Convexity of Valuation Accuracy Function: Empirical Evidence for the Canadian Economy*

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Abstract. We investigated the benefits (more accurate valuations) obtained from the use of more specific measures, such as the use of specific price indexes, rather than a general price index, to approximate the current value of historical cost accounting numbers. We found the valuation accuracy function for the Canadian economy to be highly convex; that is, marginal gains in accuracy decline sharply as the specificity of price indexes increases. A few broad price indexes yield a relatively accurate valuation of assets, and little accuracy is gained by using more specific price indexes. These results are consistent with empirical studies conducted in the United States (Hall 1982; Sunder and Waymire 1983; Shriver 1986) and Australia (Tippett 1987).

These results suggest that Canadian standard setters may be able to achieve their objective of getting relatively accurate valuations for industrial assets by adopting a simple and verifiable price index methodology, rather than resorting to costly and complicated computations such as those required by *Canadian Institute of Chartered Accountants (CICA) Handbook Section 4510.*

Résumé. Les auteurs étudient les avantages (évaluations plus exactes) de l'utilisation plus grande de mesures spécifiques – comme celles d'indices des prix spécifiques plutôt que d'un indice des prix général – pour obtenir la valeur actuelle approximative des données produites par la comptabilité au coût d'origine. Ils constatent que la fonction d'exactitude de l'évaluation pour l'économie canadienne présente une grande convexité, c'est-à-dire que les gains marginaux en exactitude déclinent abruptement lorsque la spécificité des indices de prix augmente. Quelques grands indices des prix livrent une évaluation relativement exacte de l'actif, et l'on gagne peu d'exactitude en utilisant des indices des prix plus spécifiques. Ces résultats sont conformes aux études empiriques réalisées aux États-Unis (Hall, 1982 ; Sunder et Waymire, 1983 ; Shriver, 1986) et en Australie (Tippett, 1987).

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Les résultats portent à croire que les responsables canadiens de la normalisation pourraient être en mesure d'atteindre leur objectif d'évaluations relativement exactes, dans le cas des actifs industriels, en adoptant une méthodologie d'indice des prix simple et vérifiable plutôt que de recourir à des calculs onéreux et complexes comme ceux qu'exige le chapitre 4510 du Manuel de l'I.C.C.A.

Accounting practitioners, researchers, users of accounting reports, standard setting bodies, tax authorities, and securities regulators have been concerned about the effects of changing prices on historical cost financial statements (Mattessich 1984; Lemke and Page 1992). The level of interest and concern about accounting for changing prices in both the academic and the practitioner communities rises and falls with the prevailing rate of price change. However, the time required to conduct academic research or develop generally acceptable accounting standards is so long that, by the time specific standards are developed or research studies published, the rate of price changes as well as interest in accounting for such changes often subsides.

From the mid-seventies to the early eighties, standard setters in Canada, the United States, and the United Kingdom conducted experiments to develop accounting standards for adjusting historical cost financial statements to incorporate the effects of changing prices. The results of these experiments indicate that standard setters in all three countries were unsuccessful in developing standards that incorporated accurate, verifiable, and easy to implement measurement methods (see, for example, Price Waterhouse's *Review and Perspective on Accounting Developments* 1992–93).

In the present paper, we investigate the accuracy of using price indexes to obtain a current valuation of assets in the Canadian economy. The use of price indexes to adjust the historical cost of assets is a relatively inexpensive and verifiable method of approximating the current value of firms' assets. How accurate this approximation is depends not only on how many and which price indexes are used, but also on the magnitude, variability, covariability, and errors in the price data used to construct the indexes.

A fundamental issue of concern to accounting theoreticians, standard setters, and practitioners is to understand how much cost and effort they should invest to obtain accurate measures for approximating the current value of historical cost accounting numbers. At one extreme, we could use a very coarse index (such as the producer price index or consumer price index) to adjust historical cost numbers. The use of such an index is inexpensive, simple, and easily verifiable. At the other extreme, we could hire an appraiser to determine the current value of each specific asset (or ask Statistics Canada to provide finer partitions of price indexes so that, in the extreme case, we could have a price index for each specific good in the economy). The use of an appraiser (or construction of very

fine indexes) may be very expensive, complex, and highly subjective (if an appraiser is used).

