# The Accounting Review

VOL. LI

JANUARY 1976

No. 1

# Properties of Accounting Numbers Under Full Costing and Successful-Efforts Costing in the Petroleum Industry

# Shyam Sunder

#### Introduction

NINANCIAL reporting and accounting practices used in the petroleum industry differ both among firms within the industry and also from practices of other industries in several respects. One area of difference is accounting for prediscovery costs. Because such costs are relatively large and because a large degree of uncertainty is associated with the potential benefits sought by incurrence of such costs, this area has provoked many practices; most can be grouped either as successful-efforts costing (SEC) or fullcosting (FC) practices. The practice of capitalizing only those prediscovery costs which are directly identifiable with discovery of a commercial reserve and treating all other costs as operating expense is referred to as successful-efforts costing. On the other hand, the practice of capitalizing all prediscovery costs irrespective of their result is called the full-costing method.

There has been a substantial amount of discussion and debate over the relative propriety of these two methods of accounting.<sup>2</sup> This study is an attempt to

I am grateful for the comments I have received on an earlier draft of this paper from George Foster, Nicholas Gonedes and other participants in the Accounting Workshop at the University of Chicago. Financial support for the research was provided by the National Science Foundation.

This paper was one of the winning manuscripts in the 1975 American Accounting Association Manuscript Contest.

Other major areas identified by the Accounting Principles Board are (1) Determination of the "cost center," (2) accounting for past discovery costs, (3) disposition of capitalized costs and (4) disclosure of supplementary information in financial reports. The effect of the cost center decision on various accounting numbers is the subject matter of another study by the author: "Properties of Accounting Numbers Under Various Definitions of Cost Centers in the Petroleum Exploration Industry," in Proceedings of the Southwestern Regional Meeting of the American Accounting Association, Kenneth E. Most, editor.

<sup>2</sup> Probably the best available source of position

Shyam Sunder is Assistant Professor of Accounting at the Graduate School of Business, University of Chicago.

analyze the effect of using the alternative methods (SEC and FC) on various accounting variables. It is not proposed to make a judgment as to the relative desirability of one policy over the other. An analysis of the nature of accounting numbers produced by the alternative methods, it is hoped, will help those who must choose between the two as well as those who use the numbers produced by such accounting systems. A brief background discussion of accounting practices in the petroleum industry is given in the next few paragraphs. The second section of the paper presents the analysis of accounting policies for a steady-state (unchanging) firm. This analysis is extended to the case of changing firms in the third section. Summary and conclusions are given in the fourth section.

Prediscovery costs include the costs of geological and geophysical exploration; property acquisition and carrying costs; and exploratory drilling costs. Some accountants recommend capitalizing all prediscovery costs; others would relate parts of such costs to successful and unsuccessful parts of the exploration activities to varying degrees. In the next section of this paper, a model is developed to analyze the effects of these two different accounting practices—full costing (FC) and successful-efforts costing (SEC) on various accounting variables such as income, cash flow, capitalized assets and return on assets, etc. The APB Committee on Extractive Industries, which was established to determine appropriate practices and to narrow the accounting differences in the industry, favored a position essentially close to the policy of capitalizing only those prediscovery costs which result directly in the discovery of commercial reserves and expensing the rest as incurred [APB Public Hearings, p. 3]. On the other hand, a research study commissioned by the Canadian Institute of Chartered Accountants, and conducted by W. B. Coutts, favored the full-capitalization policy [Coutts, 1963, p. 25]. Many companies use variations of these two practices.

### STEADY-STATE FIRM

Consider a firm which drills N exploratory wells each period. The probability of a successful strike  $(\theta)$  is unchanged over the years.<sup>3</sup> The nonrecoverable cost of each well is c. Each successful well yields a net operating revenue of x per period for L periods starting the period after the drilling takes place.

 $S_t$ , the number of successful wells drilled in period t, is a random variable with binomial distribution and parameters  $\theta$ and N. The probability that  $S_t$  is equal to an integer r between 0 and N is given by

$$\Pr\left(S_{t} = r \middle| \theta, N\right) = \frac{N!}{(N-r)!r!} \theta^{r} (1-\theta)^{N-r}. \tag{1}$$

The expected value and variance of  $S_i$  are  $N\theta$  and  $N\theta(1-\theta)$ , respectively. If a sufficiently large number of wells are drilled each period, this distribution can be approximated by a normal distribution of the same mean and variance.

Revenue generated by drilling efforts in period t is also a random variable. If  $S_t$  wells are successful, they will yield net operating revenues of  $S_t x$  for L periods beginning period t+1. At the end of period t

papers and discussions on this topic is APB Public Hearings on Accounting and Reporting Practices in the Petroleum Industry, Cases in Public Accounting Practice, Vol. 10 (Arthur Andersen and Co., 1972). For an analysis of various accounting issues in the petroleum industry, see W. B. Coutts, Accounting Problems in the Oil and Gas Industry (The Canadian Institute of Chartered Accountants, 1963) and Robert E. Fields, Financial Reporting in the Extractive Industries, Accounting Research Study No. 11 (American Institute of Certified Public Accountants, 1969).

<sup>&</sup>lt;sup>2</sup> In the continental United States, one in eleven new field wildcats results in the discovery of oil on average. The rate of commercially exploitable discovery for new field wildcats is about one in thirty.

when drilling is completed and the realized value of  $S_t$  becomes known, revenue for the next L periods generated by this period's exploration also becomes known with certainty. Therefore, the net cash flow in period t,  $X_t$  is

$$X_{i} = -Nc + x(S_{i-1} + S_{i-2} + \cdots + S_{i-L}).$$
 (2)

The net cash flows  $X_i$  from this operation are correlated serially. The serial correlation coefficient is

$$\rho_{(\boldsymbol{x}_{t}|\boldsymbol{x}_{t+1})} = \frac{L-1}{L}, \qquad (3)$$

and it increases monotonically from zero to one as L, the lifetime of a productive well, increases from one to infinity.

The mean of the net cash flow  $\tau$  years into the future is

$$\mathcal{E}(X_{t} | \overline{S}_{t-r-1})$$

$$= \begin{cases} -Nc + x(S_{t-r-1} + \cdots + S_{t-L}) \\ +x\tau N\theta & \text{for } \tau < L \\ -Nc + xLN\theta & \text{for } \tau \ge L, \end{cases}$$

$$(4)$$

and the variance of net cash flows  $\tau$  years into the future is

$$\operatorname{Var}(X_{t}; \overline{S}_{t-\tau-1}) = \begin{cases} x^{2\tau} N \theta (1-\theta) & \text{for } \tau < L \\ x^{2} L N \theta (1-\theta) & \text{for } \tau \geq L. \end{cases}$$

Next year's cash flow is perfectly predictable because it depends on known results of exploration. As the prediction interval is increased to L, the variance of prediction increases linearly and then levels off. It is noteworthy that the variance increases only linearly with the scale of operations—the number of exploratory wells drilled (N). Therefore, increasing the value of N is analogous to adding uncorrelated assets to the portfolio which results in a decrease in the risk of the entire portfolio through diversification.

