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# Expectations and learning under alternative monetary regimes: an experimental approach★

**Ramon Marimon**[1] **and Shyam Sunder**[2]

[1] Department of Economics, University of Minnesota, Minneapolis, MN 55455, USA, and Department of Economics, Universitat Pompeu Fabra, 08008 Barcelona, SPAIN, and Department of Applied Economics, Cambridge University, Cambridge, ENGLAND

[2] Graduate School of Industrial Administration, Carnegie-Mellon University, Pittsburgh, PA 15213, USA

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**Summary.** We design and analyze experimental versions of monetary overlapping generations economies under alternative policy regimes. Economies with a constant level of real deficit financed through seignorage, economies in which the level of deficit is adapted in order to follow a monetary policy with a target rate of inflation, and economies with preannounced changes in deficit levels are reported here. We also examine the behavior of an economy with no stationary competitive equilibrium. Our time series are compared to rational expectations equilibrium paths and to adaptive learning dynamics.

**Keywords:** Adaptive learning, monetary policy, experimental economics, indeterminacy, rational expectations.

*JEL Classification*: C62, C92, E17, E32, E44.

## 1 Introduction

Agents' expectations about future outcomes, about future prices for example, affect their current decisions which, in turn, affect future outcomes. That is, there is a mapping from beliefs, $\beta^e$, to realizations $\beta = T(\beta^e)$. The *rational expectations hypothesis* imposes the consistency condition that agents' expectations must be

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fulfilled in equilibrium; i.e., requires that beliefs, $\beta^*$, satisfy the fixed point condition $\beta^* = T(\beta^*)$. Our comprehension of economic phenomena have been greatly enhanced by the development, estimation, and calibration of rational expectations equilibrium (REE) models. However, the impact of this "revolution" on the design of economic policies has been limited, probably because in many economic regimes the REE models fail to provide a specific policy prescription. This happens when the RE models have multiple equilibria and the optimal policy varies across these equilibria. Many monetary and financial models (with an incomplete financial structure) share this feature—the set of fixed points of the $T(\cdot)$ map is large, even a continuum.

There are other shortocmings of the REE models. First, these models are mute about behavior outside equilibrium, saying little about what to expect if we observe time series that do not correspond to any recognizable equilibrium process. Second, since expectations are assumed to be fulfilled, the process of expectations formation and coordination is also left undefined. As a corollary of this simplification, the *complexity* of a particular market structure or policy is not taken into account either.

Learning models have been proposed as a way to overcome some of these shortcomings. The general idea is that if agents' beliefs depend on past observed outcomes (i.e., $\beta_t^e = \phi(\ldots, \beta_{t-1})$), then there is a dependence of current outcomes on past outcomes. That is, $\beta_t = T(\phi(\ldots, \beta_{t-1}))$. The dynamics of the composite map, $T \cdot \phi$, defines which stationary equilibria are stable. Thus a stability requirement may narrow the set of equilibria. Furthermore, outcome paths are well-defined even outside equilibrium, provided that agents' learning and choice rules are well defined. Finally, if learning and/or choice rules are affected by the institutional environment (trading rules, etc.), then institutional changes may have nontrivial real effects.

Most learning and evolutionary models, however, take as starting point specific learning or forecasting rules and assume costless maximizing behavior as agents' choice rule. Even if some models allow for some generality in the definition of the forecasting map, $\phi(\cdot)$ (see Grandmont and Laroque 1991, and Evans and Honkapohja 1992), the stability results usually are based on the existence of a representative agent whose forecasting rule satisfies certain parametric restrictions (see Marimon and McGrattan 1993 for an overview of a more general treatment in the context of repeated strategic games). Unfortunately, how agents learn in an economic environment is poorly understood.

Our approach is to generate data by conducting laboratory economic experiments with a view to help address the above questions. More precisely: are the experimental data consistent with the REE dynamics? If not, are they consistent with the REE at least in the long run? If the answer to either of these questions is affirmative, do we observe a *selection* among the set of RE equilibria? If we do, does this selection correspond to the selection achieved with adaptive learning rules? Can we explain the behavior of our subjects by means of simple adaptive learning rules? In asking these questions our work parallels some recent experimental work on learning in games (see, for example van Huyck et al. 1990). In contrast with repeated play of strategic games, our models are dynamic and therefore there is a well-defined characterization of dynamic equilibria.

In Marimon and Sunder 1993 we attempted an initial answer to these questions. We examined a monetary regime characterized by a constant level of deficit financed through seignorage. In summary, our experimental data are not consistent with non-stationary REE, but show a tendency to cluster around one of the two steady state REEs (the low inflation steady state). That is, there is a *selection* of the long-run steady state consistent with a large class of adaptive learning algorithms. However, our data also show more randomness and some biases which can not be explained on the basis of learning models that use simple forecasting rules with optimal savings decisions. Section 3 below introduces the OLG model of hyperinflation as a benchmark regime and summarizes some of our experimental results for these economies.

In Marimon, Spear and Sunder 1993 we studied these questions for an economy where there can be REE with fluctuations determined by extrinsic sources of uncertainty or "sunspots." More specifically, we posed the question of whether sunspot equilibria could emerge in an experimental laboratory. That these equilibria could be learned, and therefore could not be eliminated, had been shown by Woodford 1990 and others. We showed that emergence of sunspot equilibria is unlikely (we have never observed them in laboratory) unless sunspot "shocks" are correlated with real shocks and agents learn to predict economic fluctuations while in a "real shock" regime. That is, after the real source of uncertainty disappears, agents' behavior may show enough persistency as to sustain the sunspot fluctuations.

In this paper, we follow the same line of inquiry, but attend to some important issues that were left unaddressed. Do we observe the same patterns in different monetary regimes? Can agents *learn* through policy changes? Is the persistency or inertia observed in other laboratory economies strong enough to generate path dependencies that cannot be explained by the standard REE models? In particular, we study three new regimes, each one focuses on one of these questions.

First, since we had already observed behavior consistent with adaptive learning, we examine a monetary regime where fiscal policy adapts to monetary policy so as to achieve a given inflation target. The policy is such that if all agents share the same beliefs about a constant rate of inflation, say, $\pi^e_{i,t} = \pi^e_{i,t+1} = \beta$, then the inflation target, $\pi^*$, is achieved in one period, i.e., $\pi_t = \pi^*$. Given this adaptive government policy, nonstationary REE paths tend toward the autarkic solution with no value of money and zero deficit. This framework generalizes the stationary environment with zero deficit and zero inflation as target. We observe a tendency to converge towards the target rate of inflation, but at a slower rate and supporting higher deficits than with simple adaptive forecasting schemes (which only under certain parameter restrictions converge in some of our economies). We discuss this regime in Section 4.

Second, in the constant deficit regime, we have observed inflation rates clustering around the REE steady state with low inflation. This has been called "the classical" steady state since a decrease of the public deficit financed through seignorage reduces inflation. The transitional dynamics of an announcement of a change in fiscal policy around the classical steady state are not characterized by a nonstationary REE path, even when there are no credibility problems. In Section 5 we analyze economies in which a pre-announced change of regime (in general, a one time change in government expenditures) occurs. Again our time series are more consistent with adaptive behavior, although they are relatively volatile. Even with adaptive behavior, the question remains whether agents should learn to anticipate announced changes. We do not detect such anticipation, and more experimental work is needed in this direction.

Finally, in Section 6 we further study the persistency issue. In particular, we study an economy that generated fairly stationary data resembling other experimental economies; yet this economy has no stationary equilibrium! Our experimental data are more parsimonious than the paths suggested by the REE or the adaptive learning theory. On the other hand, our experimental data also show more local randomness than these theories predict for such a deterministic environment.

Before presenting our results, Section 2 describes how we modeled this overlapping generations environment in our laboratory.

In particular, part of our research has been to develop and study an experimental framework for analyzing dynamic macroeconomic models. The major design problems are: to give human subjects the opportunity to learn from past experience and yet be "faithful" to the theoretical model; to define rules for terminating the laboratory economy without distoring the characterization of equilibria of the infinite horizon model; to define appropriate market rules; to design experiments and collect data as to be able to study individual learning rules and to distinguish between *learning to forecast* and *learning to solve intertemporal optimization problems*. With this in mind we have also used computer assisted decision-making to create " smart" markets (see, Marimon, Spear and Sunder 1993). In summary, with our laboratory model we can study not only the relationship between alternative monetary policies and aggregate variables such as inflation, but also how individual agent's behavior is affected by- and determines the effects of- alternative policy regimes and money-market mechanisms.

## 2 Design of the experimental environment

We study experimental versions of the Overlapping Generations (OLG) model with generations that live for two periods (except the first generation that only lives for one period), where endowments and preferences are such that agents have an incentive to save, and money is the only financial asset available. Alternative monetary regimes differ only in the government's monetary policy. Changes in the level of deficit financed through seignorage is the only instrument that can be used to implement the different policies considered here. This, of course, is a fairly restrictive choice of policy instruments, but we wanted to keep changes between alternative regimes to a minimum and the analysis simple.

While the OLG model is well known and relatively simple, its implementation in an experimental environment can be fairly complex. In this section, we discuss some of the implementation problems and our solutions to them.

First, the OLG model requires a large number of participants since at each point in time markets must be competitive and each agent lives for only two periods. This not only creates the problem of requiring an infeasible number of subjects for the experiments, but also neglects the fact that it takes repeated experience in the same setting for human subjects to learn the setting they are operating in. Second, like other competitive equilibrium models, the OLG model also abstracts away from the specific trading mechanisms used to execute transactions. Experiments require us to be explicit about trading rules. Third, standard models assume that agents decide optimally and costlessly. In the laboratory we have the option to study how agents make their choices or have computer assisted decisions. When agents make savings decisions, they may take into account the endogenous uncertainty present in our laboratory setting. Finally, the overlapping generations model is an infinite period model, yet all laboratory economies must come to an end. Whenever the laboratory economies are terminated, rules of termination can not only alter the behavior in the final period of the economy but also the set of its equilibria.

### 2.1 Experimental environment

Twentyfive experimental economies, numbered chronologically for reference in this paper, have been conducted in twelve sessions.[1] A fixed number of subjects ($N$) participate in each session. Subjects know the approximate duration of the session but not of a particular economy. For each period of an economy, agents are assigned specific roles: $n$ subjects act as young consumers, $n$ as old consumers, and the remaining ($N - 2n > n$) await their turn as interested onlookers in the market. At the beginning of each period $n$ of the ($N - 2n$) players who are outside in the previous period are randomly selected to enter the market. Each player is informed whether he/she enters the market or stays out. Once an agent enters as a young consumer, he/she stays the next period as an old consumer and must spend the following period outside the market.

Consumers receive a higher endowment of chips ($\omega^1$ units) when young and they may offer to sell some or all of these chips to the old consumers. Young consumers carry the francs (label for units of fiat money in laboratory), they receive in exchange for the chips, to their old age in the next period.

Old consumers add the chips they buy to their endowment of chips $\omega^2$ ($< \omega^1$). The number of chips held at the end of the young period, $c^1$, and at the end of the old period, $c^2$, a constant and known conversion rate $k$, and the individual discount rate $\beta_i$ determine the dollar amount $k \cdot [\log c^1 + \beta_i \log c^2]$ earned by the subject when he/she leaves the market at the end of the old period. This dollar amount is accumulated and the total is paid to subjects at the end of the experiment.