It is reasonable to expect that, as the measures used for adjusting historical cost numbers become more specific (finer), a more accurate valuation will be obtained. However, we do not know what the marginal gains are from increasing the specificity (fineness) of the measures used to adjust historical cost numbers. An understanding of the benefits (more accurate valuation) obtained from use of finer measures will provide some guidance regarding the value of investing additional resources to construct such measures. Insights from portfolio theory in finance suggest that the marginal gains in accuracy of valuation are likely to decline rapidly as more specific measures are developed.

Prior research in the United States (e.g., Sunder and Waymire 1983; Hall and Shriver 1990) and Australia (e.g., Tippett 1987) indicate that the structure of prices in the United States and Australia are such that even a small number of coarse (i.e., highly aggregated) price indexes can yield relatively accurate estimates of the current value of assets. In technical terms, this means that the accuracy function for U.S. and Australian prices is highly convex. A convex accuracy function means that marginal gains in accuracy decline as the specificity of price indexes increases. A few broad price indexes yield a relatively accurate valuation of assets, and little additional accuracy is obtained by using more specific price indexes. The practical implication of this finding is that investment of resources to construct finer measures (or hire an appraiser) is not necessary. In this paper, we analyze Canadian price data from the same high inflation period studied in the United States and report that these data, too, share the convexity property.

The convexity of the accuracy function for the Canadian economy should be of interest to standard setters, preparers, and users of accounting reports. If a relatively small number of price indexes can adjust historical costs to closely approximate the total current value of a firm's assets, specific price index adjustment methods become serious contenders against the more costly and less verifiable "direct" current valuation methods in which current price quotations for individual assets are used (such as *CICA* [*Canadian Institute of Chartered Accountants*] *Handbook Section 4510*). After summarizing the theoretical structure and the empirical evidence from the extant literature in the following section, we describe the Canadian data and estimate the accuracy function from this data. The paper concludes with a discussion of our results and their implications for accountants and standard setters.

Statistical Theory of Valuation

Valuation methods, such as historical cost, general price level, and current valuation, have traditionally been thought of as discrete, mutually exclu-

sive, and competing alternatives. Detailed arguments on the merits of various approaches for adjusting historical cost data have been provided by Edwards and Bell (1961), Chambers (1966), and Sterling (1970). In contrast to this discrete approach, Ijiri (1967, 1968) and Sunder (1978) have developed a statistical approach to valuation. In this statistical approach, methods of valuation constitute a broad spectrum, extending from historical cost accounting at one end to current value accounting at the other.

The statistical approach makes it possible to quantitatively compare the precision attained by various valuation rules in any specified environment. This reframing of the problem makes it possible to use a variety of intermediate valuation rules as well as the two extreme end points. Theoretical, as well as empirical studies within this new framework (e.g., Hall 1982; Sunder and Waymire 1983; Shriver 1986; Lim and Sunder 1990, 1991; and Hall and Shriver 1990), have generated important insights into, and data about, the statistical propêrties of inflation price data and valuation rules.

The statistical theory of valuation is based on the idea that the assets of a firm can be represented as portfolios of goods in the economy. How accurately (in the sense of smaller mean square error) the price index adjusted historical costs approximate the current value of collections of assets is a function of the mean vector and covariance matrix of price changes for goods in the economy. Sunder (1978) showed that the mean squared error (MSE) associated with a valuation rule can be given by:

$$MSE(R_{ki}) = \frac{1}{\rho}\left(\omega'(\sigma + \dot{\mu}) - \sum_{\mu=1}^{k} \frac{\omega_u'}{\omega_u' e} (\Sigma_{uu} + \mu_u \mu_u')\omega_u\right)$$

Where e = vector of unit elements of appropriate length;
$\omega = E(w)$; n – vector of relative weight of n assets in the economy, $\omega' e=1$;
$\mu = E(r)$, n – vector;
$\dot{\mu}$ = n – vector of squared elements of μ;
$\Sigma = E(r - \mu)(r - \mu)'$, n x n covariance matrix of relative price changes for n assets;
$\sigma = n$ – vector of diagonal elements of Σ;
k = number of price indexes used in the valuation rule. The set of n assets is partitioned into k nonempty subsets and a price index is constructed for each subset; ω_u, μ_u, and Σ_{uu} are the subvectors and submatrix, respectively corresponding to the u-th of the k subsets;
ρ = number of multinomial trials by which the bundle of assets for individual firms is randomly drawn from the economy-wide bundle defined by ω.

