^2} \leq 0 \tag{12}$$

Using (3), (11), and (12), we get the proposer's optimal decision rule in (12A)[18]:

$$d^* = \frac{1 + a^2 + \sqrt{1 + 2a^2} - \sqrt{a^2 + a^4 + a^2\sqrt{1 + 2a^2}}}{1 + \sqrt{1 + 2a^2}}. \tag{12A}$$

Figure 5 shows the relationship between parameter a and optimal demand d^*; the upper curve is for (3) and (12A), and the lower dotted curve for (4) and (12B). Under (3), the optimal demand of the proposer, d^*, is a decreasing function of a. Intuitively, parameter a can be explained as a "toughness" parameter about the proposer's assessment of the responder's bargaining posture. Greater the value of a, "tougher" the proposer assesses the responder to be. If the proposer believes that he is playing against a "softer" responder, he expects the latter to accept a smaller share of the pie, and therefore demands more for himself. The optimal demand increases as a decreases.[19]

We consider two extreme cases to fix ideas. If a is close to zero, the responder accepts just about any positive offer from the proposer, leading to the subgame perfect equilibrium as the solution. The other extreme case is when a increases without bound, and the optimal demand of the proposer converges to the equal-split solution.[20]

We compare the optimal decisions of the proposer predicted by the estimated model against the actual decisions of the proposer. Figure 6 makes this comparison based on Slembeck's data (1999). The proposer's relative demand is plotted on horizontal axis divided into ten equal segments from 0 to 1. The frequency of the proposer's demands falling into each decile is shown by the height of the vertical bars (left hand vertical scale). The dotted line in the chart shows the expected profit (see equation 11) of the proposer from submitting a demand equal to the midpoint of the interval (0.05, 0.15, 0.25, etc.) using the MLE estimate (a = 1.40) of a for the entire Slembeck (1999) sample. (The continuous line is for LSE estimate, a = 1.49). Optimal fractional demand