TABLE 1 NOTATION

- A = capitalized value of prediscovery costs
- B = total assets .1
- $c = \cos t$  of exploration per exploratory well
- D = long-term debt
- E = owners' equity
- E' = owners' equity if all exploration costs are expensed as incurred
- G = debt-equity ratio
- I = accounting income in a given period. Superscripts F and S refer to the full-costing and successfulefforts methods of accounting
- L = average lifetime of a successful well
- N = number of exploratory wells drilled by the firm in a given period
- R = return on assets
- S = number of successful exploratory wells drilled in a given period
- $\vec{S}_t$  = results of exploration up to and including period t
- t= time subscript
- Var = variance operator

  X = total cash flow in a given period
  - x = net operating cash flow per successful exploratory well per period for L periods
- $\mathcal{E}$  = expected value operator
- ρ<sub>u.v</sub>=product moment correlation between variables *u* and *v*
- θ=probability that an exploratory well will result in discovery of exploitable reserves

The effect of the probability of successful strike  $(\theta)$  on variance of the net cash flows is also interesting.  $\theta(1-\theta)$  is a symmetric, inverted-bowl-shaped function of  $\theta$  with a minimum value zero when  $\theta$  is zero or one and a maximum value of 0.25 when  $\theta$  is 0.5. Thus, the variance is maximum when the probability of successful strike is 0.5. For realistic values of  $\theta$  between 0 and 0.5, the rate of change of variance of cash flows with  $\theta$  is positive

$$\frac{d \operatorname{Var} (X_i | \overline{S}_{i-r-1})}{d\theta} = x^2 L N(1 - 2\theta) > 0$$
(6)
for  $\theta < 0.5$ .

Therefore, an increase in the probability of successful strike results in an increase in the variance of  $X_i$ .

The effect of well life, L, on variance deserves a close look. If all other parameters were unchanged, it would appear from (5b) that the variance increases linearly with L. However, other parameters are not

unchanged when L is altered. Well life can be prolonged only by reducing the rate of production of crude roughly in the same proportion. Therefore, net operating cash flow x is inversely proportional to L, and since term  $x^2$  appears in (5), the net effect of prolonging the life of discovered reserves is to reduce proportionately the variance. Since the product of x and L is constant, let  $x \cdot L = k$  and rewrite (5b) as

$$\operatorname{Var}\left(X_{\iota}\middle|\overline{S}_{\iota^{+},-1}\right) = \frac{k^{2}}{L}\operatorname{N}\theta(1-\theta) \quad \text{for } \tau \geq L.$$

Then

$$\frac{d \operatorname{Var}}{dL} (X_{\iota} | \overline{S}_{\iota-r-1}) = -\frac{k^2}{L^2} N\theta (1-\theta) < 0.$$

The accounting variations under study here—treatment of prediscovery costs—do not affect the net cash flows. Therefore, the properties of net cash flows given above are independent of the method of accounting used. However, other accounting variables are affected by such variations, and the effect of accounting policies on income, capitalized assets and rate of return on assets, etc., is examined next.

# Successful-Efforts Costing (SEC)

Successful-efforts costing is the practice of capitalizing only those prediscovery costs which can be related directly to revenue produced from the wells on which such costs were incurred. The remaining costs are expensed as incurred. Of the total exploration outlay,  $cN_i$  in period l, an amount  $c \cdot S_i$  is capitalized as the cost of successful operations. The remaining cost  $(N-S_i)c$ , attributable to the dry holes is expensed as incurred. Capitalized assets are amortized in the following L years at a uniform rate of  $c \cdot S_i/L$  per year. Under this method of accounting, the firm's income for period l,  $l_i^s$  is:

$$I_{i}^{S} = -\text{ amortization charge} \\ -\text{ expired dryhole costs} \\ +\text{ net production cash flow}$$
(7)
$$= \left(x - \frac{c}{L}\right) \left(\sum_{j=i-L}^{i-1} S_{j}\right) - c(N - S_{i}).$$

Since the first component of income series in (7) is a moving average, this series also is correlated serially. Average income  $\tau$  periods into the future is

$$\mathcal{E}(I_{i}^{S}|\overline{S}_{i-\tau-1}) = \begin{cases}
\left(x - \frac{4c}{L}\right) \sum_{i=i-L}^{t-\tau-1} S_{i} + \tau N\theta - cN(1-\theta) \\
& \text{for } \tau \leq L \\
-Nc + xLN\theta & \text{for } \tau \geq L.
\end{cases} \tag{8}$$

Note that for L or more years into the future, the expected net cash flows are exactly equal to the expected income. The result follows from the steady-state assumption of the model. Later in this paper, this assumption is relaxed to obtain results for new, expanding and shrinking firms.

The variance of income  $\tau$  years into the future is

$$\operatorname{Var}\left(I_{t}^{S} \middle| \overline{S}_{t-\tau-1}\right)$$

$$= \begin{cases} N\theta(1-\theta) \left\{c^{2} + \left(x - \frac{c}{L}\right)^{2} \tau\right\} \\ & \text{for } \tau < L \end{cases} \tag{9}$$

$$N\theta(1-\theta) \left\{c^{2} + \left(x - \frac{c}{L}\right)^{2} L\right\} \\ & \text{for } \tau \geq L.$$

'The basis and the rate of amortization of capitalized costs is another aspect of petroleum accounting currently under investigation. I do not go into details of this issue and assume a constant rate of amortization over a fixed period of time (L) in analyzing the effect of the accounting policies under consideration.

An implicit assumption is that the life time expected revenue from an exploratory well, xL\theta exceeds its cost, c. If this condition does not hold, the operation will have a negative expected income and cash flow. Under certain conditions, peculiar risk characteristics may justify the retention of negative expected return assets in an investor's portfolio. But in the general case being considered here without reference to specific portfolios, such ex ante "ruinous" operations are ruled out from consideration in this analysis.

As is the case for the cash flows in (5), the variance of SEC income increases from zero as the prediction interval increases from 1 to L and then remains constant for further increases in the prediction interval. The effect of prolonging the lifetime of a well by slower production on variance of successful-efforts income can be seen by rewriting (9b) as

$$N\theta(1-\theta)\left\{c^2+\frac{1}{L}(xL-c)^2\right\}.$$

When the production rate is cut to increase L, xL remains unchanged. The effect of a production cut is to reduce the variance because L appears in the denominator of the second term in the brackets which is positive for a nonruinous operation.

The reduction in the variance of the income stream due to slower exploitation of discovered resources arises from stretching out the risks and benefits of exploration over a longer period of time through a smaller amortization charge. Besides any technical and other reasons which might justify an appropriate rate of production, a conceivable corporate goal of minimizing the variance of the income stream will tend to favor slower production rates over large proven reserves of petroleum. However, it may be noted that the only uncertainty explicitly accounted for in this model is with respect to the discovery of reserves, and the use of one accounting method versus another does not affect it in any way.

The variance of income increases only linearly with the scale of operations N. The coefficient of variation of income, a measure of risk, decreases as the operation grows in size. This, of course, is the familiar result of diversification in a portfolio of assets with independently distributed returns.

