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ACCURACY OF LINEAR VALUATION RULES IN INDUSTRY-SEGMENTED ENVIRONMENTS
Industry- vs. Economy-Weighted Indexes*

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The comparative ability of valuation rules using economy-weighted versus industry-weighted price indexes to estimate the unobserved economic value of a basket of assets is modelled. Industry-weighted indexes do not necessarily provide valuations of higher accuracy than economy-weighted indexes. Dominance depends on (1) the relative magnitude of the mean and variability of price changes and (2) the magnitude of errors of measurement in the current price data. Larger measurement errors favor economy-weighted indexes; larger mean and variability of prices changes favor industry-weighted indexes.

1. Introduction

Accounting valuation rules can be modelled as linear aggregations and comparisons between different valuation rules can be made using a classical econometrics approach.[1] This study extends linear aggregation model of asset valuation to industry-segmented economies. We specifically model and discuss the comparative accuracy of two valuation systems, one with economy-weighted price indexes and a second with industry-weighted price indexes.

When Statement of Financial Accounting Standards No. 33 (SFAS 33) was in effect, many firms used industry-weighted price indexes instead of economy-weighted price indexes to estimate the current cost of their assets

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[1]See, for example, Ijiri (1967, 1968), Sunder (1978), Sunder and Waymire (1984), Tippett (1987), and Lim and Sunder (1990).

[Arthur Young (1980), Perry and Searfoss (1984)]. This choice might have been driven by an intuitive belief that price indexes constructed by the use of industry-specific weights should yield more accurate valuation of firms' assets. An analysis of the statistical accuracy of such valuation rules in this paper reveals that the industry-weighted indexes do not necessarily improve accuracy.

The mean squared error between the current economic value of firms' assets and their estimated value, obtained by application of a valuation rule, is used as an inverse measure of the accuracy of the rule. This valuation error is decomposable into two additive components, one arising from the movement of prices and the other from errors in the measurement of price changes. The first component of error *decreases* when economy-weighted price indexes are substituted by industry-weighted price indexes. The second component *increases* when such substitution is made. The sign and magnitude of the net effect of such substitution on accuracy of valuation depends on the relative magnitudes of mean and variance of price changes and the variance of measurement error in price changes.

The statistical approach to valuation we use here originated in the work of Ijiri (1967, 1968) and was developed further in Sunder (1978), Sunder and Waymire (1984), and Lim and Sunder (1990). While empirical analysis of accounting data produced under the requirements of SFAS 33 has received much attention in the accounting literature, a theoretical framework for analysis has been conspicuously absent. Using a classical econometrics approach, the present study and those referenced above present a coherent system for the examination of the econometric properties of accounting valuation rules. Several empirical studies, e.g., Tritschler (1969), Hall (1982), Sunder and Waymire (1983), and Shriver (1986, 1987), have provided empirical support for several theoretical conclusions derived in the above literature.

This paper is organized as follows. Section 2 specifies the model of valuation rules as linear aggregations and defines their mean squared error as an inverse measure of accuracy in valuation. Some relevant properties of valuation rules derived in prior work are summarized. In section 3, properties of valuation rules that use industry-weighted price indexes are analyzed and compared to the properties of economy-weighted valuation rules in industry-segmented environments. The conclusions are summarized in section 4.

2. Linear aggregation model of asset valuation rules[2]

Consider an economy with n distinct goods. Let $\boldsymbol{q}^*$ be the vector of quantities of the n goods contained in a given basket or firm. Suppose under

[2]Most of the discussion in this section is based on Sunder (1978) and Lim and Sunder (1990).

a given valuation rule, P^0 is the valuation of the basket at time 0 and P^1 at time 1. The relative change in value of the basket is $R = (P^1 - P^0)/P^0$.

Let $\boldsymbol{r}$ be the n-vector of relative price changes from time 0 to time 1 for the n goods where $r_j = (p_j^1 - p_j^0)/p_j^0$, $j = 1, 2, \ldots, n$. If valuation of each good in the bundle is determined by multiplying its historical value by a price index specific to each good, the resultant number R is the relative change in the current value of the bundle. This value of R is denoted by R_{CV} and is defined as the principal aggregation,

$$R_{CV} = \boldsymbol{w}'\boldsymbol{r}, \tag{1}$$

where $w_i = p_i^0 q_i^* / (\sum_{j=1}^n p_j^0 q_j^*)$ for $i = 1, 2, \ldots, n$, p_i^0 = unit price of good i at time 0, and q_i^* = quantity of good i in the firm.

As defined above, $\boldsymbol{w}$ is the vector of relative weights of various goods in a firm. For the purpose of valuation, vector $\boldsymbol{w}$ is a complete characterization of each firm since asset valuation is expressed in terms of relative price changes.

Two subscripts are used on R to identify a specific valuation rule. The first subscript k takes integer values from 1 to n and denotes the number of mutually exclusive price indexes used to adjust the period 0 value of the goods in the basket to period 1 estimates. For the purpose of forming price indexes, all n goods could be combined into a single price index ($k = 1$), or divided into two groups ($k = 2$), or, in the extreme case, into $k = n$ groups with each good being in a group by itself. Except for $k = 1$ and $k = n$, there are multiple ways of partitioning n goods into k nonempty, mutually exclusive groups. The number of possible ways of forming k $(2 \le k \le n-1)$ price indexes from n goods in the economy, $S(n,k)$, is given by[3]

$$S(n,k) = \sum_{j=0}^{k} \{-1)^j (k-j)^n\} / \{j!(k-j)!\}.$$

Suppose that these index configurations have been arranged in some arbitrarily fixed order. The second subscript on R specifies one of these $S(n,k)$ distinct configurations used to estimate the value of the bundles of assets. R_{kl} denotes a valuation rule which uses lth of the $S(n,k)$ possible k-index configurations in an n-good economy. The corresponding partition of the set of n goods into k nonempty, nonoverlapping (orthogonal) subsets is denoted by Π_{kl}.[4] For general price level valuation ($k = 1$) and current valuation ($k = n$), there is only one way each of forming indexes, $S(n,1) = S(n,n) = 1$, and these valuation rules are denoted by $R_{1,1}$ and $R_{n,1}$, respectively.

[3] $S(n,k)$ is called Stirling Number of the Second Kind. For details, see Apostol (1967, p. 594).

[4] The indexes are assumed to be orthogonal (or mutually exclusive). Nonorthogonal indexes can be orthogonalized by, say, the Gram–Schmidt method [see Lipschutz (1974, p. 283)].

Let $\tilde{r}$, the measured price relative vector, be the sum of the true price relative r and measurement error ε: $\tilde{r} = r + \varepsilon$, where

$$\mathrm{E}(r) = \mu = (\mu_1, \mu_2, \ldots, \mu_n)',$$

$$\mathrm{var}(r) = \mathrm{E}(r - \mu)(r - \mu)' = \Sigma = [\sigma_{ij}], \qquad i, j = 1, 2, \ldots, n,$$

$$\mathrm{E}(\varepsilon) = \mathbf{0},$$

$$\mathrm{var}(\varepsilon) = \mathrm{E}(\varepsilon\varepsilon') = \Delta = [\delta_{ij}], \qquad i, j = 1, 2, \ldots, n,$$

and a tilde ($\tilde{}$) denotes variables measured with error. Thus $\tilde{R}_{kl}$ denotes an estimate of R_{kl} based on the price relative vector $\tilde{r}$ which includes errors.