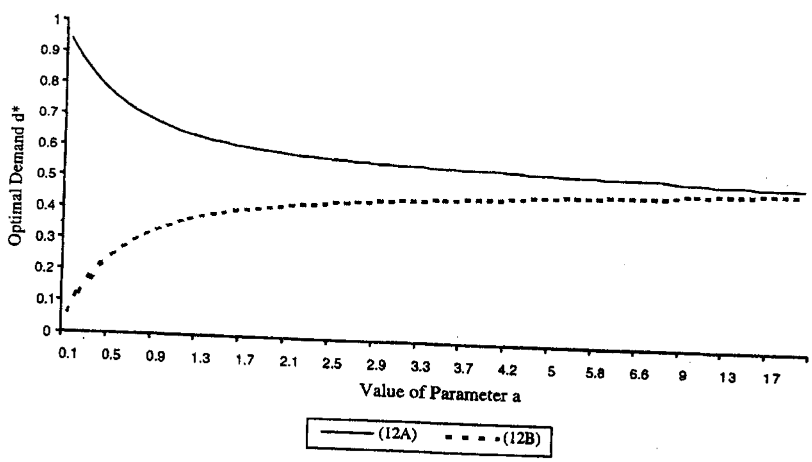


Figure 5. Relationship Between Hyperbolic Parameter a and Optimal Demand d^*

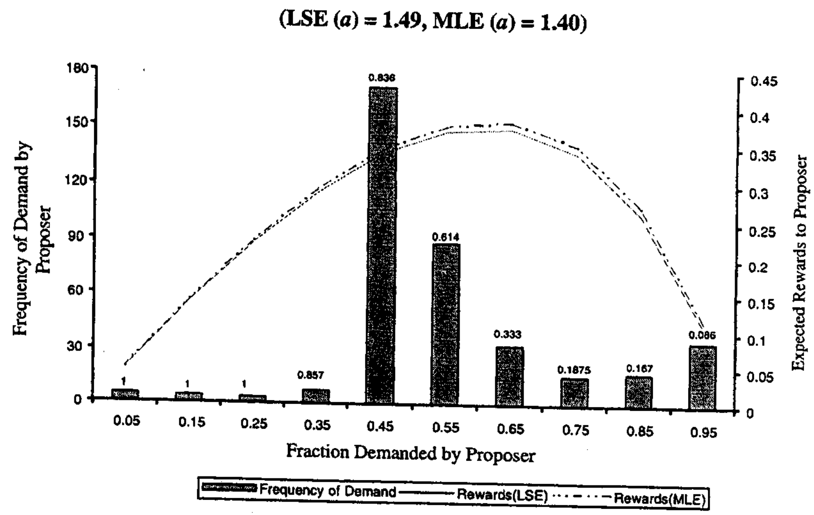


Figure 6. Frequency of Acceptance for the Full Slembeck (1999) Sample (Frequency of Acceptance at the Top of Each Bar) (LSE (a) = 1.49, MLE (a) = 1.40)

for the proposer based on MLE (LSE) estimate of a is 0.62 (0.61). Figure 6A shows the same data in the form of a cumulative frequency chart (thin line) of demands submitted by the proposer. Two thick lines (continuous line for the static model with $a_{MLE} = 1.4$, and broken line for $a = 0$ which corresponds to the assumption that the responder will accept any positive amount) in this figure are theoretical cumulative frequency charts to serve as benchmarks for comparison.

Both Figure 6 and Figure 6A suggest that the optimal demand model (12) captures the central tendency of the demand data. Perhaps we should not expect more from a static, cross-sectional single parameter model. It is also clear from these figures that the modal relative demand of the proposer at 50 percent of the pie lies well to the left of this theoretical optimum of the estimated model. About 36 percent of proposer demand lies at 50 percent and another about 24 percent lies between 50 percent and 62 percent. If the proposer stayed with the optimal demand of 0.62, he would have had an expected reward of 0.383 after considering the probability of rejection by the responder. Since the proposer deviated from this optimal, they had expected rewards of only 0.323, which is about 84 percent of the optimal expected reward. The actual ex post payoff of the proposer was 0.315 on average.

Since model (12) has no error term, there is no meaningful way of comparing the empirical relative frequencies in Figure 6 to the optimal prediction

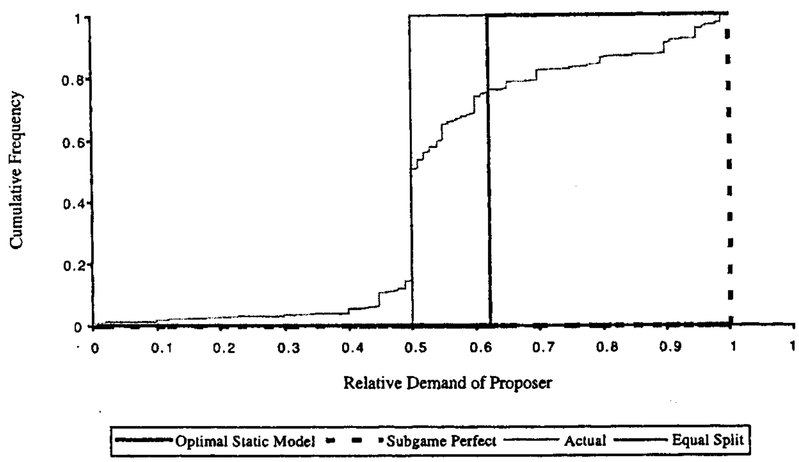


Figure 6A. Cumulative Frequency of the Proposer Relative Demand: Subgame Perfect, Static Model, Equal Split and Actual

of 0.62 (0.61). It is possible to add an error process and make comparisons using, for example, McKelvey and Palfrey's (1995) quantal response equilibrium. However, psychological and economic basis of such decision errors will require further specification and we do not include this exercise here.

6. CONCLUDING REMARKS

This paper complements theoretical derivation of outcomes from *a priori* specification of beliefs and strategies by using experimental data to model and estimate human behavior in ultimatum bargaining games. The data show that the responder's probability of acceptance of an offer from the proposer declines progressively as the proposer demands a larger share of the pie. There is only about 10 percent chance that the responder will accept an offer of less than 10 percent of the pie. The overall acceptance rate by the responder is much higher. Does the proposer, anticipating rejection of small offers, choose to offer more? Or does the proposer value a more even split of the pie against higher personal consumption? We explore the extent to which we might be able to understand the proposer behavior without adding social arguments to his preferences.