We have a fixed number of subjects in any given experimental session. A subset that is randomly selected enters the market in each period and remains in the market for two consecutive periods. When subjects re-enter the market as young in a subsequent generation they cannot use dollars from their account; they re-enter as new subjects. The total number of subjects ($N$) is chosen to be sufficiently large ($N > 3n$) to ensure that each subject sits out for a random number of periods ($\geq 1$) between leaving and re-entering the economy. In other words, our subjects live several "lives" over the many periods of a particular economy. Assets cannot be carried from one "life" to the next but memory and experience obviously are.

[1] Thirteen of these economies from seven sessions (numbered 1, 3, 5, 6, 7, 9, and 12) were reported in Marimon and Sunder 1993 under consecutive numbering 1 through 7.

### 2.2 Subjects' experience with the setting

Our experimental OLG model is more like an OLG model in which parents are not allowed to bequest assets to their children, but they may pass on their experience. Marimon and Sunder 1993, Lemma 1 prove that this repeat entry into the experimental economy does not cause a departure from the OLG model; agents behave competitively within each generation as long as there are no further opportunities for strategic behavior due to the presence of a small number of agents.

### 2.3 Trading rules

OLG models are silent on the mechanism used to exchange chips and fiat money between the young and the old. Lim, Prescott and Sunder 1994 started out using single-unit double auction with the provision that the last transaction of an old subject in any period could be exchanged for a fractional unit to enable him or her to use up all the cash for consumption. This mechanism was awkward, slow, and error prone, with many old subjects carrying money to their "graves." Cash balances left in the hands of the old caused unintended variations in the supply of money in the experimental economy.

In the economies reported here, the discrete unit double auction mechanism has been replaced by a new mechanism. The young are asked to submit a supply schedule consisting of a reservation price for each integer quantity $i$, $i = 0, 1, \ldots, \omega^1$. A continuous supply schedule is computed for each individual by linear interpolation. Individual supply functions of the young are added to calculate the market supply function.

All the cash balance in the hands of the old is used to construct a hyperbolic chip demand function. In addition, it is *common knowledge* that, for example, the experimenter buys $D = n \cdot d$ chips every period at the market clearing price and that, therefore, the amount of money (francs) in circulation grows. This experimenter or "government" demand for chips is added to the demand of the old to arrive at the market demand function.

The computer calculates the market clearing price as the point of intersection between these supply and demand functions. This price is announced and the resulting allocations are communicated to the subjects each period. The history of prices is also displayed on the computer screen.

### 2.4 Endogenous uncertainty and optimal savings

Although the model under study is deterministic, agents' errors and deviations, and the fact that the number of agents per generation is too small to properly apply the Law of the Large Numbers, introduces randomness in our experimental model. In analyzing our data, we take into account possible "deviations" which are, in fact, closer to the solution of a stochastic version of the OLG model. An alternative approach is to solicit price and/or inflation forecasts and let the computer use this information to optimally solve the individual savings problem. We have used this alternative approach in Marimon, Spear and Sunder 1993.

### 2.5 The terminal condition

The OLG model has an infinite horizon and, in a strict sense, cannot be cast in an experimental environment (see Aliprantis and Plott 1992 for implementation of a finite period special case). The experimenter's choice of a procedure to terminate the economy may affect the set of equilibria. We use a procedure introduced by Lim, Prescott and Sunder 1994. During the experiment, players outside the market play a forecasting game: At the beginning of each period, they are asked to forecast the market-clearing price for the period; the player(s) whose prediction turns out to be the best expost receive(s) a prize (in dollars) that is added to their dollar accounts. The winning forecast is announced and displayed on all computer screens at the end of each period.

Without any previous announcement, and after forecasts for the period $(T+1)$ have been submitted, the experimenter declares that the period just ended $(T)$ is the last period of the economy. It is then that the forecasting game plays a role. Money (francs) holdings of agents who entered the economy in period $T$ are converted into chips using *the average of predicted market prices for period* $T+1$ *by outside-market participants*. This procedure for ending the game is announced and explained to subjects at the outset as part of the instructions.

Since this forecasting game is not a feature of the OLG economy, one may ask if its use in the laboratory introduces an important distortion of the OLG model. Marimon and Sunder 1993 show the equivalence (of the set of equilibria) between the standard OLG model and the *anonymous* game played between agents of different generations and an outside group of forecasters. We should be alert to the strategic possibilities open to our subjects. Our laboratory implementation may depart from the OLG model only if subjects in a generation depart from competitive behavior and consider the effect of their own actions on the market price. We can scrutinize the individual data to identify such departures.

## 3 A constant deficit regime

We study a version of Cagan's model of hyperinflation (1956), an economy with an OLG structure in which fiat money is the only financial asset and the government finances a fixed level of deficit through seignorage. This model has been previously studied, among others, by Sargent and Wallace [1987].

Each generation has $n$ agents and generations born after period zero live for two periods. An agent $i$ of generation $t$, $t=1,\ldots,$ has a two-period endowment of a unique perishable good $(\omega^1_{t,i}, \omega^2_{t,i}) = (\omega^1, \omega^2)$, $\omega^1 > \omega^2 > 0$, and his preferences over consumption are represented by $u_i(c^1_t, c^2_t) = \ln(c^1) + \beta_i \ln(c^2)$ where the superscript denotes the period in the agent's life. An agent $i$ of the initial generation that exists in Period 1 only lives for one period, and is endowed with $\omega_{0,i} = \omega^2$ of the consumption good. He also has an endowment of fiat money of $h_0$ and his preferences are represented by $u_i(c_0) = \ln(c_0)$.

Given a sequence of consumption good prices $\{p_t\}^{\infty}_{t=0}$, an agent $i$ of generation $t$, $t \geq 1$, solves the problem,

$$\begin{aligned} \max \quad & \ln c^1_t + \beta_i \ln c^2_t \\ \text{s.t.} \quad & p_t(c^1_t - \omega^1) + p_{t+1}(c^2_t - \omega^2) \leq 0. \end{aligned}$$

Let $\pi_{t+1} = p_{t+1}/p_t$, and $\pi^e_{t+1} = \mathbf{E}_{t-1}\pi_{t+1}$ (i.e., expectation at the beginning of period $t$ about the rate of inflation between periods $t$ and $t+1$). If $\omega^1 - \omega^2$ is large enough, the agent's supply in the first period of his life is

$$s_{it} = (\beta_i\omega^1 - \pi^e_{t+1}\omega^2)/(1+\beta_i).$$

The per capita aggregate supply is

$$s_t = \alpha\omega^1 - \pi^e_{t+1}\gamma\omega^2 \tag{1}$$

where

$$\alpha = \frac{1}{n}\sum_{i=1}^{n}(\beta_i/(1+\beta_i)),$$

and

$$\gamma = \frac{1}{n}\sum_{i=1}^{n}(1+\beta_i)^{-1}.$$

Let $h_t$ be the *per capita* money supply in period $t$. The government finances a constant per capita level of deficit $d$ through seigniorage, and, therefore, the supply of money follows the process

$$h_t = h_{t-1} + p_t d,$$

or

$$m_t = m_{t-1}/\pi_t + d, \tag{2}$$

where $m_t = h_t/p_t$ is the per capita money supply in real terms.

The equilibrium condition is

$$m_t = s_t \tag{3}$$

Equations (1)–(3) define the equilibrium restrictions of the model. They can be integrated into the equilibrium map

$$\Phi(\pi^e_{t+1}, \pi^e_t, \pi_t) = 0, \tag{4a}$$

i.e.,

$$\pi^e_{t+1} - c - \frac{\pi^e_t - b}{\pi_t} = 0. \tag{4b}$$

where $b = \dfrac{\alpha\omega^1}{\gamma\omega^2}$ and $c = b - \dfrac{d}{\alpha\omega^2}$. Stationary solutions satisfy $T(\bar{\pi}, \bar{\pi}, \bar{\pi}) = 0$ and if $(c+1)^2 > 4b$, there are two stationary solutions $(\pi^L, \pi^H)$. Given that, for $\pi^e_t \neq b$, $\partial_3 T(\cdot) = (\pi^e_t - b)/(\pi_t)^2 \neq 0$, by the Implicit Function Theorem we have

$$\pi_t - \phi(\pi^e_{t+1}, \pi^e_t) = 0, \tag{5a}$$

where

$$\phi(\pi^e_{t+1}, \pi^e_t) = \frac{b - \pi^e_t}{c - \pi^e_{t+1}}. \tag{5b}$$

Equation (5) describes the equilibrium dynamics of the economy, actual inflation as a function of expected inflation for the current and the following period. More in general, if $H$ is the set of all possible infinite histories of inflation rates, there is a map from beliefs to realizations, $T: H \mapsto H$. That is, assuming there is a representative

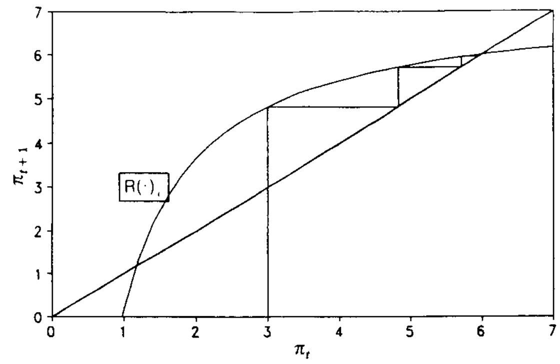


**Figure 1.** Rational expectations equilibrium paths.

agent with beliefs $\{\pi_t^e\}_{t=0}^{t=\infty}$, then the realized inflation path is given by $\{\pi_t\}_{t=0}^{t=\infty} = T(\{\pi_t^e\}_{t=0}^{t=\infty})$. Equation (5) shows that the $T(\cdot)$ map has a relatively simple structure. We can close the equilibrium condition by postulating the *rational expectations hypothesis*. That is, a fixed point of $T$:

$$\pi_t = \pi_t^e, \tag{6}$$

Then rational expectations equilibrium paths, for $\pi_t \in (0, b)$, (i.e., fixed point of $T$) are given by the difference equation

$$\pi_{t+1} = R(\pi_t), \tag{7a}$$

i.e.,

$$\pi_{t+1} = (c+1) - \frac{b}{\pi_t} \tag{7b}$$

If $\pi_0 \in (\pi^L, \pi^H)$, then the nonstationary equilibrium path $\{\pi_t\}_{t=0}^{\infty}$ satisfies $\pi_t \to \pi^H$ exponentially. Figure 1 shows the $R(\cdot)$ map and an arbitrary nonstationary equilibrium path.

As can be seen from (7), the two steady-state rates of inflation move in opposite directions when a parameter such as the real deficit $d$ is changed. The low inflation steady state $\pi^L$, known as the classical equilibrium, decreases with a decrease in deficit. On the other hand, such a decrease raises the level of the high inflation steady state, $\pi^H$.

Even if the theoretical model postulates that under rational expectations hypothesis there is a continuum of nonstationary REE paths that reach $\pi^H$ in the long-run, it does not predict which equilibria are more likely to be observed.

### 3.1 Experimental results

Table 1 shows some important features of the 25 overlapping generations economies conducted in 12 separate sessions. Gross inflation rates, per capita sale of chips and dollar earnings predicted by the Low ISS and High ISS equilibria, and by constant consumption behavior of agents in these economies are shown in Table 2.

