

# Marginal Gains in Accuracy of Valuation from Increasingly Specific Price Indexes: Empirical Evidence for the U.S. Economy

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*Statement of Financial Accounting Standards No. 33* (FASB [1979]) requires large publicly held firms to disclose certain financial data based on general price-level adjustments and current costs. General price-level data are obtained by applying a single economy-wide price index, the Consumer Price Index for All Urban Consumers, to historical cost data. Considerable flexibility is allowed in the choice of sources of information about current costs in order to encourage experimentation and learning: "An enterprise may use specific price indexes or other evidence of a more direct nature." Those who choose specific price indexes must also decide the level of specificity of the index system by balancing the gains in accuracy of valuation against the increased costs of using a more specific set of indexes. Those who choose "evidence of a more direct nature" to estimate current costs may also want to know what they can expect to gain from their efforts.

In this study we present empirical estimates of marginal gains in accuracy of asset valuation from increasing the specificity of price indexes used to adjust historical costs. Briefly, we find that the structure of prices in the U.S. economy is such that marginal gains in accuracy decline sharply as the specificity of price indexes is increased. A large proportion of the total potential gain toward the estimate of current cost is attained by a few broad indexes; additional detail adds relatively little to accuracy.

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For example, the increase in accuracy of valuation obtained by using 50 narrowly constructed price indexes instead of 10 broad indexes is smaller than the increase in accuracy from using 10 indexes instead of 5. These findings are analogous to those in modern finance theory wherein the marginal reduction of diversifiable risk of a portfolio declines with portfolio size.

The analogy to portfolio theory is more than superficial. Assets of a firm, as well as price indexes used to estimate their current value, can be conveniently represented as portfolios of goods in the economy and the accuracy (in the sense of smaller mean-squared error) of a given set of price indexes in approximating assets of a firm is a function of the mean vector and covariance matrix of price changes for goods in the economy. Previous work by Ijiri [1967; 1968], Tritschler [1969], and Sunder [1978] provided analytical results which (under appropriate assumptions) enable us to estimate the statistical accuracy of valuation rules based on various index systems directly from data on the price structure of the U.S. economy. Relative weights of goods and mean and covariance of price changes from the Producer Price Index data base are used in the study. Similar estimates could be obtained for the Consumer Price Index data base though the applicability of subindexes in this data base to current valuation of industrial assets seems to be of doubtful value.

Analytical results of Sunder [1978] and Sunder and Waymire [forthcoming], on which the empirical estimates are based, are summarized in the next section. An unbiased estimator of accuracy of valuation rules and a search algorithm for finding relatively accurate price index configurations are given in section 2. Description of data, empirical evidence, and our conclusions follow in sections 3, 4, and 5, respectively.

### *1. Framework*

The comparative analysis of alternative valuation rules has traditionally been qualitative in nature, differences among the rules being viewed as differences in principle as opposed to the extent to which a common general principle is applied (e.g., Rosenfield [1972]). A consequence of comparing valuation rules in this manner is that it becomes impossible to discriminate among alternative rules on the basis of measurable attributes upon which these alternatives could potentially be ranked according to their desirability.

Ijiri [1967; 1968] initiated an alternative approach by characterizing valuation rules as aggregation functions wherein a statistic, the linear aggregation coefficient, is defined to summarize an important property of valuation rules in a single number. Extending this line of analysis, Sunder [1978] showed that most valuation rules (e.g., historical cost, general price level, current replacement cost, exit value, and many others) may be viewed as members of a family of asset valuation rules, which he labeled "exchange valuation rules." He developed a unified scheme to

represent each rule algebraically in terms of the price index configuration employed to adjust historical cost. Various results pertaining to bias and mean-squared error of valuation rules were derived. The results of that study which are most important for the purposes of this study are discussed below. Sunder and Waymire [forthcoming] extended these findings to examine the general case of accuracy for a given index configuration with respect to any strictly finer index configuration. Hall [1982] provided empirical evidence in support of Sunder's results using data for electrical, gas pipeline, telephone, and water utilities.

