RISKY CURVES: FROM UNOBSERVABLE UTILITY TO OBSERVABLE OPPORTUNITY SETS

Shyam Sunder, Yale University

(Joint work with Daniel Friedman)

IMPRS Uncertainty Summer School, Max Planck Institute, Jena, Germany

August 6-10, 2010

Heat: Circa 1750 A.D.

- Everyone knows: heat is an element
  - In recent decades, the precise scientific term phlogiston has replaced the vague, traditional label: fire.
  - Phlogiston escapes when something burns, leaving calx
- What is the mass of this element?
- When we burn mercury, phlogiston escapes leaving powder calx behind, so the mass equation is:
- $m_{Hg} = m_{Calx} + m_p$
- But the calx weighs more than the metal…
- Imagine the theme for an 18th Century Conference:
  - How many varieties of Phlogiston in various materials?
  - And not: what does it mean to have an element of matter with negative mass?

Risk Preferences: Circa 2000 A.D.

- Everyone knows that people have risk preferences
  - In recent decades, more precise scientific labels have come into vogue: expected utility theory and the Bernoulli function
- Measuring the curvature has proved to be elusive
  - Portfolio choice etc. usually suggest concave functions
  - But gambling suggests convexity, …
  - Possibility of segmented Bernoulli functions with different curvatures, and reference point(s), and kinks
- Can we straighten it all out? Maybe.
- Can we establish "risk preference" as an explanation of observed choice behavior?
  - As opposed to a description of observed choices

Which Risk?

- There are many different concepts for which the term "risk" is widely used in different, and sometimes overlapping contexts
- Two important variations:
  - Risk in the sense of uncertainty of outcomes (for example, measured by dispersion)
  - Risk as the possibility of harm
- In economics literature on risk preferences, the first of these two meanings dominates (mathematical tractability of variance measure)
  - Although, the second meanings slips in (credit and bond ratings, insurance, health, engineering, internal controls, etc.)

What is This Owner's Bernoulli Function?

- Portfolio X: consists only of short maturity Government securities and insured CDs.
- Portfolio Y: consists only of deep out of money call options on oil futures.
- Both are owned by the same person!
  - She manages X for her great uncle.
  - She holds Y in a national contest…

Is the Bernoulli Function *u* an Intrinsic Personal Characteristic?

- Estimation from choice data is a mechanical process: entering choice data in the estimation algorithm necessarily yields an estimate of *u*
  - The specific estimate depends on the criterion used to select the "best fit"
- Existence of estimate *u* says nothing about its validity (regression estimates ≡ valid estimates?)
- How well can *u* predict the out-of-sample choice data? For a given person or population, how generalizable is *u* to new tasks and new contexts?

Degrees of Freedom

- The parameter space for increasing Bernoulli functions is infinite dimensional (unlimited flexibility)
- Often one imposes CRRA or CARA
  - 1 df, or 2 if multiple families allowed
- Friedman and Savage and Markowitz use at least 4 df
- Prospect theory uses at least 5 df:
  - location of reference point, derivatives on each side, curvatures on each side
  - Many more when you allow for probability distortions

The Best Case: *u* is Universal

- Friedman and Savage (1948) and Markowitz (1952) had hoped so.
  - both proposed rather complex functions: 3-4 segments alternately convex and concave.
- But empirics said otherwise, e.g., Ward Edwards (1953, 1955) found interpersonal differences.
- No universal alternative *u* has been found to improve on the explanatory power of a straight line *u*.

Fall back position #1

- Human population could consist of a few basic risk types
  - Such as blood types: O, A, B, etc.
  - Stable after measurement: your type doesn't change.
- Binswanger (1980, 1981, 1982) field experiment with Indian farmers is the most cited support, but it doesn't hold up.
  - Lottery choices didn't predict farming decisions.
  - Luck was the only significant explanatory variable in lottery choices!
- Evidence is unkind to the idea that human population consists of individuals of a given set of risk types.

Fall back position #2

- Observable personal characteristics knowably map into u.
  - Age, gender, wealth, education, etc.
  - E.g., Lower middle class German males of age 30-35 tend to have a Frieman-Savage type utility function with the lower inflection point near income 0 and upper near 20, and absolute risk aversion in the three segments is approx. a = 2.5, -1.2, and 2.2 respectively.
  - An upper middle income Japanese housewife of age 50-55 tends to have a CRRA utility function with $r = 3.0$
- dozens of gender studies, but inconclusive:
  - "Our findings suggest that gender-specific risk behavior found in previous survey data may be due to differences in male and female opportunity sets rather than stereotypic risk attitudes. Our results also suggest that abstract gambling experiments may not be adequate for the analysis of gender-specific risk attitudes toward financial decisions." [Schubert et al, 1999, p. 385]
  - Eckel and Grossman (2003) meta-study of 24 finds support in about half, and reverse or no effect in the other half.
- Effects of age, etc. are even more obscure.
  - Leland and Grafman (2003) undercut even ventromedial prefrontal cortex (VM) damage story of risky choices!

Binswanger and Sillers Field Studies

- Binswanger found that wealth, schooling, age, and caste were all insignificant
- Sillers (1980) field study with Filipino farmers:
  - "This chapter briefly describes an attempt to use household risk preferences, as measured in the experimental game sequence, to test the impact of household risk aversion on the rate of fertilizer applied to the dry season rice crop. This effort failed to produce a satisfactory test of the importance of this relationship or its direction…"
- Neither Binswanger nor Sillers could explain farmers' decisions in familiar tasks using risk preferences inferred from their experiments
- Our partial screening of the literature has not yet revealed a validated evidence on how the curvature of utility functions might be systematically related to some observable human characteristics

Fall back position #3

- Bernoulli functions are idiosyncratic but have some overall population distribution.
  - Population distributions of many such demographic properties have been tabulated extensively: income distribution, body sizes and weights, food and clothing preferences, etc.
  - These tables are used for production, marketing, etc.
  - "Goldman-Sachs tables" of risk preferences don't exist yet; why not?
  - However, we know of no attempts to build such tables. So the evidence is negative, but only circumstantial.