For each value of k, let L_k denote the number of distinct ways in which a set of n elements can be partitioned into k nonempty subsets. Let R_k* denote the most accurate of the L_k valuation rules (i.e., R_k* has the

smallest economy-wide average of mean squared error):

$$MSE(R_k*) \leq MSE(R_{ki}),\ i = 1,2,\ldots,L_k.$$

Let the partition of the n assets that corresponds to R_k* be denoted by Π_k*. Thus, for every value of k, there exists a partition Π_k* which yields the best accuracy (i.e., smallest mean squared error) through the valuation rule R_k*. Sunder and Waymire (1983) define $H(k)$ to be the accuracy function that gives the accuracy of the most accurate k–index valuation rule:

$$H(k) = MSE(R_k*),\ k = 1,2,\ldots n.$$

We present empirical estimates of $H(k)$ for the Canadian economy. Specifically, we are concerned with whether $H(k)$ is convex in k, as is the case for U.S. data.

Estimation of accuracy function $H(k)$

Two problems must be solved to estimate $H(k)$ from data. First, we need unbiased estimates of μ and Σ, the mean vector and covariance matrix, respectively, of relative price changes for the n assets in the economy. Second, we need to identify the index configuration with the best accuracy for each value of $k = 2, \ldots n - 1$. The total number of alternative k-partitions is very large, and thus, a search algorithm is required to obtain an approximation of the most accurate $H(k)$, given limited computational resources. In the present paper, we use an unbiased estimator and a systematic search algorithm from Shih and Sunder (1987) to deal with these two estimation concerns (see Appendix A).

The Data

We chose the Manufacturing Price Index, published by Statistics Canada, instead of the Consumer Price Index. This choice is based on the assumption that the assets included in constructing the Manufacturing Price Index are more representative of the assets held by business firms than the assets included in the Consumer Price Index.[1] The Canadian manufacturing price indexes have a two-level hierarchy: (1) an overall manufacturing price index (general price level index) and (2) 20 specific industry price indexes.

In order to estimate the mean and covariance matrix of relative price changes, we used monthly data from January 1977 to December 1982. The use of data from this time period has three advantages. First, empirical data were collected in the United States during this time period, and thus, we can make a direct comparison between Canadian and U.S. data. Second, the relatively high rate of price changes during the 1977 to 1982 time period (as opposed to the low rates of price changes being

experienced now) makes such data more relevant to understanding the effects of high rates of price changes. Third, the data are from a period when the interest in accounting for changing prices was at its peak in Canada and abroad. We can thus provide evidence of what would have happened if the CICA had chosen to use price indexes instead of the more complex procedures specified in *CICA Handbook Section 4510* (reporting the effects of changing prices). Complete time series for this period were available only for 16 of the 20 price indexes. We used these 16 indexes and renormalized the weights assigned to these 16 indexes to sum to 1 (see Table 1).

TABLE 1
Components of industry selling price indexes: Monthly data 1977–1982 (72 observations)

	Code	Description	Original Weights %	Normalized Weights %
1	D500001	Food & Beverage	19.016	23.6
2	D511200	Tobacco Products	1.084	1.4
3	D511500	Rubber & Plastics	2.417	3.0
4	D513400	Leather Industries	.840	1.0
5	D514500	Textiles	3.369	4.2
6	D516600	Knitting Mills	.846	1.0
7	D519100	Wood	4.515	5.6
8	D523200	Furniture	1.539	1.9
9	D524200	Paper	7.809	9.7
10	D527100	Primary Metals	7.970	9.9
11	D529400	Metal Fabrication	7.169	8.9
12	D532900	Machinery Industries	4.162	5.2
13	D537300	Electrical Products	6.470	8.0
14	D541400	Nonmetallic Industries	3.043	3.8
15	D544000	Petroleum & Coal	4.044	5.0
16	D545200	Chemical	6.270	7.8
		Total	80.563	100.0

All Components of the indexes for which monthly data were available for the period from January 1977 to December 1982 are included in the study. The original weights of these goods and their adjusted weights are shown in the table. The weights are determined by Statistics Canada based on surveys, questionnaires, and collection of data by other means at their disposal.

Empirical Estimates of the Accuracy Function

In our estimation of the accuracy function, we suppose that each firm in the economy consists of randomly drawn bundles of assets (multinomial distribution with parameter ω) of n = 16 assets in varying proportions. Each of these 16 assets represents the bundle of goods included in calcu-

lating the corresponding price index by Statistics Canada. To estimate the accuracy function $H(k)$, we need to find out for each value of k, which of the k-partitions have the smallest mean squared error, and the magnitude of that mean squared error.