Following the notation used previously, we use the term mean squared error (MSE) for the mean squared difference between the valuation $\tilde{R}_{kl}$ based on price indexes formed by partition Π_{kl} and the unobserved current economic valuation $R_{n,1}$,

$$\mathrm{MSE}(\tilde{R}_{kl}) = \mathrm{E}_w \mathrm{E}_r \mathrm{E}_\varepsilon (\tilde{R}_{kl} - R_{n,1})^2 = \mathrm{E}_w \mathrm{E}_r \mathrm{E}_\varepsilon (\omega^{kl\prime}\tilde{r} - w'r)^2, \tag{2}$$

where $\omega^{kl} \equiv (\omega_1^{kl}, \omega_2^{kl}, \ldots, \omega_n^{kl})'$ for partition Π_{kl} and valuation rule R_{kl}, $\omega_j^{kl} \equiv (\omega_j w_j^*)/\omega_j^*$ for $j = 1, 2, \ldots, n$, $\omega_j \equiv$ relative proportion of good j in the economy, $w_j^* \equiv$ total weight in w of all elements which are included in the same index as good j, and $\omega_j^* \equiv$ total weight in ω of all goods which are included in the same index as good j. Note that the expectation is taken with respect to the distribution of w to get the economy-wide average of mean squared error. This economy-wide average given by (2) can be used as an inverse measure of accuracy of valuation rules.[5]

Under the assumptions that (a) the vector w of relative weights of individual firms is the result of ρ multinomial[6] draws with a population parameter given by the economy-wide vector of relative proportions ω, (b) r has expected value μ and variance–covariance matrix Σ, and (c) ε has expected value zero and variance–covariance matrix Δ, the mean squared error of a valuation rule $\tilde{R}_{kl}$ can be written as the following expression:

$$\begin{aligned}\mathrm{MSE}(\tilde{R}_{kl}) = {} & (1 - 1/\rho)\omega'\Delta\omega + (1/\rho) \\ & \times \left\{ \sum_{u=1}^{k} \omega_u' \Delta_{uu} \omega_u / \omega_u' e + \omega'(\sigma + \dot{\mu}) \right. \\ & \left. - \sum_{u=1}^{k} \omega_u' (\Sigma_{uu} + \mu_u \mu_u') \omega_u / \omega_u' e \right\},\end{aligned} \tag{3}$$

[5] A numerical example of the determination of the index (weighting) configurations is found in Tippett (1987, pp. 143–145).

[6] For the properties of the multinomial distribution, see Freeman (1963, p. 129).

where

$\Delta \equiv n \times n$ covariance matrix of measurement error $\boldsymbol{\varepsilon}$,
$\Delta_{uu} \equiv n_u \times n_u$ submatrix of Δ corresponding to goods included in the uth index, where n_u is the number of goods in the uth index,
$\boldsymbol{\omega}_u \equiv$ subvector of $\boldsymbol{\omega}$ corresponding to goods included in the uth index,
$\boldsymbol{e} \equiv$ vector of unit elements with appropriate length,
$\boldsymbol{\sigma} \equiv n$-vector of diagonal elements of Σ,
$\dot{\boldsymbol{\mu}} \equiv n$-vector of squared elements of $\boldsymbol{\mu}$ where $\mathrm{E}(\boldsymbol{r}) = \boldsymbol{\mu}$,
$\boldsymbol{\mu}_u \equiv$ subvector of $\boldsymbol{\mu}$ consisting of elements included in the uth index, and
$\Sigma_{uu} \equiv n_u \times n_u$ submatrix of Σ corresponding to goods included in the uth index.

This mean squared error of a valuation rule is decomposable into two additive components: price movement error and measurement error. Price movement error decreases and measurement error increases monotonically with the fineness of the index system used in the valuation rule.[7]

Examples of valuation rules

Price indexes are formed by taking an appropriately weighted average of the elements of the vector $\boldsymbol{r}$ of relative price changes for individual goods. In this paper, we consider two weighting schemes. The first weighting scheme uses the economy-wide relative weights, $\boldsymbol{\omega}$, to form price indexes. This index set is identical for all firms in the economy. The second weighting scheme substitutes the economy-wide relative weights, $\boldsymbol{\omega}$, by an industry-specific vector of relative weights, $\boldsymbol{\omega}^I$, for the firms that belong to industry I. Accordingly, the index set used for valuation is identical for all firms within an industry but differs across industries. Since each firm is represented by the vector of relative proportions in its asset portfolio, $\boldsymbol{w}$, its valuation is a function of $\tilde{\boldsymbol{r}}$, $\boldsymbol{w}$, and $\boldsymbol{\omega}$ in the first case and of $\tilde{\boldsymbol{r}}$, $\boldsymbol{w}$, and $\boldsymbol{\omega}^I$ in the second case.

To illustrate, consider a four-good ($n = 4$) economy in which the vector of relative proportions of the goods is $\boldsymbol{\omega}' = (0.2, 0.4, 0.3, 0.1)$. Suppose over an arbitrary interval of time, the relative change in the prices of these four goods is given by $\tilde{\boldsymbol{r}}' = (0.05, 0.08, -0.02, 0.10)$. There are seven ways of forming two price indexes in a four-good economy. One of them is to combine, say, the first and the fourth goods into one index and the second and the third into another. The change in each price index of this set can be reckoned as

[7]Price index system 1 is finer than price index system 2 if and only if all the goods included in each price index in the set 1 are also included in some price index of the set 2. For a five-good set, for example, index set {(1, 2), (3), (4), (5)} is finer than {(1, 2), (3, 4), (5)} but is not finer than {(1), (2, 3), (4), (5)}.

follows:

Price index set 1

Index 1 = $(0.05 \times 0.2)/(0.2 + 0.1) + (0.1 \times 0.1)/(0.2 + 0.1) = 0.0667$,
Index 2 = $(0.08 \times 0.4)/(0.4 + 0.3) + (-0.02 \times 0.3)/(0.4 + 0.3) = 0.0371$.

Suppose a firm belongs to an industry that does not carry good 4 and the vector of relative proportions for the industry I is given by $\boldsymbol{\omega}^{I\prime} = (0.3, 0.2, 0.5, 0)$. Using these industry-specific weights and same partition of goods as used above, the two indexes take the following values:

Price index set 2

Index 1 = $(0.05 \times 0.3)/(0.3 + 0) + (0.1 \times 0)/(0.3 + 0) = 0.05$,
Index 2 = $(0.08 \times 0.2)/(0.2 + 0.5) + (-0.02 \times 0.5)/(0.2 + 0.5) = 0.0086$.

Consider a firm in industry I whose portfolio of assets is described by the vector of relative proportions $\boldsymbol{w}' = (0.25, 0.3, 0.45, 0)$. In order to determine the firm's valuation using these price indexes, each element of $\boldsymbol{w}$ is multiplied by the price index in which that good is included. The sum is the resultant valuation:

Price index set 1
$(0.25)(0.0667) + (0.3)(0.0371) + (0.45)(0.0371) + (0)(0.0667) = 0.0445$,

Price index set 2
$(0.25)(0.05) + (0.3)(0.0086) + (0.45)(0.0086) + (0)(0.05) = 0.01895$.

Furthermore, the current valuation of the firm is obtained by taking the product of $\boldsymbol{w}$ and $\tilde{\boldsymbol{r}}$

$$(0.25)(0.05) + (0.3)(0.08) - (0.45)(0.02) + (0)(0.1) = 0.0275.$$

Since $\tilde{\boldsymbol{r}}$ can have measurement errors, the current valuation is not equal to the true but unobserved current economic value of the firm's assets. In the next section we analyze and compare the statistical properties of valuation rules based on industry-specific price indexes (e.g., index set 2) and economy-wide price indexes (e.g., index set 1) and the ability of each to approximate the unobserved current economic value of assets.[8]

[8]For an alternative approach that derives error bounds in presence of measurement errors, see Tippett (1987). A second difference between our and Tippett's approach is that we take accountants' standard weights as a given in our analysis, while Tippett seeks the minimum mean squared error weights from an optimizing program for individual firms.