**Table 1.** Design of Experimental Overlapping Generations Economies (See enclosed notes for explanation)

| Economy No. | No. of Subjects in Economy and Generation $(N, n)$ | Prior Experience | Endowment | | | Govt. Deficit Per Capita | Periods | Discount Rate |
|---|---|---|---|---|---|---|---|---|
| | | | Chips | | Money | | | |
| | | | Young $\omega^1$ | Old $\omega^2$ | $h_0$ | $d$ | $T$ | $\beta$ |
| 1 | (14,4) | None | 7 | 1 | 10 | 0.5 | 1–19 | 1 |
| 2 | (13,4) | Economy 1 | 7 | 1 | 10 | 0.25<br>1.25 | 1–17<br>18–33 | 1<br>1 |
| 3 | (12,3) | 3 inexperienced<br>9 from Econ. 1 or 2 | 7 | 1 | 3.722 | 1.25 | 1–17 | 1 |
| 4 | (12,3) | Econ. 3 | 7<br>3 | 1<br>1 | 3.722 | 1.25<br>0.25 | 1–7<br>8–20 | 1<br>1 |
| 5 | (10,3) | None | 7 | 1 | 10 | 0.42 | 1–31 | 1 |
| 6A | (14,4) | 2 from Econ. 5 | 7 | 1 | 0.5 | 0.975 | 1–20 | 0.6, 0.6, 1.7, 1.7 |
| 6B | (14,4) | 12 None<br>Econ. 6A | 7 | 1 | 1 | 0.975 | 1–8 | 0.6, 0.6, 1.7, 1.7 |
| 7A | (7,2) | Econ. 5 or 6 | 6 | 1 | 1 | $(1/3)h_{t-1}/p_{t-1}$ | 1–12 | 0.6, 1.0 |
| 7B | (12,3) | 7 from 7A<br>5 Inexperienced | 6 | 1 | 1 | 1 | 1–28 | 0.6, 1.0, 1.7 |
| 8A, C | (12,3) | Econ. 5, 6 or 7 | 7 | 1 | 1 | 1.3<br>0.1 | 1–6<br>>6 | 1 |
| 8B, D | (12,3) | Econ. 5, 6 or 7 and 8A | 7 | 1 | 1 | 0.1<br>1.3 | 1–6<br>>6 | 1 |
| 9A | (8,2) | 7 Experienced<br>1 None | 7 | 1 | 1 | $0.355h_{t-1}/p_{t-1}$ | 1–19 | 1 |
| 9B | (15,4) | 8 Econ. 9A | 7 | 1 | 0.1 | 1.3 | 1–14 | 1 |
| 9C | | 7 None | 7 | 1 | 0.1 | 1.3 | 1–17 | 1 |
| 10A | (13,4) | 7 Experienced | 7 | 1 | 0.1 | 1.5 | 1–19 | 1 |
| 10B | | 6 None | 7 | 1 | 0.1 | 1.5 | 1–15 | 1 |
| 11A, C | (12,3) | Experienced | 7 | 1 | 1 | 1.3<br>0.1 | 1–6<br>>6 | 1 |
| 11B, D | (12,3) | Experienced | 7 | 1 | 1 | 0.1<br>1.3 | 1–6<br>>6 | 1 |
| 12A | (14,4) | Experienced | 6 | 1 | 1 | $(2/3)h_{t-1}/p_{t-1}$ | 1–14 | 1 |
| 12B | (14,4) | Econ. 12A | 6 | 1 | 6.225 | $(2/3)h_{t-1}/p_{t-1}$ | 1–6 | 1 |
| 12C | (14,4) | Econ. 12A, B | 6 | 1 | 1 | 1 | 1–18 | 1 |

Econ. 2: At the end of period 13, experimenter announced a change in deficit $d$ from 0.25 to 1.25 to become effective at the beginning of period 18, and that no further changes will occur until the end.

Econ. 3: At the outset of this economy, subjects were informed that there will be no parameter changes between the beginning and termination.

Econ. 4: At the outset of this economy, experimenter announced that a change in $\omega^1$ from 7 to 3, and a change in $d$ from 1.25 to 0.25 will be effective beginning period 8, and that there will be no further parameter changes until termination. Low ISS inflation rate before the change was equal to the high ISS inflation rate after the change.

Econ. 6: One of the four possible values of $\beta_i$ (0.6, 0.6, 1.667, 1.667) was randomly assigned to the four members of each generation. While the discount rate was random for each individual, it was the same for every generation.

**Table 1.** Footnote (*Continued*)

| | |
|---|---|
| 7A: | Per capita real deficit was adjusted each period using formula $d_t = h_{t-1}(\pi^* - 1)/\pi^* p_{t-1}$, where $h_t$ is per capita money supply in period $t$ and $\pi^*$ is the target rate of inflation. $\pi^*$ was set equal to 1.5, the same as constant consumption rate of inflation in Economy 7B with $d = 1$. In Economy 7A, first period deficit was set $d_1 = 1$. |
| 8ABCD: | In Economies 8A and 8C, real deficit was 1.3 per capita during the first 6 periods and was lowered to 0.1 beginning period 7. The planned change in deficit was announced at the beginning. In Economies 8B and 8C, the real deficit was 0.1 during the first 6 periods and was raised to 1.3 beginning period 7. The planned change was announced at the beginning. |
| 9A: | This economy was similar to 7A except that the target inflation rate $\pi^*$ was 1.55 same as the constant consumption inflation in Economies 9B and C. Fist period deficit in 9A was $d_1 = 1.3$. |
| 11ABCD: | Repeat of Economies 8A, B, C and D respectively. |
| 12A: | This economy is similar to 7A and 9A except that the target rate of inflation $\pi^*$ was equal to the High ISS rate of 3.00. First period deficit in 12A was $d_1 = 1$. |
| 12B: | This economy is effectively a continuation of 12A. $H_0$ in this economy was set to one millionth of the value of $h_{14}$ in economy 12A. Also, initial deficit $d$ was set to 1.115 on the basis of price and money supply in the final period of 12A. |

**Table 2.** Stationary Equilibria of Experimental Economics

| Economy No. Period | Low ISS $\pi$ | $S$ | $u$ | High ISS $\pi$ | $S$ | $u$ | Constant Consumption $\pi$ | $S$ | $u$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 (1–19) | 1.21 | 2.90 | 3.73 | 5.79 | 0.60 | 2.66 | 1.18 | 3.25 | 3.75 |
| 2 (1–17)** | 1.09 | 2.95 | 3.87 | 6.41 | 0.30 | 2.65 | 1.09 | 3.13 | 3.88 |
| (18–33) | 2.00 | 2.50 | 3.18 | 3.50 | 1.75 | 2.81 | 1.53 | 3.63 | 3.38 |
| 3 (1–17) | 2.00 | 2.50 | 3.18 | 3.50 | 1.75 | 2.81 | 1.53 | 3.63 | 3.38 |
| 4 (1–7)** | 2.00 | 2.50 | 3.18 | 3.50 | 1.75 | 2.81 | 1.53 | 3.63 | 3.38 |
| (8–20) | 1.50 | 0.75 | 1.84 | 2.00 | 0.50 | 1.77 | 1.29 | 1.13 | 1.88 |
| 5 | 1.17 | 2.92 | 1.78 | 6.00 | 0.50 | 0.015 | 1.15 | 3.21 | 1.80 |
| 6A&B | 1.56 | 2.72 | 1.91 | 4.49 | 1.25 | 0.40 | 1.39 | 3.49 | 1.95 |
| 7A* | — | — | — | — | — | — | 1.50 | 3.00 | 2.57 |
| 7B | 2.00 | 2.00 | 1.97 | 3.00 | 1.50 | 0.99 | 1.5 | 3.00 | 2.52 |
| 8A&C (1–6)** 8B&D (> 6)** | 2.16 | 2.42 | 1.64 | 3.24 | 1.88 | 0.73 | 1.55 | 3.65 | 2.36 |
| 8B&D (1–6)** 8A&C (> 6)** | 1.035 | 2.98 | 4.01 | 6.77 | 0.12 | 0.00 | 1.034 | 3.05 | 4.01 |
| 9A* | — | — | — | — | — | — | 1.55 | 3.65 | 2.36 |
| 9B&C | 2.16 | 2.42 | 1.63 | 3.24 | 1.88 | 0.72 | 1.55 | 3.65 | 2.36 |
| 10A | Nonexistence | | | Nonexistence | | | 1.67 | 3.75 | 1.76 |
| 11A&C (1–6)** 11B&D (> 6)** | 2.16 | 2.42 | 1.64 | 3.24 | 1.88 | 0.73 | 1.55 | 3.65 | 2.36 |
| 11B&D (1–6)** 11A&C (> 6)** | 1.035 | 2.98 | 4.01 | 6.77 | 0.12 | 0.00 | 1.034 | 3.05 | 4.01 |
| 12A&B* | — | — | — | 3.00 | 1.50 | 0.94 | — | — | — |
| 12C | 2.00 | 2.00 | 2.30 | 3.00 | 1.50 | 0.94 | 1.5 | 3.00 | 3.24 |

* Economies 7A and 9A targeted the inflation rate corresponding to the constant consumption inflation associated with the deficit level used in period 1 ($d_1$). Economies 12A and B targeted $\pi^H$ associated with $d_1$.

** Deficit levels ($d_t$) in Economies 8A&C and 11A&C were changed from 1.3 to 0.1 in period 7. In Economies 8B&D and 11B&D, the switch was from 0.1 to 1.3.

Futher details about the design and rationale of these economies are discussed in the following section along with the results. A summary of procedures and instructions for subjects used in one of the economies (Economy 3) is enclosed as Appendix I to this paper. Instructions used in all other economies were variations on this basic form. Complete details are available from the authors on request.

Imrohoroglu 1993 econometrically estimated the above equilibrium model for the German interwars hyperinflation. He inferred that the German economy followed a nonstationary REE path towards the high inflation steady state, $\pi^H$. Under this steady state ($\pi^H$), increased deficit is the appropriate prescription for reducing inflation.

We examined the same model in an experimental environment. Figure 2 summarizes the data from nine economies.[2] Our results are in striking contrast to the results of the econometric studies because we observed no signs of nonstationary REE paths in the data. We did observe nonstationary paths but they tended towards-or somewhat below-the Low ISS, $\pi^L$. For example, in Economies 9B and 9C, the High ISS is 224 percent and the Low ISS is 116 percent. Figure 3 shows two nonstationary paths (solid squares) converging from below towards the Low ISS.

Given the initial price, $P_1$, the indeterminacy problem disappears because there is a unique REE path through $P_1$ (at time 1). Figure 3 also shows the theoretical REE path (marked by $x$) when agents coordinate their beliefs consistently with $P_1$ and the rational expectations hypothesis.

How does one explain the difference between the results of econometric studies of the German hyperinflation and our experiments? Is the behavior of our subjects (students at the University of Minnesota and at Carnegie Mellon University) very different from the behavior of Germans before the war? We do not believe that there

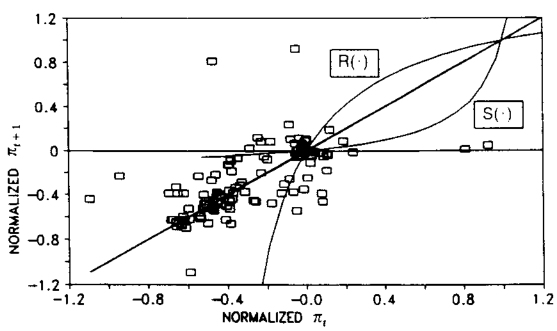


**Figure 2.** Inflation patterns (economies 1, 3, 5, 6A, 6B, 7B, 9B, 9C, and 12C).

[2] We have normalized inflation rates using the transformation $\hat{\pi}_t = (\pi_t - \pi^L)/(\pi^H - \pi^L)$. The $S(\beta)$ and $R(\beta)$ maps are obtained from the normalized values of Economy 6. Table 1 summarizes the description of the experimental economies.