Fall back position #4

- Bernoulli functions are idiosyncratic but at least are stable at the individual level.
- Harlow and Brown (1990) is the most favorable evidence we could find.
  - weak but significant correlations across four choice and physiological risk measures for male Ss; no relation for the female Ss; artifact indications even for males.
- Isaac and James (2000) is a blow to this position
  - strong *negative* correlation between risk aversion measured in (a) 1P-IPV auction vs. (b) BDM task.

Rock-Bottom Position: Can't Fall Any Further

- Bernoulli Functions are person *and* context specific
  - Contexts: investment, insurance, sports, gambling, health, etc…
- These functions vary across *N* persons and *M* contexts to yield *N*M* functions; still have plenty of degrees of freedom in choice data to estimate so many functions
- Evidence: no such functions have been discovered so far
- No room for further retreat within the realm of science
  - Unless we postulate that risk preferences are unique to each act of choice
  - One or more parameters need to be estimated from each observed choice (not enough degrees of freedom)
  - In what sense can we call such a process science?

If Not Micro, Do We Have Macro Evidence?

- The micro-level evidence so far is not palatable to non-linear utility
- If non-linear utility u shifts arbitrarily even for a given individual and context, it seems to have little scientific value (attributing choices to themselves!) or arbitrary moods or spirits (outside the domain of science)

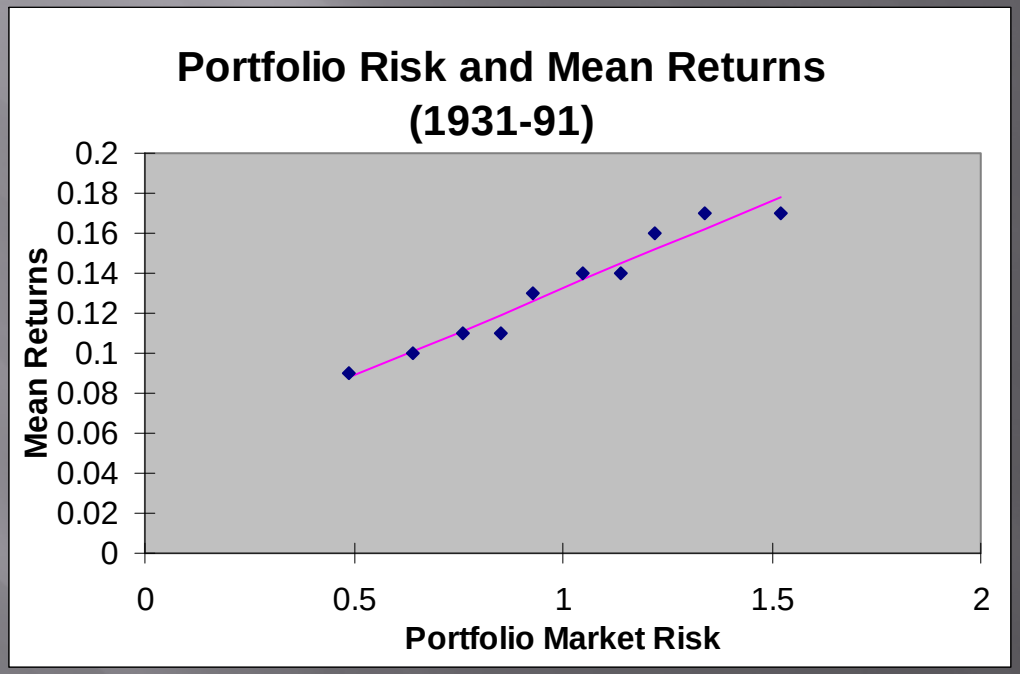
Macro-Level Insights?

- Even if we cannot measure them at individual (micro) level, perhaps Bernoulli functions for "representative agents" (or population distribution across heterogenous agents) can give us macro-level insights into important aspects of the economy and society, e.g., in
  - Stock Market
  - Bond Market
  - Insurance
  - Gambling