For example, consider $k = 5$. We need to consider the many ways of partitioning 16 assets into 5 nonempty subsets and find out which of these partitions yields the smallest mean squared error. This is a combinatorial problem, and we cannot apply the usual methods of calculus to find the minima. In addition, the number of possible partitions is very large, and exhaustive enumeration is not feasible. We therefore resort to using heuristics based on the results of Shih and Sunder (1987) to search for a close approximation of the accuracy function. Using this method, we identified 15 partitions (and the corresponding valuation rules) for each value of k= 2, ... 15 (for both $k = 1$ and $k = 16$, the partition is unique). The findings are shown in Figures 1 and 2.

Figure 1 Mean and minimum MSE of 15 valuation rules for each value of k

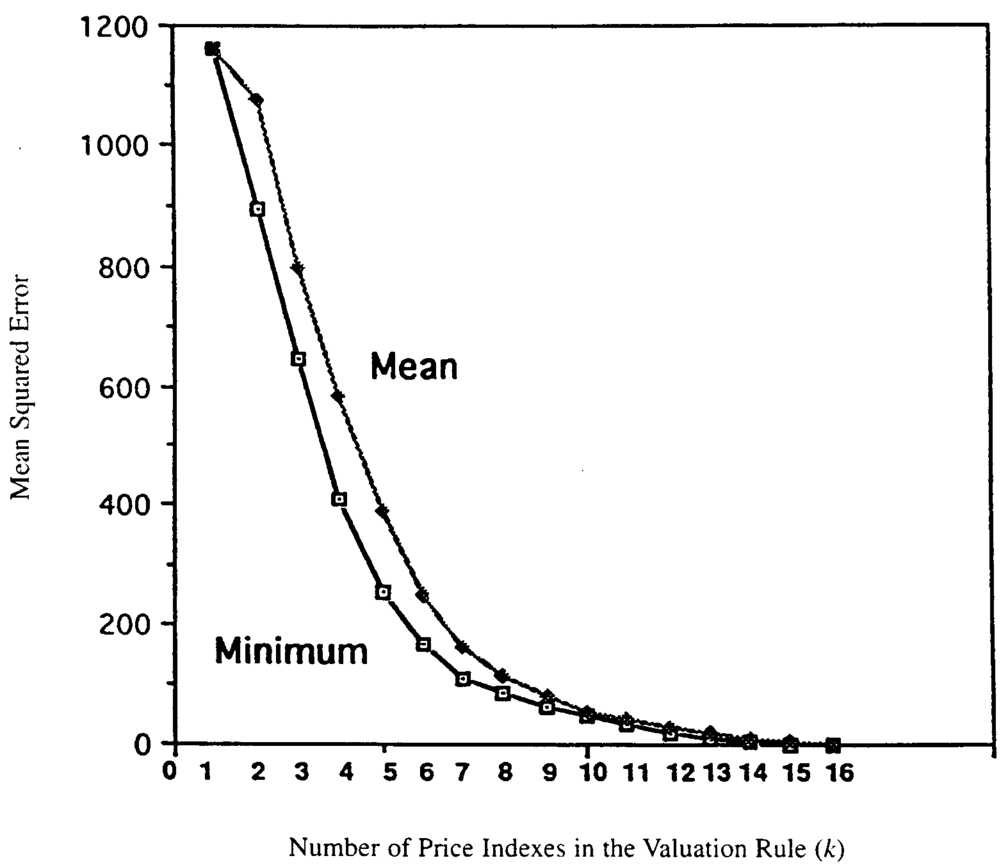
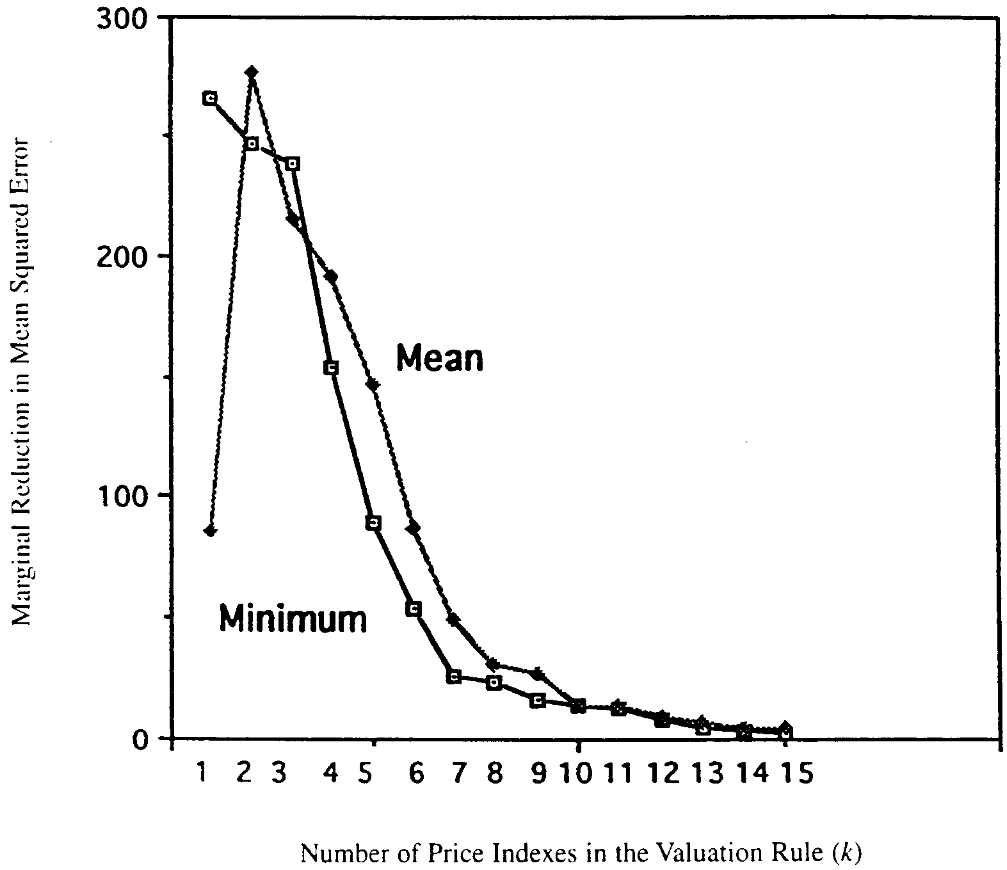