3. Valuation in an industry-segmented economy

Consider an economy which has n goods and let the relative proportions of the ith good in the economy be given by the ith element of vector $\boldsymbol{\omega}$, where $\omega_i = p_i^0 q_i / \sum_{j=1}^{n} p_j^0 q_j$ for $i = 1, 2, \ldots, n$, $\boldsymbol{p}^0$ is the vector of prices at time 0, and $\boldsymbol{q}$ is the unchanging vector of quantities of n goods in the economy.[9]

Each industry, I, in the economy is characterized by a vector of relative proportions, $\boldsymbol{\omega}^I$, of n assets in the industry. We assume that $\boldsymbol{\omega}^I$ is a random vector of relative proportions that results from ρ_I independent multinomial draws with population parameter $\boldsymbol{\omega}$. From properties of multinomial distribution, the expectation and variance of $\boldsymbol{\omega}^I$ are given by

$$\mathrm{E}(\boldsymbol{\omega}^I) = \boldsymbol{\omega},$$

$$\operatorname{cov}\left(\omega_i^I, \omega_j^I\right) = \begin{cases} (1-\omega_i)\omega_i/\rho_I & \text{if} \quad i=j, \\ -\omega_i \omega_j/\rho_I & \text{if} \quad i \neq j. \end{cases} \tag{4}$$

As ρ_I increases, the variance of $\boldsymbol{\omega}^I$ decreases and the asset composition of the industry gets closer to the asset composition of the economy. Therefore, ρ_I can be described as the diversification parameter for the industry; as ρ_I increases without bound, $\boldsymbol{\omega}^I$ approaches $\boldsymbol{\omega}$ in the limit.

Each firm in the economy belongs to one of the industries and is characterized by its vector $\boldsymbol{w}$ of relative asset proportions. We assume that $\boldsymbol{w}$ for a firm in industry I is a random vector of relative proportions that results from ρ_F independent multinomial draws with population parameter $\boldsymbol{\omega}^I$. Again, from the properties of multinomial distribution, conditional expectation and variance of $\boldsymbol{w}$ are given by

$$\mathrm{E}(\boldsymbol{w} | \boldsymbol{\omega}^I) = \boldsymbol{\omega}^I,$$

$$\operatorname{cov}\left(w_i, w_j | \boldsymbol{\omega}^I\right) = \begin{cases} \left(1-\omega_i^I\right)\omega_i^I/\rho_F & \text{if} \quad i=j, \\ -\omega_i^I \omega_j^I/\rho_F & \text{if} \quad i \neq j. \end{cases} \tag{5}$$

As ρ_F increases without bound, $\boldsymbol{w}$ approaches $\boldsymbol{\omega}^I$ in the limit. Accordingly ρ_F can be characterized as the diversification parameter for firms. The larger

[9]We assume that all quantities and vectors of proportions such as $\boldsymbol{w}$, $\boldsymbol{\omega}$, and $\boldsymbol{\omega}^I$ remain unchanged over the time interval considered. Imposition of this static assumption permits us to explore asset valuation in industry-segmented economies.

the value of ρ_F, the less dissimilar is the asset composition of firms within each industry.

Consider a k-index asset valuation rule that uses a partition Π_{kl} of n goods into k nonempty subsets. We compare the accuracy of valuation of firms' assets under two different ways of assigning weights to individual goods in formation of price indexes. One is the relative abundance of goods in the portfolio of the respective industries to which firms belong. Expressions (6) and (7) approximate[10] the economy-wide average of the mean squared error of the valuation $\bar{R}_{kl}$ under industry weights (MSE^I) and under the economy weights (MSE^E), respectively, using $\boldsymbol{\delta}$ for the vector of diagonal elements of Δ,

$$\begin{aligned}\mathrm{MSE}^I(\bar{R}_{kl}) \cong\ & (1 - 1/\rho_F - 1/\rho_I + 1/\rho_F\rho_I)\boldsymbol{\omega}'\Delta\boldsymbol{\omega} \\ & + \boldsymbol{\omega}'(\boldsymbol{\sigma} + \dot{\boldsymbol{\mu}})/\rho_F + (1/\rho_I - 1/\rho_F\rho_I)\boldsymbol{\omega}'\boldsymbol{\delta} \\ & - (1/\rho_F)\sum_{u=1}^{k}\sum_{i\in u}\sum_{j\in u}\gamma_{ij}^u(\sigma_{ij} + \mu_i\mu_j - \delta_{ij}),\end{aligned} \tag{6}$$

where (i) for $i = j$, $j \in u$,

$$\begin{aligned}\gamma_{ij}^u = (1/\omega_i^*)\{\omega_i/\rho_I + (1 - 1/\rho_I)\omega_i^2\} \\ \times\Big[2 + (1-\omega_i^*)/\rho_I\omega_i^* \\ - \Big\{1 + (\rho_I - 1)(\rho_I - 2)\omega_i^2 + 3(\rho_I - 1)\omega_i \\ + \sum_{t\neq i;\, t\in u}((\rho_I - 1)(\rho_I - 2)\omega_i\omega_t + (\rho_I - 1)\omega_t)\Big\} \\ /\{\rho_I + \rho_I(\rho_I - 1)\omega_i\}\omega_i^*\Big],\end{aligned}$$

[10] The need to add 'approximation' qualification here and elsewhere in the paper arises from the Pearson's (1897) approximation of the expectation of the ratio of two random variables. For a discussion of the magnitude of the approximation error in (6), see footnote 13 in the appendix.

(ii) for $i \neq j$ and $i, j \in u$,

$$\gamma_{ij}^{u} = (1/\omega_i^*)\{(1 - 1/\rho_I)\omega_i\omega_j\} \times \left[2 - (1/\rho_I\omega_i^*)\left\{1 + \omega_i^* + (\rho_I - 2)(\omega_i + \omega_j) + \sum_{t \neq i, j, t \in u} (\rho_I - 2)\omega_t\right\}\right],$$

and

$$\begin{aligned}\mathrm{MSE}^E(\tilde{R}_{k1}) = {} & \{1 - 1/\rho_F - 1/\rho_I + 1/\rho_F\rho_I\}\boldsymbol{\omega}'\Delta\boldsymbol{\omega} \\ & + (1/\rho_F + 1/\rho_I - 1/\rho_F\rho_I) \\ & \times \left\{\boldsymbol{\omega}'(\boldsymbol{\sigma} + \dot{\boldsymbol{\mu}}) - \sum_{u=1}^{k} \boldsymbol{\omega}_u'(\Sigma_{uu} + \boldsymbol{\mu}_u\boldsymbol{\mu}_u' - \Delta_{uu})\boldsymbol{\omega}_u/\boldsymbol{\omega}_u'\boldsymbol{e}\right\}.\end{aligned} \tag{7}$$

Proofs for the above expressions are given in the appendix. It is also shown that the excess of mean squared error of valuation from economy-weighted indexes over valuation from industry-weighted indexes is given by

$$\begin{aligned}& \mathrm{MSE}^E(\tilde{R}_{kI}) - \mathrm{MSE}^I(\tilde{R}_{kI}) \\ & \cong (1/\rho_I - 1/\rho_F\rho_I)\boldsymbol{\omega}'(\boldsymbol{\sigma} + \dot{\boldsymbol{\mu}} - \boldsymbol{\delta}) - (1/\rho_F + 1/\rho_I - 1/\rho_F\rho_I) \\ & \quad \times \sum_{u=1}^{k} \boldsymbol{\omega}_u'(\Sigma_{uu} + \boldsymbol{\mu}_u\boldsymbol{\mu}_u' - \Delta_{uu})/\boldsymbol{\omega}_u'\boldsymbol{e} \\ & \quad + (1/\rho_F)\sum_{u=1}^{k}\sum_{i \in u}\sum_{j \in u} \gamma_{ij}^{u}(\sigma_{ij} + \mu_i\mu_j - \delta_{ij}),\end{aligned} \tag{8}$$

where γ_{ij}^{u} is defined in (6).