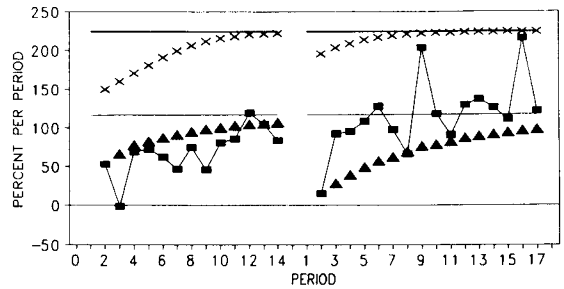


**Figure 3.** Inflation (economies 9B and 9C). Legend: —■— actual; —— low ISS; —— high ISS; × RE path; ▲ LSQ path.

are errors in either the econometric work, or in our own experiments. Nor do we believe the source of differences to be cultural.

There is crucial difference between the work of the econometrician and of the experimenter. In testing a rational expectations equilibrium model the econometrician imposes equilibrium restrictions to estimate the parameters of the model.

In contrast, the experimenter knows the parameters of the model and does not impose equilibrium restrictions.[3] In the model of inflation studied here, if one imposes RE equilibrium restrictions on nonstationary data with well defined stationary asymptotic properties, one is left with only one class of equilibrium processes – the nonstationary REE paths converging to $\pi^H$. If one were to perform the same econometric exercise on (possibly longer time series) generated by our Economy 9, it too might estimate a different model with $\pi^H \simeq 116$ percent. Alternatively, an econometrician could estimate an adaptive model for which adaptive paths converge to the Low ISS, and with the (longer) time series generated by our Economy 9 he would then arrive at a new model with $\pi^L \simeq 116$ percent[4]. The experimenter is not subject to this estimation problem, and it is in this sense that our data can shed some light on which equilibria are most frequently observed.

As discussed in Marimon and Sunder 1993 the data seem to be consistent with adaptive learning models for which inflation paths with initial conditions in the region $(0, \pi^H)$ converge to $\pi^L$. Nevertheless, we tend to observe randomness and certain tendency of the young agents to oversupply the consumption good. These features do not appear in learning models where agents automatically solve their maximization problems. Learning to make good predictions seems to come faster than learning to submit competitive supplies.

[3] It is not our intention to make a sweeping argument about the superiority of one method over the other. Each has its strengths and weaknesses.

[4] As Marcet and Sargent 1989 have pointed out, the asymptotic properties of these models make their estimation a nontrivial exercise.

## 4 Economies with a targeted rate of inflation

We consider now a regime where the monetary policy defines a level of inflation as its target and the fiscal policy adapts to this target. We preserve the OLG structure of the previous section. More precisely, given an inflation target $\pi^*$, at the beginning of each period the government determines the level of per capita government expenditures for the period according to

$$d_t = \left(\frac{\pi^* - 1}{\pi^*}\right)\frac{h_{t-1}}{p_{t-1}}.$$

That is, if $r^* = \frac{\pi^* - 1}{\pi^*}$, and recalling that per capita real balances at $t$ are denoted by $s_t$, then the fiscal policy follows the rule:

$$d_t = r^* s_{t-1} \tag{8}$$

We can substitute (8) in (2), and solve the equilibrium system (1), (2), (3) and (8) as we did in the previous section. Provided that $\pi_t^e \neq b$, we also obtain that current inflation is a function of expected inflation for the current and the next period. That is,

$$\pi_t - \phi(\pi_{t+1}^e, \pi_t^e) = 0, \tag{9a}$$

where

$$\phi(\pi_{t+1}^e, \pi_t^e) = \frac{b - \pi_t^e}{\frac{b}{\pi^*} - (\pi_{t+1}^e - \pi_t^e r^*)} \tag{9b}$$

or

$$\phi(\pi_{t+1}^e, \pi_t^e) = \pi^* \frac{b - \pi_t^e}{(b - \pi_t^e) - \pi^*(\pi_{t+1}^e - \pi_t^e)}. \tag{9c}$$

Equation (9) characterizes the $T$ map between expected and realized inflation paths. The fixed points of this map, i.e., the rational expectations equilibria, are characterized by the following difference equation:

$$(1 - r_{t+1})^{-1} = b(r_t - r^*) + (1 - (r_t - r^*))(1 - r_t)^{-1} \tag{10}$$

where $(1 - r_t)^{-1} = \pi_t$. Equation (10) can be written as $\pi_{t+1} = Q(\pi_t)$. It has two stationary solutions given by $\pi_t = \pi^*$ and $\pi_t = b$, $(b = \alpha\omega^1/\gamma\omega^1)$. Figure 4 shows the $Q(\cdot)$ map. The stationary solution $\pi^* = b$ corresponds to the autarkic solution $s_t = 0$. That is, nonstationary rational expectations paths tend to a long-run equilibrium in which money has no value. As we have said, this model is a general version of the stationary environment with zero deficit (i.e., $\pi^* = 1$).

The study of this particular adaptive policy has been inspired by the experimental results from economies with constant inflation. Agents' behavior in such economies is consistent with *adaptive learning*. The adaptive policy studied here has the following property. Suppose all agents believe inflation will take some constant value (not necessarily the target value) in the following two periods. That is, agents

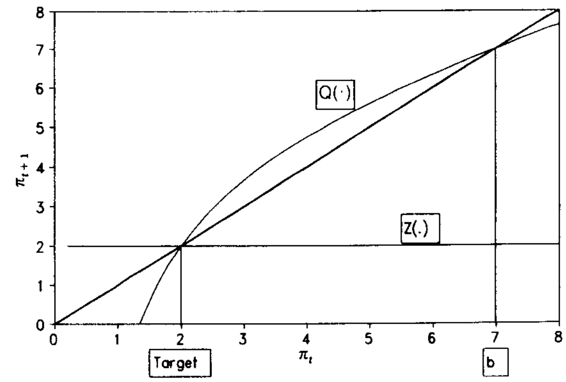


**Figure 4.** Dynamics with target inflation.

form their expectations according to

$$\pi_t^e = \pi_{t+1}^e = \beta. \tag{11}$$

Then (2) and (8) result in

$$\pi_t = \pi^*. \tag{12}$$

That is, if $Z(\cdot)$ maps the current expected rate of inflation into the current realized inflation, then

$$Z(\beta) = \pi^*. \tag{13}$$

It is a constant map. In other words, if agents coordinate their beliefs in a particular inflation rate $\beta$, then the realized inflation rate is the target inflation $\pi^*$. It is in this sense that the government follows an adaptive economic policy.

We can consider instead that agents forecast inflation rates using a first order adaptive scheme. That is,

$$\pi_{t+1}^e = \pi_t^e + \alpha_t(\pi_{t-1} - \pi_t^e). \tag{14}$$

Notice that we take into account that agents know $\pi_{t-1}$, but not $\pi_t$, when they have to forecast $\pi_{t+1}$. This general adaptive scheme encompasses many specific learning algorithms. Least squares learning takes this form with $\alpha_t$ converging to zero at the rate of $1/t$ if agents forecast inflation rates, and $\alpha_t$ converging to a constant, $\alpha \in (0, 1)$, if agents forecast prices. Now, by (9), we obtain,

$$\pi_{t+1}^e = \pi_t^e + \alpha_t(\phi(\pi_t^e, \pi_{t-1}^e) - \pi_t^e). \tag{15}$$

We can study the local stability of the above difference equation around the steady state $\pi^*$. Let $\hat{\pi}_t^e = \pi_t^e - \pi^*$, then we have,

$$\hat{\pi}_{t+1}^e = (1-\alpha_t)\hat{\pi}_t^e + \alpha_t\left[\pi^* \frac{\pi^*(\hat{\pi}_t^e - \hat{\pi}_{t-1}^e)}{(b - (\hat{\pi}_{t-1}^e + \pi^*)) - \pi^*(\hat{\pi}_t^e - \hat{\pi}_{t-1}^e)}\right].$$

Linearizing this difference equation system at the steady state $\pi^*$ we obtain

$$\begin{bmatrix} \hat{\pi}^e_{t+1} \\ \hat{\pi}^e_t \end{bmatrix} = \begin{bmatrix} (1-\alpha_t) + \alpha_t \dfrac{(\pi^*)^2}{b-\pi^*} & -\alpha_t \dfrac{(\pi^*)^2}{b-\pi^*} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{\pi}^e_t \\ \hat{\pi}^e_{t-1} \end{bmatrix}$$

As we can see, the stability properties depend on the parameter $b = \frac{\omega^1}{\omega^2}$, the target inflation rate $\pi^*$, and, possibly, the adaptive parameter $\alpha_t$.

We design four economies (7A, 9A, 12A and 12B) with the above government policy (which is publicly known) with two purposes in mind. First, we wish to study whether the findings described in Section 3 extend to this class of economies, and, second, to give our subjects experience with specified inflation levels.[5]

Figures 5 and 6 show the inflation patterns and evolution of deficits for the experimental economies. Figure 5 also shows the REE path characterized by equation (10) and passing through the first realized inflation rate, $\pi_1$, and an adaptive path characterized by equation (15) with arbitrary initial values.[6] Our experimental economies clearly differ from the REE nonstationary paths and show more volatility than simple adaptive learning algorithms. Although inflation rates tend toward the prespecified targets, deficit levels- shown in Figure 6- are larger than the predicted by simple adaptive dynamics. Part of this bias may be due to the fact that our experimental economies is not a "representative agent" deterministic economy. That is, there is endogenous uncertainty due to the iteraction of a relatively small number of agents who submit savings schedules. In contrast, in the law of motion described by equation (15) there is no uncertainty: point expectations generate competitive supplies which determine realized inflation rates which, in turn, are used to form adaptive expectations. In a stochastic environment with a unique asset, savings also play an insurance role, therefore the presence of uncertainty results in higher savings and since, under the fiscal policy considered here, deficits are proportional to past savings, it follows that deficits should also be higher as a result of uncertainty.

## 5 Policy changes: adaptation and anticipation

The results of the preceding sections provide a strong empirical argument in favor of "classical" prescriptions. In particular, in the constant deficit environment of Section 2, as long as the economy settles around the Low ISS, a reduction in the government deficit will lower the stationary level of inflation. But to analyze policy

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[5] It is always an issue whether subjects use their past experience when they are "born" ("reborn"), in an experimental economy, e.g., whether U.S. students might behave differently than Argentinian students. Target inflation experiments provided the subjects with "common experience" around some critical inflation values (see Table 2).

[6] Economies 7A and 9A are locally stable, around $\pi^*$, for all values of $\alpha$, while Economies 12A and 12B are locally stable if, either $\alpha < 0.5$ or $\alpha_t$ decreases fast enough. For the adapted paths simulated in Figures 5 and 6 we have taken the following values: Economies 7A and 9A $\alpha = 0.5$, $\pi^e_0 = 100$, $\pi^e_1 = 80$; Economies 12A and 12B $\alpha = 0.2$, $\pi^e_0 = 220$, $\pi^e_1 = 180$.