Figure 2 Marginal gains in accuracy of valuation rules for each value of k (average and minimum of 15 valuation rules)



For each value of $k = 1, 2, \ldots 16$, the gray line in Figure 1 plots the mean squared error associated with the k-index valuation rules averaged over the 15 different valuation rules. The minimum MSE over the 15 rules is shown by the dark line. The dark line can be thought of as an estimate of the valuation accuracy function for the Canadian economy. Because the accuracy function is defined as the global minimum and our estimate is the minimum of only the 15 valuation rules identified by our heuristic, we know that this estimated function is positively biased. Further experimentation with this algorithm (and more computer resources) may enable us to obtain an even better (lower) estimates of the accuracy function. Still, it is clear that the accuracy function is highly convex. Even the average of MSE for the 15 valuation rules (the gray line) is highly convex.

Figure 2 plots the marginal reduction in accuracy from Figure 1 as k increases. Convexity of the accuracy function requires the marginal curve to be monotonically decreasing. All except one marginal change is

decreasing in k. After the first four or five indexes, the marginal reduction in mean squared error becomes quite small.

Discussion

The empirical evidence presented earlier suggests that the accuracy function of the Canadian economy is highly convex. This implies that the marginal gain in accuracy of valuation declines sharply as the number and specificity of price indexes used for valuation increases. In Canada, as in the United States, use of a small number of price indexes to adjust historical costs will yield most of the benefits of current valuation in the sense of reducing the mean squared error of valuation. Addition of more and more indexes yield smaller and smaller additional benefits in the form of reduced mean squared error.

Due to the high inflation rates experienced in the 1960's and early 1970's, standard setting bodies in Canada, the United Kingdom, and the United States issued standards for supplementary disclosure of information on the effects of changing prices on financial statements. In Canada, the CICA issued *Handbook Section 4510* (reporting the effects of changing prices) in December 1982, requiring companies to disclose the effect of changing prices on inventory, property plant and equipment, a financing adjustment, and the gain or loss in general purchasing power resulting from holding net monetary items. *Section 4510* was introduced as an experiment, and companies were encouraged to volunteer to comply with its requirements. In the United States, companies were allowed, but not required, to use price indexes to estimate the current value of their assets.

The experiment was unsuccessful. Compliance rates with *Section 4510* were very low. In 1986 only four percent of eligible companies voluntarily complied with *Section 4510* (Hanna 1988). The CICA, thus, terminated the experiment, and *Section 4510* was withdrawn from the *CICA Handbook*. A sharp decrease in inflation rates during the experiment period has been suggested as one explanation for the failure of the experiment (Hanna 1988). This explanation is consistent with the decline in interest in accounting for changing prices when the rate of price changes is low.