If industry-weighted indexes yield more accurate valuation than economy-weighted indexes, expression (8) must be positive. When ρ_I approaches infinity, i.e., each industry is completely diversified, so its asset composition becomes identical with the economy, mean squared errors (6) and (7) converge to equality and to expression (3) which is derived under the assumption that there is no industry segmentation of the economy.[11,12]

When the price data used to construct indexes are measured without error ($\Delta = 0$) and ρ_I is relatively large, the excess mean squared error (8) is nonnegative. Conversely, when prices of goods remain unchanged ($\boldsymbol{\mu} = \mathbf{0}$, $\Sigma = 0$) with relatively large ρ_I, the expression (8) is nonpositive. This result leads to Theorems 1 and 2.

Theorem 1. When price data used to form price indexes are measured without error ($\Delta = 0$), *a valuation rule based on industry-weighted price indexes has mean squared error which is approximately as small or smaller than a valuation rule using economy-weighted price indexes provided that both index sets are based on the same partition of n goods in the economy and the assets of the industry are well-diversified* (ρ_I *is relatively large).*

Theorem 2. When prices do not change ($\boldsymbol{\mu} = \mathbf{0}$, $\Sigma = 0$) *but measurement error is present, industry-weighted price indexes have approximately as large or larger mean squared error of valuation as the economy-weighted price indexes provided that both index sets are based o the same partition of n goods in the economy with relatively large* ρ_I.

As shown in the appendix, when $k = 1$ (general price level valuation) and $k = n$ (current valuation), we can do away with approximation in (6) and derive exact expressions for $\mathrm{MSE}^I(\tilde{R}_{k1})$. The stronger results for these special cases are stated in Theorems 3 and 4.

[11]As ρ_I approaches infinity, from the expresion (6),

$$\gamma_{ij}^u = \begin{cases} \left\{\omega_i^2\left(2 - \omega_i/\omega_i^* - \sum_{t \neq i;\, t \in u} \omega_t/\omega_i^*\right)\right\}/\omega_i^* & \text{for } i = j, \quad i, j \in u, \\ \left\{\omega_i\omega_j(2 - (\omega_i + \omega_j)/\omega_i^* - \sum_{t \neq i, j;\, t \in u} \omega_t/\omega_i^* \right. & \text{for } i \neq j, \quad i, j \in u, \end{cases}$$

$$= \begin{cases} \omega_i^2/\omega_i^* & \text{for } i = j, \quad i, j \in u, \\ \omega_i\omega_j/\omega_i^* & \text{for } i \neq j, \quad i, j \in u. \end{cases}$$

Therefore, with a large ρ_I, $\mathrm{MSE}^I(\tilde{R}_{kl})$ is equivalent to (3).

[12]Also note that (7) for MSE^E can be derived from (3) if ρ is replaced by $\rho_I\rho_F/(\rho_I + \rho_F - 1)$ (denoted ρ^*). Thus, diversification parameter ρ for an unsegmented economy is equivalent to ρ^* in an industry-segmented economy. Moreover, ρ^* is necessarily less than or equal to ρ_I as well as ρ_F. Suppose the diversification parameter in an unsegmented economy is $\rho = 1{,}000$. Now consider a segmented economy with $\rho_I = 2{,}000$. In order for a firm in this segmented economy to be as diversified as in the unsegmented economy, ρ_F will have to be equal to $\rho(\rho_I - 1)/(\rho_I - \rho) = 1{,}999$.

Theorem 3. *Under the general price level or the current valuation, when price indexes are measured without error ($\Delta = 0$), the mean squared error of industry-weighted price indexes is smaller than or equal to that of economy-weighted price indexes.*

Theorem 4. *Under the general price level or the current valuation, when price indexes are measured with errors but prices do not change ($\boldsymbol{\mu} = \mathbf{0}$, $\Sigma = 0$), the mean squared error of industry-weighted price indexes is larger than or equal to that of economy-weighted price indexes.*

Proofs of the above theorems are in the appendix. They state the sufficient conditions for dominance of economy- vs. industry-weighted index systems. The necessary conditions for dominance remain to be derived.

A numerical example

An economy with four goods a, b, c, and d is considered ($n = 4$). Assume that parameters $\boldsymbol{\omega}$ (economy-wide relative weights used in index construction), $\boldsymbol{\mu}$ (expectation of relative price change), Σ (covariance of relative price change), Δ (covariance of price measurement error of four goods), and ρ_F and ρ_I (diversification parameters) are as follows:

$$\boldsymbol{\omega}' = (0.2, 0.4, 0.3, 0.1),$$

$$\boldsymbol{\mu}' = (0.3, 0.4, -0.1, 0.2),$$

$$\Sigma = \begin{vmatrix} 0.0600 & 0.0624 & 0.0219 & 0.0441 \\ 0.0624 & 0.0800 & -0.0316 & 0.0453 \\ 0.0219 & -0.0316 & 0.0500 & -0.0313 \\ 0.0441 & 0.0453 & -0.0313 & 0.0400 \end{vmatrix},$$

$$\Delta = \begin{vmatrix} 0.0300 & -0.0122 & -0.0104 & 0.0039 \\ -0.0122 & 0.0200 & 0.0141 & -0.0190 \\ -0.0104 & 0.0141 & 0.0400 & 0.0179 \\ 0.0039 & -0.0190 & 0.0179 & 0.0500 \end{vmatrix},$$

and

$$\rho_F = \rho_I = 100.$$

The errors calculated from the above assumptions and expressions (6) and (7) are shown in figs. 1 through 3. The differences between mean squared errors under economy-weighted indexes and industry-weighted indexes can be seen from the figures. The numbers are consistent with Theorems 1 and 2.

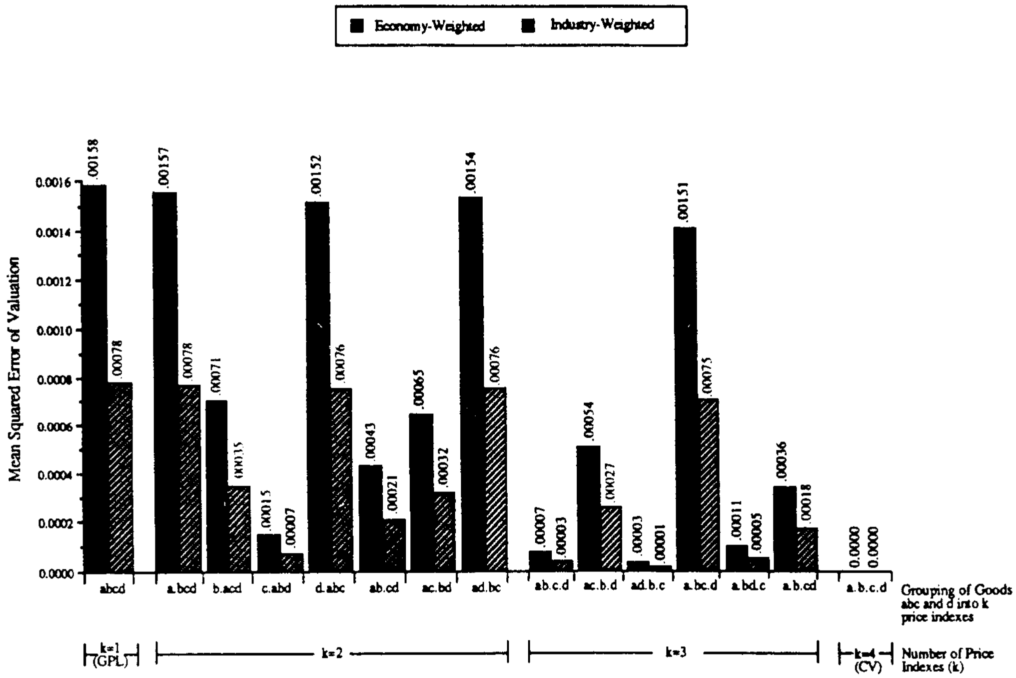

Fig. 1. Comparison between movement error of valuation rules based on economy- vs. industry-weighted indexes.