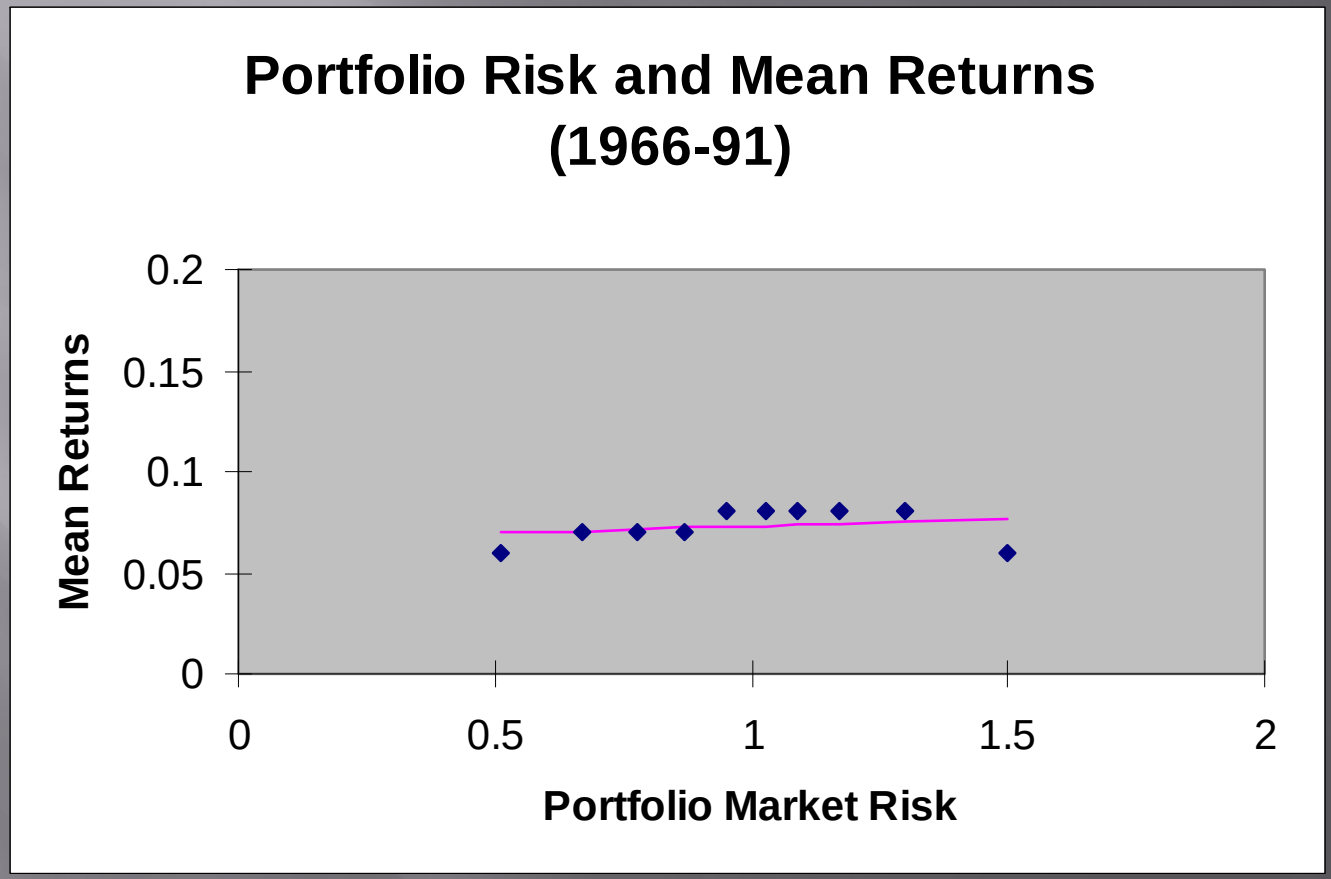
Portfolio Theory and Stock Market

- Portfolio theory built on the assumption of risk preferences (Markowitz)
- A special interpretation of risk (dispersion of outcomes) which is at variance with the interpretation of risk in virtually every other field (harm and uncertainty)
- This interpretation of risk was probably chosen for mathematical convenience
- In equilibrium, mean return on a security is proportion to its market risk (Sharpe 1964, Lintner 1965)
- Risk-return trade-off is a foundation result in finance taught to students for almost five decades
- What is the supporting evidence?

Evidence from Stock Market on Risk and Return (Source: Prepared by authors from data in Black (1992, Exhibits 3 and 4)



Evidence from Stock Market on Risk and Return (Source: Prepared by authors from data in Black (1992, Exhibits 3 and 4)



Current Consensus in Finance?

- Our tests do not support the most basic prediction of the Sharpe-Lintner-Black model, that average stock returns are positively related to market betas. (Fama and French 1992, p. 428)
- Since William Sharpe published his seminal paper on CAPM (*capital asset pricing model*), researchers have subjected the model to numerous empirical tests. Early on, most of these tests seem to support the CAPM's main predictions. Over time, however, evidence mounted indicating that the CAPM had serious flaws (Smart, Magginson and Gitman 2004, pp. 210-212).
- What is going on here? It is hard to say. …One thing is for sure. It will be very hard to reject the CAPM beyond all reasonable doubt (Brealy & Myers, pp. 187-8).
- …do not feel that the evidence for discarding beta is clear cut and overwhelming (Chan and Lokanishok 1993)
- Hardly a ringing endorsement of portfolio theory
- No doubts about the existence of concave utility functions

Equity Risk Premium Puzzle

- Predictions of equilibrium equity risk premium are an order of magnitude smaller than what has been observed during the past 100 years (Mehra and Prescott 1985)
- Recommendations of equity risk premiums vary widely across finance text books (Fernandez 2010)
- No explanation/consensus in finance, behavioral or otherwise

The Bond Market

- Unlike the stock market, most analyses of risk in the bond market still use the "possibility of harm" concept of risk
- Popular bond ratings (Moody's and Standard & Poor's) not based on dispersion measure of risk
- Even with (dispersion) risk neutral investors, one should expect to see a higher promised yield on higher risk (lower rated) bonds; higher yields do not imply concave utility
- Fisher (1959) model, based on default risk and marketability explains 74 percent of the variability of bond yields in excess of treasury securities; no role for concavity of utility function
- Altman (1989) finds monotonic link between bond ratings and returns net of defaults which is not consistent with CAPM; multiple explanations, most of them do not require concave utilities
- Evidence from bond markets does not favor curved (concave or convex) over linear utilities