We specify and estimate two global static models of proposer behavior.[21] The estimates suggest that while the responder rejects smaller offers with greater frequency, and the proposer behaves as if he knows this, and offers a fraction of the pie to the responder which approximates the amount that will maximize the proposer's expected reward. The data organizing power of the global static model dominates the predictions of the equal split (fairness) doctrine, as well as subgame perfect equilibrium derived from an extreme assumption about the responder behavior (this extreme assumption could have been, but is not chosen, by the estimation process). However, the model is far from organizing the data perfectly. We are currently developing and evaluating models that drop the global assumption in favor of person-specific parameters and drop the static assumption in favor of a dynamic adjustment process for the responder's rejection threshold and for the proposer's expectation of the rejection threshold.

If both the proposers and responders are driven purely by consideration of fairness in bargaining, 100 percent of the offers made by the proposer will be 50/50 splits, and all of them would be accepted, leading to 100 percent efficiency (total money made by the two bargaining agents as a percent of the maximum possible sum of money the two agents could have made).[22] In subgame perfect equilibrium based on the expectation that the responder will accept any positive amount, efficiency will again be 100 percent. The actual efficiency in Slembeck's (1999) data is 61.9 percent, which also matches the efficiency of the static model because it is fitted to the data. See Figure 7.

Models have different distributive consequences. In the subgame perfect equilibrium based on expectation of acceptance of minimal rewards by the responder, virtually 100 percent of the wealth will end up in the hands of the proposer and nothing with the responder. In a world driven by consideration of fairness alone, each player will receive 50 percent of total. If the world were described perfectly by our static model, only 38.3 percent of the maximum possible wealth will be in the hands of proposers and 23.6 percent in the hands of the responders (the two percentages add up to the efficiency of 61.9 percent). The actual distribution of wealth observed in Slembeck's data is more even, 31.5 percent to the proposer and 30.4 percent to the responder. See Figure 8.

If we return to Figure 6A that compares the cumulative frequencies of the actual proposer decision with the three models, it is clear that the equal split captures the modal behavior, and the static model covers the central tendency of the data. The subgame perfect equilibrium based on extreme assumption about the behavior of the responder lies at the right extreme, and does not seem to be a serious contender to organize the data.

One can defend selfishness as the motive for the proposer's "generosity;" if the proposer believes, for whatever reason, that the responder will reject an offer smaller than, say, fifty percent, selfishness is a better explanation of why he offers that much to the responder. It is not so much that the proposer wants

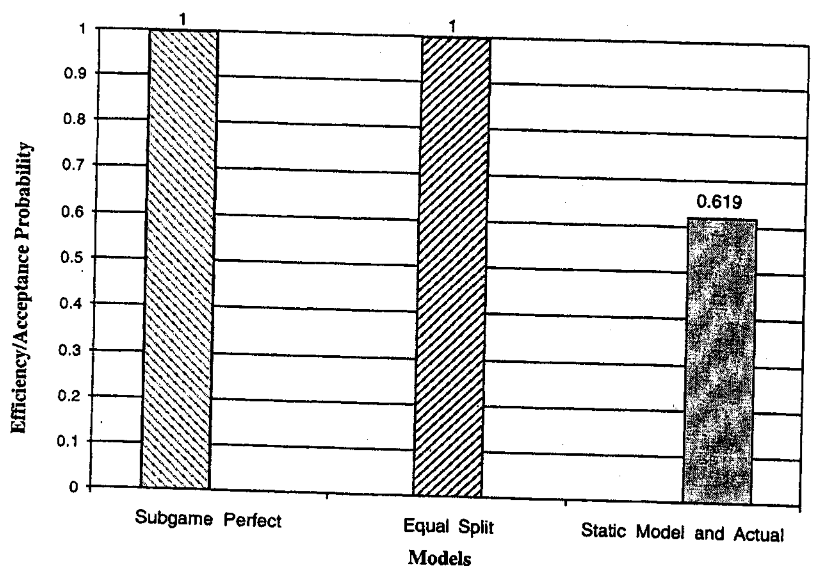


Figure 7. Efficiency and Probability of Acceptance by Responder

to be fair to the responder, but the proposer may expect that the responder may not want to be at the short end of the stick, and is willing to impose discipline on the proposer by denying herself the amount offered.