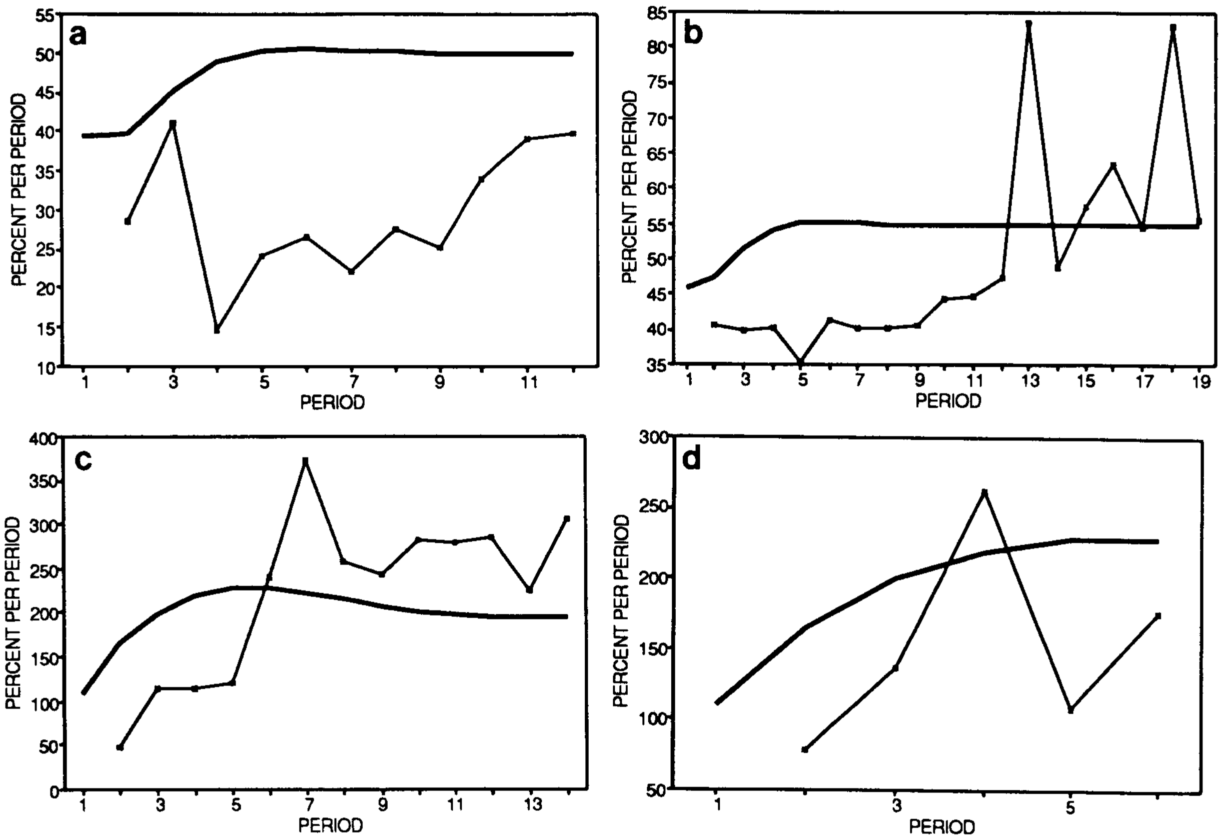


**Figure 5.** **a** Inflation paths in target economy 7A (target inflation 50 percent). **b** Inflation paths in target economy 9A (target inflation 55 percent). **c** Inflation paths in target economy 12A (target inflation 200 percent). **d** Inflation paths in target economy 12B (target inflation 200 percent). Legend: —●— actual; —— adaptive path.

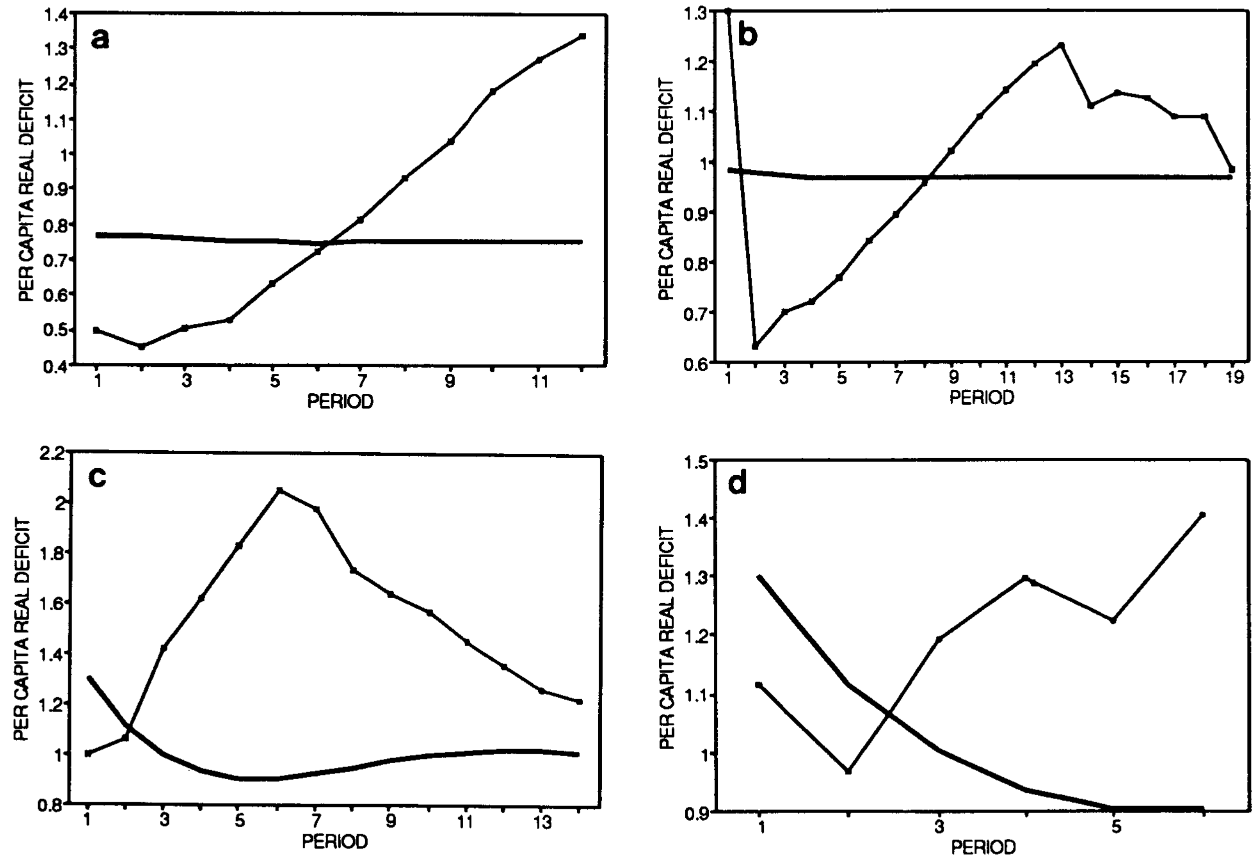


**Figure 6.** **a** Deficit paths in target economy 7A (target inflation 50 percent). **b** Deficit paths in target economy 9A (target inflation 55 percent). **c** Deficit paths in target economy 12A (target inflation 200 percent). **d** Deficit paths in target economy 12B (target inflation 200 percent). Legend: —●— actual; —— adaptive path.

changes we must carry out such changes and/or describe the paths from an old regime to a new regime.

We can expand the analysis of Section 2 to incorporate changes in government policies. Here we are mainly interested in changes that are announced in advance and that, for the sake of simplicity, occur only once in an economy. Under the rational expectations hypothesis some form of anticipation should be observed along an equilibrium path that takes into account the announced changes. In Appendix II we analyze REE paths with announced changes. In particular, it is

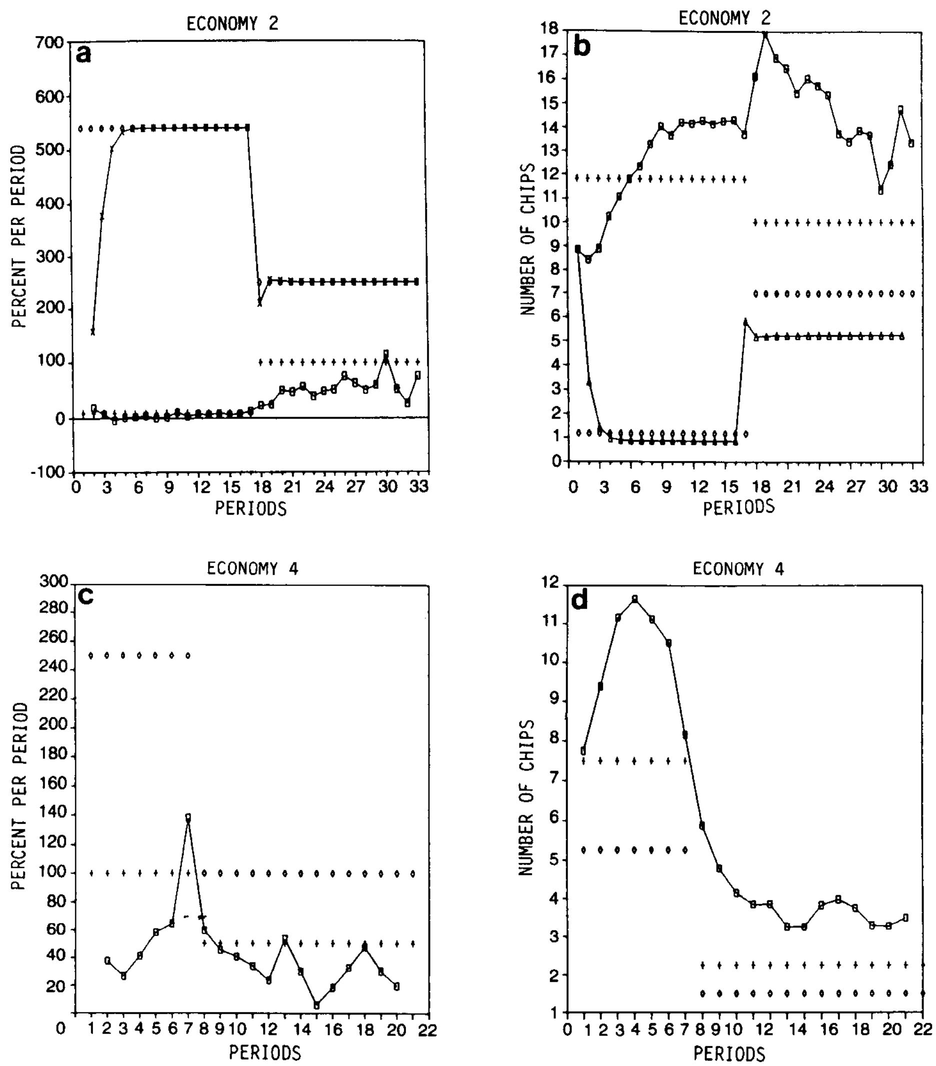


**Figure 7.** **a** Inflation (economy 2). Legend: □ actual; + low ISS; ◇ high ISS; × RE path. **b** Volume of trade (economy 2). Legend: □ actual; + low ISS; ◇ high ISS; △ RE path. **c** Inflation (economy 4). Legend: □ actual; + low ISS; ◇ high ISS. **d** Volume of trade (economy 4). Legend: □ actual; + low ISS; ◇ high ISS.

possible to describe the nonstationary paths that converge to the high ISS of the after-the-change economy. From Section 2 (see also Appendix II), there is no REE path describing a transition between the before- and after-the-change low ISSs. That is, a "classical" policy prescription based on a deficit reduction cannot be thought of as a prescription for following a given equilibrium path. Nevertheless, even if agents follow adaptive learning rules (hence, converge to the Low ISS), it is possible that they may also learn to anticipate the effect of well defined- and previously experienced- policy changes.