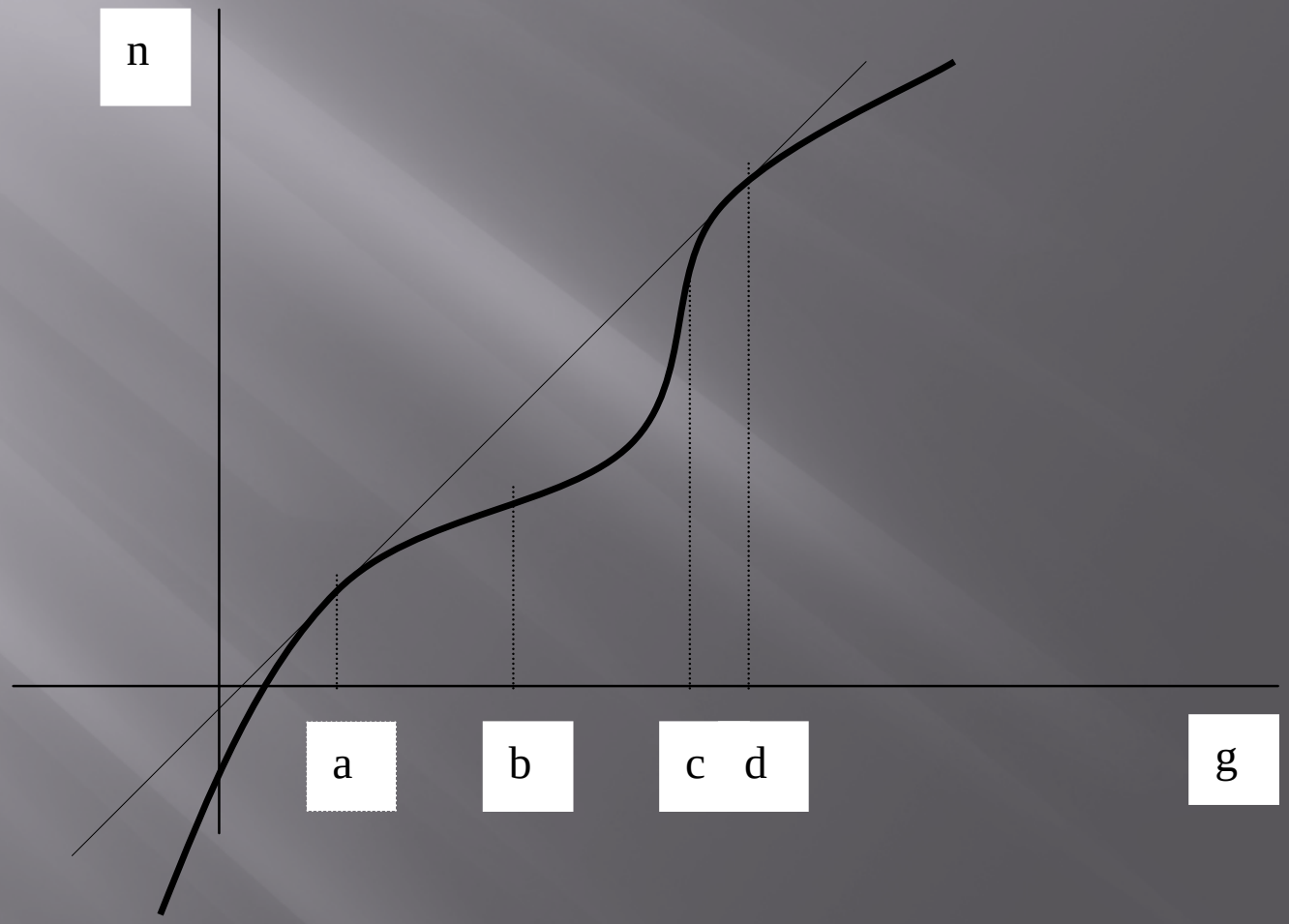
Insurance

- Poster boy for theories of risk aversion
- Like bond markets, it is the "possibility of harm" concept of risk that dominates insurance
- Risk (dispersion) aversion is only one possible explanation for why people buy insurance
- Alternatives include
  - Concave net payoffs (discussed later)
  - Legal requirements
  - Social pressure and conventions (not seen in primitive societies, insurance sales and marketing programs)
  - Simplifying planning for contingencies (chop off branches of DT)
- Amazingly—insurance is inconsistent with the currently popular prospect theory that predicts that people would pay to bear (dispersion) uncertainty in loss domains!
- One sided nature of insurance suggests appropriateness of applying option theory here

Gambling

- Extensive use of curved utility functions to generate facile but inconsistent explanations of gambling
- The same people also buy insurance
- We could not find estimates or predictions of actual individual or aggregate behavior using Friedman and Savage or Markowitz type of utility functions with multiple points of inflection; no identification of parameters or laws that determine these parameters
- Marshall (1984), Figure 6: Optimal fair bet involves only two outcomes *a* and *d* in FS the utility function
- Nonmonetary motivations behind gambling (emotional, social, addictive, compulsive, entertainment aspects of gambling)
- Utility curvature rarely appears in serious scientific analyses of gambling in any field (or in design of gambling systems)
  - In economics, gambling is just assumed away to have been generated by convex but unobservable utility function (effectively treating utility function as a plug for observed behavior)

Figure 6: Smoothed Payoff and the Optimal Bet



Where Does This Leave Risk Attitudes as a Scientific Concept?

- At individual level, curved utility functions remain beyond scientific measurement and validation (from any given perspective—universal, a few basic types, demographic, population distribution, idiosyncratic, or idiosyncratic context dependent)
- At macro level, few phenomena are explained by an assumption of a universal risk attitude of "representative" agents or population distribution of heterogeneous agents
- What, then, is the substance behind the concept of risk attitudes? In what sense is it "real" or scientific?

Occam's Razor

- Utility functions are neither deduced from fundamental propositions, nor observed directly
- They are inferred from observed behavior
- Scientific value of inferred functions derives from their usefulness in organizing and predicting out of sample observations
- Sufficient stability and consistency is necessary for scientific value
- Nonlinear Bernoulli functions, in spite of multiple degrees of freedom, have not yet exhibited an advantage over simple linear function
- Principle of parsimony (Occam's Razor) favors linear function, *ceteris paribus*

What Can We Do?

