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Why do biased heuristics approximate Bayes rule in double auctions?

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Abstract

Jamal and Sunder [Journal of Economic Behavior and Organization, 31 (1996) 273] showed that the median prices in double auctions populated by zero-intelligence (ZI) traders whose trading limits are set by two biased heuristics tend to converge to the same equilibrium as if their trading limits were set by applying Bayes' rule. This note provides an analytical explanation of why the repeated use of biased heuristics approximates Bayes rule. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Jamal and Sunder (1996) reported the performance of double auction asset markets populated by simulated computer traders who use two biased heuristics, called representativeness and anchor-and-adjust heuristics in the psychology literature. Evidence from their computational experiment indicated that the central tendency of the aggregate outcomes of markets populated by agents who repeatedly use these heuristics is the same as the central tendency of markets populated by Bayesian traders. We provide a simple explanation of this phenomenon.

Briefly, the representativeness heuristic provides only a biased starting point for the process of adjustment. As the process adjusts to new observations, the importance of the starting point gradually fades, and becomes irrelevant in the long run.

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Simulated traders in Jamal and Sunder's double auctions use a two-part anchor-and-adjust process, one to update their trading limits after each transaction, and the other to update these limits after the dividend is realized at the end of each period. We show that the dividend adjustment process alone is sufficient to cause the trading limits to converge to the Bayesian posterior expected dividends conditional on the observed signal. This convergence of trading limits causes the heuristic individual behavior, and the consequent market outcomes to coincide with the behavior and outcomes under the Bayesian assumption.

2. Representativeness heuristic

Computer traders use the representativeness heuristic to select an initial aspiration level conditional on the observed signal. The representativeness heuristic uses the most probable outcome under uncertainty to infer the underlying state. For example, if the state space $\Theta = \{X, Y\}$, and probability $(X) >$ probability (Y), then X, being more likely, is considered more representative outcome of the process. Suppose the subject receives an imperfect signal $s \in \{G, B\}$ such that probability $(G|X) >$ probability $(G|Y)$. Under this heuristic, a subject who sees signal G infers the state to be X because this state is more likely to generate that signal. Accordingly, the trader assigns an initial aspiration level under signal G to be $\mathrm{CAL}^i_{G1} = D_{Xi}$ where D_{Xi} is trader i's dividend under state X. Conversely, if probability $(G|X) <$ probability $(G|Y)$, subject i who sees signal G infers the state to be Y, and starts with the initial aspiration level under signal G to be $\mathrm{CAL}^i_{G1} = D_{Yi}$. Similar conditions apply to signal B. The key feature of the representativeness heuristic is its insensitivity to base rates (the probabilities of X and Y). We shall show in Section 3 that in the limiting case, this choice of heuristic has no effect, though, by imparting an initial bias, it does affect the path of convergence of prices.

3. Anchor-and-adjust heuristic

The anchor-and-adjust heuristic captures the idea that subjects make gradual adjustments (usually insufficient compared to Bayesian adjustment) by adding a weight (α) to new observations n_t, and discounting the past (p_t) by a factor of $(1-\alpha)$. This process can be represented statistically as a first order adaptive process

$$P_{t+1} = (1-\alpha)P_t + \alpha n_t$$

If p_1 is the initial value of p_t, we can rewrite this expression as

$$P_{t+1} = (1-\alpha)^t P_1 + \alpha((1-\alpha)^{t-1} n_1 + (1-\alpha)^{t-2} n_2 + \cdots + (1-\alpha) n_{t-1} + n_t) \qquad (1)$$

In the report of Jamal and Sunder, this adaptive process is applied to aspiration levels conditional on observed signal at two levels — at the end of the period when the dividend is realized, and after each transaction within every period.

4. Dividend adjustment process

Signal-contingent aspirations of each type of trader are adjusted by an adaptive parameter δ, $(0 < \delta < 1)$, in the direction of the realized value of the state-contingent dividend. Given the probability of the occurrence of dividends of trader i conditional on observed signal, probability (state/signal), this aspiration level (CAL^i_{st}, $s \in \{G, B\}$) gradually converges to the posterior expected value of the dividend for trader i (D_{si}, $s \in \{X, Y\}$) conditional on the relevant signal

$$\mathrm{CAL}^i_{Gt} \Rightarrow D_{Xi}\,\mathrm{probability}(X|G) + D_{Yi}\,\mathrm{probability}(Y|G)$$

$$\mathrm{CAL}^i_{Bt} \Rightarrow D_{Xi}\,\mathrm{probability}(X|B) + D_{Yi}\,\mathrm{probability}(Y|B)$$

This can be seen as follows. Let subscript τ index the periods in which a given signal s is observed, and therefore, the CAL conditional on this signal is updated at the end of each of these periods.

$$\mathrm{CAL}^i_{(\tau+1)} = (1-\delta)\,\mathrm{CAL}^i_{s\tau} + \delta d^i_\tau = (1-\delta)^\tau\,\mathrm{CAL}^i_{s1} + (1-\delta)^{\tau-1}\delta d^i_1 + (1-\delta)^{\tau-2}\delta d^i_2 + \cdots + (1-\delta)^0\delta d^i_\tau, \quad (2)$$

where d^i_τ is the dividend realized by trader i in the τth of the periods in which signal s is observed.