Consider an economy with  $n$  assets. Let  $\mathbf{q}^*$  be the vector of quantities of the  $n$  assets contained in a given bundle. Suppose that under a given rule, valuation of the bundle is  $P^0$  at time 0 and  $P^1$  at time 1; the relative price change is  $R = (P^1 - P^0)/P^0$ . Let  $\mathbf{r}$  be the  $n$ -vector of relative price changes from time 0 to time 1 for the  $n$  assets. If valuation of each of the  $n$  assets in the bundle is arrived at by applying a price index specific to each asset, the resultant value of  $R$  is the relative price change in current value of the bundle ( $R_{cv}$ ) and is defined to be the principal aggregation:

$$R_{cv} = \mathbf{w}'\mathbf{r} \tag{1}$$

where:

$$w_i = \frac{P_i^0 q_i^*}{\sum_{j=1}^n P_j^0 q_j^*} \text{ for } i = 1, 2, \dots, n$$

and:

$$P_i^0 = \text{unit price of asset } i \text{ at time 0.}$$

$\mathbf{w}$  represents the vector of relative weights characterizing a given bundle of assets, that is, the firm.

$R$  is used as a generic symbol for valuation rules and two modifiers are added to identify a specific rule.  $R_{ki}$  represents a valuation rule which uses  $k (\leq n)$  different price indexes to adjust the beginning of the period valuation of all  $n$  assets. Since, in most cases, there is more than one way of forming  $k$  price indexes from the  $n$  assets,  $R_{ki}$  represents the valuation obtained by using the  $i$ th of  $L_k$  possible configurations of  $k$  indexes.<sup>1</sup> Because there is only one way to form  $n$  indexes from  $n$  assets,  $R_{cn} \equiv R_{n1}$ .

Sunder [1978] defined the accuracy of a valuation rule to be the economy-wide average of the mean-squared error of valuation for individual firms,  $R_{ki}$ , with respect to the principal aggregation  $R_{n1} (\equiv R_{cv})$ .<sup>2</sup>

<sup>1</sup>

$$L_k = \sum_{j=0}^{k-1} \frac{(k-j)^n}{j!(k-j)!} (-1)^j.$$

See Sunder [1978, p. 347].

<sup>2</sup> Note that mean-squared error is an inverse measure of accuracy. The valuation system becomes increasingly accurate as the mean-squared error decreases.

Each individual bundle of assets (representing one firm) is characterized by its vectors of relative weights,  $\mathbf{w}$ , which is assumed to be generated randomly from the economy-wide bundle of relative proportions  $\omega$ , using a constant number ( $\rho$ ) of multinomial trials. This accuracy measure is given by:

$$\begin{aligned} A(R_{ki}) &= E_{\mathbf{w}} E_{\mathbf{r}} (R_{ki} - R_{n1})^2 \\ &= \frac{1}{\rho} (\omega'(\sigma + \dot{\mu}) - \sum_{u=1}^k \frac{\omega_u'}{\omega_u' \mathbf{e}} (\Sigma_{uu} + \mu_u \mu_u') \omega_u) \end{aligned} \quad (2)$$

where  $\mathbf{e}$  = vector of unit elements of appropriate length;

$\omega = E(w)$ ;  $n$ -vector of relative weights of  $n$  assets in the economy,  $\omega' \mathbf{e} = 1$ ;

$\mu = E(\mathbf{r})$ ,  $n$ -vector of expected relative price changes for  $n$  assets;

$\dot{\mu} = n$ -vector of squared elements of  $\mu$ ;

$\Sigma = E(\mathbf{r} - \mu)(\mathbf{r} - \mu)'$ ,  $n \times n$  covariance matrix of relative price changes for  $n$  assets;

$\sigma = n$ -vector of diagonal elements of  $\Sigma$ ;

$k$  = number of price indexes used in the valuation rule. The set of  $n$  assets is partitioned into  $k$  nonempty subsets and a price index is constructed for each subset.  $\omega_u$ ,  $\mu_u$ , and  $\Sigma_{uu}$  are the subvectors and submatrix, respectively, corresponding to the  $u$ th of the  $k$  subsets;

$\rho$  = number of multinomial trials by which the bundle of assets for individual firms is randomly drawn from the economy wide bundle defined by  $\omega$ .

