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Tracking the Invisible Hand: Convergence of Double Auctions to Competitive Equilibrium

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Abstract. Economics is the science of want and scarcity. We show that want and scarcity, operating within a simple exchange institution (double auction), can be sufficient for an economy consisting of multiple inter-related markets to attain competitive equilibrium (CE). We generalize Gode and Sunder's (1993a,b) single-market finding to multi-market economies, and explore the role of the scarcity constraint in convergence of economies to CE. When the scarcity constraint is relaxed by allowing arbitrageurs in middle markets to enter speculative trades, prices still converge to CE, but allocative efficiency of the economy declines. Optimization by individual agents, often used to derive competitive equilibria, is unnecessary for an actual economy to approximately attain such equilibria. From the failure of humans to optimize in complex tasks, one need not conclude that the equilibria derived from the competitive model are descriptively irrelevant. We show that even in complex economic systems which are highly inefficient, such equilibria can be attained under a range of surprisingly weak assumptions about agent behavior.

Key words: competitive equilibrium, convergence, zero-intelligence traders, minimal rationality economics

1. Introduction

Economics is the science of want and scarcity. We show that want and scarcity, operating within a simple exchange institution (double auction), can be sufficient for an economy consisting of multiple inter-linked markets to attain competitive equilibrium (CE). Gode and Sunder (1993a,b) reported similar findings for a single market. We use multi-market economies to generalize their results and to explore the role of the *scarcity* condition in convergence of economies to CE. In a second set of detailed simulations, allowing arbitrageurs in multiple markets to enter speculative trades weakens the scarcity constraint. With relaxation of scarcity condition, prices still converge to CE, but allocative efficiency of the economy drops.

Double auction is often used to organize stock, bond, option, and commodity markets as well as internet markets in recent years. The idea of want is implemented in our experimental economies by the willingness of agents to exchange

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what they have for more. The idea of scarcity is implemented by restricting their actions to the set defined by their resources. Within the opportunity set defined by these two constraints, individual agents behave randomly. There is no memory, learning, optimization, evolution, or selection of individuals over time. Rules of the double auction mechanism, in conjunction with random individual actions, yield the market outcomes.

The paper does not intend to describe how real markets work. Nor are the simulations intended to be a representation of individual decision making. The paper deliberately creates an extreme environment to ask whether the competitive predictions can be descriptively relevant when markets are populated by minimally rational agents. In broad terms we confirm what Frank Knight anticipated many years ago – market institutions can make up for human fallibility (e.g., Knight, 1947). Specifically, we explore whether and how the constraint of opportunity sets of agents (i.e., scarcity) can transmit derived demand and supply functions across markets, and how well CE predicts the prices and allocations in such multi-market economies.

This is done in three steps. First, we verify that Gode and Sunder results (1993a,b) that agent optimization is unnecessary for competitive equilibrium are replicated in a multi-market set-up. Second, we show that the efficiency results depend on the tightness of the constraints faced by market agents. Third, we show that in a multi-market environment with subjects making random decisions, where intermediaries can operate short and long, and in which contracts are not enforceable, allocative efficiency of markets suffers, but the competitive price and quantity predictions can still be sustained.

2. Markets

As shown in Figure 1, we consider an economy consisting of m markets labeled 1 through m. In the first market, a number of buyers with exogenously specified demand functions (see below) can buy goods. In the mth market, a number of sellers with exogenously given supply functions (specified later) can sell goods. The first and the last markets are linked through $(m-1)$ mutually exclusive sets of arbitrageurs[1] (see below), who can sell in Market i and buy in the adjacent Market $(i+1), i = 1, 2, \ldots, (m-1)$. In other words, m markets constitute a supply chain with buyers at one end, sellers at the other, and $(m-1)$ sets of middlemen in between. These sets of middlemen can be thought of as regional wholesalers, local distributors, and retailers, etc.

All markets are organized as double auctions. Any buyer can submit a bid (to buy) for a single unit of the good at any time in one market in which it is assigned to the buyer role. For a bid to be valid, two conditions must be fulfilled. First, the bid must exceed any existing bid that may be present in that market. Second, the originator of the bid must have the ability to settle the transaction if that bid were accepted. In other words, the proposed transaction must be in the opportunity set

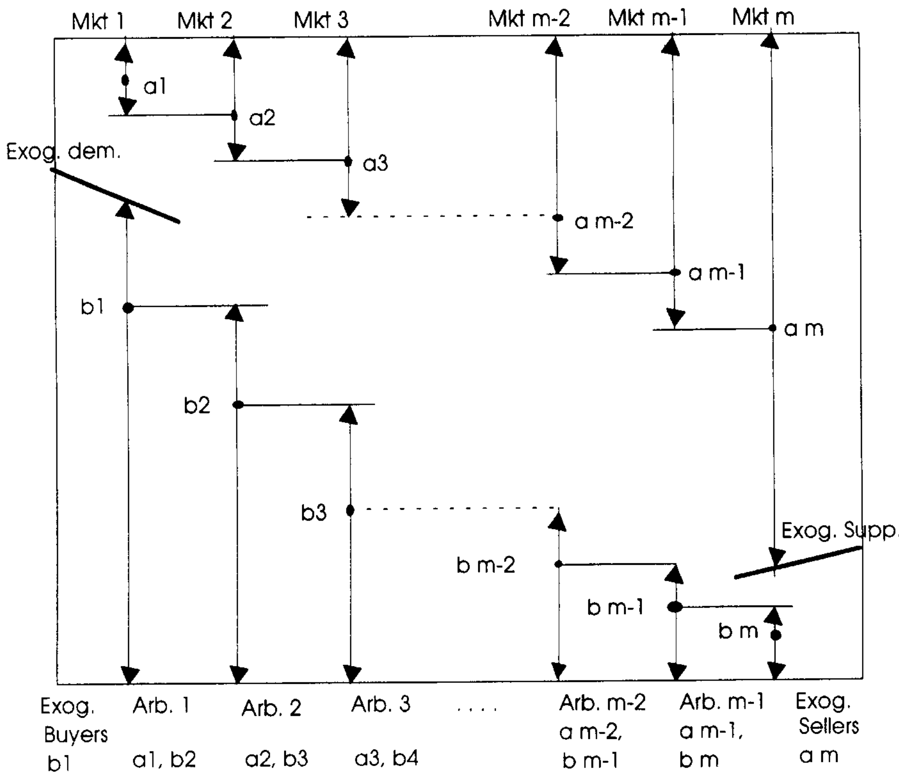


Figure 1. Arrangements of markets, buyers, sellers, arbitrageurs, and their opportunity sets.

of the trader. Similarly, any seller can submit an ask (to sell) for a single unit of the good at any time. Again, for an ask to be valid in a market, two analogous conditions must be fulfilled: the ask must be lower than any existing ask in that market, and the proposed transaction must not lie outside the opportunity set of the trader who makes the offer.

A higher bid becomes the current bid and a lower ask becomes the current ask in the market. Current bid and ask are the only ones standing in the market at any time; all superseded bids and asks are canceled, i.e., there is no queue. If the current bid and the current ask cross at any time, a transaction is immediately executed. The transaction is recorded at the mean of the crossing bid and ask.

The markets are open for a specified period of time. In case of a single machine simulation each market is open for a specified number of cycles. A cycle occurs every time one of the $(m + 1)$ groups (buyers, sellers, or $m - 1$ sets of arbitrageurs) is picked randomly, with equal probability, and then one of the traders within that group is picked randomly, again with equal probability. If the trader is a buyer (seller), a bid (ask) is solicited. If the trader is an arbitrageur, both a bid and an ask are solicited. The number of cycles is chosen so that it does not constrain the number of potential transactions. To make sure that this is the case, we verify that doubling the number of cycles does not result in any additional transactions.

2.1. EXOGENOUS BUYERS

There are n buyers in Market 1; each has the right to buy one unit. Redemption value of the ith buyer is v_i, and without loss of generality, we shall assume that the buyers have been indexed so that $v_i \geq v_{i+1}$, $i = 1, 2, \ldots, (n-1)$. Collectively, v constitutes the market demand function in Market 1. The right to buy a single unit restricts the opportunity set of each buyer to a single purchase transaction; redemption values limit the opportunity set of buyer i to transactions at or below price v_i.

We implement the idea of opportunity set-constrained zero-intelligence buyer as follows: until buyer i is able to buy this unit, it generates a uniformly and independently distributed random number in the range $(0, v_i)$ every time it has an opportunity to submit a bid. Alternatively, the idea can be implemented by letting each buyer generate a uniformly and independently distributed random number in the range $(0, M)$, M being some arbitrary upper limit, and allowing the market to reject those bids that fall beyond the ability of the buyer to settle the transaction (i.e., greater than v_i). As soon as the buyer exhausts its right to buy a single unit by entering in a transaction, it stops making further proposals. This constraint, too, can be implemented at either the individual level or at the market level by letting the market reject proposals that fall outside the opportunity sets of their originators.