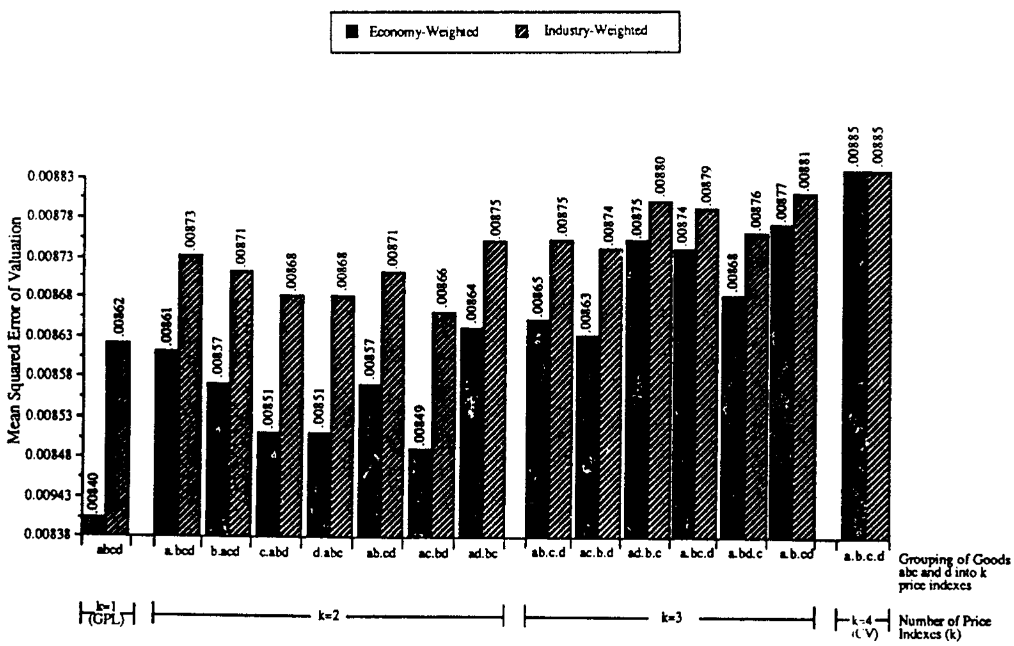

Fig. 2. Comparison between measurement error of valuation rules based on economy- vs. industry-weighted indexes.

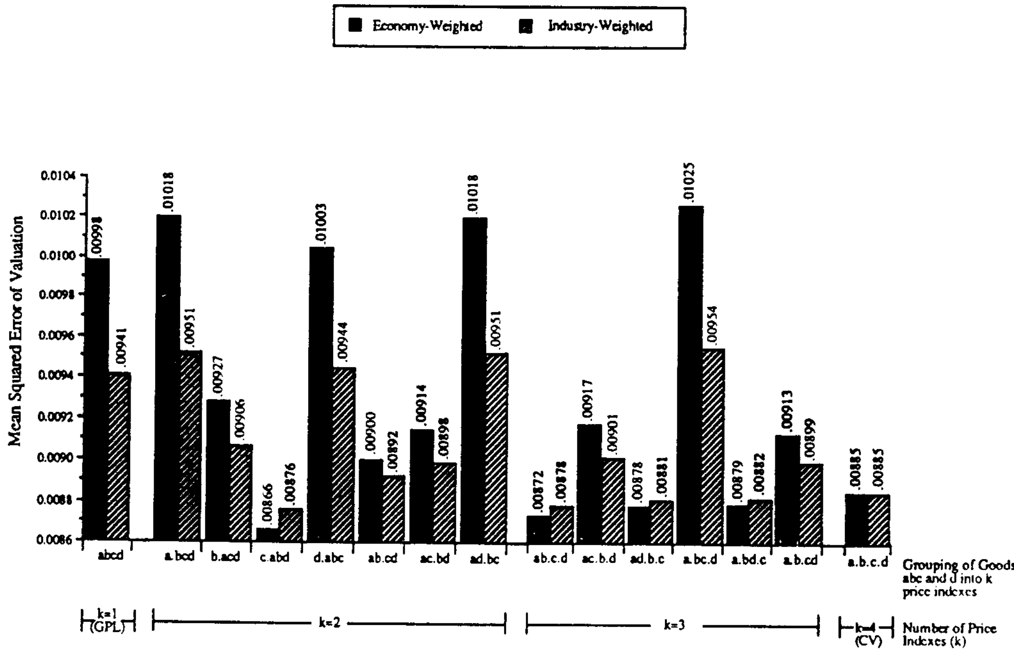

Fig. 3. Comparison between total error of valuation rules based on economy- vs. industry-weighted indexes.

For example, take the index partition (ac, b, d). From fig. 1, in the absence of measurement error its mean squared error under the industry-weighted index system (0.00027) is less than its mean squared error under the economy-weighted index system (0.00054). From fig. 2, without price movement error the mean squared error under industry-weighted indexes (0.00874) is larger than under economy-weighted indexes (0.00863). The same is true for all other index partitions.

When $k = 1$ (general price level valuation) or $k = 4$ (current valaution) we have the exact mean squared errors shown in parentheses in figs. 1 through 3. The figures are also consistent with Theorems 3 and 4. We found that our approximation of (6) holds to the fifth significant digit.

From fig. 3, it can be seen that under some index partitions the industry-weighted index system incurs less total error than the economy-weighted index system but under other partitions the latter performs better. Therefore, for a given set of parameters, neither index system is uniformly superior to the other. For any given index partition, whether one index system is better depends on the relative magnitudes of the parameters in (8).

4. Concluding remarks

This paper is a part of an effort to develop a unified econometric theory of asset valuation rules in accounting. The linear aggregation approach used here will help integrate a variety of valuation rules into a single framework and facilitate direct comparison of their statistical properties. We have shown:

(1) When price data used in formation of price indexes are measured without error, industry-weighted price indexes yield a more accurate valuation of assets than do economy-weighted price indexes.
(2) At the other extreme, when prices do not change, the industry-weighted price index system yields a less accurate valuation of assets than does the economy-weighted index system.
(3) Whether the industry-weighted or the economy-weighted indexes yield more accurate valuation of assets depends on $\boldsymbol{\Sigma}$, $\boldsymbol{\mu}$, $\boldsymbol{\Delta}$, ρ_I, ρ_F, and $\boldsymbol{\omega}$ discussed above. The expression (8) can be used to determine which weighting system is better where both index sets are based on the same partition of n goods in the economy.

Appendix

To prove Theorems 1 through 4, we first derive expressions (6) and (7) for the mean squared errors associated with industry- and economy-weighted index systems in an industry-segmented environment.

A.1. Industry-segmented environment

Each industry is defined by its vector of relative proportions of assets, $\boldsymbol{\omega}^I$. Vector $\boldsymbol{\omega}^I$ is the result of ρ_I multinomial draws (with replacement) with population parameter $\boldsymbol{\omega}$, the economy's vector of asset proportions. ρ_I is a measure of diversification of asset portfolio of industries and not of their size and is assumed to be identical for all industries. The first and the second moments of the distribution of $\boldsymbol{\omega}^I$ in (4) are derived from properties of multinomial distributions [see Freeman (1963, p. 129)].

Individual firms are defined by their vector of relative proportions $\boldsymbol{w}$ which is assumed to be the result of ρ_F multinomial draws (with replacement) with parameter $\boldsymbol{\omega}^I$ for firms in industry I. ρ_F, the diversification parameter for individual firms, is assumed to be the same for all firms. The conditional moments of $\boldsymbol{w}$ are given in (5).