Full-Cost Accounting

I shall now examine the properties of income series under the full-costing method of accounting and compare the results with the successful-efforts method. The full-cost method refers to the practice of capitalizing the cost of all exploratory efforts and amortizing the costs over the discovered reserves on a pro-rata basis. For a steady-state company, the amortization charge will equal the exploration costs incurred in any given year. Therefore, the full cost income in year t is

$$I_{i}^{F} = -N \cdot c + x(S_{i-1} + \cdots + S_{i-2}). \quad (10)$$

Comparison with expression (2) reveals that a steady-state firm's full-cost income series is identical to its cash flow series. Accordingly, the expectation, variance and serial correlation coefficient of FC income series are given by

$$\mathcal{E}(I_{t}^{F}|\overline{S}_{t-\tau-1}) \\
= \begin{cases}
-Nc + x(\tau N\theta + S_{t-\tau-1} + \cdots + S_{t-L}) \\
\text{for } \tau < L \end{cases} \\
-Nc + xLN\theta \quad \text{for } \tau \ge L$$

$$\text{Var } (I_{t}^{F}|\overline{S}_{t-\tau-1}) \\
= \begin{cases}
x^{2}\tau N\theta(1-\theta) & \text{for } \tau < L \\
x^{2}LN\theta(1-\theta) & \text{for } \tau \ge L
\end{cases} \\
\rho_{(I_{t-1}^{F},I_{t-1}^{F})} = \frac{L-1}{L}$$
(13)

Comparison between FC and SEC Income Series

From expressions (7) and (10), it is seen that the difference between FC and SEC income is

$$I_i^F - I_i^S = \frac{c}{L} \left( \sum_{i=i-L}^{t-1} S_i \right) - cS_i.$$
 (14)

Note that we are considering the changes in variance arising solely out of a single source of uncertainty—the results of exploratory operations. Therefore, such statements are conditional upon perfect knowledge of other parameters involved. The difference, of course, arises from the delay in expensing the costs incurred on dry holes when the full-costing method is used. The expected value of this difference  $\tau$  years into the future is

$$\mathcal{E}(I_{i}^{F} - I_{i}^{S}) = \mathcal{E}I_{i}^{F} - \mathcal{E}I_{i}^{S}$$

$$= \left\{ \frac{c}{L} \left[ \sum_{i=t-L}^{t-\tau-1} S_{i} + \tau N\theta \right] - cN\theta \text{ for } \tau < L \right. (15)$$

$$= \begin{cases} 0 & \text{for } \tau > I \end{cases}$$

The long-run expectation of FC and SEC income is the same. In the shorter term (< L) the two expectations differ depending on the results of the recent exploration efforts. Full-costing income is expected to be higher (lower) if the recent exploratory efforts have yielded better (worse) than average results. For average results, the two terms in expression (15a) cancel out, and expectations of both accounting series are the same.

The variance of the full-cost income series  $\tau$  periods into the future is given in expression (12). The uncertainty about the results of exploration activities enters the FC income only in the form of the operating cash flows from an uncertain number of successful wells. On the other hand, the variance of SEC income (see expression (9)) includes the additional uncertainty of the dry hole costs which are expensed as incurred. Additional variance of the successful-efforts income stream can be seen clearly by rewriting expression (9a) as

$$Var (I_{\iota}^{S}|\overline{S}_{\iota-\tau-1}) = Var (I_{\iota}^{F}|\overline{S}_{\iota-\tau-1}) + N\theta(1-\theta) \left\{c^{2} + \frac{c^{2}\tau}{L^{2}} - 2\frac{cx\tau}{L}\right\}.$$

$$(16)$$

Therefore, the difference in income variance under the two accounting methods is

$$\operatorname{Var}\left(I_{t}^{S}|\overline{S}_{t-\tau-1}\right) - \operatorname{Var}\left(I_{t}^{F}|\overline{S}_{t-\tau-1}\right) = cN\theta(1-\theta)\left\{c\left(\frac{L^{2}+\tau}{L^{2}}\right) - 2xL\frac{\tau}{L^{2}}\right\}. \tag{17}$$

Variance of the SEC income is greater if

the term on the right-hand side is positive, that is,

$$\frac{xL}{c} < \frac{L^2}{2\tau} + 0.5. \tag{18}$$

Since xL is the lifetime net operating cash flow from a successful well and c is the cost per exploration well, the ratio xL/c must be no less than  $1/\theta$  in order that the results of the operation pay off at least the costs of exploration and production on average. Therefore, the ratio  $((L^2/2\tau)+0.5)$  would have to be greater than  $1/\theta$ . For variance of the next year's income  $(\tau=1)$ , L must be at least  $\sqrt{2(xL/c)-1}$  (about 5 for  $\theta=0.09$ ) for the SEC income to have a greater variance than the FC income. For predictive variance after L years or more  $(\tau=L)$ , L would have to be at least

$$\sqrt{\frac{2xL}{c}-1} \quad (\simeq 21 \text{ for } \theta = 0.09)$$

before SEC variance can be higher. (See Appendix I.)

In summary, the relationship between the variance of accounting income under the two methods depends on the relative profitability of operation, the time taken to deplete the discovered reserves and the prediction intervals. An increase in the relative profitability of operations increases the FC variance relative to the SEC variance. On the other hand, an increase in L, the time span of exploitation, results in an increase in the variance of the SEC income. For a sufficiently high value of L, SEC income variance always will exceed the FC income variance. (See Appendix I for the proof.)

The serial correlation coefficient of the FC income series is always higher than for the SEC series. It can be seen by rewriting (7) as

$$\rho(I_{i'}^{SS}) = \rho = (I_{i'}^{F}I_{i-1})/(Lp+1)$$
where  $p = \frac{c^2}{(\pi L - c)^2}$  (19)

and the denominator is always greater than 1.

### Correlation of Income with Cash Flow

For a steady-state firm, FC income is identical to the cash flow as can be confirmed readily by comparing expressions (2) and (10). This identity between income and cash flow is probably the main appeal of the full- cost method. The identity of cash flow and income also implies perfect correlation. Therefore,

Cov 
$$(X_i, I_i^F; \overline{S}_{i-\tau-1})$$

$$= \begin{cases} x^2 \tau N \theta (1-\theta) & \text{for } \tau < L \\ x^2 L N \theta (1-\theta) & \text{for } \tau \ge L \end{cases}$$
(20)

and

$$\rho_{X_t,I_t}^F = 1$$
 for all values of  $\tau$ . (21)

Covariance of cash flow and successfulefforts income is

$$\operatorname{Cov}\left(X_{t}, I_{t}^{S} \middle| \overline{S}_{t-\tau-1}\right) = \begin{cases} x\left(x - \frac{c}{L}\right) \tau N\theta(1-\theta) & \text{for } \tau < L \end{cases}$$

$$= \begin{cases} x\left(x - \frac{c}{L}\right) \tau N\theta(1-\theta) & \text{for } \tau < L \end{cases}$$

$$= \begin{cases} x\left(x - \frac{c}{L}\right) \tau N\theta(1-\theta) & \text{for } \tau < L \end{cases}$$

$$= \begin{cases} x\left(x - \frac{c}{L}\right) \tau N\theta(1-\theta) & \text{for } \tau < L \end{cases}$$

$$= \begin{cases} x\left(x - \frac{c}{L}\right)\tau N\theta(1-\theta) & \text{for } \tau < L \\ x\left(x - \frac{c}{L}\right)LN\theta(1-\theta) & \text{for } \tau \ge L. \end{cases}$$

A comparison between (20) and (22) indicates that the covariance of cash flow with the successful-efforts income is always less than its covariance with the fullcost income. Similarly, the correlation coefficient is:

$$\rho_{X_{i},I_{i}}^{S} = \begin{cases}
\frac{1}{\sqrt{1 + \frac{L^{2}/\tau}{\left(\frac{xL}{c} - 1\right)^{2}}}} < 1 \\
& \text{for } \tau < L \\
\frac{1}{\sqrt{1 + \frac{L}{\left(\frac{xL}{c} - 1\right)^{2}}}} < 1
\end{cases}$$

$$\text{for } \tau \ge L$$

Therefore, correlation between cash flow and successful-efforts income is always positive, but less than one. As prediction interval  $\tau$  increases to L, the correlation steadily approaches its limiting value given by expression (23b). As the profitability ratio xL/c increases, the value of correlation also increases.