Let  $\Pi_{ki}$  denote the  $i$ th of  $L_k$  distinct partitions that can be used to form  $k$  price indexes for  $n$  assets. Similarly,  $\Pi_{k+mj}$  is the  $j$ th of  $L_{k+m}$  partitions that can be used to form  $(k + m)$  indexes from the  $n$  assets,  $m = 1, 2, \dots, n - k$ . Sunder [1978] proved that if  $\Pi_{k+mj}$  is a strictly finer partition of the  $n$  assets than  $\Pi_{ki}$ , then  $A(R_{k+mj})$  must be less than  $A(R_{ki})$ :

$$\Pi_{k+mj} \subset \Pi_{ki} \Rightarrow A(R_{k+mj}) < A(R_{ki}).$$

In other words, the economy-wide average of mean-squared error of valuation is a monotonically decreasing function of the fineness of the partitions used to form price indexes for  $n$  goods.  $A_{ki}$ , however, is not monotonic in  $k$ , the number of price indexes employed.

For each value of  $k$ , let  $R_k^*$  denote the most accurate of the  $L_k$  valuation rules (i.e.,  $R_k^*$  has the smallest economy-wide average of mean-squared error):

$$A(R_k^*) \leq A(R_{ki}), \quad i = 1, 2, \dots, L_k.$$

Let the corresponding partition of the  $n$  assets be denoted by  $\Pi_k^*$ . Thus, for every value of  $k$ , there exists a partition  $\Pi_k^*$  which yields the best accuracy (i.e., smallest mean-squared error) through the valuation rule

$R_k^*$ . We define  $H(k)$  to be the accuracy function which gives the accuracy for the most accurate  $k$ -index valuation rule:

$$H(k) = A(R_k^*), k = 1, 2, \dots n. \tag{3}$$

The remainder of this paper is devoted strictly to examining the properties of  $H(k)$  for the United States economy. Specifically, we are concerned with whether  $H(k)$  is convex in  $k$ . We know already from the results in Sunder [1978] that  $H(k)$  is strictly decreasing in  $k$ . For any partition  $\Pi_k^*$  (for any  $k < n$ ) along the accuracy function, we can generate a strictly finer partition of the  $n$  assets into  $k + 1$  indexes (denoted  $\Pi_{k+1j}$ ). Since  $\Pi_k^*$  and  $\Pi_{k+1j}$  are comparable with respect to fineness,  $A(R_{k+1j})$  must be less than  $A(R_k^*)$ . By applying this argument for every value of  $k$  strictly less than  $n$ , we can conclude that  $H(k)$  is strictly decreasing in  $k$ .

Convexity of the accuracy function,  $H(k)$ , implies that the marginal gain in accuracy (i.e., reduction in mean-squared error) declines as  $k$  increases. The convexity properties of the accuracy function are important from a practical viewpoint. A highly convex accuracy function implies that use of only a few broad indexes achieves a large proportion of total potential gain toward accurate estimates of current cost, and further gains in accuracy may not be worth the additional costs associated with using a more detailed set of price indexes or direct measurement of current cost of individual assets.

## 2. Estimation of Accuracy Function $H(k)$

Two problems must be solved to estimate the accuracy function,  $H(k)$ , from data. First, the accuracy measure  $A(R_{ki})$  for index configuration  $\Pi_{ki}$  is a function of  $\mu$  and  $\Sigma$ , the mean vector and covariance matrix, respectively, of relative price changes for the  $n$  assets in the economy. Since these parameters are unknown, they must be estimated from data. Sampling errors bias estimates of  $A(R_{ki})$  upward if (2) is applied. We present an estimator which corrects for the bias. Second, even for moderate values of  $n$ , the set of all possible valuation rules is very large, making an exhaustive search over the set infeasible. Estimated accuracy function depends on the search procedures used to identify the most accurate price index systems for each value of  $k$ . We present an algorithm which systematically searches the set to identify relatively accurate index systems.

Consider first the issue of sampling errors in  $\mu$  and  $\Sigma$ . If unbiased parameter estimates (denoted  $\hat{\mu}$  and  $\hat{\Sigma}$ ) are employed in (2), the estimated accuracy for valuation  $R_{ki}$  based on partition  $\Pi_{ki}$  is:

$$\hat{A}(R_{ki}) = \omega'(\hat{\sigma} + \hat{\mu}) - \sum_{u=1}^k \frac{\omega_u'}{\omega_u' \mathbf{e}} (\hat{\Sigma}_{uu} + \hat{\mu}_u \hat{\mu}_u') \omega_u. \tag{4}$$

Sunder and Waymire [1983] show that the presence of sampling errors in  $\mu$  biases (4) upward, while sampling errors in estimating the covariance matrix,  $\Sigma$ , have no effect. The estimator to correct for this bias in (4) is (heretofore referred to as the unbiased estimator):

$$\hat{A}(R_{ki}) = \omega' \left[ \left( \frac{T-1}{T} \right) \hat{\sigma} + \hat{\mu} \right] - \sum_{u=1}^k \frac{\omega_u'}{\omega_u' \mathbf{e}} \left[ \frac{T-1}{T} \hat{\Sigma}_{uu} + \hat{\mu}_u \hat{\mu}_u \right] \omega_u \quad (5)$$

where  $T$  represents the number of relative price change observations used to estimate  $\mu$  and  $\Sigma$ .