Since the expected value of dividend conditional on signal s is given by D_{Xi} probability$(X|s)+$ D_{Yi} probability$(Y|s)$, the expected value of $\mathrm{CAL}^i_{s(\tau+1)}$ is given by

$$\begin{aligned} E(\mathrm{CAL}^i_{s(\tau+1)}) &= (1-\delta)^\tau\,\mathrm{CAL}^i_{s1} + ((1-\delta)^{\tau-1} + (1-\delta)^{\tau-2} + \cdots \\ &\quad + (1-\delta)^0)\delta(D_{Xi}\,\mathrm{probability}(X|s) + D_{Yi}\,\mathrm{probability}(Y|s)) \\ &= (1-\delta)^\tau\,\mathrm{CAL}^i_{s1} + (1-(1-\delta)^\tau)(D_{Xi}\,\mathrm{probability}(X|s) \\ &\quad + D_{Yi}\,\mathrm{probability}(Y|s)) \end{aligned} \quad (3)$$

Since $0 < \delta < 1$, and CAL^i_{s1} is a constant, as τ increases indefinitely, this expected value of the aspiration level converges to (D_{Xi} probability$(X|s)$ + D_{Yi} probability$(Y|s)$) which is identical to the aspiration levels of Bayesian traders.

Next, consider the variance of the aspiration levels. This is given by $\mathrm{var}(\mathrm{CAL}^i_{s(\tau+1)}) = E(\mathrm{CAL}^i_{s(\tau+1)})^2 - (E(\mathrm{CAL}^i_{s(\tau+1)}))^2$. Substituting from Eq. (2), taking expectations, and considering that CAL^i_{s1} are constants and dividends d_τ are drawn independently of one another, we get

$$\begin{aligned} \mathrm{var}(\mathrm{CAL}^i_{s(\tau+1)}) &= \mathrm{var}(d|s)\delta^{2\tau}(1-(1-\delta)^{2\tau})/(1-(1-\delta)^2) \\ &= (D^2_{Xi}\,\mathrm{probability}(X|s) + D^2_{Yi}\,\mathrm{probability}(Y|s) \\ &\quad - (D_{Xi}\,\mathrm{probability}(Y|s))^2)\delta^2(1-(1-\delta)^{2\tau})/(1-(1-\delta)^2) \end{aligned}$$

Again, since $0 < \delta < 1$, as τ increases without limit,

$$\begin{aligned} \mathrm{var}(\mathrm{CAL}^i_{s(\tau+1)}) &\Rightarrow (D^2_{Xi}\,\mathrm{probability}(X|s) + D^2_{Yi}\,\mathrm{probability}(Y|s) \\ &\quad - (D_{Xi}\,\mathrm{probability}(X|s) + D_{Yi}\,\mathrm{probability}(Y|s))^2)\delta/(2-\delta) \end{aligned} \quad (4)$$

This limiting value depends on the adaptive parameter δ. At one extreme, if the adaptive parameter is zero, the variance is also zero because the realized dividends have no impact on aspiration levels. At the other extreme, if the parameter is 1, the aspiration level is always set equal to the most recent dividend realized in the state, and the limiting variance of aspiration levels is identical to the variance of dividends conditional on the signal, $\mathrm{var}(d/s)$.

Note that the bias caused by the representativeness heuristic is included in the first term of expression Eq. (3). In the limit, this term goes to zero, and the effect of this bias gradually disappears.

5. Transaction adjustment process

While the transaction adjustment process is not necessary for convergence, it may speed up the adjustment. Suppose that at the start of trading in a period under a given signal *s*, the aspiration level of two groups of traders is given by CAL_{s1}^{1} and CAL_{s1}^{2}, respectively. The traders use their aspiration levels as lower limits for their asks and as upper limits for their bids. Therefore, all transactions will take place at prices between the highest and the lowest aspiration levels. The transaction-by-transaction adjustment after the *u*th transaction of the period is defined by

$$\mathrm{CAL}_{s(u+1)}^{i} = (1-\gamma)\,\mathrm{CAL}_{su}^{i} + \gamma P_u \qquad (5)$$

where P_u is the *u*th transaction price in a period.

This adjustment process pulls the aspirations of *all* traders towards each other, and closer to actual transaction prices. The dividend adjustment process, on the other hand, pulls the beginning of the period aspiration of each type of trader towards the signal-conditioned expected dividend of that trader type. While the latter process brings aspirations into line with Bayesian expected values, the former brings the aspirations closer to the actual transaction prices, reducing the variance of transaction prices. Both these processes, together, generate the results of Jamal and Sunder's computational experiment in which the central tendency of transaction prices in markets populated by biased heuristic traders converges to the same level as the markets populated by Bayesian traders.

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