Also note that the above expressions and statements about the covariance and correlation between successful-efforts income and cash flow also apply to the covariance and correlation between the SEC and the FC income because the cash flow is identical to the FC income.

# Current Income as a Predictor of Future Cash Flow

The Study Group on Objectives of Financial Statements stated in its report that one of the objectives of the financial statements is to enable the users to predict the cash flows (cash consequences of decisions). In this section, I compare the performance of current income as a predictor of future cash flows. Analysis in the preceding section indicated that FC income gives more information about (has higher correlation with) the contemporaneous cash flow than does the successful-efforts income. Whether such superiority holds for the future predictions is examined next.

Ability of current income numbers  $I_i^F$ and  $I_i^s$  to predict the future cash flow  $X_{i+r}$  can be evaluated by comparing the covariances, correlation coefficients and the mean squared error. Covariances are given by

$$\operatorname{Cov}(I_{\iota}^{F}, X_{\iota+\tau}) = \begin{cases} x^{2}N\theta(1-\theta)(L-\tau) & \text{for } \tau < L \\ 0 & \text{for } \tau \geq L \end{cases}$$

$$\operatorname{Cov}(I_{\iota}^{S}, X_{\iota+\tau}) = \begin{cases} x^{2}N\theta(1-\theta)(L-\tau) + N\theta(1-\theta)xc\tau \\ & \text{for } \tau \leq L \\ 0 & \text{for } \tau \geq L. \end{cases}$$

$$(24)$$

The covariance of future cash flows with the current SEC income is clearly higher than with the current FC income. The difference between the two covariances is reduced to zero as the prediction interval  $\tau$ approaches L, the time span of exploita-

Correlation coefficients between cash flow and the two income series are given by

$$\rho_{T_{i}.X_{i+\tau}}^{F} = \begin{cases} (L-\tau)/L & \text{for } \tau < L \\ 0 & \text{for } \tau \ge L. \end{cases}$$

$$\rho_{T_{i}.X_{i+\tau}}^{S} = \begin{cases} \frac{L-\tau}{L} + \frac{c\tau}{xL^{2}} \\ \sqrt{\left(\frac{xL-c}{xL}\right)^{2} + \frac{c^{2}L}{x^{2}L^{2}}} \\ & \text{for } \tau < L \end{cases}$$

$$0 & \text{for } \tau \ge L.$$
(26)

For L < (2xL/c) - 1, the correlation between cash flow and lagged successfulefforts income is higher than the correlation between cash flow and full-cost income with a similar lag.

Finally, the mean squared error of prediction when current income is used as a predictor of future cash flow is given by

$$\operatorname{Var}(d_{\tau}^{P}) = \operatorname{Var}(I_{\tau}^{P} - X_{t+\tau}) \qquad \qquad \mathcal{E}(A_{\tau}^{S} | \overline{S}_{t-\tau-1})$$

$$= \begin{cases} 2x^{2}\tau N\theta(1-\theta) & \text{for } \tau < L \\ 2x^{2}LN\theta(1-\theta) & \text{for } \tau \geq L \end{cases} \qquad \qquad \begin{cases} c \begin{cases} N\theta \frac{\tau}{L} \left(\frac{1+2L-\tau}{2}\right) \\ + \sum_{i=t-L}^{t-\tau-1} S_{i} \left(\frac{t-\tau-L}{L}\right) \\ -\sum_{i=t-L}^{t-\tau-1} S_{i} \left(\frac{t-\tau-L}{L}\right) \end{cases}$$

$$= \begin{cases} \left[ \left(\frac{c}{L} - x\right)^{2}\tau + \frac{c^{2}}{L^{2}}(L-\tau) \\ + x^{2}(\tau-1) + (x-c)^{2} \right] N\theta(1-\theta) \end{cases} \qquad \operatorname{Var}(A_{\tau}^{S} | \overline{S}_{t-\tau-1}) \end{cases} \qquad \operatorname{Var}(A_{\tau}^{S} | \overline{S}_{t-\tau-1})$$

$$= \begin{cases} \left[ \left(\frac{c}{L} - x\right)^{2}L + x^{2}L + c^{2} \right] N\theta(1-\theta) \\ -\sum_{i=t-L}^{t-\tau-1} S_{i} \left(\frac{t-\tau-L}{L}\right) \\ -\sum_{i=t-L}^{t-\tau-1} S_{i} \left(\frac{t$$

The predictive mean square error of cash flow by using SEC income is not uniformly

greater than or less than the predictive mean square error with FC income. For example, a comparison of expressions (28) and (29) indicates that for  $\tau > L$ , successful efforts predictive mean square error is greater if ((1+L)/2) is greater than xL/c, the recovery ratio; otherwise, the predictive mean square error with full costing is greater.

# Stock Variables under the Two Accounting Methods

The difference in capitalization policy for prediscovery costs influences not only the flow variables discussed above but also two stock variables which appear on the balance sheet—the capitalized assets and owners' equity. The properties of capitalized assets under the full-cost and successful-efforts methods are examined next.

Under SEC, the capitalized value of assets at the beginning of any period t is  $A_{i}^{s} = c(S_{i-1} + S_{i-2} + \cdots + S_{i-L})$ 

$$I_{i} = c(S_{i-1} + S_{i-2} + \cdots + S_{i-L})$$

$$-\frac{c}{r} (S_{i-2} + 2S_{i-3} + \cdots + (L-1)S_{i-L}).$$
(30)

The expected value and variance of capitalized assets  $\tau$  years into the future are

$$= \operatorname{Var}(I_{t} - X_{t+\tau}) \qquad \mathcal{E}(A_{t} | S_{t-\tau-1})$$

$$= \begin{cases} 2x^{2}\tau N\theta(1-\theta) & \text{for } \tau < L \\ 2x^{2}LN\theta(1-\theta) & \text{for } \tau \geq L \end{cases} \qquad \qquad \begin{cases} c \left\{ N\theta \frac{\tau}{L} \left( \frac{1+2L-\tau}{2} \right) \right. \\ + \sum_{i=t-L}^{t-\tau-1} S_{i} \left( \frac{t-\tau-L}{L} \right) & \text{for } \tau < L \end{cases} \end{cases}$$

$$= \begin{cases} \left[ \left( \frac{c}{L} - x \right)^{2}\tau + \frac{c^{2}}{L^{2}} (L-\tau) \right. \\ + x^{2}(\tau-1) + (x-c)^{2} \right] N\theta(1-\theta) \\ = \begin{cases} \left[ \left( \frac{c}{L} - x \right)^{2}L + x^{2}L + c^{2} \right] N\theta(1-\theta) \\ \text{for } \tau > L. \end{cases} \qquad \qquad \begin{cases} c \left( \frac{N\theta}{L} + \frac{1}{2} \right) & \text{for } \tau < L \\ - \frac{c^{2}(L+1)(2L+1)}{L^{2}} \left( \sum_{i=L-\tau+1}^{t-\tau-1} t^{2} \right) & \text{for } \tau < L \end{cases} \end{cases}$$

$$= \begin{cases} c^{2} \frac{N\theta(1-\theta)}{L^{2}} \left( \sum_{i=L-\tau+1}^{t-\tau-1} t^{2} \right) & \text{for } \tau < L \\ - \frac{c^{2}(L+1)(2L+1)}{6L} & N\theta(1-\theta) & \text{for } \tau \geq L. \end{cases}$$