From inspection of equation (5), it is evident that the unbiased estimator differs from equation (4) only by the term  $(T-1)/T$ , which is multiplied by the diagonal elements in the covariance matrix and the covariance submatrix for each of  $k$  indexes. Estimator (4) converges to the unbiased estimator (5) as  $T \rightarrow \infty$  and is, therefore, asymptotically unbiased. The values of  $A(R_{ki})$  reported in this paper have been calculated using the unbiased estimator (5).

The second estimation problem concerns selection of an appropriate procedure for searching over the set of valuation rules. Identification of accuracy function  $H$  requires that for each value of  $k = 2, \dots, n-1$ , the index configuration with best accuracy (i.e., minimum mean-squared error) be identified and an unbiased estimate of its accuracy be obtained by using (5). This requires optimization of (5) with respect to alternative index systems defined by  $k$  partitions of the set of  $n$  assets, a problem we have not been able to solve analytically. The total number,  $L_k$ , of alternative  $k$ -partitions of the set of  $n$  elements is extremely large, even for moderate values of  $n$ . An exhaustive search over such a large set is infeasible and efficient search algorithms must be devised to obtain an approximation of the accuracy function  $H(k)$  under the constraint of limited computer resources.

Our analysis employs two alternative search procedures to estimate  $H(k)$ . The first is referred to as the *random search procedure*. For each value of  $k = 2, \dots, n-1$ , 100 random index partitions are drawn from the population of all  $k$ -index partitions without replacement. For each  $k$ , the partition which yields the lowest value of  $A_{ki}$  is used as the estimate for  $H(k)$ . The second search procedure uses the algorithm described in Appendix A. This *systematic search procedure* algorithm exploits previous analytical results to identify those index partitions which are likely candidates for inclusion in the accuracy function. Likely candidates for the most accurate  $k$ -partition are those which are strictly coarser than the most accurate  $(k+1)$ -partition and strictly finer than the most accurate  $(k-1)$ -partition. The algorithm exploits this conjecture and concentrates search in the fine-coarse neighborhood of the most accurate index partitions identified at any given point in the execution of the algorithm. In estimating  $H(k)$  using systematic search, we set  $t_1$ ,  $t_2$ , and  $t_3$  (see Appendix A for definition) equal to 100, 20, and 20, respectively.

3. The Data

The Producer Price Index (*PPI*) data contain several hierarchical levels of price indexes. We limited our estimates to the first three hierarchical levels on the *PPI* magnetic tape file dated December 1978 for two reasons: (1) the computing costs when working with large numbers of price indexes are high, and (2) we are uncertain about whether the less-aggregated *PPI* price index data are strictly comparable in fineness to the coarser levels of aggregated data in the data base. The first three hierarchical levels within the *PPI* data base are characterized as two-, three-, and four-digit classifications, respectively. After the single-producer price index, the coarsest level is the two-digit classification containing the 15 commodity indexes listed in Appendix B. Commodities in each of these 15 indexes are further classified into three-digit classes. In turn, each three-digit index is partitioned further into four-digit index classifications.

To clarify, an example of *PPI* classification is provided in figure 1. The example shows various indexes in the *PPI* data base under the heading of Rubber and Plastic products (two-digit index 07). Within this class, there are two three-digit indexes, Rubber Products (071) and Plastic Products (072). Each of these is partitioned further into four-