- Return to the first principles: DM chooses the most preferred available opportunity
- Simplest possible assumption about utility—linear utility so EU ≡ EV.
- Careful analysis of opportunity sets and choose the highest expected value alternative
- Estimate the out-of-sample explanatory power of this fundamental rule across a wide range of decision contexts and populations
- Demand better explanatory power before yielding to those who come hawking "better" solutions with hidden free parameters and special contexts

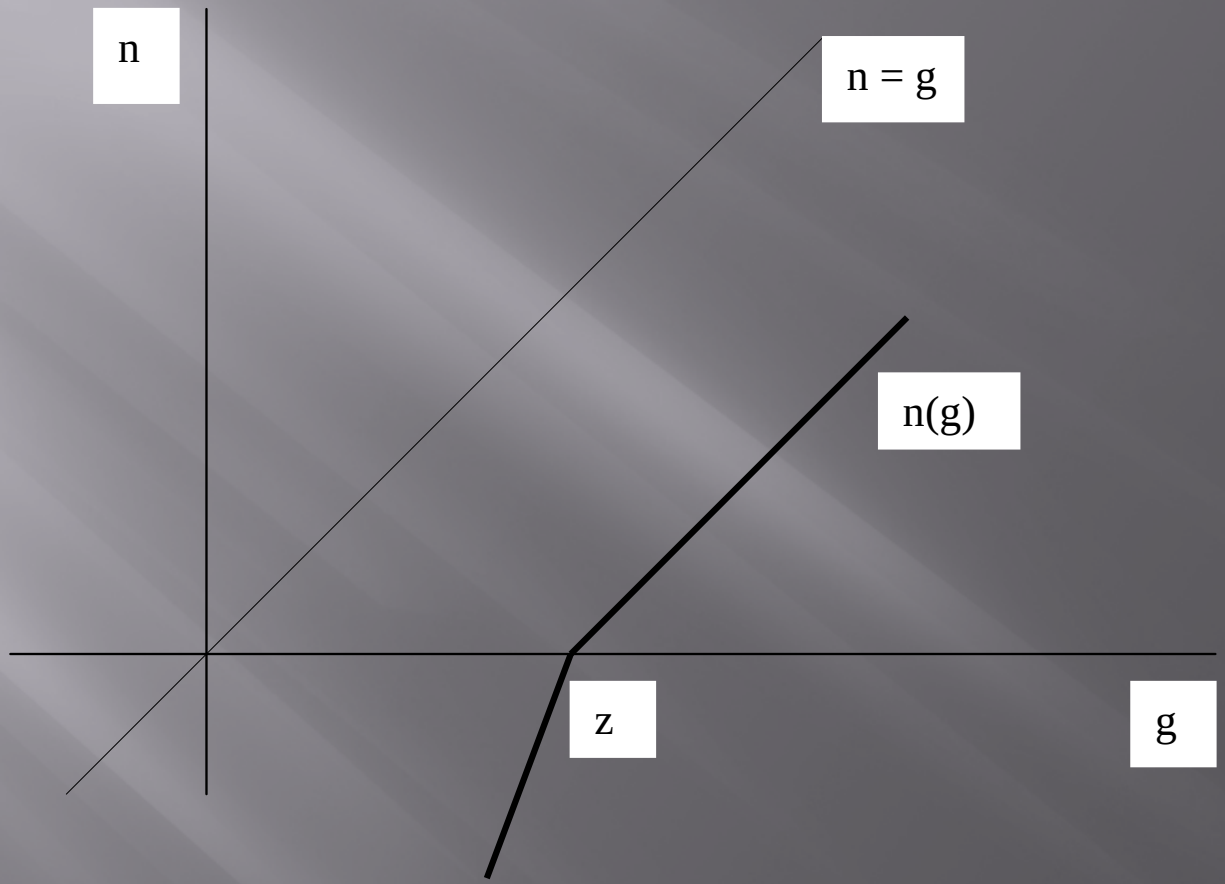
Gross and Net Payoffs

- Net payoff is what DM values
- Gross payoff is the lottery prizes
- There is often a gap between the two
- How far can examination of net payoffs as a function of gross payoffs in the context of linear utility help?
- This puts nonlinearities in directly observable opportunities rather than preferences
  - More parsimonious, observed not inferred

Example 1: Concave Net Payoffs

- DM has some obligation $z > 0$ (e.g., credit card balance, payroll of a firm, bond indentures)
- DM has random cash flow g to meet the obligation
- Failure to meet the obligation brings a penalty proportional to the shortfall ($a^*(z-g)$ if $g < z$)
- Net payoff as a function of gross is in Fig. 1
- It is strictly concave for $a > 0$
- Also, progressive income taxes, fiduciary duties due to penalties and legal costs
- Also, turning down a superior job offer may be due to the option value of waiting for an even better offer, and not concave utility (Dixit Pindyck)
- An ignorant outsider may not be able to distinguish between risk averse DM with linear net payoff and risk neutral DM with concave net payoff (specification error)

Figure 1: Concave Net Payoff



Example 2: Convex Net Payoffs

- Opposite specification error: An outsider infers risk neutral DM with convex net payoff to be risk loving with linear net payoff
- E.g., tournament payoffs: only the highest g gets a prize P; each of the K >0 contestants draws gross payoff independently from a cumulative distribution G then the expected net payoff is $n(g) = PG(g)^K$ whose concavity increases with K (Figure 2)
- Other examples: decisions under shadow of bankruptcy with shortfalls passed to creditors (Figure 3): net payoff n(g) = g-z for g>z but n(g) = (1-a)(g-z) for g<z where 0<a<1 is the share of shortfall borne by others
- Bank bailouts create similar convex net payoffs which induce DM to choose riskier gambles