Uncertainty in capitalized values of assets in the SEC method arises from their dependence on the results of exploration activities because only the costs of successful exploration are capitalized. On the other hand, under FC, the capitalized assets are independent of the results of exploration and are determined completely by the size of the exploration program. This can be seen by writing the expression for capitalized assets under full costing:

$$A_{i}^{F} = \frac{NC}{L} [L + (L - 1) + \dots + 1]$$

$$= \frac{Nc}{2} (L + 1).$$
(33)

Af is equal to its own expectation since its variance is zero. Therefore, under the full-cost method, the capitalized assets are stable over time when compared to the successful-efforts method. In addition, average capitalized assets are considerably higher under the full-cost method than under the successful-efforts method. From expressions (31) and (33), the ratio of expected values of capitalized exploration costs is

$$\frac{\mathcal{E}A_{i}^{F}}{\mathcal{E}A_{i}^{S}} = \frac{Nc(L+1)/2}{Nc\theta(L+1)/2} = \frac{1}{\theta}$$
 (34)

If  $\theta$  is 0.1, this ratio is equal to 10. The smaller the probability of a successful strike  $(\theta)$ , the greater is the discrepancy between the capital base of firms using different methods of accounting.

# Return on Assets under Different Accounting Methods

In order to analyze the effect of full-cost and successful-efforts method on the rate of return on assets, let  $B_t$  be the assets of the firm other than those arising out of capitalization of exploration costs. Thus, the total assets in period t are  $(A_t + B_t)$  and return on assets is

$$R_t = \frac{I_t}{A_t + B_t} \tag{35}$$

For the full-cost firms, return is

$$R_{i}^{F} = \frac{I_{i}^{F}}{A_{i}^{F} + B_{i}}$$

$$= \frac{-Nc + x \sum_{i=i-L}^{i=1} S_{i}}{B_{i} + Nc \frac{(L+1)}{2}}$$
(36)

Expected value of return on assets for such firms is

$$\mathcal{E}(R_i^F) = \frac{-Nc + xLN\theta}{B_i + Nc\frac{(L+1)}{2}}$$

$$= \frac{\frac{2}{L+1} \left(\frac{xL\theta}{c} - 1\right)}{1 + \frac{2B_i}{Nc(L+1)}}$$
(37)

For the successful efforts firms, the corresponding expressions are

$$R_{i}^{S} = \frac{I_{i}^{S}}{A_{i}^{S} + B_{i}}$$

$$= \frac{\left(x - \frac{c}{L}\right)\left(\sum_{i=i-L}^{L-1} S_{i}\right) - c(N - S_{i})}{B_{i} + \frac{c}{L}\sum_{i=1}^{L} S_{i-i}(L - i + 1)}$$

$$\mathcal{E}(R_{i}^{S}) = \frac{-Nc + xLN\theta}{B_{i} + cN\theta \frac{(L + 1)}{2}}$$

$$= \frac{\frac{2}{L+1}\left(\frac{xL\theta}{c} - 1\right)}{\theta + \frac{2B_{i}}{Nc(L+1)}}$$
(39)

A comparison of the average rate of return on assets under the FC and SEC methods indicates that the latter is always higher because  $\theta$ , the probability of a successful strike, is no greater than one. The

ratio of expected returns under the two methods is

$$\frac{\mathcal{E}(R_i^p)}{\mathcal{E}(R_i^s)} = \frac{\theta + \frac{2B_i}{Nc(L+1)}}{1 + \frac{2B_i}{Nc(L+1)}}$$
(40)

A decrease in the probability of finding petroleum also results in a relative reduction in the FC rate of return on assets. The second term in numerator and denominator is the ratio of other assets to assets represented by fully capitalized exploration costs. As this ratio increases, i.e., other assets become dominant in the total assets of the firm, the ratio of the two expected rates of return on assets moves toward 1.

Capital Structure under Different Accounting Methods

Define

 $D_i \equiv \text{long-term debt of the firm at time } t$  $E_i \equiv \text{owners' equity at time } t$ 

 $E_t' \equiv$  owners' equity at time t if all exploration costs are expensed as incurred.

For FC and SEC firms,  $D_t$  and  $E_t'$  are identical, but the owners' equity depends on capitalization policy. Therefore, for a full-cost firm in steady state, owners' equity at any time t is

$$E_t' + Nc \frac{(L+1)}{2}$$
(41)

Given a value of  $E_t$  and the program for exploration, there is no uncertainty in owners' equity of a full-cost firm. Its debtequity ratio,  $G_t^F$ , is

$$G_{i}^{F} = \frac{D_{i}}{E_{i}' + Nc(L+1)/2} \cdot (42)$$

For a successful-efforts firm, the value of owners' equity at time t is

 $E_{i} = E_{i}' + \frac{c}{L} \sum_{i=1}^{L} S_{i-i}(L-i+1),$  (43)

and its debt-equity ratio,  $G_i^s$  is

$$G_{i}^{S} = \frac{D_{i}}{E_{i}' + \frac{c}{L} \sum_{i=1}^{L} S_{i-i}(L - i + 1)}$$
 (44)

The ratio of the debt-equity ratios of the firms is

$$\frac{G_{i}^{F}}{G_{i}^{S}} = \frac{E_{i}' + \frac{c}{L} \sum_{i=1}^{L} S_{i-i}(L - i + 1)}{E_{i}' + Nc(L + 1)/2}, \quad (45)$$

and the average value of this ratio is

$$\mathcal{E}\left(\frac{G_{\iota}^{F}}{G_{\iota}^{S}}\right) = \frac{E_{\iota}' + cN\theta(L+1)/2}{E_{\iota}' + cN(L+1)/2} = \frac{1 + \theta \cdot \frac{cN(L+1)}{2E_{\iota}'}}{1 + \frac{cN(L+1)}{2E_{\iota}'}}.$$
 (46)

Since  $\theta$  is always less than 1, expression (46) is always less than 1. If two companies are identical in all respects except in the accounting policy for capitalization of exploration costs, the full-cost firms will have a higher capitalized value and lower debtequity ratios. If term

$$\frac{cN(L+1)}{2E.'},$$

the ratio of fully capitalized exploration costs to other parts of owners' equity, is small, capitalization policy will have little effect and ratio (46) will be close to one If, on the other hand,

$$\frac{cN(L+1)}{2E'}$$

is large, the effect of the accounting policy on capital structure can be quite large. The following values will illustrate the point for  $\theta = 0.1$ . (See Table 1.)

TABLE 2		
$\frac{(1)}{cN(L-1)} = \frac{cN(L-1)}{2Ec}$	$\frac{(2)}{E_{i'}}$ $\frac{E_{i'} + \frac{cN(L+1)}{2}}{2}$	$\mathcal{E}\left(\frac{G_{\iota}^{F}}{G_{\iota}^{S}}\right)$
1.1 .25 .5	.91 .80 .67	.92 .82 .70

For small values of  $\theta$ , (46) can be approximated by:

$$\varepsilon\left(\frac{G_i^F}{G_i^S}\right) \simeq \frac{E_i'}{E_i' + \frac{cN(L+1)}{2}}$$
 (47)

A comparison between columns (2) and (3) of Table 2 will give an idea of how well the approximation works. The right-hand side of (47) is simply the ratio of owners' equity other than capitalized exploration costs to the owners' equity with fully capitalized exploration costs. This ratio can be used to transform the debt-equity ratios of FC firms to SEC accounting and vice versa for the purpose of standard comparisons.