2.2. EXOGENOUS SELLERS

Sellers are defined analogously to buyers. There are a total of n sellers, each with the right to sell one unit. Cost of the ith seller is c_i, and without loss of generality, we shall assume that the sellers have been indexed so $c_i \leq c_{i+1}$, $i = 1, 2, \ldots, (n-1)$. Collectively, c constitutes the market supply function in the mth market. The right to sell a single unit restricts the opportunity set of each seller to a single sale transaction; costs limit the opportunity set of seller i to transactions at or above price c_i.

We implement the idea of opportunity set-constrained zero intelligence seller as follows: until seller i is able to sell this unit, it generates a uniformly and independently distributed random number in the range (c_i, M), M being an arbitrary upper limit, every time it has an opportunity to submit an ask. Alternatively, the idea can be implemented by letting each seller generate a uniformly and independently distributed random number in the range $(0, M)$, and allowing the market to reject those asks that fall beyond the ability of the seller to settle (i.e., less than c_i). As soon as the seller exhausts its right to sell a single unit by entering in a transaction, it stops making further proposals. This constraint, too, can be implemented at either the individual level or at the market level by letting the market reject proposals that fall outside the opportunity sets of their originators.

2.3. ARBITRAGEURS

Figure 1 shows the configuration of markets, exogenous buyers, exogenous sellers and arbitrageurs. There are $(m-1)$ sets of arbitrageurs, each includes a traders. The first set sells to the exogenous buyers in Market 1, and buys from the second set of arbitrageurs in Market 2. The second set buys in Market 3 from the third set of arbitrageurs, and sells in Market 2 to the first set of arbitrageurs, and so on, until the $(m-1)$th set of arbitrageurs buys from the exogenous sellers in the mth market and sell to the $(m-2)$th set of arbitrageurs in Market $m-1$. Each arbitrageur is permitted to engage in an unlimited number of purchase and sale transactions within the constraints of its opportunity set.

The opportunity set constraints on the arbitrageurs subject them to a market discipline similar to that imposed on the buyers and the sellers. This constraint can be defined in terms of money or units of goods. For now, we choose to impose an opportunity set constraint on all arbitrageurs by requiring them to make only those bids and offers that can be immediately and simultaneously executed at a profit. Simultaneous purchase and sale transactions hold their inventory at zero at all times, and shield them from any risk of price shifts.

Figure 1 illustrates simple constraints imposed on arbitrageurs to implement the idea of scarcity. Consider the set of arbitrageurs who buy in Market $j+1$ and sell in Market j. Let b_j be the current bid in Market j and let a_{j+1} be the current ask in Market $j+1$. Knowing that they can sell immediately in Market j at b_j, they submit a bid in Market $j+1$ which is a uniformly distributed random variable between 0 and b_j, $b_{j+1} \sim U(0, b_j)$. If bids and asks cross in any market ($b_j \geq a_j$ in any Market j), a 'primary' transaction is recorded at the mean of the crossed bid and ask in that market.

Since arbitrageurs can hold no inventory ($L = 0$), any 'primary' transaction in any market must be immediately followed by 'complementary' transactions in all other markets so a unit of good is transferred from the only net source of goods (the exogenous suppliers in Market m) to the only net buyers (i.e., the exogenous buyers in Market 1). This takes place as follows. The first transaction must occur in Market m, since this is the only market where an ask has been placed by the exogenous seller. It happens whenever $b_m > a_m$. At this point, the arbitrageur who has just acquired this unit, immediately places an ask in Market $m-1$ equal to its bid in Market m: $a_{m-1} = b_m$. Since, by construction, $b_{m-1} > b_m$, a transaction occurs in Market $m-1$, recorded at the mean of the two, $0.5(b_m + b_{m-1})$. The same process is continued all the way to Market 1.

Under these conditions, it makes no difference whether there are only one or more arbitrageurs in each set. To simplify things, we have reduced the number of arbitrageurs, a, in each set to one. The arbitrageur in set 1, for example, bids in

Table I. Simulation design for opportunity-set constrained economies.

No. of markets and arbitrageurs in the economy	Number of replications
2-markets	
1 set of arbitrageurs (1 in each set)	10
5-markets	
4 sets of arbitrageurs (1 in each set)	10
10-markets	
9 sets of arbitrageurs (1 in each set)	10

Market 2 below the bid it observes in Market 1, and, eventually, if a transaction takes place in Market 2, asks in Market 1 at the bid it placed in Market 2. Thus, it remains entirely within its opportunity set, eliminating any risk of a loss or any non-zero inventory. It can enter into a transaction in one market knowing full well that it can profitably execute the other side of the transaction at any time. Depending on the degree of patience we wish to endow into these arbitrageurs, the second side of the transaction could take place immediately at the mean of bid/ask in the appropriate market, or the arbitrageur could be permitted to engage in further bidding, using the first transaction price as the limit. In the spirit of their zero intelligence, we model the arbitrageurs to be impatient, and take the former of these two options.

3. Simulation Design

The simulation design is shown in the first column of Table I. The number of markets (m) takes three different values, two, five and ten. In the two market economy, sellers sell to a set of arbitrageurs in Market 2 who, in turn, sell to the buyers in Market 1, or buyers purchase from arbitrageurs in Market 1 who, in turn, purchase from sellers in Market 2. Similarly, buyer and sellers in the five-market economy are linked through four different sets of arbitrageurs, and in the ten-market economy by nine sets of arbitrageurs.

The number of buyers and sellers is set at 100. Each buyer has the right to buy up to one unit of the good from arbitrageurs in Market 1. The redemption value of the good for the first buyer, v_i, is 125, and declines in steps of 1 down to 26 for the last buyer. Similarly, each seller has the right to sell one unit. The cost of this unit for the first seller, c_i, (incurred only if the unit is sold) is 5, and rises in steps of 1 to 104 for the last seller. These market demand and supply functions are shown in Figure 2.

Competitive equilibrium price of these economies is 65, and the volume is 60–61. The sum of consumer and producer surplus is 3660. If the profits of all traders

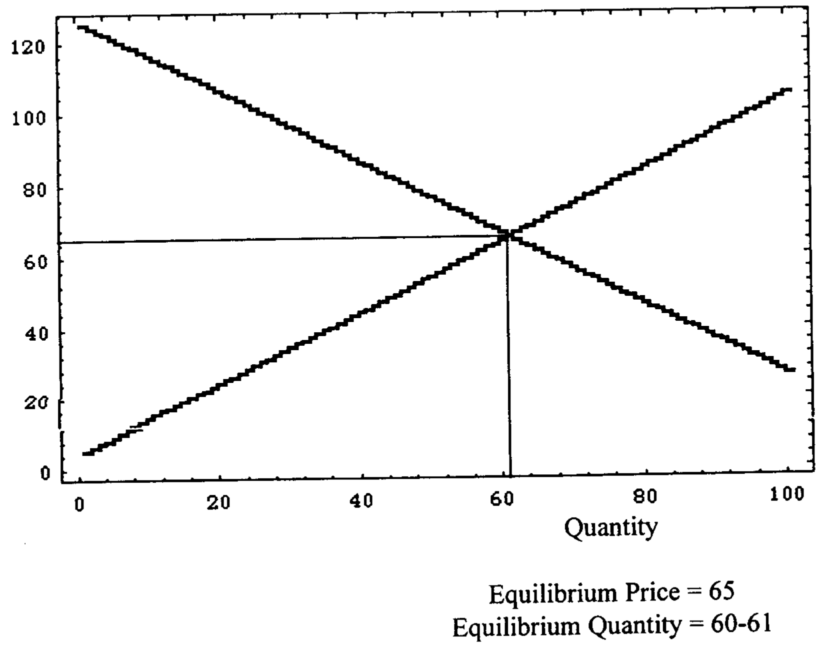


Figure 2. Exogenous supply and demand.

in the economy add up to 3660, the economy will be 100% efficient in extracting the total surplus. Given the symmetrical supply and demand functions, the equilibrium distribution of this surplus is equal between the buyers and the sellers. Since arbitrageurs are present, we expect them to capture some of these profits.[2]

4. Economies under Opportunity Set Constraint

4.1. PRICE CONVERGENCE

The middle column in Figure 3 shows the paths of observed transaction prices for the three economies, with different number of markets, corresponding to the three rows of Table I. Prices for each market are plotted vertically against the transaction sequence number. Each graph in this column was chosen arbitrarily from a collection of 10 different runs of the same economy for each cell in Table I. In the right column, we plot the *average* price of each transaction number in each of the markets.