The argument for the responder is more complicated and less clear. First, there is some evidence in the part of the Slembeck (1999) data we did not analyze here that the responder has a "tougher" posture (higher probability of rejection) when she plays repeatedly against the same proposer than in playing against randomly drawn opponents in each round. Second, establishment of social norms may be driven by the "big-picture" self-interest of individuals. The responder may find that making a personal sacrifice by punishing the greedy proposer now in order to gain longer-term benefits of desirable social norms is well worth the investment. As Harkavay and Benson (1992) put it, "altruism pays." Even if the laboratory game is played only once, it is not unreasonable to recognize the externalities that may exist between even carefully designed laboratory games and the life outside the lab. Indeed experimental economics depends on such externalities between lab and the outside world for its value and validity of results. The responder's rejection of small offers can be seen as an investment in the establishment of social norms, a public good. Third, the responder may have competitive preferences along the lines suggested by Bolton where a large payoff to the proposer hurts her more

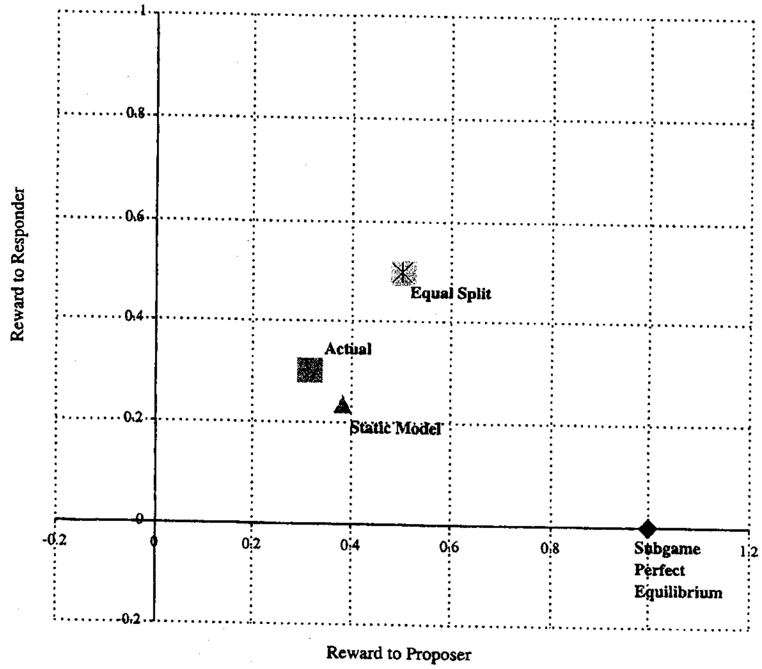


Figure 8. Model Comparison: Distribution of Rewards Between the Two Players

than zero payoff to herself. Fourth, the responder may act emotionally, not calculatively to maximize personal goals. Perhaps direct observations of brain or heart physiology may help enlighten us in this regard. In any case, in spite of repeated observations that the responder rejects nontrivial sums of money, it is not obvious that fairness is valued for itself, instead of being a means to more private goals.

One possibility is to develop models in which it is costly to formulate a set of strategies and to choose one from the set. This cost may arise not only from cognitive effort, but also from the time and observations necessary to accumulate data to evaluate the consequences of using a given strategy.[23] Given a positive cost, agents would be inclined to reuse a strategy from their tried and true arsenal in situations that may yield behavior close to Simon's (1955) satisficing under bounded rationality. Samuelson (2001) explicitly defines complexity cost as the limitation on reasoning resources. Without spelling out where this cost function comes from, he assumes complexity costs, as a function of the number of states and a shift parameter, directly and additively enter into individual payoff.