# DYNAMIC MODELS FOR CHANGING FIRMS

The behavior of certain accounting variables under FC and SEC methods of accounting for prediscovery costs has been examined in the preceding section for steady-state firms. The analysis is conducted under the assumption that the firm has reached a stable size and will remain at this level in the future. While such analysis is useful in obtaining insights into the behavior of accounting variables, it is clearly not sufficient. In this section, the steady-state assumption will be relaxed to analyze new, expanding and shrinking firms. This analysis is applicable to firms

which may not necessarily be new, expanding or shrinking in size but whose petroleum exploration operations can be characterized as such.

#### New Firms

Consider a new firm which plans to drill N exploratory wells per period at the cost of c per well. The probability of success is  $\theta$ and a successful well yields a lifetime net operating cash flow of xL, where L is the number of periods over which the discovered reserve is exploited at a uniform rate to yield net operating cash flow of xper period. This firm will arrive in steady state in period t = L if it starts its operations in period t=1. Therefore, for t>L, the analysis of the preceding section is applicable and the firm can be characterized as a new firm only for the period (0 < t < L). Using various definitions from the previous section, cash flow in period

$$X_t = -Nc + x(S_1 + S_2 + \cdots + S_{t-1}),$$
  
for  $t \le L.$  (48)

Expectation and variance of cash flow  $\tau$  periods into the future is

$$\mathcal{E}(X_{t}|\overline{S}_{t-\tau-1}) = -Nc + x \sum_{i=1}^{t-\tau-1} S_{i} + x\tau N\theta$$

$$\text{for } \tau < t \le L$$

$$\text{Var } (X_{t}|\overline{S}_{t-\tau-1}) = x^{2}\tau N\theta(1-\theta) \quad (49)$$

$$\text{for } \tau < t \le L.$$

Therefore, the mean and variance of cash flow forecasts  $X_t$  for a new firm made at period 1  $(\tau = t - 1)$  are  $xN\theta(t-1) - Nc$  and  $x^2N\theta(1-\theta)(t-1)$  respectively until  $t \le L$ . The firm enters a steady state after t > L.

FC and SEC income for the new firms, given by  $I_i^F$  and  $I_i^S$ , respectively, is

$$I_{t}^{F} = x \sum_{i=1}^{t-1} S_{i} - \frac{Nc}{L} (t-1) \quad \text{for } t \le L$$
 (50)

$$I_{i}^{S} = \left(x - \frac{c}{L}\right) \sum_{i=1}^{t-1} S_{i} - Nc + cS_{t}$$
for  $t \le L$ . (51)

The difference between full-cost and successful-efforts income is

$$I_{t}^{F} - I_{t}^{S} = \frac{c}{L} \sum_{i=1}^{t-1} S_{i} - cS_{t}$$

$$-\frac{Nc}{L} (t-1-L)$$
for  $t < L$ .
$$(52)$$

At the beginning of the firm (t=1), the expectation of the difference between full-cost and successful-efforts income in each year is

$$\mathcal{E}(I_{\iota}^{P} - I_{\iota}^{S}) = \frac{c}{L} \sum_{i=1}^{\ell-1} N\theta - cN\theta$$
$$-\frac{Nc}{L} (\ell - 1 - L)$$
$$= \frac{Nc}{L} (L + 1 - \ell)(1 - \theta)$$
 (53)

for  $t \leq L$ .

For a new firm, therefore, the expected difference between  $I_i^F$  and  $I_i^S$  is always positive and it approaches zero as  $t \rightarrow L$ . Finally when t = L + 1, the difference is zero and the firm is no longer defined to be a new firm.

This relationship explains the substantial support found among relatively new and small petroleum exploration firms in favor of the full-costing method. As noted above, the average difference between full-cost and successful-efforts income moves to zero as the firm matures. But for the new firms, the difference is substantial; and the newer the firm, the greater is the effect of the accounting method on its average income. During its infancy, a firm is most susceptible to the evaluations made by outsiders about its performance and viability. It is true that in a perfect market the method of accounting alone adds noth-

ing to the information available to outsiders about the firm. However, if there are any imperfections in the market, they will seem to help survival of the full-cost firm as compared to the successful efforts firm

It is important to note the difference between expressions (14) and (15) for mature firms and (52) and (53) for the new firms. For mature firms, the difference between full-cost and successful-efforts income is zero on average, although the two income streams hardly ever coincide. For new firms, the FC income is greater than the SEC income, on average. The difference lasts as long as t is less than or equal to L. The longer the life span of exploitation of discovered reserves, the longer will the income bias in favor of the FC income continue. Since the value of L can be quite large in the petroleum industry, the firm will remain "new" for a much longer period of time than firms in most other industries.

While a larger value of L will cause the bias to last for a longer period of time, the magnitude of bias towards the full-cost income gets smaller as L gets larger. To see this, expression (53) can be rewritten as  $\mathcal{E}(I_{\cdot}^{F}-I_{\cdot}^{S})$ 

$$= Nc(1-\theta) + \frac{Nc}{L}(1-t)(1-\theta).$$

The first term on the right-hand side is independent of L, but the second term gets smaller as L becomes larger.

Expression (53) also illustrates the effect of  $\theta$ , the probability of a successful strike on income bias for a new firm. The greater, the value of  $\theta$ , the smaller is the income bias and, therefore, the motivation for the firm to adopt the full-cost method. In other words, when the failure rate  $(1-\theta)$  is high, the comparative income advantage of a new FC firm over a similar SEC firm is also higher.

Capitalized Assets and Return on Assets for New Firms

The capitalized value of exploration costs for the new firm under the FC and SEC methods can be rewritten from expressions (30) and (33), respectively.

$$A_{i}^{S} = \frac{c}{L} \sum_{i=1}^{t-1} (L+i-t+1)S_{i}$$
 for  $t \le L$  (54)

$$A_{i}^{F} = \frac{Nc}{L} \sum_{i=L-t+2}^{L} i = \frac{Nc}{L} (L+1-t/2)(t-1)$$
 for  $t \le L$  (55)

At the beginning (t=1), the expected value of capitalized assets at time t for a SEC firm is

$$\mathcal{E}(A_{i}^{S}) = \frac{Nc\theta}{L} (L + 1 - t/2)(t - 1)$$
for  $t < L$ . (56)

Therefore, the capitalized values of the prediscovery costs for a SEC firm is  $\theta$  times smaller than the corresponding number for an FC firm on average. This result is identical to the result obtained above in expression (34) for the mature firms.

It is not clear whether the return on assets is higher for the full-cost firms also. For the mature firms, expression (40) indicates that the answer is no. In general, the expected return on assets is higher for the successful-efforts firms, the other things being equal. This is not necessarily true of the new firms, as can be seen from the following expressions:

$$\frac{\mathcal{E}(I_t^F)}{B_t + \mathcal{E}(A_t^F)} = \frac{xN\theta(t-1) - \frac{Nc}{L}(t-1)}{B_t + \frac{Nc}{L}(L+1-t/2)(t-1)}$$
for  $t \le L$ .

$$\frac{\mathcal{E}(I_{t}^{S})}{B_{t} + \mathcal{E}(A_{t}^{S})} = \frac{\left(x - \frac{c}{L}\right)N\theta(t-1) - cN(1-\theta)}{B_{t} + \frac{Nc\theta}{L}(L+1-t/2)(t-1)}$$

$$\text{for } t \ge L$$

Both the numerator and denominator of ratio (58) are less than the corresponding quantities in (57). However, whether ratio (58) is greater or less than (57) depends on a complex relationship between various parameters of this model and is not pursued here.