We designed several economies for which there was an announcement of a single change in regime (except for Economy 4, the change was simply a change in government deficit; in Economy 4 $\omega^1$ was also changed). This change was always announced in period zero (except for Economy 2, in which the announcement was made at the end of period 13). For Economies 2 and 4 our aim was twofold: to study behavior under announced policy changes, and to expose our subjects – in case they behave adaptively – to some critical inflation values. In particular, the change in Economy 4 is designed so "before-the-change Low ISS" was equal to the "after-the-change High ISS." Figure 7 shows the inflation patterns of Economies 2 and 4. In both economies, we observe a change in the behavior of prices (inflation) and quantities (volume of trade) before and after the change of regime. The more experienced subjects of Economy 4 seemed to react faster to the change. (There is no clear evidence that the decrease in volume of trade in period 7 is due to an anticipation effect. In contrast, it seems that the less experienced subjects of Economy 2 found themselves "oversupplying" when prices suddenly increased following the increase in deficit.)

Sessions 8 and 11 yielded a sequence of four economies each, labeled 8A, B, C and D, and 11A, B, C and D respectively. In each economy there is a once-only change in the level of deficit that is announced in period zero. There are two possible levels of per capita deficit (0.1 and 1.3) and in every sequence the deficit at the beginning (before the change) of a new economy is the same as the deficit at the end (after the change) of the preceding economy. With this stationary formulation of announcements and changes we tried to capture possible learning patterns, especially whether agents learn to anticipate changes. Given the "short life span" of the generations, the anticipation effect can only be detected one period before the deficit changes. Except that when there is a change from high to low deficit (i.e., an increase in the High ISS) there is no monetary rational expectations equilibrium that converges to the ex-post High ISS. In particular, the entering generation before the change that foresees a high enough inflation prefers not to sell any chips.

Figure 8a shows the behavior of inflation in these economies and compares the observed data with the REE path that, starting at the observed initial price, converges to the ex-post High ISS (when this equilibrium path exists). The LSQ path from the observed initial path that adapts to the change once it has taken place is shown in Figure 8b, along with the Low ISS and the data.

Unfortunately, a design that requires a sequence of economies within an experimental session results in relatively short individual economies. While we observe substantial changes, and in particular, ex-post adaptation of inflation rates, the data are volatile. The economies are too short, even for the LSQ

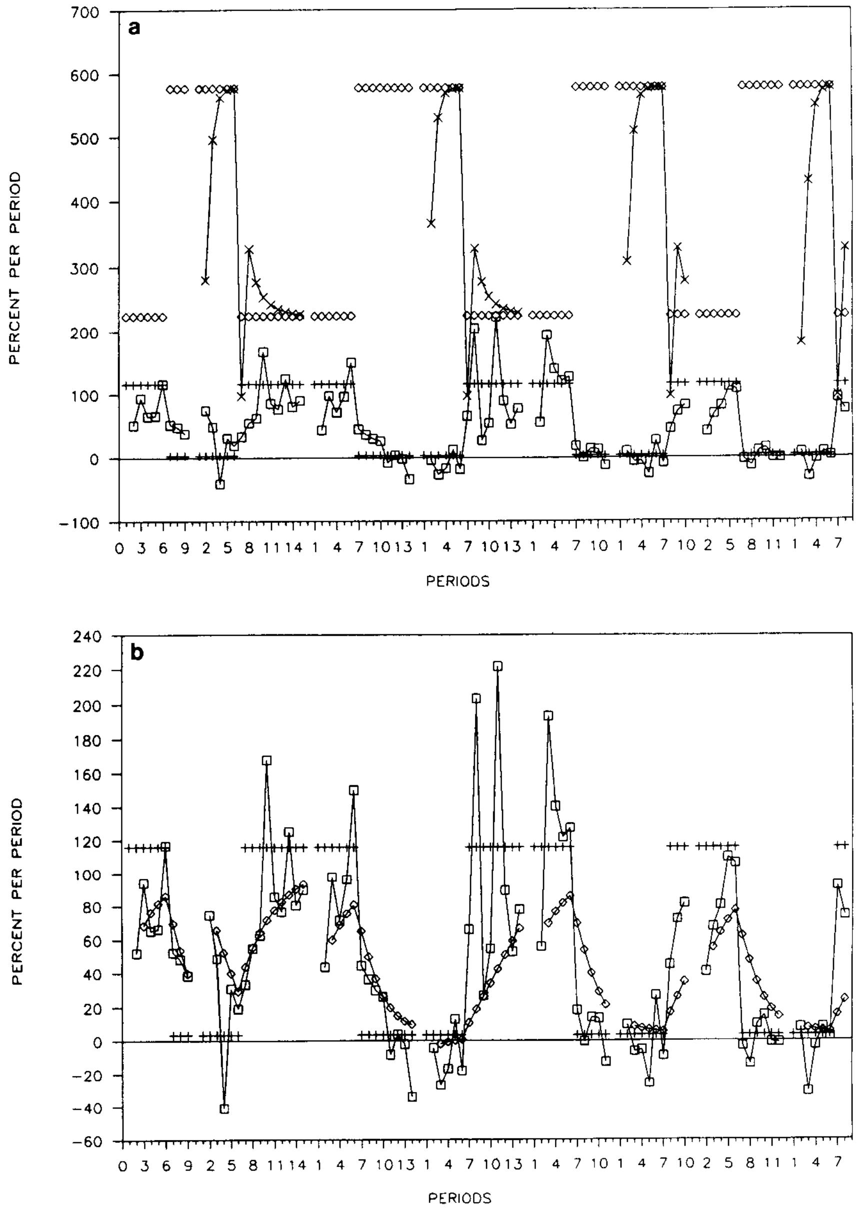


**Figure 8.** **a** Inflation with rational expectations path (economies 8 and 11). Legend: □ actual inflation; + low ISS; ◇ high ISS; × RE path. **b** Inflation with least squares path (economies 8 and 11). Legend: □ actual inflation; + Low ISS; ◇ LSQ path.

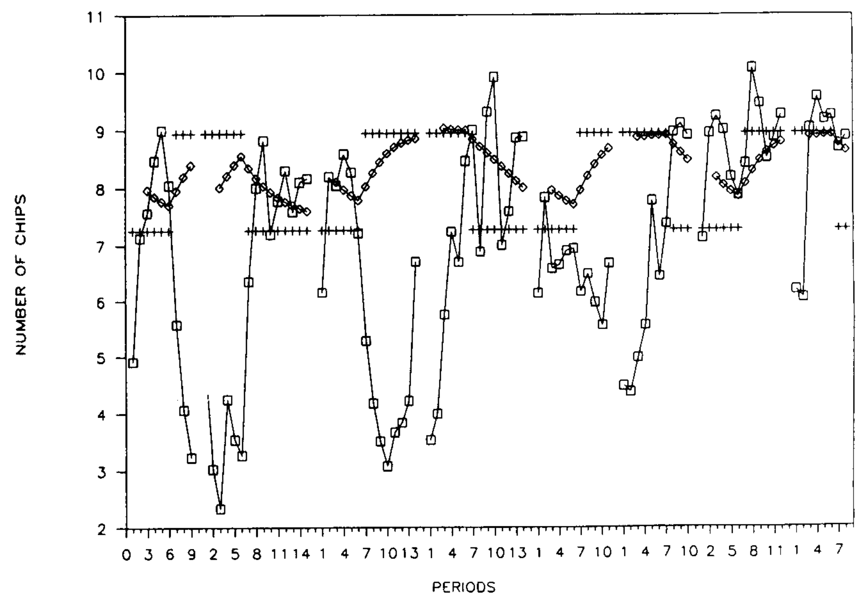


**Figure 9.** Volume of trade (economies 8 and 11). Legend: □ actual; + low ISS; ◇ LSQ path.

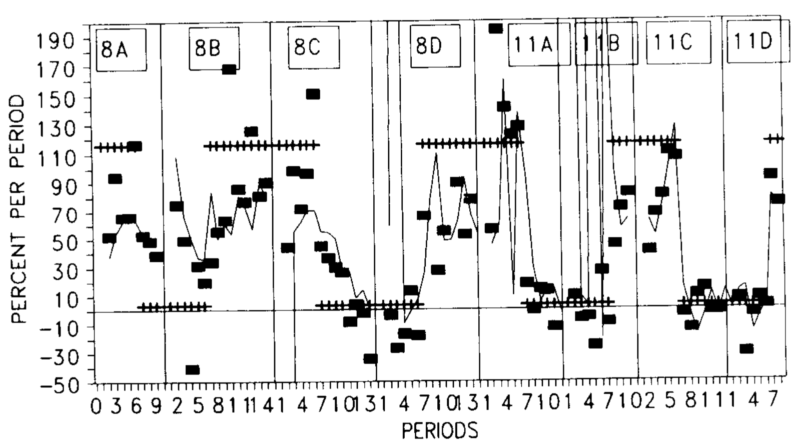


**Figure 10.** Inflation with subjects' predictions (economies 8 and 11). Legend: ■ actual; + low ISS; — mean prediction.

paths to reach the corresponding stationary levels. Inflation patterns are qualitatively consistent with adaptive behavior–such as LSQ on prices. Nevertheless, the evolution of volume of trade, a good place to observe anticipation effects, is fairly erratic and only at the end of the sequence does Economy 11 show a fairly consistent adaptive behavior in Figure 9.

Figure 10 shows subjects' predictions of inflation rates (they were only asked about the next period prices). The predictions rapidly adapt to changes and are fairly close to the ex-post observations.

## 6 Parsimonious behavior in an inflationary environment

Figure 11 shows the inflation paths of two realizations for two different economies (Economies 9B and C and 10A and B). These are the type of data that an econometrician confronts (often with longer time series). Although inflation rates in Economy 10B are somewhat more volatile, it will be difficult to reject the null hypothesis that both pairs of realizations come from fairly similar economies. There is, however, a fundamental difference between them. While Economies 9B and C have a sustainable level of deficit (hence two stationary REE, see Table 2), the level