In all three economies, the sequence of transaction prices in each market gets gradually though noisily closer to the equilibrium price (represented by a horizontal line). These results show that the competitive equilibrium price is a good predictor of the tendency of prices. In analytical models, equilibrium is arrived at by assuming that a large number of price-taking agents engage in profit maximizing behavior. Empirically, the equilibrium price so derived is a good predictor of the tendency of prices in an economy consisting of several inter-related markets that

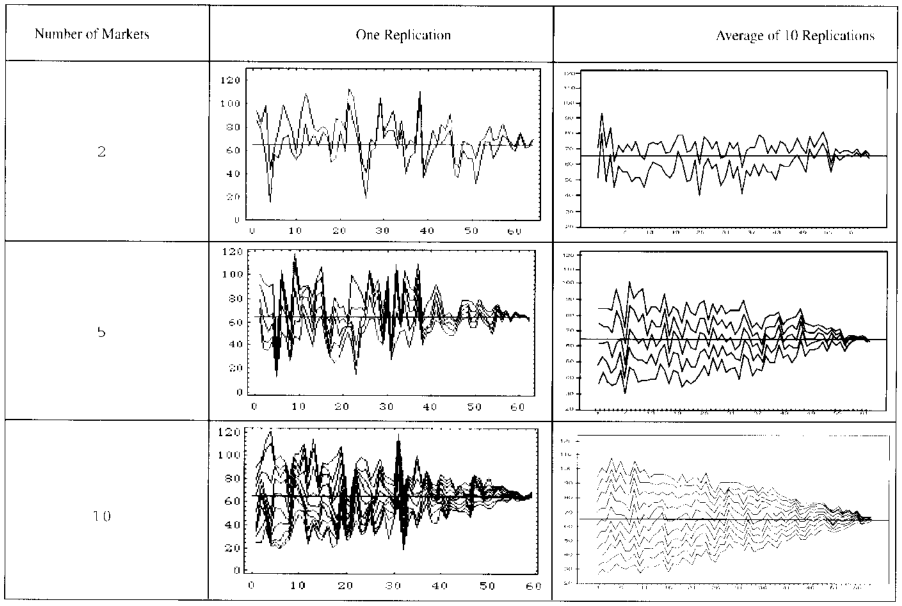


Figure 3. Price paths for one replication from each cell of Table I and average of 10 replications.

operate under continuous double auction trading mechanism populated by a small number of agents who make random ask and bid decisions within the boundaries defined by their opportunity sets.

4.2. TRANSACTIONS VOLUME

Competitive equilibrium prediction of transaction volume is 60–61. This prediction is quite accurate. As can be seen from Figure 4, in the 2-market economies, the number of transactions (62 to 67) slightly exceeds the competitive equilibrium prediction. In 5-market economies, most of the markets in most of the ten replications have exactly the number of transactions predicted by the competitive model. When the number of markets increases to 10, the number of transactions (58–59) is slightly under the CE prediction of 60. An explanation for the decrease in the number of transactions as the number of markets in the economies increase is that the number of cycles required for all potential transactions to take place may be larger than was allowed in the simulations. As mentioned earlier, the amount of time allowed to the markets has been established so that doubling this amount does not result in any further transaction. This may not guarantee, especially in more complex economies, that a small number of potential transactions may not be left out. More significant is the overshooting, compared to the CE prediction, of the number of transactions in the 2-markets economies.

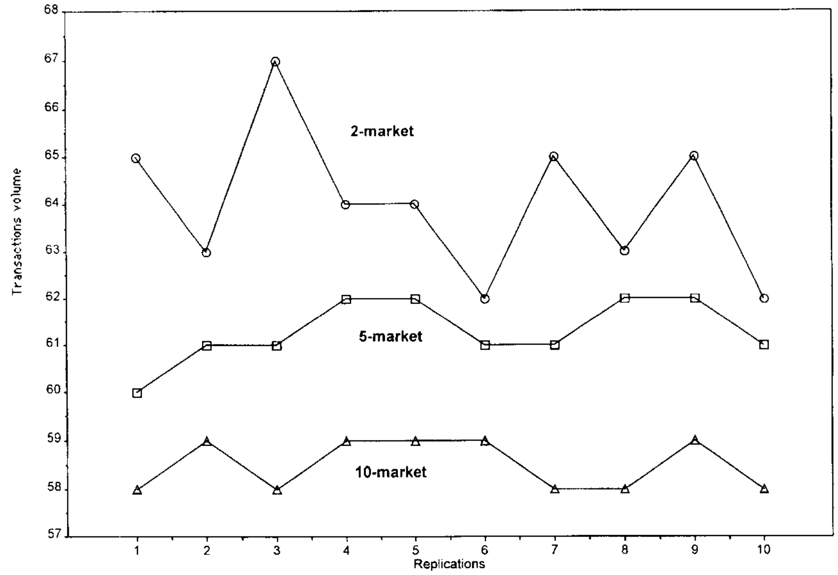


Figure 4. Transactions volume in individual replications of n-market. (In each replication, the volume is the same in all the markets of one economy).

Notice also that in the 2-market economies the CE price prediction is less accurate. This happens because the double auction mechanism used in these simulations *shrinks the opportunity sets* of arbitrageurs in the middle markets and, therefore, confines the transaction prices in these markets in a narrow band around the CE price. Consequently, the more 'middle markets' there exist, the narrower is this band, and more accurate are the CE predictions.

4.3. EFFICIENCY

Efficiency measures the extraction of social surplus. Following Plott and Smith (1978), efficiency of a market is the total profit actually earned by all traders divided by the maximum total profit that could have been earned by all traders. Participants in a fully efficient system will extract the theoretical maximum social surplus.

Using this measure we observe relatively high efficiencies, specially as the number of markets increases. Average efficiency across 10 runs for 2, 5, and 10 markets is 98.2, 99.7 and 99.6% respectively (see Figure 5). As Gode and Sunder (1993b, 1997) explain, a small loss of efficiency occurs because double auction has no recontracting. Intra-marginal buyers and sellers have a non-zero probability of being replaced by extra-marginal buyers and sellers, reducing the amount of surplus extracted through the trading process. As observed in discussing the price and quantity predictions, the probability of extra-marginal transactions decreases as the number of 'middle markets' increases, resulting in some efficiency gain.

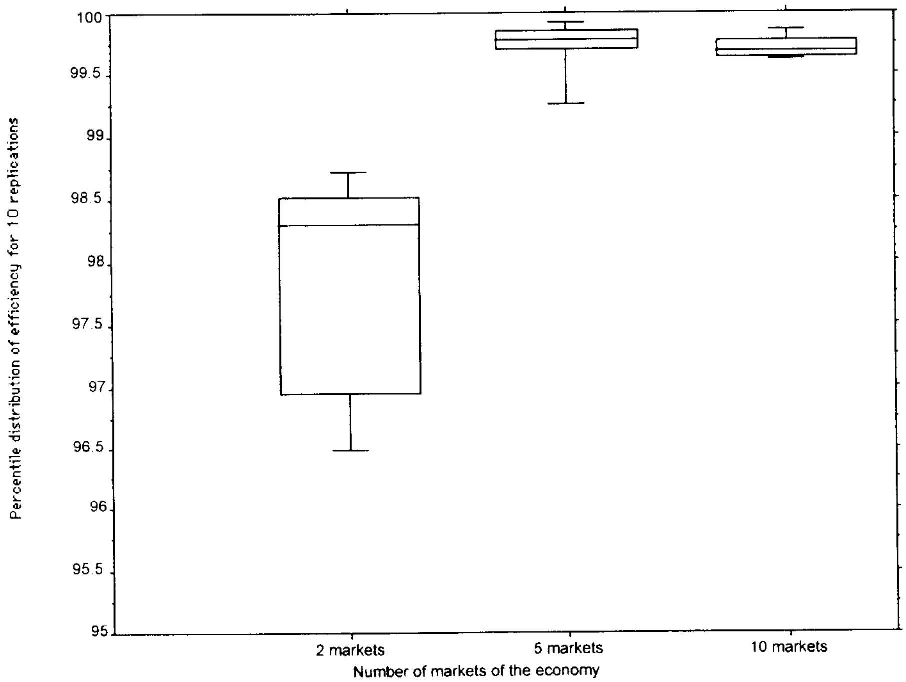


Figure 5. Distribution of allocative efficiency of n-market economies over 10. (Median, interquartile range and range).

5. Relaxing the Opportunity Set Constraint

Can the convergence of our experimental economies close to CE be attributed to the imposition of scarcity or opportunity set constraint on trader behavior as we have implied so far? One way of answering this question is to relax the opportunity set constraint, and compare the behavior of such economies to the benchmark behavior described above. We designed and ran a larger set of economies shown in Table II for this purpose.

5.1. ARBITRAGEURS' INVENTORY

Recall that the opportunity set constraint means that no trader can make a proposal from which it cannot turn a profit if the proposal were accepted by some counterpart. For arbitrageurs, this constraint required them to trade simultaneously in two markets, buying in one and selling in the other. Arbitrageurs never held any inventory. Holding inventory means taking the risk of incurring loss on a trade, and thus violating the opportunity set constraint.

We parameterized relaxation of the opportunity set constraint in the form of the units of inventory the arbitrageurs were permitted to hold. The absolute magnitude of the arbitrageurs' inventory constraint L was allowed to vary from 1 to 3; the larger the value of L, the greater is the relaxation of the opportunity set constraint.