A characteristic feature of the new and expanding businesses is the cash shortage. The expected cash flow for a mature firm is  $N(xL\theta-c)$  from expression (4). For a nonruinous operation, this value is positive. For new firms, on the other hand, the expected cash flow is  $N(x\theta(t-1)-c)$  from expression (48). For small values of t, the expected net cash flow is almost certainly negative. In order to meet the cash requirements, the new firms need to raise capital, and their capacity to raise capital depends to an extent on their financial position. As discussed earlier, for mature firms, the debt equity ratio for the full-cost firms is more favorable. The same holds for new firms as can be seen by revising expression (46) as applied to the new firms:

$$\varepsilon\left(\frac{G_{\iota}^{F}}{G_{\iota}^{S}}\right) = \frac{E_{\iota}' + \frac{Nc\theta}{L} \sum_{i=1}^{\iota-1} (L - i + 1)}{E_{\iota}' + \frac{Nc}{L} \sum_{i=1}^{\iota-1} (L - i + 1)} < 1$$
for  $\iota \le L$ .

Given the cash needs of the new firms, combined with their ability to show better income and more favorable capital structure under the full-cost method, it is

hardly surprising that such firms tend to prefer this method of accounting in the absence of other overriding considerations.<sup>7</sup>

### Expanding Firms

The results obtained above for the new firms can be generalized qualitatively to the expanding firms. Expanding firms are defined as those which are increasing the scale of their exploratory operations; i.e., the number of exploratory wells drilled (N) is increasing through time. Let  $N_i$  be the number of exploratory wells drilled in period t. For simplicity, linear additive expansion can be defined by

$$N_t = a + bt \tag{60}$$

where the number of wells drilled in each period increase by b over the number drilled in the previous period

$$N_{i} = N_{i-1} + b. (61)$$

Expressions for net cash flow, income and other accounting variables can now be revised for expanding firms using FC and SEC.

$$X_i = -N_i c + x \sum_{i=i-L}^{i-1} S_i$$
 (62)

where  $S_i$  has binomial distribution with parameters  $\theta$  and  $N_i$ .

$$\mathcal{E}(X_i) = -N_i c + x\theta \left( \sum_{i=i-L}^{i-1} N_i \right)$$

$$= -N_i c + x\theta L N_i - \frac{L(L+1)}{2} x\theta b.$$
(63)

The expected net cash flow from expanding firms is smaller than from a steady-state firm with the same current level of explorations  $(N=N_i)$  by an amount  $(\theta x L(L+1)/2)b$ . This expression points out the extra cash requirements of an expanding business. The cash shortfall is directly proportional to the rate of

growth b, expected lifetime net operating revenue from a successful well  $\theta x L$  and the time span of exploitation (L+1).

Variance of net cash flow is

$$Var (X_i) = x^2 \theta (1 - \theta) \sum_{i=i-L}^{i-1} N_i$$

$$= x^2 \theta (1 - \theta) \left( N_i L - \frac{L(L+1)}{2} b \right)$$

Period t earnings of an SEC firm can be given by

$$I_{i}^{S} = -c(N_{i} - S_{i}) + \left(x - \frac{c}{L}\right) \sum_{i=1}^{i-1} S_{i} \quad (65)$$

with an expected value of

$$\mathcal{E}(I_{i}^{S}) = -cN_{i} + L\theta x N_{i}$$

$$-\frac{\theta L(L+1)}{2} \left(x - \frac{c}{L}\right) b.$$
 (66)

A comparison with (8) indicates that expression (8) is a special case of (66) with b=0. Since the third term in the latter expression is negative for a nonruinous operation  $(x\theta > c/L)$ , for any given level of current operations,  $N_i$ , expectation of income of a firm using the successful-efforts method is lower for an expanding firm than for a steady-state firm.

For a full-cost expanding firm, periodic income and its expected value are:

$$I_{i}^{F} = \sum_{i=t-L}^{t-1} \left( xS_{i} - \frac{c}{L} N_{i} \right)$$

$$= x \sum_{i=t-L}^{t-1} S_{i} - cN_{i} + cb \frac{(L+1)}{2}$$
(67)

$$\mathcal{E}(I_{i}) = x\theta L N_{i} - c N_{i}$$

$$-\frac{(L+1)}{2} b(Lx\theta - c). \tag{68}$$

<sup>7</sup> For a survey of the current accounting practices of 296 firms registered with the SEC, see Ginsburg, Feldman and Bress, Comments of the Ad Hoc Committee (Petroleum Companies) on Full Cost Accounting (Securities and Exchange Commission, 1973), pp. 28-31.

Again, for a nonruinous operation  $(Lx\theta-c)$  must be positive, and therefore the expected income of a full-cost firm under expansion is less than the expected income of a mature firm at the same current level of operations and using the full-cost method. The reason for this negative difference is that when a firm is growing (b>0), the income expected to be realized from past discoveries is smaller than for a steady-state firm. However, this difference is smaller at

$$\frac{(L+1)b}{2}(Lx\theta-c)$$

for the full-cost firms than for the successful-efforts firms—

$$\frac{(L+1)b}{2}\left(xL\theta-c\theta\right).$$

The difference between expected income of full-cost and successful-efforts firms is

$$\mathcal{E}(I_{i}^{F} - I_{i}^{S}) = (L+1)\frac{bc}{2}(1-\theta) > 0$$
for  $b > 0$ .

Unlike a steady-state firm for which the expected difference between income streams for the two accounting methods is zero, a growing firm will report higher earnings on average with the full-cost method. The difference is directly proportional to the life span of exploitation (L+1), probability of failure  $(1-\theta)$ , growth rate b and cost of exploration per well c. Earlier remarks about the need for large amounts of cash by new firms are also applicable to the expanding firms. Besides the better earnings figure, the use of full-cost method also results in a more favorable debt-equity ratio for the expanding firms. Expressions for the effects of different accounting methods on capitalized value of exploration costs, return on assets and debt equity ratio of expanding firms can be developed on the same lines as done earlier for the steady state and new firms.

# Shrinking Firms

Analysis of the two accounting methods for the growth firms is also applicable to the shrinking firms; the only difference is that b is negative for them. Several, but not all, of the results for growth firms are simply reversed for the shrinking firms. While no upper limit is placed on the value of  $N_{\rm eff}$  it cannot be allowed to go below zero, and a firm cannot shrink forever at a constant linear rate.

Net cash flow from a shrinking firm with the same current level of exploration is higher than the cash flow from a steadystate firm by the amount

$$\frac{L(L+1)}{2} x\theta b.$$

Thus, a shrinking firm appears to have more cash as compared to a steady-state firm.

Income of a shrinking company is greater than the income of a steady-state firm by amounts

$$\frac{L+1}{2}(xL\theta-c\theta)b$$
 and  $\frac{L+1}{2}(xL\theta-c)b$ 

for the firms using the SEC and FC methods, respectively. The excess income is greater for the successful-efforts method by

$$\frac{L+1}{2}cb(1-\theta).$$

It had been noted earlier that the expected income of successful-efforts firms is less than the income of full-cost firms by a similar amount during periods of growth. Thus, the net effect of using the successful-efforts method on the time series of income of the firm is to shift income recognition from years of growth to the years of decline.