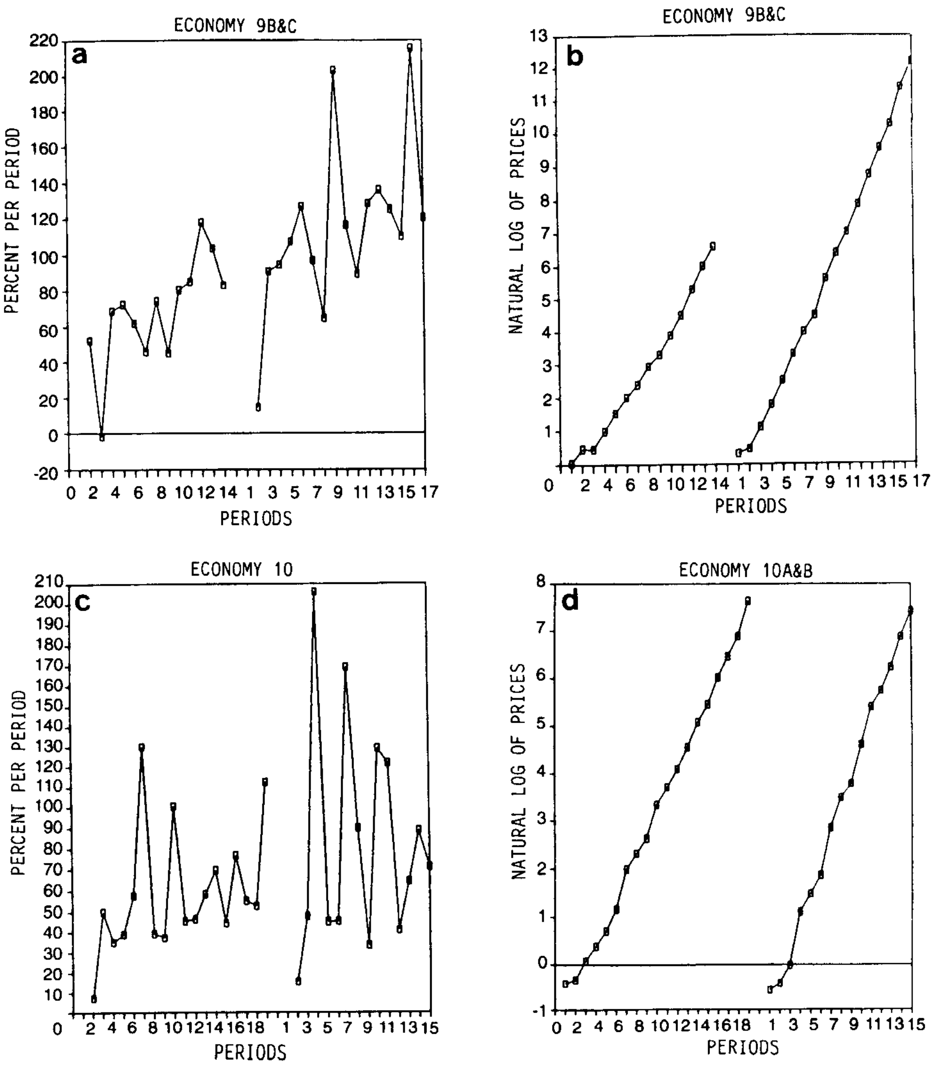


**Figure 11.** **a** Inflation (economies 9B and 9c). **b** Price level (money/chip for economies 9B and 9C). **c** Inflation (economies 10A and B). **d** Price level (money/chip for economies 10A and 10B).

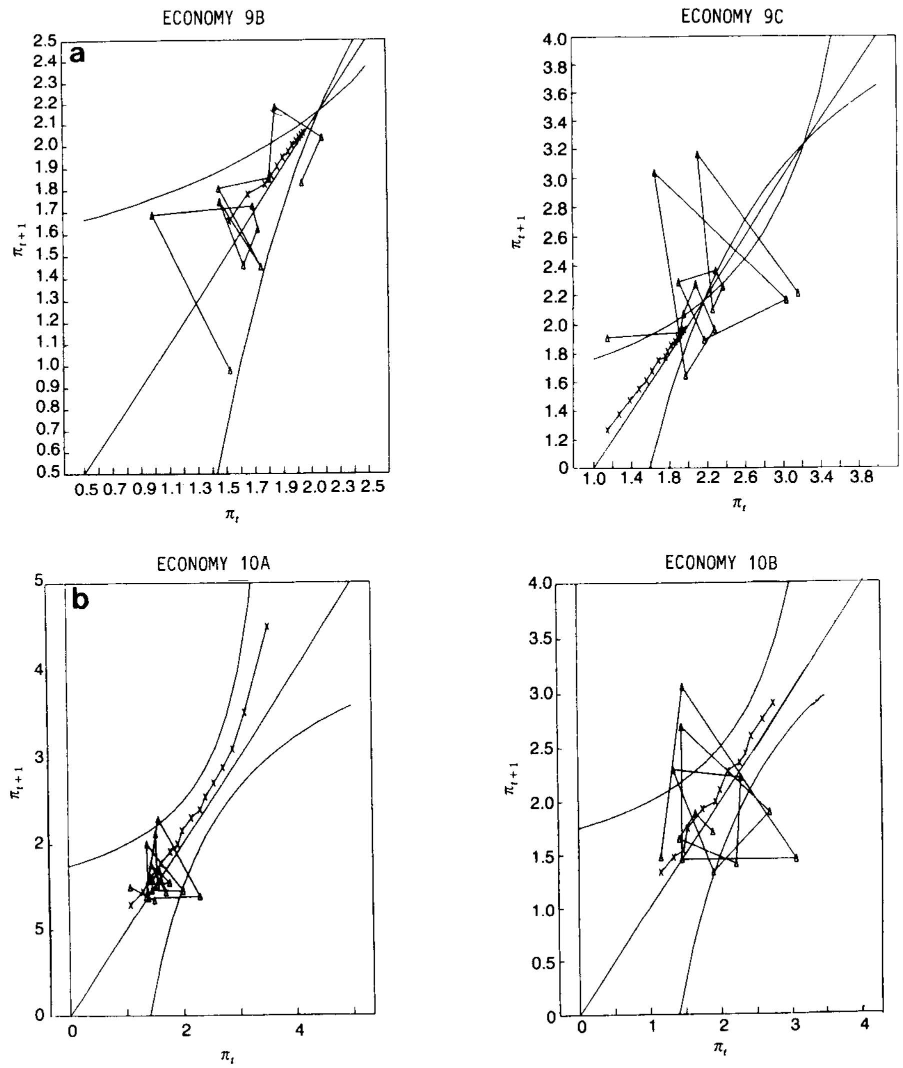


**Figure 12.** **a** Inflation patterns and least squares paths (economies 9B and 9C). **b** Inflation patterns and least squares paths (economies 10A and 10B). Legend: △ inflation pattern, × LSQ path

of deficit in Economies 10A and B ($d = 1.5$) is not sustainable at any stationary REE (i.e., there is no stationary equilibrium in these economies). With no stationary equilibria, both the REE paths and the LSQ paths, are unstable,. Nevertheless, as can be seen in Figure 11, observed data are fairly parsimonious.

It is interesting to compare Figure 12b for Economy 10A and B with Figure 12a for Economy 9B and C. In the latter, around the low inflation steady state, the experimental data show more variability than the deterministic- convergent LSQ process. In contrast, in economies with no stationary state, it is the experimental data that show more stationarity. It is as if our experimental economies enjoy ergodicity properties which are not present in the simple difference equation models (REE and LSQ), due to the accumulation of mismatched expectations, errors in individual optimizations and a supply mechanism that allows for some self-insurance. Note that in Economy 10 inflation rates tend to cluster between the constant consumption inflation rate (67 percent) and what would have been the unique stationary equilibrium at the maximum sustainable level of deficit (i.e., 165 percent at $d = 1.354$). It is not clear, however, whether the inflationary process would remain in this region if the economy were to last for many more periods. Evans and Ramey 1992 have suggested that the type of persistence shown in this data may be consistent with a model where it is costly for agents to revise their expectations.

*Universitat Pompeu Fabra, University of Minnesota, Cambridge University CPER and NBER and Carnegie Mellon University.*

## APPENDIX I

### A brief description of experiment

These experiments were conducted on student subjects at the University of Minnesota and Carnegie Mellon University. Economies 1–4 were conducted manually, with computer assistance to determine market clearing price and allocations; all other economies were conducted on a network of computers with each student seated at a terminal to receive information on the video screen and to enter his/her decisions through the keyboard.

$N > 3n$ subjects were recruited for each session. Of these, $n$ subjects each played the role of "young" and "old" generations respectively, while the remaining $N - 2n > n$ subjects waited as interested onlookers. At the beginning of each period, $n$ subjects were randomly picked from this waiting pool to enter before the subjects exiting in the previous period were added to the waiting pool. This process ensured that every subject had to wait a random number of periods (minimum 1) between exiting the economy and reentering it as young again.

After reading and explaining the instructions (instructions for Economy 1 follow this narrative), the subjects participated in four periods of a trial economy. Fiat money was labeled " francs" and the consumption good was labeled "chips." The number of chips "consumed" were converted into U.S. dollars at the end of the exit period of each subject. Total dollars accumulated in this manner were paid to subjects at the end of the session. Most sessions lasted for about three hours, and subjects took home 25–30 dollars on average from each session.

All onlookers were asked to predict the market clearing price (francs per chip) at the beginning of each period. Entering subjects received an endowment of $\omega^1$ chips and no francs and were asked to specify a seven or eight point supply function for chips. Computer constructed a market supply function from the individual supplies of the young, a market demand function from the money balances of the old and the real deficit of the government, computed the market clearing price and allocations, and distributed this information. Most accurate predictor of market clearing price received a one or two dollar prize each period. When experimenter terminated an economy without advance warning, franc balances were converted into chips at the average predicted price for the next (unplayed) period.

**Instructions**

This is an experiment in decision-making. Various research foundations have provided funds for this research. The instructions are simple, and if you follow them carefully and make good decisions, you might earn a considerable amount of money which will be paid to you in cash.

In this experiment, we are going to have a market in which you may buy and sell chips in a sequence of market periods. Attached to these instructions you will find sheets labeled Information and Record Sheet, Selling Offer Sheet and Market Price Prediction Sheet which help you record your decisions and determine their value to you.

The type of currency used in this market is francs. The only use of this currency is to buy and sell chips. It has no other use. The money you take home with you is in dollars. The procedures for determining the number of dollars you take home with you is explained later in these instructions.

You will participate in the market for two consecutive periods at a time. Let us call the first of these periods your entry period (because you begin your participation then) and the second of these periods your exit period (because you end your participation in the market). Diffferent individuals may have different entry and exit periods and the experimenter will inform you about when you will enter and exit the market. You may be asked to enter and exit more than once depending on the number of periods for which the market is operated.

At the beginning of your entry period, you will receive 7 chips from the experimenter and at the beginning of your exit period you will receive 1 chip. In your entry period, you may keep these chips or sell chips to others. In your exit period, you can buy more chips from others but you cannot sell. Buying and selling of chips will occur in francs according to the rules to be explained later.

The product of the number of chips you hold at the end of trading each period determines the amount of money you earn for that pair of entry-exit periods. The experimenter will calculate the square root of the product and multiply it by $1.25 to calculate the amount of dollars you earn. Thus, suppose you hold 5 chips at the end of your entry poriod and 3.5 chips at the end of your exit period. The product of these two numbers is $5 \times 3.5 = 17.5$.The square root of 17.5 which is multiplied by $1.25 to yield $5.22 as your earning in these two periods. Note that the higher the product of the numbers of chips held by you at the end of entry and exit periods, the higher is the profit you earn. Also note that if you hold zero chips at the end of either period, your profits will be zero because the product of zero with any other finite number is zero. All chips are returned to the experimenter at the end of each period.

The first period of the market will be an entry period for some of you (as described above). For some of you, however, this first period itself will be an exit period and you will receive the exit period endowment of 1 chip at the beginning of this period. In addition, each of you for whom the first period is an exit period will receive 10 francs from the experimenter at the beginning of this period. You have to use all these francs to buy chips during the exit period because the francs you hold at the end of an exit period are worthless; they cannot be converted into dollars directly.

When you sell chips, your holding of chips decreases and your holding of francs increases by the amount of the price of the chips. Similarly, when you buy chips, your holding of chips increases and your holding of francs vanishes. At the end of each period, all your chips on hand are used up to earn profits in dollars and thus returned to the experimenter. The francs you have on hand at the end of the entry period are carried over to the exit period and used to buy chips in this latter period.

All outside-market players participate in the market directly. At the beginning of each period, each outsider-market player predicts the market price of the period. The average of the predicted price will be used to convert the francs held by the entry-period players to chips at the end of the experiment. A $2.00 prize will be given to the player whose prediction is the closest to the actual market price. If there is a tie, the prize will be split.

*Trading and recording rules*

(1) All entry-period players are sellers and all exit-period players are buyers.

(2) Every exit-period player must pay all his francs to entry-period players in exchange for chips at a market price determined below.