Table II. Simulation design for economies with relaxed opportunity-set constraint.

No. of Markets in the economy	Number of Arbitrageurs in the Economy	Inventory constraint $L = 0$	Inventory constraint $L = 1$	Inventory constraint $L = 2$	Inventory constraint $L = 3$
2-markets	1 set of arbitrageurs (5 in each set)	10 replications	10 replications	10 replications	10 replications
2-markets	1 set of arbitrageurs (25 in each set)	10 replications	10 replications	10 replications	10 replications
2-markets	1 set of arbitrageurs (50 in each set)	10 replications	10 replications	10 replications	10 replications
5-markets	4 sets of arbitrageurs (5 in each set)	10 replications	10 replications	10 replications	10 replications
5-markets	4 sets of arbitrageurs (25 in each set)	10 replications	10 replications	10 replications	10 replications
5-markets	4 sets of arbitrageurs (50 in each set)	10 replications	10 replications	10 replications	10 replications
10-markets	9 sets of arbitrageurs (5 in each set)	10 replications	10 replications	10 replications	10 replications
10-markets	9 sets of arbitrageurs (25 in each set)	10 replications	10 replications	10 replications	10 replications
10-markets	9 sets of arbitrageurs (50 in each set)	10 replications	10 replications	10 replications	10 replications

In order to permit the arbitrageurs to hold inventory, the auction procedure was modified. Recall that in zero-inventory, opportunity set-constrained economies of the preceding section, whenever an arbitrageur entered into a transaction in any market, a chain of complementary transactions in all other markets were immediately executed because no arbitrageur could hold any long or short positions. For example, in a ten-market economy, if bid in Market 4 exceeded the ask in that market, a transaction in Market 4 was accompanied by a transaction between high bidders in market pairs (4–3), (3–2), and (2–1) and between low askers in market pairs (4–5), (5–6), (6–7), (7–8), (8–9), and (9–10). In other words, a unit was transferred all the way from the low asker in Market 10 to the high bidder in Market 1. When inventory were permitted, these immediate complementary transactions in other markets of the economy were not necessary. In the context of our example, a cross between high bid and low ask in Market 4 resulted in the high bidder and the low asker in that market changing their inventory positions by +1 and −1 respectively.

Also recall that in zero-inventory markets, whenever an arbitrageur was picked (randomly) to make a proposal, it entered a bid as well as an ask in the appropriate markets. In a second modification to implement non-zero inventory markets, when a trader hit the limit of its long position in the market in which it was allowed to buy, it could no longer submit a bid in that market. Similarly, if an arbitrageur hit the limit of its short position in the market in which it was allowed to sell, it was no longer allowed to submit an ask in that market. Whenever a binding inventory constraint prevented an arbitrageur from submitting a proposal, another arbitrageur from the same set was picked randomly to submit that proposal. This meant that we could no longer assume a single arbitrageur in each set. Therefore Table II shows our $3 \times 3 \times 3$ design (number of markets $m = 2, 5$, or 10; inventory constraint $|L| = 1, 2$, or 3; and the number of arbitrageurs $a = 5, 25$, or 50). As before, we conducted 10 replications of each cell in the table.

Multiple, distinct arbitrageurs within each group allowed the possibility that some members of a group might hold positive inventory while the others hold short positions after the m markets close. To clear this inventory, we considered opening a set of $(m - 1)$ subsidiary markets for a specified period of time (or for a specified number of cycles in case of a single machine simulation). In each of these markets, arbitrageurs within the same set could trade with one another subject to the same set of double auction rules. These subsidiary markets, analogous to after-the-hours crossing networks operated by the New York Stock Exchange, Instinet, and many other exchanges, would, in principle, enable the individual arbitrageurs to clear their inventories at the end of each period. But, for reasons we explain later, hardly any trades occur in these subsidiary markets. (In future work we plan to eliminate the price discovery function in the subsidiary markets, and use them purely for the purpose of clearing arbitrageurs' inventories at closing price).

5.2. ARBITRAGEURS

The market-based constraints on arbitrageurs described in Section 4 were modified to take into account their inventory positions. In the following explanation of how arbitrageurs function under the relaxed opportunity set constraint, we focus on the first and the last set (who deal with the exogenous buyers and exogenous sellers respectively), and then on one of the intermediate sets of arbitrageurs.

Each arbitrageur is permitted to engage in an unlimited number of purchase and sale transactions within the constraints of its opportunity set. When buying to increase their inventory, arbitrageurs use borrowed funds that must be paid back through subsequent inventory liquidation. Similarly, when selling to expand their short position (negative inventory), arbitrageurs use borrowed goods that must be returned through subsequent purchases to liquidate the short position. (The subsidiary market described above would have served this function of settling up the accounts among each set of arbitrageurs at the end of each period).

The opportunity set constraints can be defined in terms of money or units of goods. For now, we choose to impose an opportunity set constraint on all arbitrageurs by requiring them to hold their long or short inventory positions to no more than L units. In other words, an arbitrageur can take any inventory position from $-L$ to $+L$. This limit represents the extent to which arbitrageurs can borrow to finance their trading activity. This constraint is analogous (but not identical) to the single unit constraint imposed on our buyers and sellers.

In addition to the inventory constraint, the arbitrageurs must also be subject to a constraint on their bids and asks in a manner similar to the constraint on the range of bids of buyers and asks from sellers. These constraints can be specified as follows:

Let I_i be the inventory position of arbitrageur i from the first set of arbitrageurs who sell in Market 1 at any time during trading. Consider the following cases:

$I_i = 0$: Since it has no inventory, this arbitrageur can bid as well as ask at the same time. Its asks (in Market 1) must be constrained from below by the current ask in Market 2 at the time: $a_1 \sim U(a_2, M)$. Its bids (in Market 2) must be constrained from above by the current bid in Market 1 at the time: $b_2 \sim U(0, b_1)$.

$I_i = +L$: Since it has reached its upper inventory limit, this arbitrageur cannot bid in Market 2 but can still ask in Market 1. It will be selling from its inventory. Let p be the price of the lowest cost unit in its inventory. It must not propose to sell below this cost. Therefore: $a_1 \sim U(p, M)$. When it does sell from inventory, it is assumed that the lowest cost unit is sold first.

$I_i = -L$: Since it has reached its lower inventory limit, this arbitrageur can bid in Market 2, but cannot ask in Market 1. Let p be the price of the highest priced unit in its short position. It must not propose to buy units at a price above p. Therefore: $b_2 \sim U(0, p)$. When it does buy to cover a short position, it is assumed that the highest priced unit is covered first.

$-L < I_i < 0$: Since it has negative inventory, but has not reached the limit on its short position, this arbitrageur can bid as well as ask at the same time. However, its sales will add to its short position, and therefore its asks in Market 1 must be constrained from below by the current ask in Market 2: $a_1 \sim U(a_2, M)$. Its purchases will liquidate its short position. If p is the price of the highest priced unit in its short position, its bids (in Market 2) must be constrained from above by $p : b_2 \sim U(0, p)$.

$0 < I_i < +L$: Since it has a positive inventory, but has not reached the limit on its long position, this arbitrageur can bid as well as ask at the same time. However, its sales will be from inventory. If p is the price of the lowest cost unit in this inventory, it must not propose to sell below this cost in Market 1: $a_1 \sim U(p, M)$. Its purchases will increase its long position and it must not propose to buy at a price above the current bid in Market 1: $b_2 \sim U(0, b_1)$.

For the second set of arbitrageurs, the same rules apply by increasing the subscripts of asks (a_i) and bids (b_i) by one. The same procedure continues until we reach the last, $(m-1)$th, set of arbitrageurs. Figure 1 schematic still applies except that it does not graphically depict the constraints based on the price of units in arbitrageurs' inventories.

For each value of m and a, there are three cells in Table II, one for each of the values of inventory constraint on the arbitrageurs, $L = 1$, 2, and 3. Opportunity set constrained economies of the previous section have been labeled as $L = 0$. As explained above, arbitrageurs' inventories must remain within the range $-L$ to $+L$.