# Transition from the Growth to the Shrinking Phase

In analyzing the behavior of shrinking firms, it has been assumed that the firm has been reducing its exploratory operations at a uniform rate for at least the last L years. If the value of L is large, the past L years may include the period of change from positive to negative growth. L years following the hump will be the years of transition. During this period, the accounting variables will change their nature from the growth state to the shrinking state. Simple relationships given above between cash flows, income and capitalized assets of successful-efforts and full-cost firms do not apply in this period.

### Changes in Other Parameters of the Model

Besides changes in the level of exploration activity,  $N_{ij}$ , other changes can occur in the parameters of the models. For example, the probability of a successful strike,  $\theta$ , may change over time. It is a fair assumption that the value of  $\theta$  will decline for a given geographical region as more and more exploration is conducted in the area. Chances of finding new reserves decrease with time in an actively explored area. For the United States considered as a single area of exploration,  $\theta$  does appear to have declined during the past. Among the new field wildcats, the percentage of successful wells declines from 11.31 in 1958 to 10.31 in 1966. Over the same period of time, the percentage of successful wells in all exploratory drilling declined from 18.98 to 15.59 [Petroleum Facts and Figures, 1967]. Similarly, the discovery of commercially profitable oil fields as a percentage of new field wildcats declined from 2.07 in 1945 to 0.60 in 1960. The corresponding decline in gas field discoveries was from 1.69 percent to 1.09 percent. Where a secular decline in the value of  $\theta$  is expected,

it would be useful to account for such changes by modifying the model.

The cost-benefit relationships of exploratory drilling also may change over time. As the value of  $\theta$  decreases, the exploratory wells will have to be drilled deeper and deeper to find oil. Average depth of all exploratory wells drilled in the United States has increased only slightly from 4,921 feet to 5,095 feet during the period 1958–1966 [Petroleum Facts and Figures, 1967]. Deeper wells generally cost more to drill. Changes in average depth of hole drilled will require changes in the value of parameter c over time.

Inflation and changes in factor and product market environment also can cause changes in the cost-benefit parameters. Over the course of time, inflation will affect the exploration costs and revenues from production. If any such changes are anticipated, they also can be included in the model.

The introduction of changing parameters N,  $\theta$ , c and x, will result in a considerably more complex model, and if all modifications are introduced to the model at the same time, we cannot get many generalizable results like those obtained for simpler models earlier. Detailed analysis of changing and even uncertain parameters is conducted more easily through a simulation. A computer simulation program can be written to analyze various relationships among the accounting variables and how they are affected by the behavior of the above-mentioned parameters.

## CONCLUDING REMARKS

A simple analytical model has been developed above to examine the effects of accounting policy with respect to capitalization of exploration costs in the petroleum industry on several accounting variables. Under simplifying assumptions, some prop-

erties and interrelationships of these variables have been analyzed. Such an analysis can be used by the producers of the accounting statements to understand and predict the implications of alternative accounting policies for the resultant statements. Similarly, the users of accounting statements can use such an analysis to examine the implications of published inancial statements for the economic condition of the firm when they have the information about the accounting procedure used to prepare the statements.

Of necessity, several simplifying assumptions had to be made in order to obtain some generalizable results in closed-form expressions. Considerably more complex models of the behavior of accounting variables under alternative accounting policies can be made on similar lines and analyzed with the help of computer simulation [Eskew, 1973]. However, greater realism of simulation results cannot be realized without sacrificing a degree of generalizability in the process.

Since the second and third sections of this article contain a large number of results about the properties of the two various accounting methods in reasonably condensed form, I shall not try to summarize them again. A few remarks about the data requirements for implementation of this model seem to be in order here.

Variables and parameters used in the model are listed in Table 1. The parameters are (1)  $\theta$ , the probability of a successful strike, (2) x, net operating cash flow per successful well per period. (3) c, exploration cost per well, (4) L, life of a successful well and (5) N, the number of exploratory holes drilled each period. The values of these parameters can be estimated from the past data available for the United States as a whole as well as for each state published in Petroleum Facts and Figures by the American Petroleum Institute. Ac-

counting variables in the model are either balance sheet quantities or derivatives of the model itself. Thus, the data requirements for implementation of the model do not seem to be too demanding.

#### APPENDIX I

DIFFERENCE IN VARIANCE OF SUCCESSFUL EFFORTS AND FULL-COST INCOME

From expression (16) the difference between the variance of successful-efforts and full-cost income is

$$V(I_{i}^{S}/\cdot) - V(I_{i}^{F}/\cdot) = N\theta c^{2}(1-\theta) \left\{ 1 + \frac{\tau}{L^{2}} - \frac{2xL}{cL^{2}} \tau \right\}.$$
 (A.1)

Sign of the difference depends on the quantity H where

$$H = \left(1 + \frac{\tau}{L^2}\right) - 2\frac{xL}{cL^2}\tau. \tag{A.2}$$

xL/c is relative profitability of the operation, denote this by p

$$H = 1 + \frac{\tau}{L^2} - 2\frac{\rho\tau}{L^2}$$
 (A.3)

$$\frac{dH}{d\rho} = -\frac{2\tau}{L^2} \,. \tag{A.4}$$

Since  $\tau$  and L are positive quantities, increase in profitability results in relative increase in variance of full-cost income, and the latter exceeds the variance of successful-efforts income when p is greater than  $(L^2+\tau)/2\tau$ .

$$\frac{dH}{dL} = -\frac{2\tau}{L^3} + \frac{4p\tau}{L^3} = \frac{2\tau}{L^3} (2p - 1). \quad (A.5)$$

As discussed in the text, quantity p is of the order of ten and, therefore, the derivative of H with respect to L is positive. For values of L greater than  $((2p-1)\tau)^{1/2}$ , successful-efforts accounting will result in greater income variance. The maximum possible differences as,  $L \rightarrow \infty$  is  $c^2 N\theta(1-\theta)$ .

To examine the effect of prediction interval  $\tau$  on the difference in income variance, differentiate H with respect to  $\tau$ 

$$\frac{dH}{d\tau} = 1 + \frac{1}{L^2} - \frac{2p}{L^2}$$
 (A.6)

The derivative is positive when  $L > (2p-1)^{1/2}$ ; the difference H increases with  $\tau$  in this range. Otherwise, changes in prediction interval  $\tau$  have the effect of decreasing the difference H.

#### REFERENCES

- APB Public Hearings on Accounting and Reporting Practices in the Petroleum Industry, Cases in Public Accounting Practice, Vol. 10 (Arthur Anderson and Co., 1972).
- Coults, W. B., Accounting Problems in the Oil and Gas Industry (The Canadian Institute of Chartered Accountants, 1963).
- Eskew, Robert, "An Empirical Examination of the Interaction between Accounting Alternatives and Share Price in the Extractive Petroleum Industry" Ph.D. Diss, (Purdue University, 1973).
- Fields, Robert E., Financial Reporting in the Extractive Industries, Accounting Research Study, No. 11 (AICPA, 1969).
- Ginsburg, Feldman and Bress, Comments of the Ad Hoc Committee (Petroleum Companies) on Full Cos. Accounting (SEC, 1973), pp. 28-31.
- Petroleum Facts and Figures (American Petroleum Institute, 1967), p. 17.
- Sunder, Shyam, "Properties of Accounting Numbers under Various Definitions of Cost Centers in the Petroleum Exploration Industry," in Kenneth E. Most, ed., Proceedings of the Southwestern Regiona Meeting of the American Accounting Association (Texas A&M University, 6-7 March 1975).
- Sunder, Shyam, "Comparability of Divergent Financial Statments in the Petroleum Exploration Industry." Presented at the Conference on Topical Research in Accounting (New York University, 23 May 1975).