A second explanation proposed by Price Waterhouse is that the standard was very complex, and the costs of preparing the data exceeded the benefits (Price Waterhouse, *Review and Perspective on Accounting Developments* 1992-93). This explanation suggests that devising effective accounting standards for changing prices requires that the standards should be simple, verifiable and should yield accurate valuation of assets.

The major result from the current study is that valuation rules based on a few broad price indexes yield a simple, verifiable, and relatively accurate valuation of assets. There appears to be relatively little gain from obtaining more specific asset valuations. This is an important finding because it suggests that simple adjustments to historical cost data will

result in a relatively accurate valuation of assets, and thus, complicated computations, such as those required by *CICA Handbook Section 4510*, may not be worth the additional effort required. Our results indicate that the structure of price changes in Canada is sufficiently similar to the United States and Australia for the key propositions and findings of the statistical approach to valuation to hold for the Canadian economy.

The analysis conducted in this paper has focused on errors of valuation rules that arise from price changes over time. Lim and Sunder (1991) label this source of errors as "price movement errors." Lim and Sunder, however, have identified and modeled a second source of valuation errors that arise from errors of measurement in price data used to construct price indexes (measurement error). Lim and Sunder (1991) then conducted a simulation to show that when both sources of error are taken into account, the number of price indexes needed to construct an efficient valuation rule decreases even further. This suggests that the conclusions of this paper would be strengthened if we could obtain data on measurement errors. Because there is no known source of Canadian data on price measurement errors, such analysis has not been undertaken. However, the theory suggests that such additional empirical data, whenever it becomes available, will further strengthen our results.

The results suggest that, in the future, Canadian standard setters should allow companies to adjust historical cost financial statements using highly aggregated price indexes. These adjustments will yield a relatively accurate, inexpensive, and verifiable method of approximating the current value of a firms assets. More complicated, costly, or subjective adjustment methods should be mandated only if the standard setters can demonstrate that there are large measurement errors in constructing price indexes for certain classes of assets in the Canadian economy.

Appendix A

Unbiased Estimator

The unbiased estimator of mean squared error constructed by Sunder and Waymire (1984, 403) is:

$$A(\hat{R}_{ki}) = \omega'((\frac{T-1}{T})\hat{\sigma}+\hat{\mu}) - \sum_{\mu=1}^{k} \frac{\omega'\mu}{\omega'\mu e}(\frac{T-1}{T})\hat{\Sigma}\mu\mu+\hat{\mu}_u\hat{\mu}_u)\omega_u$$

where T represents the number of relative price change observations used to estimate $\hat{\mu}$ and $\hat{\Sigma}$.

Search Algorithm

(1) Collect time series of price data from Statistics Canada Database that have complete monthly data over the six-year period from January 1977 to December 1982. This resulted in selection of data from 16 industries.

(2) Compute time series of relative price changes, $r = (P^1 - P^0) / P^0$.
(3) Estimate mean $(\hat{\mu})$ and variance-covariance matrix $(\hat{\Sigma})$ of each time series of relative price changes.
(4) Obtain the relative weight vector ω for the Statistics Canada price index data base and renormalize relative weights to the sum of weights for the time series to 1.
(5) Prepare an n x n matrix E, where

$$e_{ij} = \omega_i\omega_j(\hat{\sigma}_{ii} + \hat{\sigma}_{jj} - 2\hat{\sigma}_{ij} + \hat{\mu}_i^2 + \hat{\mu}_j^2 - 2\hat{\mu}_i\hat{\mu}_j).$$

(6) To construct a more accurate k-index system, find a set J of (n - k + 1) goods for which

$$\frac{\sum_{i,j\epsilon J}\sum_{i\neq j} e_{ij}}{\sum_{i\epsilon j} \omega_i} \text{ is small.}$$

Shih and Sunder (1987) present theorems (1) to (4), which formalize the intuition that accurate valuation rules require placing assets with large variances, very large or very small means, large weights, and small covariances each in a single good index. These four criteria were applied heuristically to generate 15 relatively accurate valuation rules for each value of K.

Endnotes

1 Even if we had used the Consumer Price Index data in this study, we doubt if our results would have been significantly different. However, a definite answer to this question can be provided only by an empirical analysis of that data.

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