For the data at issue, our simple model works well, though its generalizability remains to be explored. Costa-Gomes and Zauner (2001) estimate a simple model of preferences, which is a linear combination of the monetary payoff of both the proposer and the responder and an error term, with Roth et al. (1991) data gathered in Israel, Japan, Slovenia and U.S. Unlike our study that takes the behavior of the responders as given, their model is more general in yielding a nested set of hypotheses about both the proposer as well as the responder behavior. They fail to reject the hypotheses that (1) the regard for the other player's payoff is independent of the role, proposer or responder, assigned to individuals; and (2) the proposers in Israel and Japan have no regard for responder payoff. They reject the hypotheses that (3) proposers are altruistic in any country; (4) the proposers in Slovenia and U.S. have no regard for responder payoff; and (5) responders have no regard for payoffs of proposers. Their result (2) is consistent with our approach, and result (4) contradicts it. They seem to have estimated and validated their model on the same data and we cannot tell if validating it separately would have made a difference. Our own results would not have changed much by being validated on the same sample.

In summary, for organizing laboratory data on proposer behavior in single-round ultimatum games, it may not be necessary to abandon or weaken the basic assumptions of individual preferences, and subgame perfect equilibrium. Using more realistic proposer beliefs about responder behavior seems to help. It has often been suggested that we must use social and cultural factors to understand bargaining behavior. People's beliefs and expectations of what others will do in a given situation are, perhaps, a good definition of culture and social norms in social sciences including economics.

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Notes

1. See, for example, Guth et al. (1982), Binmore et al. (1985), Guth and Tietz (1988), Neelin et al. (1988), and Ochs and Roth (1989).

2. See Thaler (1988), Guth and Tietz (1990), Roth (1995), and Bolton (1998) for some reviews.
3. The four broad approaches are not exclusive. They are defined in terms of the research focus on ultimatum games.
4. In the concluding section, we compare and discuss Costa-Gomes and Zauner (2001), which also used data for modeling behavior in ultimatum games.
5. While twenty rounds of repeated play against fixed opponents imparts certain features to this data not shared with single round games (e.g., Figure 1 versus Figure 2F), these data do allow us to conduct some preliminary investigations. These investigations will have to be followed by analysis of single play data.
6. Note that the probability density of responders' cutoff points being the first derivative of acceptance probability, is an equivalent specification. See Henrich and McElreath (2000), and Henrich and Smith (2000) for analysis based on a two-parameter truncated gausssian distribution of cutoff points.
7. This assumption implies that the cutoff points of responders are all concentrated in the narrow demand interval (1-ε, 1).
8. This assumption implies that the cutoff points of responders are distributed uniformly in interval (0,1). Also see Rapoport et al. (1996) for modeling of behavior in ultimatum games with uncertainty.
9. A simpler, deterministic version of this assumption is that the proposer's belief about the cutoff demand beyond which the responder will reject the proposal equals the cutoff point the proposer would use if he were a responder instead.
10. Note that the two sub-samples from this scheme of dividing the sample are not independent of each other. We repeated the analysis using mutually independent sub-samples consisting of 10 odd-numbered pairs (henceforth called the first independent sub-sample) for estimation and 9 even-numbered pairs (henceforth called the second independent sub-sample) for Chow's test. Results were essentially the same.
11. While the figure shows relative frequencies for 10 intervals only, estimation of a was carried out using 190 individual observations as specified above.
12. Estimates of a from the first independent sub-sample (see footnote 11 above) are $\hat{a}_{\text{LSE}}$ = 1.62 and $\hat{a}_{\text{MLE}}$ = 1.53; and $\hat{a}_{\text{LSE}}$ = 1.38 and $\hat{a}_{\text{MLE}}$ = 1.28 for the second independent sub-sample.
13. In the estimating sub-sample, first part of the data set, $n_1 = 190$. After regressing ϕ on d_i, we obtain residual sum of squares $RSS_1 = 32.73$. We apply the estimate from the first sub-sample to the entire data set of 380 observations to obtain $RSS_R = 66.12$. Calculate the F-statistics as follows:

$$|F| = \left|\frac{(RSS_R - RSS_1)/(380-n_1)}{RSS_1/(n_1-k)}\right| = \left|\frac{(66.12-32.73)/(380-190)}{32.73/(190-1)}\right| = 1.01$$