The economy-weighted indexes use weights $\boldsymbol{\omega}$ to form indexes. For example, if $\omega_1 = 0.02$, $\omega_2 = 0.06$, and an index consists of these two goods, then their relative weights in the index will be $0.02/(0.02+0.06) = 0.25$ and $0.06/(0.02+0.06) = 0.75$, respectively. More generally, if ω_j^* is the total weight in $\boldsymbol{\omega}$ of all goods included in the same index as good j, the relative weight of good j within the index it is included in is ω_j/ω_j^*. In constructing the industry-weighted indexes for industry I, weight ω_j/ω_j^* is substituted by ω_j^I/ω_j^{I*}.

A.2. Industry-weighted indexes

Mean squared error under the industry-weighted index system is derived in two steps.

(a) *Mean squared error for firms within industry I*

The industry-wide average mean squared error of valuation rule $\tilde{R}_{kl}$ for firms within industry I using industry-weighted indexes is obtained by substituting $\boldsymbol{\omega}^I$ for $\boldsymbol{\omega}$ in (3),

$$\begin{aligned} \mathrm{E}_{w,r,\varepsilon}\left\{(\tilde{R}_{kl}-R_{n,1})^2 | \boldsymbol{\omega}^I\right\} &= (1-1/\rho_F)\boldsymbol{\omega}^{I\prime}\Delta\boldsymbol{\omega}^I + (1/\rho_F) \\ &\quad\times\left\{\sum_{u=1}^{k}\boldsymbol{\omega}_u^{I\prime}\Delta_{uu}\boldsymbol{\omega}_u^I/\boldsymbol{\omega}_u^{I\prime}\boldsymbol{e} + \boldsymbol{\omega}^{I\prime}(\boldsymbol{\sigma}+\dot{\boldsymbol{\mu}})\right. \\ &\quad\left. -\sum_{u=1}^{k}\boldsymbol{\omega}_u^{I\prime}(\Sigma_{uu}+\boldsymbol{\mu}_u\boldsymbol{\mu}_u')\boldsymbol{\omega}_u^I/\boldsymbol{\omega}_u^{I\prime}\boldsymbol{e}\right\}. \end{aligned} \tag{A.1}$$

(b) *Mean squared error for all firms in an industry-segmented economy using the industry-weighted indexes*

In order to obtain the economy-wide average mean squared error of valuation rule $\tilde{R}_{kl}$ for all firms in the economy, we must take the expectation of (A.1) with respect to the distribution of $\boldsymbol{\omega}^I$ given in (4).

For calculation of this expectation, (A.1) can be seen as the sum of the elements of an $n \times n$ matrix, say C, whose elements are given by

$$c_{ij} = \begin{cases} (1-1/\rho_F)\omega_i^I\delta_{ii}\omega_i^I + \omega_i^I\delta_{ii}\omega_i^I/\rho_F\omega_i^{I*} + \omega_i^I(\sigma_{ii}+\mu_i\mu_i)/\rho_F \\ -\omega_i^I(\sigma_{ii}+\mu_i\mu_i)\omega_i^I/\rho_F\omega_i^{I*} \\ \quad\text{for}\quad i=j, \\ (1-1/\rho_F)\omega_i^I\delta_{ij}\omega_j^I + \omega_i^I\delta_{ij}\omega_j^I/\rho_F\omega_i^{I*} \\ -\omega_i^I(\sigma_{ij}+\mu_i\mu_j)\omega_j^I/\rho_F\omega_i^{I*} \\ \quad\text{for}\quad i\neq j,\quad i,j\text{ in the same index,} \\ (1-1/\rho_F)\omega_i^I\delta_{ij}\omega_j^I \\ \quad\text{for}\quad i\neq j,\quad i,j\text{ not in the same index.} \end{cases} \tag{A.2}$$

$E(\omega_i^I\omega_j^I/\omega_i^I*)$ is the expectation of a ratio of random variables. Using (4) and Pearson's (1897, p. 492) approximation of the ratio of two random variables,[13]

$$E\left(\omega_i^I\omega_j^I/\omega_i^I*\right) \cong E\left(\omega_i^I\omega_j^I\right)/E\left(\omega_i^I*\right)\cdot\Big[1+\mathrm{var}\left(\omega_i^I*\right)/\left\{E\left(\omega_i^I*\right)\right\}^2$$

$$-\mathrm{cov}\left(\omega_i^I\omega_j^I,\omega_i^I*\right)/E\left(\omega_i^I\omega_j^I\right)E\left(\omega_i^I*\right)\Big]. \qquad (A.3)$$

The covariance term in (A.3) is obtained by using mixed factorial moment of multinomial distribution [see Johnson and Kotz (1969, p. 284)],

$$\mathrm{cov}\left(\omega_i^I\omega_j^I,\omega_i^I*\right)=\begin{cases}\{(\rho_I-1)(\rho_I-2)\omega_i^3+3(\rho_I-1)\omega_i^2+\omega_i\}/\rho_I^2\\ +(1/\rho_I^2)\displaystyle\sum_{t\neq i,\,t\in u}\{(\rho_I-1)(\rho_I-2)\omega_i^2\omega_t\\ +(\rho_I-1)\omega_i\omega_t\}\\ -\{\omega_i/\rho_I+(1-1/\rho_I)\omega_i^2\}\omega_i^*\\ \quad\text{for}\quad i=j,\\ \{(\rho_I-1)(\rho_I-2)\omega_i^2\omega_j+2(\rho_I-1)\omega_i\omega_j\\ +(\rho_I-1)(\rho_I-2)\omega_i^2\omega_j\}/\rho_I^2\\ +(1/\rho_I^2)\displaystyle\sum_{t\neq i,j,\,t\in u}(\rho_I-1)(\rho_I-2)\omega_i\omega_j\omega_t\\ -(1-1/\rho_I)\omega_i\omega_j\omega_i^*\\ \quad\text{for}\quad i\neq j,\quad i,j\in u.\end{cases} \qquad (A.4)$$

[13]The referee pointed out to us a potential pitfall in using Pearson's approximation which ignores the higher-order terms of the Taylor series expansion. Since ω_j^* is asymptotically normal and since the product of two normal variates has a 'scaled' Bessel distribution [Craig (1942)], dropping the higher-order terms can only be justified if the moments of the ratio of a 'scaled' Bessel and a normal variate exist. Unfortunately, the existence of such moments remains unverified, especially when the normal random variables involved are correlated [Geary (1930), Hinckley (1969), and the referee's own investigations communicated to us through the review of this paper]. Being unable analytically to show the insignificance of the remainder term, we ran a computer simulation with the parametric values used in the numerical example of the paper. The difference between the approximation and the sample mean was found to be less than two-tenths of a percent.

From (4), (A.3), and (A.4), the expectation of the elements of matrix C is given by

$$\mathrm{E}_{\omega^I}(c_{ij}) = \begin{cases} (1/\rho_I - 1/\rho_F\rho_I)\omega_i\delta_{ii} \\ +(1 - 1/\rho_F - 1/\rho_I + 1/\rho_F\rho_I)\omega_i^2\delta_{ii} \\ +\{\omega_i(\sigma_{ii} + \mu_i^2) - \gamma_{ii}^u(\sigma_{ii} + \mu_i^2 - \delta_{ii})\}/\rho_F \\ \quad \text{for} \quad i = j, \quad i \in u, \\ (1 - 1/\rho_F - 1/\rho_I + 1/\rho_F\rho_I)\omega_i\omega_j\delta_{ij} \\ -\gamma_{ij}^u(\sigma_{ij} + \mu_i\mu_j - \delta_{ij})/\rho_F \\ \quad \text{for} \quad i \neq j, \quad i, j \in u, \\ (1 - 1/\rho_F - 1/\rho_I + 1/\rho_F\rho_I)\omega_i\omega_j\delta_{ij} \\ \quad \text{for} \quad i \neq j, \quad i, j \text{ not in the same index,} \end{cases} \tag{A.5}$$

and γ_{ij}^u is defined in (6).