(3) At the beginning of each period, every entry-period player must state the following prices on the Selling Offer Sheet and submit it to the experimenter. If the prices you submit are not nondecreasing in the number of chips offered, we shall make them so.

Price below which you don't want to sell any chips_francs/chip:
Price at which you are willing to sell up to 1 chip_francs/chip:
Price at which you are willing to sell up to 2 chips_francs/chip:
Price at which you are willing to sell up to 3 chips_francs/chip:
Price at which you are willing to sell up to 4 chips_francs/chip:
Price at which you are willing to sell up to 5 chips_francs/chip:
Price at which you are willing to sell up to 6 chips_francs/chip:
Price at which you are willing to sell up to 7 chips_francs/chip:

(4) The experimenter collects the Selling Offer Sheets from all entry-period players and buys 2 chips for himself each period. After considering the amount of francs available from the exit-period players, offers made by the entry -period players and his own need for 2 chips each period, he computes and announces the market clearing price. Exit-period players and the experimenter pay this price for each chip they buy. Each entry-period player will be informed of the number of chips he/she has been able to sell at the market price, and each exit-period player will be told of the number of chips that he/she has been able to buy with his/her francs on hand.

Note that if you (entry-period player) do not specify a price for zero chip, up to one chip of yours may be sold at zero francs. If you do not want to sell more than a specified number of chips under any circumstances, specify a very high price. This is the only way you have of not wanting to sell. The actual number of chips you sell will almost always be in fractions, depending on the market clearing price. The way the market clearing mechanism works, if you are willing to sell, say two units at unit price x and 3 units at unit price y, you may end up selling, say 2.4 units at a price between $x$ and $y$.

(5) At the beginning of each period, each outside-market player writes down a predicted market price on the Market Price Prediction Sheet which is collected by the experimenter. At the end of each period, the experimenter announces average predicted market price and the winner(s)- the outside-market players whose prediction was the closest to the actual market price. This player records $2.00 prize on the Market Price Prediction Sheet. But, when there is more than one winner, the prize is split. All other outside-market players record $0 prize on the sheet.

(6) After the transaction information is received from the experimenter, each entry-period player computes the chips remaining on hand and the francs received from sale and records them on the Information and Record Sheet.

(7) Each exit-period player records the number of chips purchased on the Information and Record Sheet. Then the experimenter computes the product of the number of chips held by each exit-period player at the end of entry and exit periods respectively, takes the square root of the product and multiplies by $1.25. This amount is the profit of the exit-period player who records this profit on the Information and Record Sheet. At the conclusion of the experiment, the experimenter will pay each player the total amount of profits made.

(8) The francs received by the entry-period players in the entry period will be used to buy chips in the exit period which follows immediately. So, carry your francs on hand forward to the exit period by entering them in the column Beginning-Francs on Hand on the Information and Record Sheet.

(9) At the end of the experiment, francs held by all entry-period players are converted into chips using the average of predicted market prices by outside-market players.

(10) At the end of the experiment, add up the profit column of your Information and Record Sheet. The experimenter will pay you this amount of money.

## Appendix II

### Rational expectations paths with announced changes

In Economies 4, 8ABCD, and 11ABCD changes in parameters were announced several periods before they took effect. As a reference, we analyze the set of rational expectations equilibria when changes are announced in advance. The notation is as in Section 3. Recall that the evolution of prices and money holdings is given by

$$p_t = b_t^{-1} p_{t+1} + c_t h_t, \tag{16}$$

and

$$h_t = h_{t-1} + d_t p_t. \tag{17}$$

If there are no changes in endowments, $b_t = b$ and $c_t = c$, then equation (16) takes the form

$$p_t = c(1 - b^{-1}L^{-1})^{-1}h_t + k(b)^t \tag{18}$$

where $L$ is the lag operator and $k$ is a constant. That is,

$$p_t = c\sum_{n=0}^{\infty} b^{-n}h_{t+n} + k(b)^t \tag{19}$$

If the level of deficit is also unchanged, then equation (17) takes the form

$$p_t = d^{-1}(1 - L)h_t. \tag{20}$$

Combining (19) with $k = 0$ and (20) we obtain the equilibrium equation for money holdings,

$$(1 + b^{-1} - dc)h_t - b^{-1}h_{t+1} - h_{t-1} = 0 \tag{21}$$

where $h_{-1}$ is given. To obtain stationary solutions we must solve the quadratic equation

$$b^{-1}L^{-1}[-1 + (1 + b - bdc)L - bL^2]h_t = 0. \tag{22}$$

Suppose now that at the end of period $T - \tau$ it is announced that a new set of parameters, e.g., a new level of deficit, will describe the economy from period $T$ on, and that no further changes will take place.

If $\tau < T$ then, for $n = 0, \ldots, T - \tau - 1$, the economy can be considered an economy where there are no changes in parameters, assuming that agents believe this to be the case. This may not be an adequate assumption if agents have experienced unexpected announcements in the past. Without loss of generality we take $\tau = T$; that is, announcements are made in period zero.[7] Denote by $\tilde{b} = \tilde{\omega}^1/\tilde{\omega}^2$, $\tilde{c} = 2/\tilde{\omega}^1$, and $\tilde{d}$ the underlying parameters at $t = 0, \ldots, T - 1$, and $b$, $c$, and $d$, the corresponding parameters at $t \geq T$.

For $t \geq T$, equation (21) describes the evolution of money balances. For $t < T$, we see from (16) that

$$p_t = (\tilde{b}^{-1})^{T-t} \cdot p_T + \tilde{c}\sum_{n=0}^{T-1-t} \tilde{b}^{-n}h_{t+n}, \tag{23}$$

and substituting (19) we can express $p_t$ as

$$p_t = (\tilde{b})^{-(T-t)} \cdot c\sum_{n=0}^{\infty} b^{-n}h_{T+n} + \tilde{c}\sum_{n=0}^{T-1-t} \tilde{b}^{-n}h_{t+n}. \tag{24}$$

Let $R = c\sum_{n=0}^{\infty} b^{-n}h_{T+n}$, then substituting (24) in (17) we obtain

$$(1 - \tilde{d}\tilde{c})h_t - h_{t-1} - \tilde{d}\tilde{c}\sum_{n=1}^{T-1-t} \tilde{b}^{-n}h_{t+n} - d\tilde{b}^{-(T-t)} \cdot R = 0, \tag{25}$$

[7] Only in Economy 2 $\tau < T$, but it seems reasonable to assume that before $T - \tau$, agents believed that such announcement had zero probability.

and, similarly,

$$(1-\tilde{d}\tilde{c})h_{t+1}-h_t-\tilde{d}\tilde{c}\sum_{n=1}^{T-1-(t+1)}\tilde{b}^{-n}h_{t+1+n}-d\tilde{b}^{-(T-t-1)}\cdot R=0. \tag{26}$$

Multiplying (26) by $\tilde{b}^{-1}$ and subtracting from (25) results in

$$(1+\tilde{b}^{-1}-\tilde{d}\tilde{c})h_t-\tilde{b}^{-1}h_{t+1}-h_{t-1}=0. \tag{27}$$

Notice that for $t=T-1$ (25) takes the form

$$(1-\tilde{d}\tilde{c})h_{T-1}-h_{T-2}-\tilde{d}c\tilde{b}^{-1}\cdot\sum_{n=0}^{\infty}b^{-n}h_{T+n}=0, \tag{28}$$

and for $t=T$, (16) and (17) give

$$(1-dc)h_T-h_{T-1}-dc\sum_{n=1}^{\infty}b^{-n}h_{T+n}=0. \tag{29}$$

Now multiplying (29) by $\tilde{b}^{-1}$ and subtracting from (28) results in

$$(1+\tilde{b}^{-1}-\tilde{d}\tilde{c})h_{T-1}-(\tilde{b}^{-1}+(\tilde{d}-d)c\tilde{b}^{-1})h_T-h_{T-2}-(\tilde{d}-d)c\tilde{b}^{-1}=0. \tag{30}$$

That is, *a sequence of per capita money holdings $\{h_t\}_{t=0}^{\infty}$ defines a rational expectations equilibrium with initial money holdings $h_{-1}$ when there is a change of parameters from $(\tilde{\omega}^1,\tilde{\omega}^2,\tilde{d})$ to $(\omega^1,\omega^2,d)$, at period T and announced at the beginning of period zero, if it satisfies*:

For $t>T$,

$$h_{t+1}-ah_t+bh_{t-1}=0, \tag{31}$$

for $t=0,\ldots,T-2$

$$h_{t+1}-\tilde{a}h_t+\tilde{b}h_{t-1}=0, \tag{32}$$

$$(1-c\cdot\Delta d)h_T-\tilde{a}h_{T-1}+\tilde{b}h_{T-2}-c\cdot\Delta d\sum_{n=1}^{\infty}b^{-n}h_{T+n}=0, \tag{33}$$

and $h_{0-1}=h_{-1}$, where $b=\dfrac{\omega^1}{\omega^2}$, $a=1+b\cdot\dfrac{2d}{\omega^2}$, (similarly for $\tilde{b}$ and $\tilde{a}$) and $\Delta d=(d-\tilde{d})$.

By repeatedly substituting (31) into (33) we can express (33) in terms of $h_T$, $h_{T-1}$, $h_{T-2}$. Notice that

$$\sum_{j=0}^{n}b^{-j}h_{T+j}=h_T\sum_{m=1}^{n}\sum_{r=\max\{0,m-(n-m)\}}^{m}\binom{m}{r}(-1)^{(m+r)}\cdot b^{-m}a^r$$
$$-h_{T-1}\sum_{m=0}^{n-1}\sum_{r=\max\{0,m-(n-1-m)\}}^{m}\binom{m}{r}(-1)^{(m+r)}\cdot b^{-m}a^r,$$

or

$$\sum_{j=0}^{n}b^{-j}h_{T+j}=(h_T-h_{T-1})\sum_{m=1}^{n-1}\sum_{r=\max\{0,m-(n-1-m)\}}^{m}\binom{m}{r}(-1)^{(m+r)}\cdot b^{-m}\cdot a^r$$
$$+h_T\sum_{m=1}^{n}\chi\{m-(n-m)\geq 0\}\binom{m}{2m-n}(-1)^{3m-n}b^{-m}a^{2m-n}-h_{T-1}, \tag{35}$$

where $\chi\{m-(n)\geq 0\}$ is one if $m-(n-m)>0$ and it is zero otherwise.

Provided that (35) has a well defined limit as $n \to \infty$, we can write $\sum_{j=0}^{\infty} b^{-j} h_{T+1} = Ah_{T-1}$, that is,

$$(1 - c \cdot \Delta d(1 + A))h_T - (\tilde{a} - c \cdot \Delta dB)h_{T-1} + \tilde{b}h_{T-2} = 0. \tag{36}$$

Given an equilibrium path for $t \geq T$, for example one of constant inflation, (36) and (32) characterize the "anticipated reaction" to the announced changes. Of particular interest, given our empirical results, is whether there are equilibrium paths with constant inflation for $t = 0, \ldots, T-1$ that converge to a new level of constant inflation after the change has taken place.

An immediate consequence is that if $\Delta d > 0$, as in many of our experimental economies, and there is path of constant inflation for $t = 0, \ldots, T-1$ corresponding to the lower root of (32) then there is no converging path after $T$. More generally, (36) shows that if, for $t = 0, \ldots, T-1$, $h_t = \pi^t \cdot h_0$ then $h_T \neq \pi^T h_0$ whenever $\Delta d \neq 0$. For example, in Economy 4 even if 2.00 is the lower root of (31) and the higher root of (32) there is no equilibrium path of constant inflation through the change in parameters.

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