Figure 2: Tournament Payoff

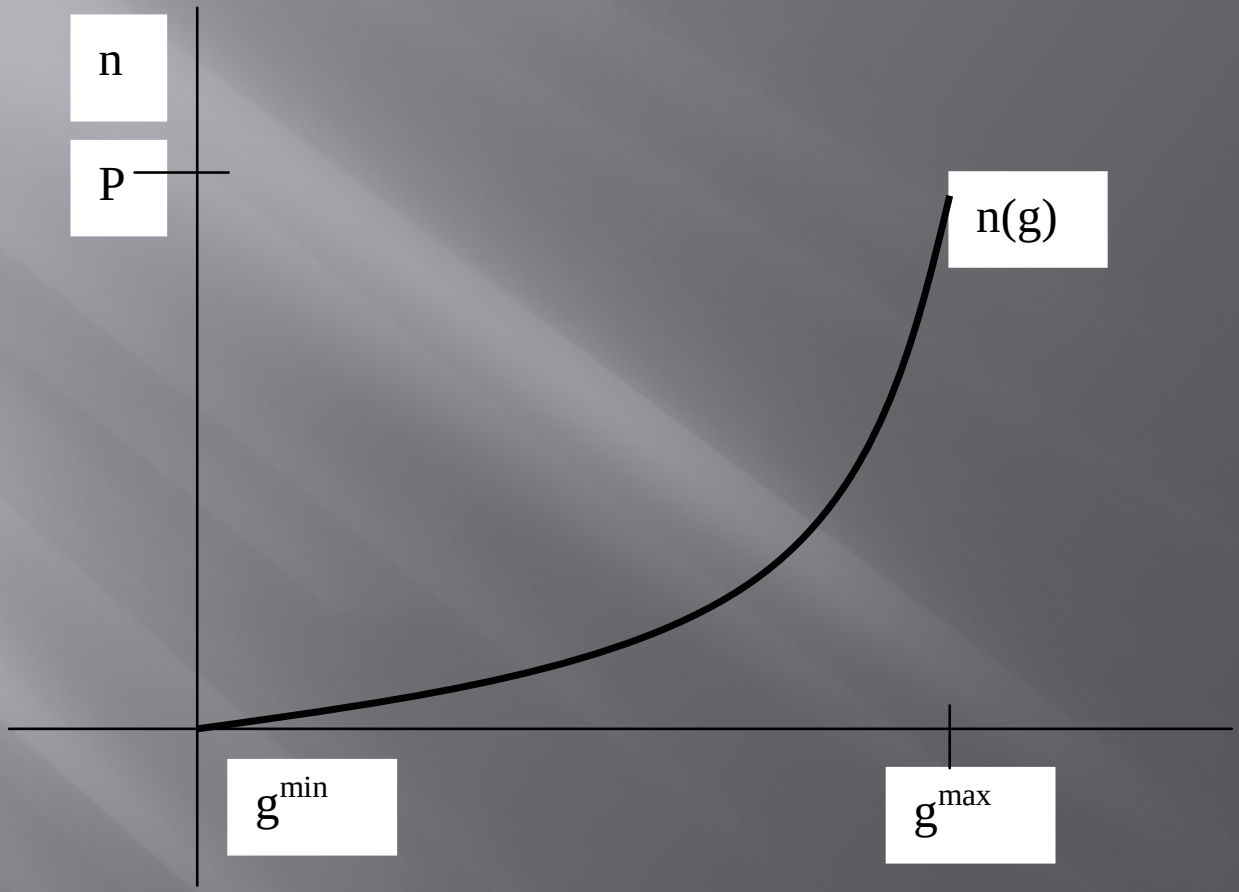
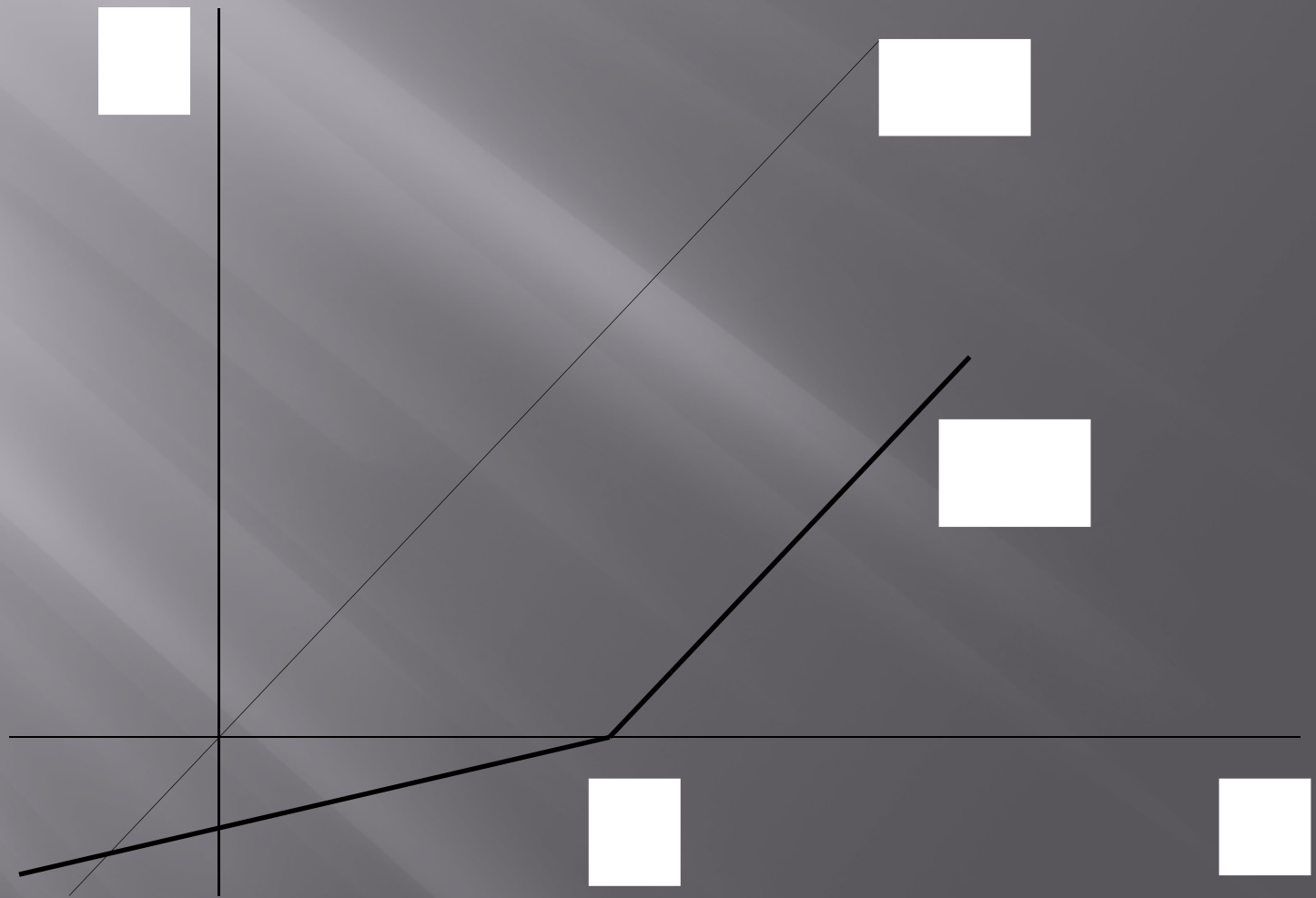