Summation of the expected values of the elements of matrix C in (A.5) yields the mean squared error of valuation rule $\tilde{R}_{kl}$ for all firms in an industry-segmented economy using the industry-weighted indexes as follows:

$$\begin{aligned} \mathrm{MSE}^I(\tilde{R}_{kl}) &= \mathrm{E}_{\omega^I, w, r, \varepsilon}(\tilde{R}_{kl} - R_{n,1})^2 \\ &\cong (1 - 1/\rho_F - 1/\rho_I + 1/\rho_F\rho_I)\boldsymbol{\omega}'\Delta\boldsymbol{\omega} \\ &\quad + \boldsymbol{\omega}'(\boldsymbol{\sigma} + \dot{\boldsymbol{\mu}})/\rho_F + (1/\rho_I - 1/\rho_F\rho_I)\boldsymbol{\omega}'\boldsymbol{\delta} \\ &\quad - (1/\rho_F)\sum_{u=1}^{k}\sum_{i \in u}\sum_{j \in u}\gamma_{ij}^u(\sigma_{ij} + \mu_i\mu_j - \delta_{ij}), \end{aligned} \tag{A.6}$$

where γ_{ij}^u is defined in (6).

(c) *Mean squared error for the general price level valuation*

For the general price level valuation ($k = 1$) and the current valuation ($k = n$) we do not need Pearson's approximation because ω_i^{I*} drops out from (A.2). Therefore, exact derivations are possible. First, the mean squared error for the general price level valuation under the industry-weighted index system is derived from (3),

$$\begin{aligned} \mathrm{E}_{w,r,\varepsilon}\{(\tilde{R}_{1,1} - R_{n,1})^2 | \boldsymbol{\omega}^I\} &= \boldsymbol{\omega}^{I\prime}(\boldsymbol{\sigma} + \dot{\boldsymbol{\mu}})/\rho_F + \boldsymbol{\omega}^{I\prime}\Delta\boldsymbol{\omega}^I \\ &\quad - \boldsymbol{\omega}^{I\prime}(\Sigma + \boldsymbol{\mu}\boldsymbol{\mu}')\boldsymbol{\omega}^I/\rho_F. \end{aligned}$$

This is the sum of the elements of an $n \times n$ matrix whose (ij)th element c_{ij} is

$$c_{ij} = \begin{cases} (\omega_i^I)^2 \delta_{ii} + \left\{\omega_i^I - (\omega_i^I)^2\right\}(\sigma_{ii} + \mu_i^2)/\rho_F & \text{for} \quad i = j, \\ \omega_i^I \omega_j^I \delta_{ij} - \omega_i^I \omega_j^I (\sigma_{ij} + \mu_i \mu_j)/\rho_F & \text{for} \quad i \neq j. \end{cases}$$

From the distribution of $\boldsymbol{\omega}^I$ given by (4),

$$\mathrm{E}_{\omega^I}(c_{ij}) = \begin{cases} \{\omega_i/\rho_I + (1 - 1/\rho_I)\omega_i^2\}\delta_{ii} \\ \quad + \{\omega_i - \omega_i/\rho_I - (1 - 1/\rho_I)\omega_i^2\}(\sigma_{ii} + \mu_i^2)/\rho_F \\ \quad \text{for} \quad i = j, \\ (1 - 1/\rho_I)\omega_i \omega_j \delta_{ij} - (1/\rho_F - 1/\rho_F \rho_I)\omega_i \omega_j (\sigma_{ij} + \mu_i \mu_j) \\ \quad \text{for} \quad i \neq j. \end{cases}$$

Summation of $\mathrm{E}_{\omega^I}(c_{ij})$'s for all i and j yields the mean squared error for the general price level valuation rule $(k = 1)$ for all firms in an industry-segmented economy using the industry-weighted indexes as follows:

$$\mathrm{MSE}^I(\tilde{R}_{1,1}) = (1/\rho_I)\boldsymbol{\omega}' \Delta \boldsymbol{\omega} + \boldsymbol{\omega}'\boldsymbol{\delta}/\rho_I + (1/\rho_F - 1/\rho_F \rho_I)$$
$$\times \{\boldsymbol{\omega}'(\boldsymbol{\sigma} + \dot{\boldsymbol{\mu}}) + \boldsymbol{\omega}'(\Sigma + \boldsymbol{\mu}\boldsymbol{\mu}')\boldsymbol{\omega}\}. \tag{A.7}$$

(d) *Mean squared error for the current valuation*

From (3),

$$\mathrm{E}_{w,r,\varepsilon}\left\{(\tilde{R}_{n,1} - R_{n,1})^2 | \boldsymbol{\omega}^I\right\} = (1 - 1/\rho_F)\boldsymbol{\omega}^{I\prime} \Delta \boldsymbol{\omega}^I + \boldsymbol{\omega}^{I\prime}\boldsymbol{\delta}/\rho_F.$$

Then by (4) we take expectation of $\mathrm{E}_{w,r,\varepsilon}\{(\tilde{R}_{n,1} - R_{n,1})^2 | \boldsymbol{\omega}^I\}$ with respect to $\boldsymbol{\omega}^I$ to get the economy-wide average. Thus, the mean squared error of current valuation $(k = n)$ for all firms in an industry-segmented economy using the industry-weighted indexes is

$$\mathrm{MSE}^I(\tilde{R}_{n,1}) = (1 - 1/\rho_F - 1/\rho_I + 1/\rho_F \rho_I)\boldsymbol{\omega}' \Delta \boldsymbol{\omega}$$
$$+ (1/\rho_F + 1/\rho_I - 1/\rho_F \rho_I)\boldsymbol{\omega}'\boldsymbol{\delta}. \tag{A.8}$$

A.3. Economy-weighted indexes

When the economy-weighted indexes are used for valuation in an industry-segmented environment, valuations $\tilde{R}_{kl}$ and $R_{n,1}$ are

$$\tilde{R}_{kl} = \sum_{j=1}^{n} w_j^* \omega_j \tilde{r}_j / \omega_j^* \quad \text{and} \quad R_{n,1} = \sum_{j=1}^{n} w_j r_j .$$

Distribution of w conditional on ω^I is given by (5), but the distribution of ω^I is given by (4). Therefore, the unconditional distribution of w is

$$\mathrm{E}(w) = \omega,$$

$$\operatorname{cov}(w_i, w_j) = \begin{cases} (1/\rho_F + 1/\rho_I - 1/\rho_F\rho_I)\,\omega_i(1-\omega_i) & \text{for} \quad i = j, \\ -(1/\rho_F + 1/\rho_I - 1/\rho_F\rho_I)\,\omega_i\omega_j & \text{for} \quad i \neq j, \end{cases}$$

$$\mathrm{E}(w_i w_j) = \begin{cases} (1/\rho_F + 1/\rho_I - 1/\rho_F\rho_I)\,\omega_i(1-\omega_i) + \omega_i^2 & \text{for} \quad i = j, \\ -(1/\rho_F + 1/\rho_I - 1/\rho_F\rho_I)\,\omega_i\omega_j + \omega_i\omega_j & \text{for} \quad i \neq j. \end{cases} \tag{A.9}$$

Note that in the covariance of (w_i, w_j), the parameter value ρ of an unsegmented economy is replaced by the inverse of $(1/\rho_F + 1/\rho_I - 1/\rho_F\rho_I)$.