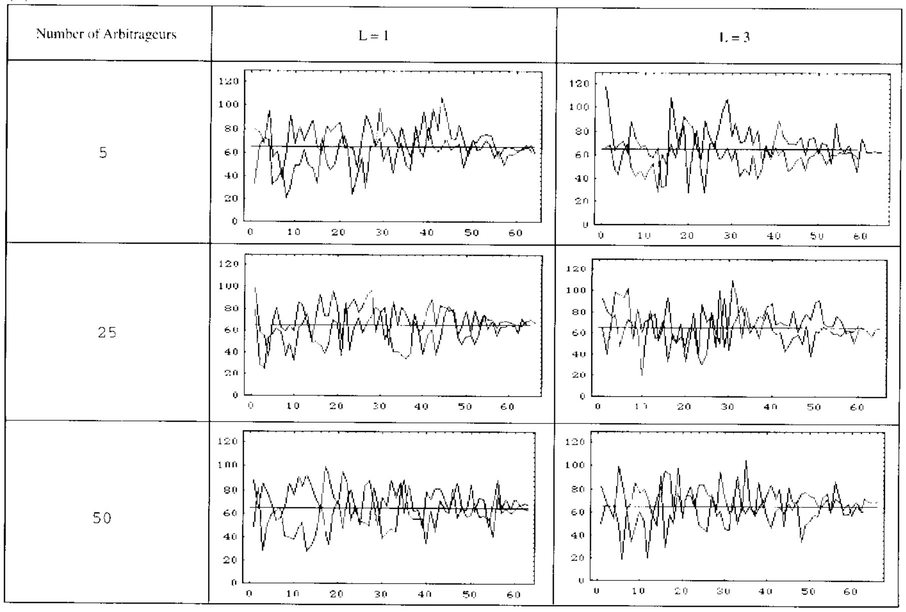
As before, all these economies have 100 buyers and 100 sellers. Seller costs vary from 5 to 124 and buyer values vary from 125 to 26 in steps of 1. Competitive equilibrium price of these economies is at 65, and the volume is 61–62.

5.3. PRICES

Figures 6a–c show the prices paths for different multi-market economies, one from each cell of columns $L = 1$ and $L = 3$ of Table II. Again, each graph was chosen arbitrarily from a collection of 10 different (but similar looking) replications of the same market.

Two features of these graphs are worth noting: (1) in all cases, the sequence of transaction prices in each markets gets gradually and noisily closer to the equilibrium price (represented by a horizontal line); and (2) the larger the number of markets, tighter is the *appearance* of convergence. The first result can be confirmed from the graphs. The second relates to the particular market mechanism we used, to which we return in Section 6.

(a)



(b)

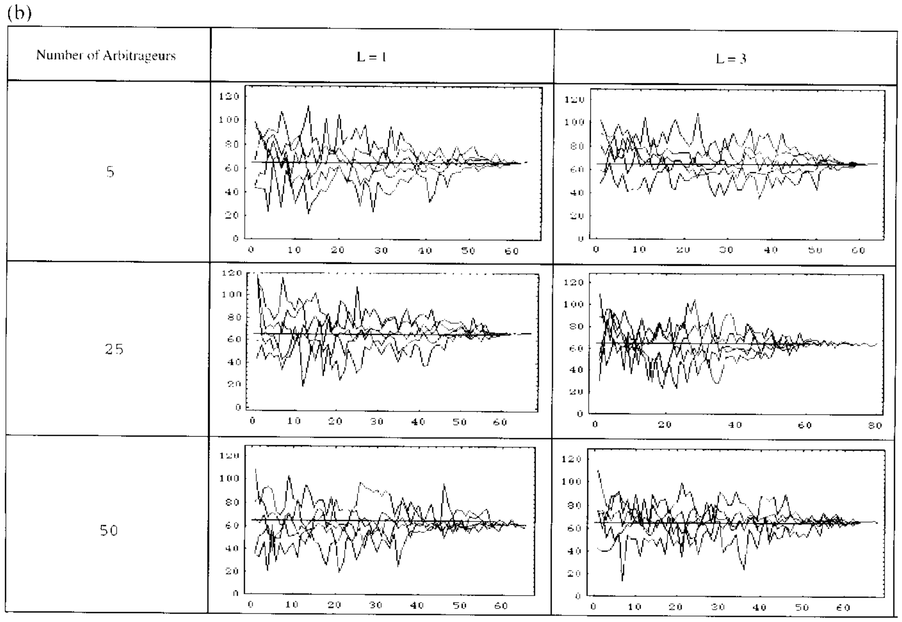


Figure 6a,b.

(c)

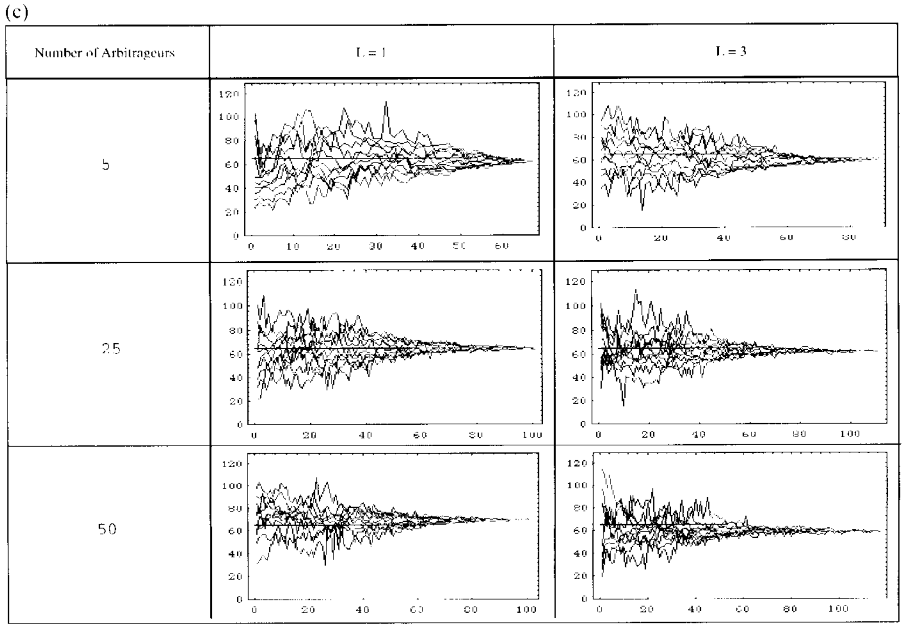


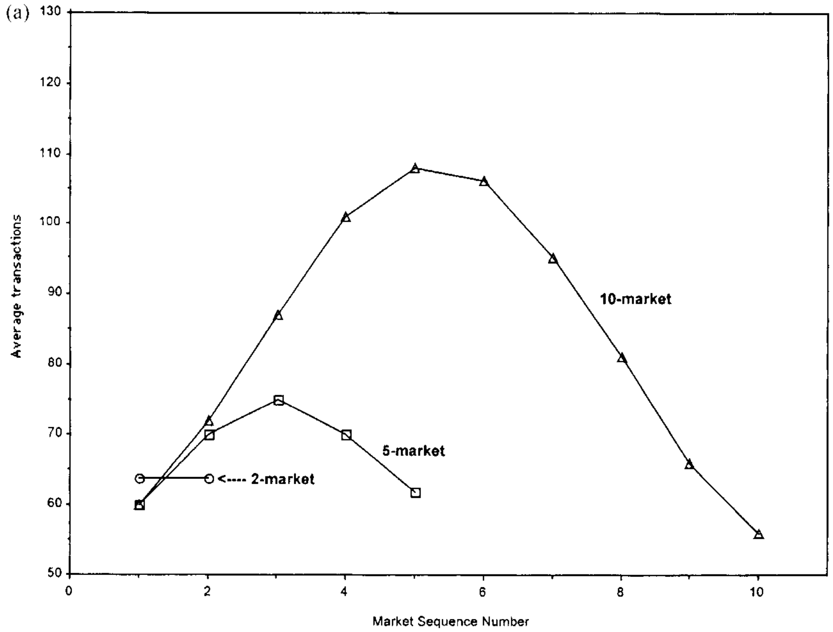
Figure 6c.

Figure 6. Price paths for one replication from the cell of Table II corresponding to (a) 2 markets $L = 1$, $L = 3$; (b) 5 markets $L = 1$, $L = 3$; (c) 10 markets $L = 1$, $L = 3$.

5.4. NUMBER OF TRANSACTIONS

When $L > 0$, results for the 2-market economy are essentially unchanged. In some runs a market might allow for a number of transactions slightly above the number observed with no inventory. As the number of markets (and therefore the number of groups of arbitrageurs) increases, the competitive equilibrium quantity still is an excellent predictor of volume in Markets 1 and m. However, in the intermediate markets, the trading volume increases with the number of markets, the number of arbitrageurs and the limit on inventory arbitrageurs are allowed to hold. Prices are consistently close to the CE prediction, but the quantities traded are not. Allowing arbitrageurs to take long or short positions increases the trading volume as should be expected. But the number of arbitrageurs is not a neutral variable, either. In the intermediate markets the number of transactions increases with the number of arbitrageurs. The highest trading volume occurs in the markets near the center of the chain as can be seen in Figures 7a–c.

In summary, relaxing the opportunity set constraint on arbitrageurs affects the number of transactions among arbitrageurs but it does not affect the high predictive power of the competitive equilibrium model in the markets of final sellers or buyers.