14. From analysis of the second independent sub-sample, we get $|F| = 1.019$ which yields the same statistical inference.
15. This specification is equivalent to assuming that the responders' cutoff points are distributed uniformly over interval (a^P,1).
16. Estimates of a^P from the first independent sub-sample (see footnote 11 above) are $\hat{a}^P_{\text{LSE}}$ = 0.321 and $\hat{a}^P_{\text{MLE}}$ = 0.4499, and $\hat{a}^P_{\text{LSE}}$ = 0.3629 and $\hat{a}^P_{\text{MLE}}$ = 0.4999 for the second independent sub-sample.
17. Alternative split of the sub-samples (see footnote 11) yields similar results, $|F|$ = 0.9739.
18. If, we used (4) instead of (3), the optimal decision rule would be given by:

$$d^* = \frac{-1 - a^2 + \sqrt{1+2a^2} + \sqrt{a^2 + a^4 - a^2\sqrt{1+2a^2}}}{-1+\sqrt{1+2a^2}}. \qquad (12B)$$

19. On the other hand, if we use (4), the proposer assesses the responder to be softer by assigning bigger value of a. Therefore, with small value of a, the proposer could only get smaller demand accepted by the responder. The optimal demand is decreasing as a decreases.
20. On the other hand, when we use (4), the optimal demand of the proposer is zero at $a = 0$ and converges to equal split as a increases without bound. See Figure 5.
21. Qualifier global indicates that the models assume that parameter is identical across players; static indicates that the parameter is assumed not to change with experience.
22. If the responders demanded fairness to the extent that they were willing to reject all offers of less than 50/50, the proposers who know this about responders would be forced to make 50/50 offers by their self-interest alone. If proposers valued fairness but the responders do not, we will see 50/50 splits even when responders accept smaller sums. Thus the extreme attachment to fairness on part of either or both players implies equal-split offers.
23. See Johnson et al. (2001) for evidence of insufficient data gathering and errors by untutored subjects in playing games in which they could do better by doing backward induction. Instruction in backward induction has significant impact on their behavior.

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APPENDIX 1

By the definition of p-value, we have

P-value (1.01) = Pr (x > 1.01)

Notice here x ~ F(190, 189)

P-value (1.01) = 1 − Pr (x ≤ 1.01)

$$\Pr(x \le 1.01) = \int_{-\infty}^{1.01} \frac{\Gamma\left(\frac{m+n}{2}\right) m^{\frac{m}{2}} n^{\frac{n}{2}}}{\Gamma\left(\frac{m}{2}\right)\Gamma\left(\frac{n}{2}\right)} \frac{x^{\frac{m}{2}-1}}{(mx+n)^{\frac{m+n}{2}}} dx$$

$$= \frac{\Gamma\left(\frac{190+189}{2}\right) * 190^{\frac{190}{2}} * 189^{\frac{189}{2}}}{\Gamma\left(\frac{190}{2}\right)\Gamma\left(\frac{189}{2}\right)} \int_{-\infty}^{1.01} \frac{x^{\frac{190}{2}-1}}{(190x+189)^{\frac{190+89}{21}}} dx$$

$$= \frac{\Gamma\left(\frac{379}{2}\right) * 190^{\frac{190}{2}} * 189^{\frac{189}{2}}}{\Gamma\left(\frac{190}{2}\right)\Gamma\left(\frac{189}{2}\right)} * 4.282985089554 * 10^{-490}$$

$$= 0.527$$

P-value (1.01) = 1 − 0.527 = 0.473

Similarly,

$$\Pr(x \le 0.9804) = \frac{\Gamma\left(\frac{379}{2}\right) * 190^{\frac{190}{2}} * 189^{\frac{189}{2}}}{\Gamma\left(\frac{190}{2}\right)\Gamma\left(\frac{189}{2}\right)} \int_{-\infty}^{0.9804} \frac{x^{\frac{190}{2}-1}}{(190x+189)^{\frac{190+89}{21}}} dx$$

$$= 0.4458$$

P-value (0.1093) = 1 − 0.4458 = 0.5542