By substituting $(1/\rho_F + 1/\rho_I - 1/\rho_F\rho_I)$ for $1/\rho$ in (3), we get the mean squared error of valuation rule $\tilde{R}_{kl}$ for all firms in an industry-segmented economy using the economy-weighted indexes,

$$\begin{aligned} \mathrm{MSE}^E(\tilde{R}_{kl}) = {} & \{1 - 1/\rho_F - 1/\rho_I + 1/\rho_F\rho_I\}\,\boldsymbol{\omega}'\Delta\boldsymbol{\omega} \\ & + (1/\rho_F + 1/\rho_I - 1/\rho_F\rho_I) \\ & \times \left\{ \boldsymbol{\omega}'(\boldsymbol{\sigma} + \dot{\boldsymbol{\mu}}) - \sum_{u=1}^{k} \boldsymbol{\omega}_u'(\Sigma_{uu} + \boldsymbol{\mu}_u\boldsymbol{\mu}_u' - \Delta_{uu})\boldsymbol{\omega}_u / \boldsymbol{\omega}_u'\boldsymbol{e} \right\}. \end{aligned} \tag{A.10}$$

From (A.10) and (A.6), the difference between the mean squared errors associated with the economy-weighted and the industry-weighted indexes in valuation rules, both based on the same partition Π_{kl}, is

$$\begin{aligned}
&\mathrm{MSE}^E(\tilde{R}_{kl}) - \mathrm{MSE}^I(\tilde{R}_{kl}) \\
&\cong (1/\rho_I - 1/\rho_F\rho_I)\boldsymbol{\omega}'(\boldsymbol{\sigma} + \dot{\boldsymbol{\mu}} - \boldsymbol{\delta}) - (1/\rho_F + 1/\rho_I - 1/\rho_F\rho_I) \\
&\quad \times \sum_{u=1}^{k} \boldsymbol{\omega}_u'(\Sigma_{uu} + \boldsymbol{\mu}_u\boldsymbol{\mu}_u' - \Delta_{uu})/\boldsymbol{\omega}_u'\boldsymbol{e} \\
&\quad + (1/\rho_F) \sum_{u=1}^{k} \sum_{i\in u} \sum_{j\in u} \gamma_{ij}^u(\sigma_{ij} + \mu_i\mu_j - \delta_{ij}),
\end{aligned} \tag{A.11}$$

where γ_{ij}^u is defined in (6).

A.4. Proofs of Theorems 1–4

Proof of Theorem 1. Substituting $\Delta = 0$ in (A.11), we get

$$\begin{aligned}
&\mathrm{MSE}^E(\tilde{R}_{kl}) - \mathrm{MSE}^I(\tilde{R}_{kl}) \\
&\cong (1/\rho_I - 1/\rho_F\rho_I)\boldsymbol{\omega}'(\boldsymbol{\sigma} + \dot{\boldsymbol{\mu}}) - (1/\rho_F + 1/\rho_I - 1/\rho_F\rho_I) \\
&\quad \times \sum_{u=1}^{k} \boldsymbol{\omega}_u'(\Sigma_{uu} + \boldsymbol{\mu}_u\boldsymbol{\mu}_u')\boldsymbol{\omega}_u/\boldsymbol{\omega}_u'\boldsymbol{e} \\
&\quad + (1/\rho_F) \sum_{u=1}^{k} \sum_{i\in u} \sum_{j\in u} \gamma_{ij}^u(\sigma_{ij} + \mu_i\mu_j) \\
&= (1/\rho_I - 1/\rho_F\rho_I)\left\{\boldsymbol{\omega}'(\boldsymbol{\sigma} + \dot{\boldsymbol{\mu}}) - \sum_{u=1}^{k} \boldsymbol{\omega}_u'(\Sigma_{uu} + \boldsymbol{\mu}_u\boldsymbol{\mu}_u')\boldsymbol{\omega}_u/\boldsymbol{\omega}_u'\boldsymbol{e}\right\} \\
&\quad + (1/\rho_F) \sum_{u=1}^{k} \left\{\sum_{i\in u} \sum_{j\in u} \gamma_{ij}^u(\sigma_{ij} + \mu_i\mu_j) \right. \\
&\quad \left. - \boldsymbol{\omega}_u'(\Sigma_{uu} + \boldsymbol{\mu}_u\boldsymbol{\mu}_u')\boldsymbol{\omega}_u/\boldsymbol{\omega}_u'\boldsymbol{e}\right\},
\end{aligned}$$

where γ_{ij}^u is defined in (6).

The term in the first curly brackets is the same as the expression of mean squared error without measurement errors and is therefore nonnegative. As shown in footnote 11, $\omega_i\omega_j/\omega_i^*$ approximates γ_{ij}^u for i, j in the same index u with a large ρ_I, which means the term in the second curly brackets is approximately zero. Therefore, given a large ρ_I,

$$\mathrm{MSE}^E(\tilde{R}_{kl}) - \mathrm{MSE}^I(\tilde{R}_{kl}) \geq 0. \quad \text{Q.E.D.}$$

Proof of Theorem 2. Substituting $\boldsymbol{\mu} = 0$ and $\Sigma = 0$ in (A.11), we have

$$\begin{aligned}
&\mathrm{MSE}^E(\tilde{R}_{kl}) - \mathrm{MSE}^I(\tilde{R}_{kl}) \\
&\cong -(1/\rho_I - 1/\rho_F\rho_I)\left\{\boldsymbol{\omega}'\boldsymbol{\delta} - \sum_{u=1}^{k} \boldsymbol{\omega}_u'\Delta_{uu}\boldsymbol{\omega}_u/\boldsymbol{\omega}_u'\boldsymbol{e}\right\} \\
&\quad +(1/\rho_F)\sum_{u=1}^{k}\left(\boldsymbol{\omega}_u'\Delta_{uu}\boldsymbol{\omega}_u/\boldsymbol{\omega}_u'\boldsymbol{e} - \sum_{i\in u}\sum_{j\in u}\gamma_{ij}^u\delta_{ij}\right),
\end{aligned}$$

where γ_{ij}^u is defined in (6).

As discussed above, the term in the last parentheses is approximately zero with a large ρ_I and the term inside the curly brackets is nonnegative. So, the above expression is nonpositive. Q.E.D.

Proof of Theorem 3. Suppose $\Delta = 0$. From (A.7) and (A.10), letting $k = 1$, we have

$$\mathrm{MSE}^E(\tilde{R}_{kl}) - \mathrm{MSE}^I(\tilde{R}_{kl}) = \{\boldsymbol{\omega}'(\boldsymbol{\sigma} + \dot{\boldsymbol{\mu}}) - \boldsymbol{\omega}'(\Sigma + \boldsymbol{\mu}\boldsymbol{\mu}')\boldsymbol{\omega}\}/\rho_I,$$

which is nonnegative as shown in the proof of Theorem 1.

When $k = n$, by (A.8) and (A.10), the following is obvious:

$$\mathrm{MSE}^E(\tilde{R}_{kl}) = \mathrm{MSE}^I(\tilde{R}_{kl}) = 0. \quad \text{Q.E.D.}$$

Proof of Theorem 4. Substituting $\Sigma = 0$, $\boldsymbol{\mu} = \mathbf{0}$, and $k = 1$ in (A.7) and (A.10), we have

$$\mathrm{MSE}^E(\tilde{R}_{kl}) - \mathrm{MSE}^I(\tilde{R}_{kl}) = -(\boldsymbol{\omega}'\boldsymbol{\delta} - \boldsymbol{\omega}'\Delta\boldsymbol{\omega})/\rho_I,$$

which is nonpositive since the numerator in the above is the same as the expression of measurement-error-only mean squared error.

When $k = n$, if $\Sigma = 0$ and $\boldsymbol{\mu} = \mathbf{0}$, (A.8) and (A.10) are equivalent. Thus,

$$\mathrm{MSE}^E(\tilde{R}_{kl}) - \mathrm{MSE}^I(\tilde{R}_{kl}) = 0. \quad \text{Q.E.D.}$$

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