(a)


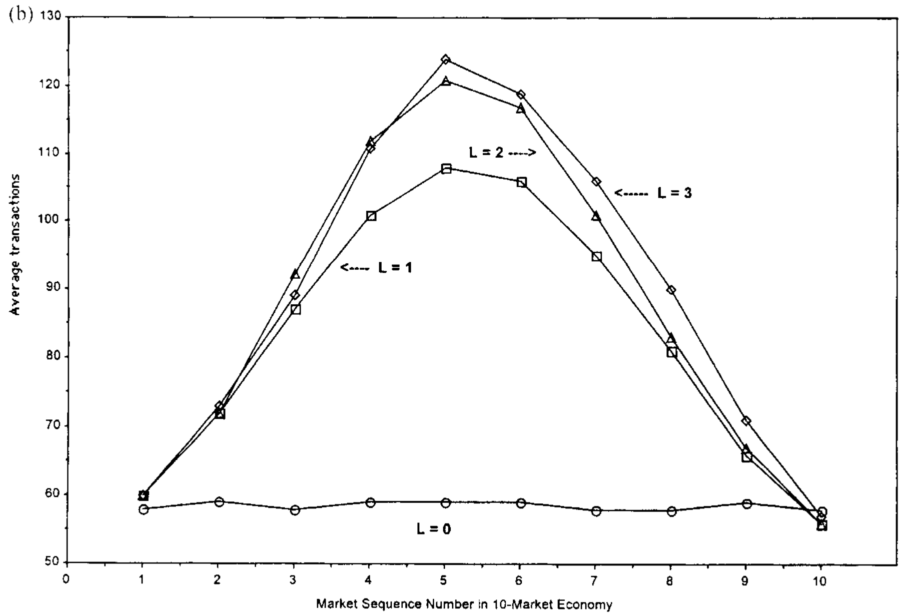
(b)


Figure 7a,b.

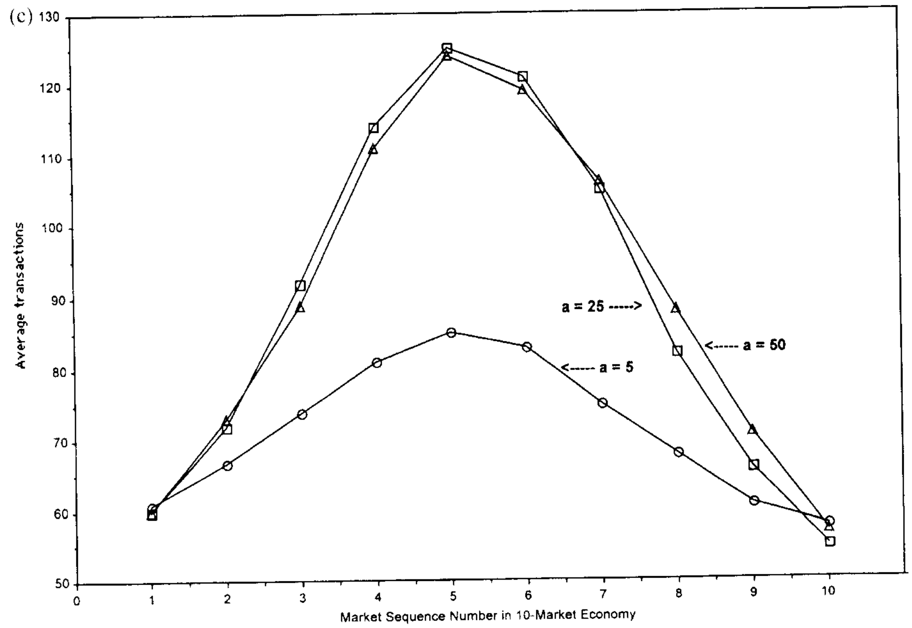


Figure 7c.

Figure 7. Average number of transactions with (a) 50 arbitrageurs, inventory constraint $L = 1$, and three different number of markets; (b) 50 arbitrageurs and four levels of inventory constraint (L) in 10-market economies; (c) inventory constraint $L = 3$ and three different numbers of arbitrageurs in 10-market economies.

5.5. EFFICIENCY

Efficiency is almost 100% with any number of markets or arbitrageurs, when inventories allowed are zero. As Figure 8 shows, when short selling and long buying are allowed, efficiency drops sharply as inventory limit and the number of markets is increased. In 5 arbitrageur economies, efficiencies drop from 99, 82 and 48% (for 2, 5 and 10 markets with an inventory limit of |1|) to 85, 53 and 32 when the inventory limit is raised to |3|. In 25 arbitrageur markets, these drops are from 97 to 85, from 60.3 to 57, and from 19 to 11 when moving from inventories of |1| to inventories of |3| for 2, 5 and 10 markets. Similarly, in 50 arbitrageur markets, these changes are from 95 to 88, from 41 to 55, an odd result, and from 19 to 5 respectively. When the complexity of the environment is large enough, the drop in efficiency resulting from allowing just a |1| unit inventory is so sharp, that the analogy with a phase transition is brought to mind.

Arbitrageurs who are left holding inventory at the end of a period incur losses. If competitive forces are at play, such inventory-holding arbitrageurs will be pushed out of the market sooner or later, leaving only the zero-inventory arbitrageurs in the field.[3]

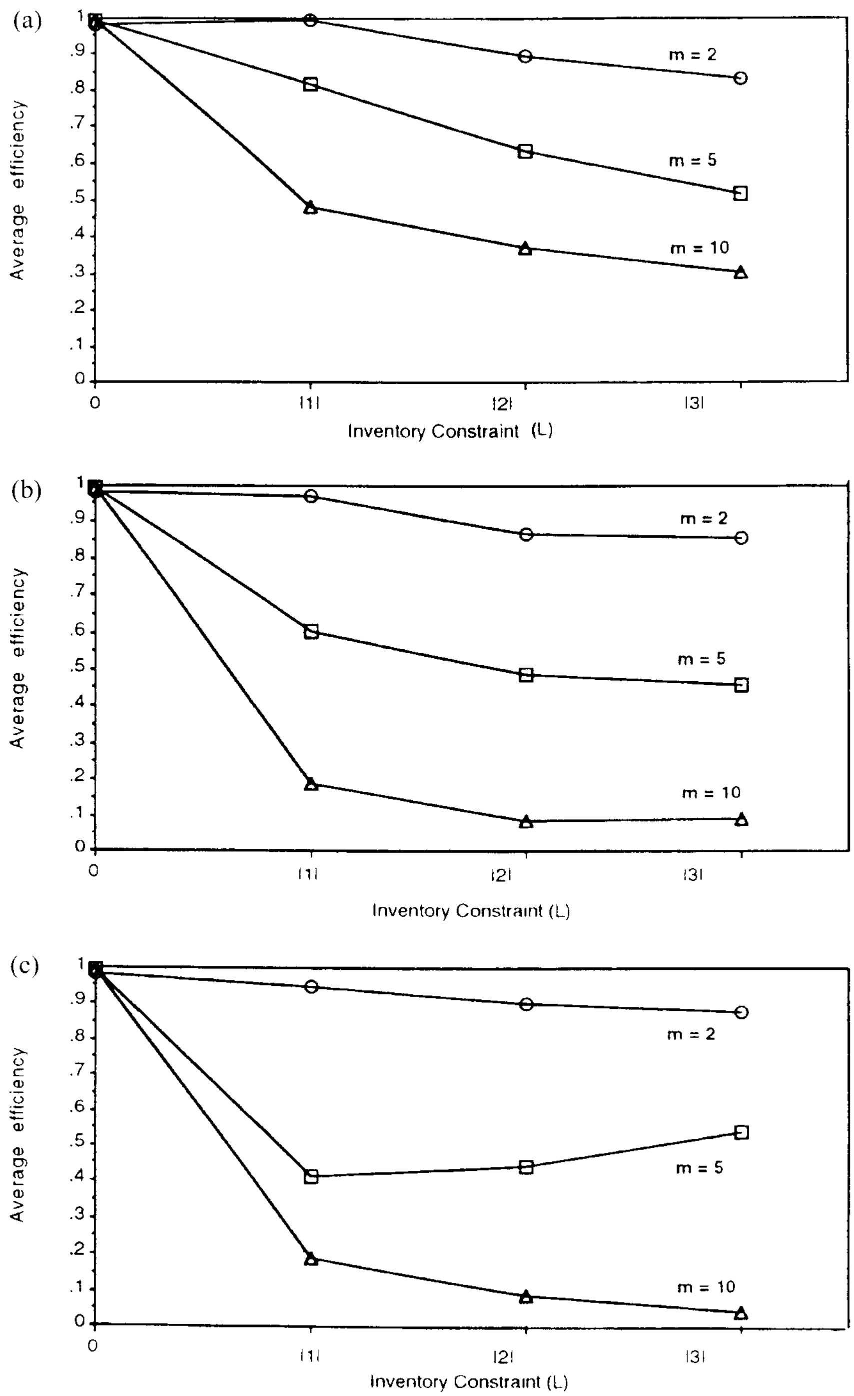


Figure 8. Mean allocative efficiency with 5 arbitrageurs over 10 replications.

6. Tracking the Invisible Hand

How and why the relaxation of the opportunity set constraint causes efficiency to drop so sharply?