Figure 3: Convex Net Payoff



More Complex Net Payoffs

- Income below certain level gets housing subsidy and income above certain level makes the person ineligible for subsidy makes an outsider infer a Markowitz type utility function (Figure 4) when in fact it is linear with Markowitz type net payoff function
- Friedman-Savage story of upward mobility yields a similar inference about utility in presence of costs of private schools and other cash flows that might be associated with the move to a better neighborhood (Figure 5)
- Also see Masson (1972); assuming linear utility for net payoffs, induced utility in g is concave or strictly concave, Markowitz and Friedman-Savage type induced utility functions over gross payoffs

Figure 4: Net Payoff with Means-Tested Subsidy

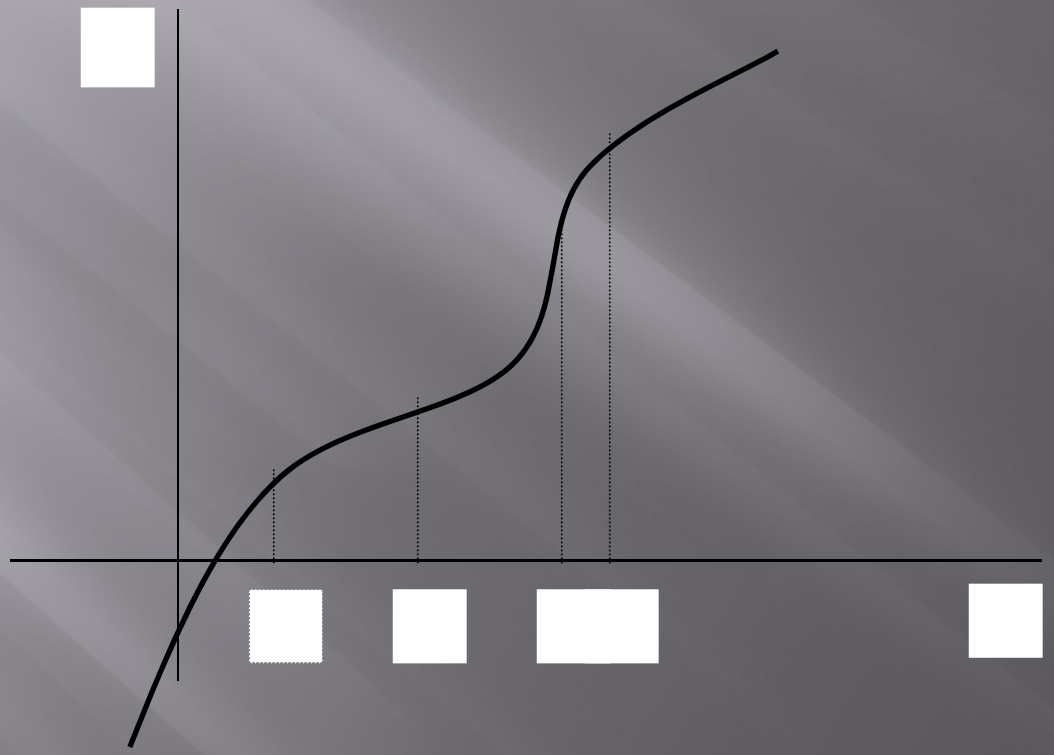
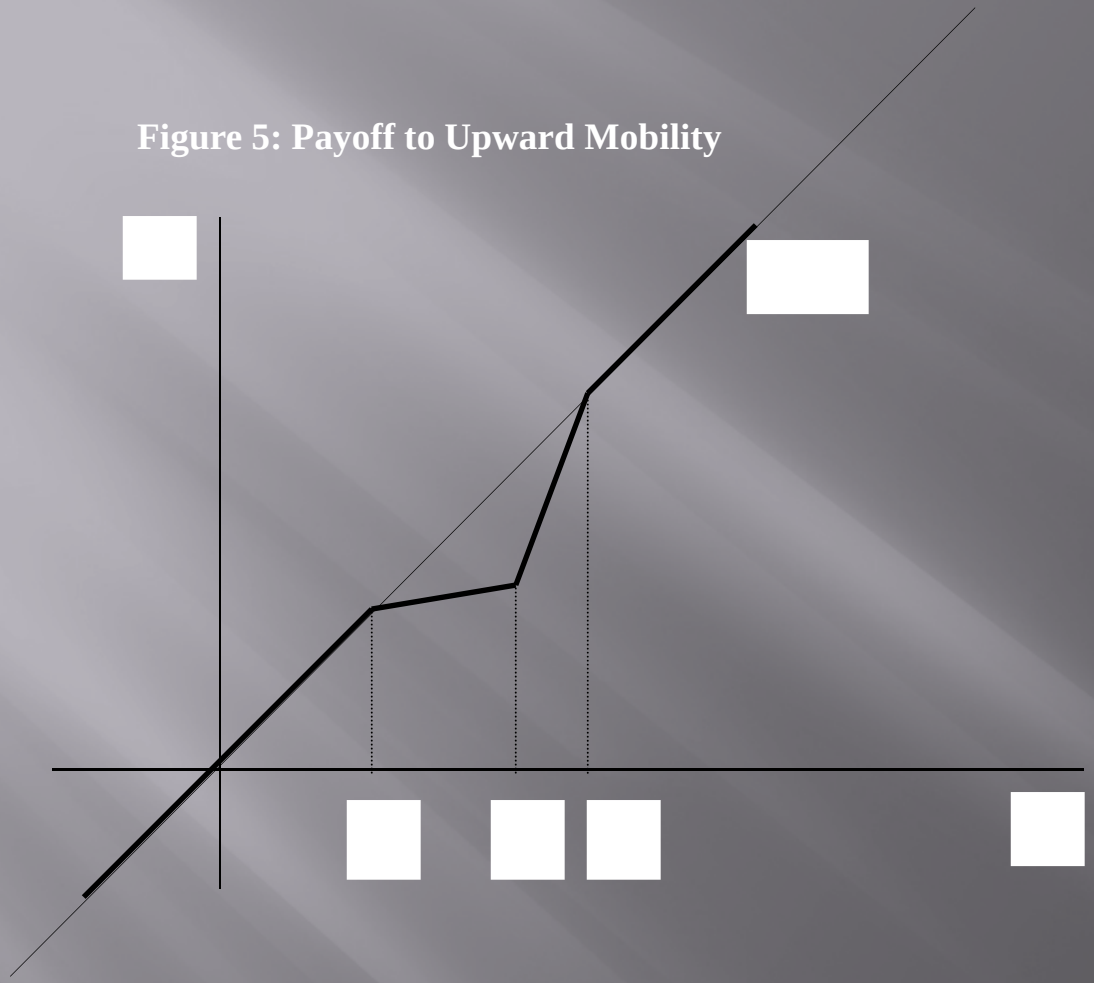


Figure 5: Payoff to Upward Mobility



Phlogiston Goes Up in Smoke

- Is non-linear utility phlogiston of economics (proposed by Gabriel Cramer and Daniel Bernoulli (1738)) entered mainstream with vNM in 1944
- It has not yet fulfilled its early promise: no stable consistent estimates of individual utility functions or explanations of macroeconomic phenomena, has not aged well.
- Lavoisier's radical oxidation/reduction theory took hold in the 1780s and was orthodox by 1800 (oxygen =antiphlogiston; or phlogiston = antioxygen).
- J. Priestly and friends never accepted it, but phlogiston died with them.
- A radical new theory of risky choice could yet emerge eventually (e.g., from neuroscience).

Has Behavioral Economics Provided a Better Alternative?

- Behavior economics might seem to be a promising source for a better alternative
- But it takes nonlinear utility as a given, and is preoccupied with finding specific non-linear functions that might fix the empirical inadequacies of the EUT.