6.1. MOVEMENT OF UNITS ACROSS MARKETS AND EFFICIENCY

Consider three scenarios for start of a chain of transactions (i.e., exchange of a new unit of the good for the first time, perhaps only nominally). The first possibility is that the starting trade occurs in Market 1. The exogenous buyer buys at some price below its reservation price and therefore makes a potential profit on the trade. It is prudent to call this *potential* profit because the unit purchased cannot yet be delivered to the buyer by the arbitrageur of Set 1; the latter sells this unit but may not itself have the unit to deliver from its inventory. At this point the profit of the arbitrageur who sells the unit is indeterminate until it actually buys the unit from someone else. In fact the profit of all arbitrageurs who hold inventory in any market remains indeterminate until the short sale is covered at the time this unit is actually sold by a seller in Market m. No increase in allocative efficiency can occur until the unit traded is transmitted right across the m markets of the economy, since arbitrageurs have no consumption value for the unit.

In the second scenario, the starting point is at the mth market, so that a seller actually sells one unit to an arbitrageur of set $(m-1)$. This case is exactly analogous to the preceding paragraph. Again, no social surplus is extracted until all links in the chain of transactions that transmit this unit right across the m markets are executed.

The third possibility is that trade starts between two arbitrageurs in a market different from 1 and m. In this case also, no net surplus is extracted at the time of the transaction. In summary, as long as this unit does not reach *both* end markets, i.e., as long as the unit is not sold by an exogenous seller *and* bought by an exogenous buyer, nothing can be added to the measure of efficiency. Only those units that have both a seller in Market m and a buyer in Market 1 should enter our computation of efficiency.

This helps understand why efficiency drops dramatically with relaxed opportunity set constraint when the number of markets and arbitrageurs increases. Recall from Section 5 that with the relaxation of the opportunity set constraint, our arbitrageurs acquire speculative inventory positions, but they cannot liquidate these inventory positions at a loss. They can only sell their positive inventory at prices above what they paid to buy it, and buy out of their short positions at prices below what they received for the short sales. The combined effect of allowing them to acquire speculative positions, but not allowing them to liquidate these positions at a loss is that some units get stuck in the hands of the arbitrageurs, and do not contribute to efficiency. Under these market rules, all transactions initiated in some market do not necessarily reach both ends of the economy to the exogenous buyers and sellers.

The above helps to clarify an element that may not have been sufficiently stressed in the literature. The efficiency in Gode and Sunder (1993a, b) occurs because of two particular rules. A no-loss constraint and the guarantee that all profitable transactions take place. As long as these two rules are maintained, loss of efficiency is entirely due to extra-marginal transactions. But change one of the rules, as we do in this part of the experiment in which not all-possible transactions actually take place, and efficiency declines.

6.2. ARBITRAGE AND TRANSACTIONS VOLUME

Recall that we have m markets. In Market 1 each buyer cannot buy more than one unit even if arbitrageurs can take short and long positions. Therefore, the number of transactions cannot exceed the number of buyers. In Market 2, the number of units that arbitrageurs can buy cannot be greater than the number of units that can be sold to the buyers in Market 1 (i.e., exactly the number of buyers) plus the amount of stock that they are allowed to have. Therefore, the number of transactions has to be less than or equal to the number of transactions in the previous market plus the inventory they are entitled to hold. In Market 3 the same argument applies. The number of transactions cannot be greater than the number of transactions in Market 2 plus the amount of inventory that arbitrageurs are allowed to hold. Therefore in Market $k = 1, \ldots, m$, the number of units traded cannot be greater than $n + (k - 1)aL$.

If we start looking at the market transactions from the other end, from Market m downstream, we observe a similar pattern. In Market m, each seller cannot sell more than one unit. The number of transactions, therefore, cannot exceed the number of sellers. In Market $m-1$, the maximum numbers of units that arbitrageurs can sell is equal to the number of units that can be bought form sellers in Market m (i.e., exactly the number of sellers in Market m) plus the amount of short selling that they are allowed to do. Therefore, number of transactions has to be at most equal to the number of transactions in the previous market plus the number of units they are entitled to short sell. And so on. Therefore in Market k, the number of units traded cannot be greater than $n + (m - k)aL$.

Consequently, the maximum number of units that can be bought keeps increasing upstream from Market 1 to Market m, while the maximum number of units that can be sold keeps increasing downstream from Market m to Market 1. The number of transactions in every market has to be less than or equal to the smaller of the two which is $n + aL \min(k - 1, m - k)$. This limit is lower at the end markets than in the middle markets, which helps explain the single-peaked shape of the graph of transaction volume across m markets in Figures 7a–c.

This also explains why the upstream arbitrageurs (in markets near the exogenous buyers) hold mostly short positions and downstream arbitrageurs (in markets near the exogenous sellers) tend to hold long positions at the end of trading. Since the long and short positions of arbitrageurs are held in different markets, opening local subsidiary markets at the end of the session (in which arbitrageurs within a group can trade with each other) do not see much action.

6.3. LINK BETWEEN TRANSACTION VOLUME AND PRICE CONVERGENCE

The above analysis also helps explain why prices appear to converge more spectacularly to the competitive equilibrium price when the economy consists of more markets. With more markets, the number of transactions in the middle markets is larger than in the end markets. In addition, as observed in Section 4 and seen in Figure 1, the double auction institution used in these simulations allows smaller opportunity sets for the arbitrageurs in the middle markets. This keeps prices in the middle markets within a smaller interval centered around the equilibrium price, as can be observed in Figures 3 and 6. The middle markets have more transactions at prices closer to the CE price.[4] In the end markets, there are fewer transactions, and prices converge towards the equilibrium level from further away. Short price paths starting away from equilibrium combined with longer ones closer to the equilibrium accentuate the appearance of sharp convergence.

Efficiency drops with increased number of markets in the economy because fewer transactions get through. Fewer transactions get through a longer chain of markets because there is greater opportunity for units to get trapped in arbitrageurs' inventory in a longer chain of markets. Why are more units trapped when the number of arbitrageurs increases or when the short and long selling allowed is increased?

As the number of arbitrageurs increases (and the number of buyers stays the same), the probability that an arbitrageur who has purchased one unit cannot sell it at a price above its purchasing price also increases. Why is this so? The probability that an arbitrageur gets another chance to make a proposal in the market after its purchase decreases as the number of arbitrageurs increases. Therefore, with large numbers of arbitrageurs operating in the same market, when its turn arrives again, it may very well find that the reference bid on which the arbitrageur based its purchase may have been substituted by another one. If the new bid were higher, this would allow it to sell anyway. But the average bid declines in the later part of the period just as the average offer rises. This happens because a buyer with a high reservation price and a seller with a low cost is more likely to succeed early in the market.[5] Since at any moment of time all bids have to be below the remaining reservation prices, bids will tend to become lower. A similar argument explains why asks tend to become higher and why an arbitrageur that has short sold one unit may discover when its turn to cover that unit arrives, that current asks are above the price at which he short sold the unit. In addition, the larger is the inventory allowed, the greater is this effect. The result is that as the number of markets, arbitrageurs and inventories increases, the difference between the number of transactions occurring in the different markets of one economy also increases.

As the number of markets in the economy increases, the number of commitments made by arbitrageurs that cannot be met grows as well. The interesting part is that, in spite of this, the number of commitments to sell made to the buyers in Market 1 and the number of sales made in Market m, are close to the number of transactions predicted by the competitive equilibrium model. Moreover, prices tend to converge towards the vicinity of the CE price. As the number of markets in the economy increases, relaxation of opportunity constraint can cause it to become deadlocked. Commitments to sell are made at a price too low, and commitments to buy are made at a price too high, so that no subsidiary market can be of any help. And yet, *even in such an economy*, the CE model is a good predictor of prices in all markets and of quantities in the end markets, even if few actual units of the good are exchanged all the way from exogenous sellers to exogenous buyers.

7. Concluding Remarks

Since Adam Smith, economists have generally accepted that the roots of the economic order lie in the individual striving for self-interest. Are the imperfections of individual behavior mirrored in the aggregate economy? Our simulations show that social order (market equilibrium and efficiency) emerges in these markets even with little individual rationality. We say that it emerges because equilibrium and efficiency are hardly the motives that drive individual actions (want). This emergence does, however, require *scarcity*. The scarcity constraint, of which the world has aplenty, imposes order on an otherwise random system. These results build on Becker (1962) who showed that agents who choose randomly within their opportunity set defined by the budget constraint give rise to a downward sloping demand and upward sloping supply function.[6] They also support North's (1990) view of our social and economic institutions as human artifacts that structure our interactions and solve problems of coordination by defining and limiting individual choice sets.

Our conclusion that scarcity is one source of this order is confirmed by evidence that order progressively breaks down as the scarcity constraint is relaxed in the economy. *Efficiency* of the economy tumbles under random bidding in double auction as larger (long and short) inventory positions are allowed, and markedly so as the number of arbitrageurs and markets increase. With a sufficiently large number of arbitrageurs, markets and inventories, economy becomes inefficient because commitments cannot be met.