- Therefore most behavior economics is subject to our critique and is included above; no better theoretical alternatives offered yet by behavioral economics
- KT (1979) prospect theory is central to behavioral economics (S-shaped function similar to Markowitz', convex below an inflection point z and concave above
- It has at least four free parameters and slope and shape parameters for upper and lower ranges of outcomes compounding the problems of inconsistency
- Yet it is inconsistent with insurance for losses
- Additional use of probability weighting and other theories (regret, rank-dependent, etc.) for ex post rationalization of any observations
- Yet to show powers of prediction based on observable explanatory variables

Way Forward

- Until something better comes along, it seems wiser and defensible to stick to what we can directly observe: analysis of opportunity sets of decision makers
  - Use simplest possible (linear) preferences; EV=EU
  - Careful examination of observable opportunity sets and how possible outcomes interact with future opportunities and past commitments
  - Insurance: examine observable commitments and embedded options
  - Gambling: potentially observable bailout options

Concluding Remarks

- What would it take to vindicate empirically non-linear Bernoulli functions?
  - Consistency at the individual level: not transient.
  - Predictability in new tasks and contexts
- What is the radical new approach?
  - Who knows? Not prospect theory…
- What to do in the meantime?
  - Conservative program: focus on observable opportunities in gross versus net payoffs.
  - Let's not base a theory of choice on unobservable concepts and untestable propositions
  - Fifty years of concentrated efforts to validate curved utility to explain risky choices has not worked
  - Try option theory instead of risk attitudes as the foundation

THANK YOU.

Shyam.sunder@yale.edu

www.som.yale.edu/faculty/sunder/research