Even when efficiency drops, one hypothesis dear to economists is sustained: *prices* tend to converge to the level at which supply equals demand. The predictions of the law of supply and demand are accurate, even if agents act randomly in markets interrelated in a complex manner, as long as the agents want to exchange and they trade within some bounds.

Our data show that the price that equates supply and demand is descriptively relevant, but it has little to do with individual maximization. This issue has been a challenge in economics (Simon 1978, 1996). The behavior of individual 'human subjects in the laboratory often violate the canon of rational choice when tested as isolated individuals, ... in the social context of exchange institutions serve up decisions that are consistent (as though by magic) with predictive models based on individual rationality' (Smith, 1991). 'In spite of its mathematical complexity, the competitive model is very crude when placed in the context of ... interactive markets and behaviors. Nevertheless, if the assumptions of the model are applied with an "as if" interpretation the resulting model is very powerful. ... In essence, the mathematical problem was solved quickly and without all the relevant information existing in a single place. ... Some sort of parallel processing appears to be taking place but its form remains a mystery' (Goodfellow and Plott, 1990).[7]

Our simulations indicate that no substantial mathematical problem necessarily has to be solved in the process of bidding and asking. Even more, they indicate that no learning needs to take place in order to approximate the competitive outcome. What problem there is gets solved when agents act within the constraints of their reservation prices and unit costs (in terms of aggregates, once there exists the market supply and demand). As it were, the real action is off the stage. Given market supply and demand, want (i.e., bidding, asking and exchanging) and scarcity (i.e., opportunity set constraints) combined with the appropriate institutional mechanism do the trick.[8] This trick consists in funneling transaction prices towards the CE price, not in solving a complex mathematical problem through parallel processing algorithms. Nor does there have to be any sort of learning process to guide agents to act as if they were in complete information Nash equilibrium.[9] On the other hand, except for a trivial case, a certain degree of heterogeneity of reservation values is necessary to create the funnel. This is required for well-behaved aggregated demand curves with irrational subjects (see, for example, Trockel, 1984; Hildenbrand, 1994), an intuition that Cournot and Marshall already had.

If buyers are enticed to buy, but are constrained to do it at prices below some reservation price, and if sellers are enticed to sell, but are constrained to do it at price above some specified cost, and arbitrageurs have similar constraints, then, under the appropriate institutional arrangements, transactions converge toward the price at the intersection of the frontiers of the reservation prices and marginal costs, known as demand and supply. Hayek (1945) and North (1990) point to this critical informational and problem solving role of market institutions in society. This can happen even in complex environments with many intermediate markets, with arbitrageurs entitled to hold short and long inventory positions and allowed not to honor their contracts. In such an adverse environment efficiency declines but the competitive price and quantity predictions still hold. The opportunity set constraints on actions of economic agents are capable of transmitting demand and supply across inter-linked markets in the form of derived demand and supply.

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Appendix

VIEWING THE SIMULATION LIVE ON YOUR OWN COMPUTER

You can get a better sense of the dynamics of the markets reported in this article, by looking at the actual simulations on your own computer by following the instructions given in this appendix. We also describe what you will see on your computer screen.

HARDWARE REQUIREMENTS:

IBM/clone with VGA or Enhanced VGA monitor.

HOW TO GET THE SOFTWARE:

Download a copy of file simul2.exe from http://www.som.yale.edu/faculty/sunder/zisoft.html and store it on your hard drive or a floppy.

HOW TO RUN THE SIMULATION:

1. Enter the directory in which file simul2.exe is stored, type simul2, and press the enter key.
2. The computer prompts your for two integers, lag% and nolag%. You should enter the two numbers separated by a comma, and press the enter key. The parameters you choose will have the following effects. Make your choice accordingly:
   If you respond with 0,1 the program will run automatically without a break. If you respond with 1,0 it will run automatically but with one second pause after each step so you can see most steps of the process. If you respond with 0,0 the program will run in small steps, and you have to hit the space bar after each step. Try them in this order. If the program does not seem to be doing anything for a while, it is probably waiting for you to hit the space bar. All other parameters are picked by the computer as preset values or random draws. Sit back and enjoy the show.

3. The program will show you a 5-market economy with strict budge constraint ($L = 0$). As described in the paper, there are four sets of arbitrageurs. There are eight exogenous buyers, eight exogenous sellers, and eight arbitrageurs in each set.

WHAT YOU SEE ON THE SCREEN:

The first row of boxes: The five boxes from the left show the five markets, beginning with market 1 on the left. Market 1 has exogenous buyers and their demand function is shown in empty red circles. The second from right is the last market with exogenous sellers and their supply function is shown in filled purpule circles. Horizontal dotted line is the equilibrium price range.

When the market runs, you will see moving black circles in each market to indicate current asks, and moving red circles to indicate current bids in each market.

If you run the program with 1,0 or 0,0 in response to the first query, you will also see the following (it is lost in speed of the computer if you run this with 0,1). When a bid and an ask cross in a market, a green line connecting that bid and ask appears in the box for the appropriate market. The computer chalks out in blue the transfer path of one unit from the appropriate seller on the right (whose circle becomes hollow) to the appropriate buyer on the left (whose circle becomes filled) after the transaction is completed in each market in sequence.

Please note that the bids of eight buyers in each market are arranged from left to right beginning from the left had side wall of the box corresponding to that market. Asks of the eight sellers in each market are also arrange from left to right, ending at the left-hand side of the wall of the box corresponding to that market.

Second row of boxes: As each transaction is completed in each market, the transaction price path in each market is plotted in red circles in the second row of boxes. Again, the dotted line is the equilibrium price range.

Third row of boxes: As each trader earns profit, its amount is plotted in the third row of boxes. The exogenous buyers are arranged to the right of the left wall of the first box. All other groups of traders are arranged to the left of the right wall of the appropriate boxes.

Fourth row of boxes: The first box in the fourth row plots the efficiency (the cumulative percent of the maximum possible surplus extracted) with each transaction. If the last bar does not reach the top of the box, the market is less than 100% efficient.

Second box in the fourth row is the counter. Each horizontal bar represents a round. The proportion of the bar completed shows the proportion of total iterations for the round completed. The program is set to run 10 times, and each run completes a horizontal strip in this box.

Last column of boxes: The last column to the right gives summary data accumulated over multiple runs of this market. The graphs in the right column are updated refreshed after every round.

In the first row, you can see the percent of runs in which each of the exogenous buyers (from let to right in red) and each of the exogenous sellers (from left to right in purple) trades its unit. This gives you an idea of the relative frequency with which the intra and extra-marginal traders get to trade.

In the second row, you see the average of the transaction price paths in each of the five markets, averaged over the rounds run so far. The blue bars under the average price path are the number of transactions from which the average was computed. This explains why the last one or two points on the price path can have a large deviation from the equilibrium price level (because they are computed from fewer observations.)

In the third row, you see six vertical bars. The left bar shows the percent of maximum possible total profit earned by the exogenous buyers. The rightmost bar is for the exogenous sellers, and the other bars are for the various groups of arbitrageurs. The total height of these bars adds up to average efficiency of all rounds so far plotted in the right side of the rightmost box in the fourth row.

In the fourth row, you will see a round-by-round graph of efficiency of these markets. The right most thick line is the average value of this graph.

Have fun. Let us know if there are any problems at shyam.sunder@yale.edu.

Notes

1 Their role may be better understood as market makers, as in a dealership market. But calling them arbitrageurs helps remind us that each of them operates in two different markets and that buyers and sellers can also quote bids and asks.

2 To save space, we report the results for symmetric demand and supply only. Asymmetric supply and demand affect the direction of price convergence path but have little effect on efficiency or price convergence (Gode and Sunder, 1993a). The Appendix to this paper includes a simulation program for the reader to verify the results for asymmetric demand and supply.

3 In the foreign currency markets, most deals are made on telephone, and the currency brokers are known to hold one side to an imminent deal on hold, while they negotiate with the other side on a second line. This simultaneous dealing allows brokers to rid themselves of risk in volatile and unpredictable markets. In the New York Stock Exchange, specialists are required to take the risk of holding inventories of their own. They are compensated for taking this risk by grant of a profitable monopoly on the exchange floor in making market for a given security.

4 In all these examples the short selling and long buying allowed is symmetrical. When it is not, an asymmetry is observed in the number of transactions that occur upstream and downstream as the previous explanation leads us to expect.

5 There is an *efficient order* (see Wilson, 1985) but obviously not due to any impatience.

6 Also see Leijonhufvud (1993), Hardle and Kirman (1995), Luo (1995) and Evans (1997).

7 Also see Plott (1986, p. 306), and D. Friedman and J. Rust (1993, p. xix).

8 Possibly, as mentioned by Rothschild (1994), the success of the invisible hand requires good institutions and good norms. In the single market case, Gode and Sunder (1993c, 1997) have already

observed that not all institutional arrangements lead to the same degree of efficiency. In this sense, the present simulations may be showing that good rules are a substitute for rich behavior.

[9] See Friedman (1993), p. 15.

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