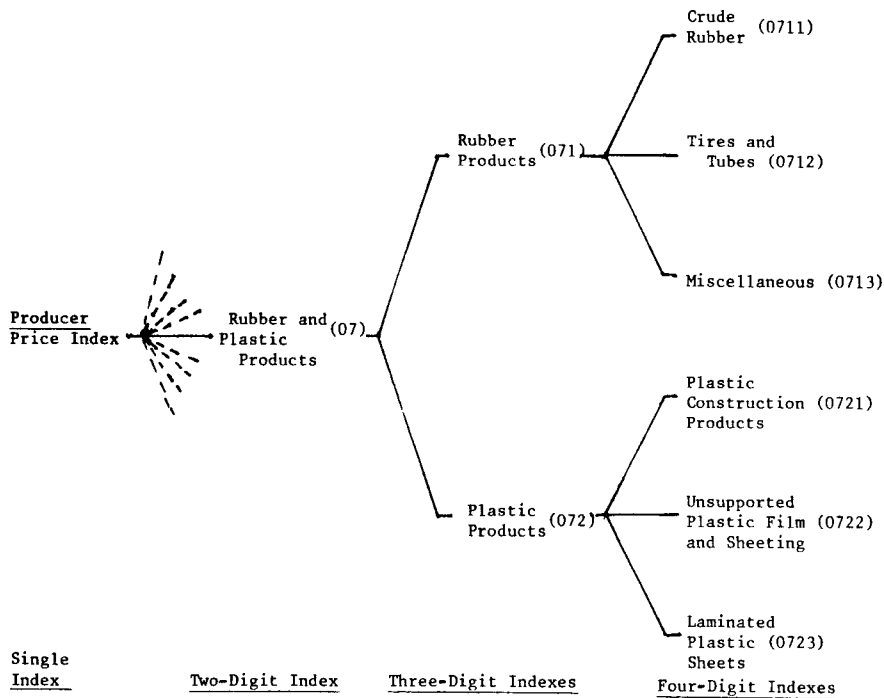


FIG. 1.—Producer Price Index (*PPI*) classification example.

digit indexes. For example, the indexes for Crude Rubber (0711), Tires and Tubes (0712), and Miscellaneous Rubber Products (0713) are all components of the three-digit index Rubber Products (071). In all, there are 87 three-digit indexes and 292 four-digit indexes in the data base (see Appendix B).

We used fewer indexes than the total available in the data base. The sample was determined in the following manner. First, we included only those commodities in the four-digit class which have annual price observations available for each of the 29 years during 1947-75.<sup>3</sup> This criterion resulted in the selection of 199 four-digit indexes. These 199 indexes were then aggregated according to the *PPI* classification scheme into 68 three-digit indexes and 15 two-digit indexes.

Each of these classifications can be thought of as partitions of the 199 four-digit commodities into 15, 68, and 199 indexes according to the *PPI* classification scheme. We denote partitions as  $\Pi_{15}^{PPI}$ ,  $\Pi_{68}^{PPI}$ , and  $\Pi_{199}^{PPI}$  ( $= \Pi_{199,1}$ ). By construction, the partitions are strictly comparable with respect to fineness, that is,  $\Pi_{15}^{PPI} \supset \Pi_{68}^{PPI} \supset \Pi_{199}^{PPI}$ . Further, the overall Producer Price Index corresponds  $\Pi_1^{PPI}$  to that which is coarser than all other partitions.

#### 4. Empirical Estimates of Accuracy Function

We estimated accuracy functions for three approximations of the U.S. economy. The first of these approximate economies is assumed to consist of only 15 assets, each of which is represented by one of the 15 two-digit indexes in the *PPI* data base, and relative abundance of these assets in the economy is given by the relative weights of the 15 assets in the *PPI* data base. Suppose all firms in the economy consisted of randomly drawn bundles of these 15 assets in varying proportions and these assets were valued using only, say, 8 ( $=k$ ) price indexes. Which particular combination of 15 assets into 8 price indexes can be expected to yield the most accurate valuation of individual firms on average? And how accurate is this 8-price index system? The three curves shown in figure 2 answer the second question. We randomly combined 15 assets into 8 price indexes in 100 different ways. Point X (0.00122) shows the mean accuracy for these 100 8-index valuation rules. Point Y (0.00059) shows the accuracy of the most accurate of these 100 8-index valuation rules. Point Z (0.00021) shows the accuracy of the most accurate 8-index system we could find with the systematic search algorithm shown in Appendix A. Note that in this 15-asset economy, 15 price indexes ( $k = 15$ ), one for each good, automatically yield the perfectly accurate current cost of every firm and therefore the accuracy measure for the 15-index system in figure 2 is shown to be zero. At the other end of the scale, valuation based on a single price index ( $k = 1$ , the *PPI* itself) is least accurate. Also note that

<sup>3</sup> This is done in order to ensure identical data availability for all  $n$  goods in estimating the covariance matrix of relative price changes.



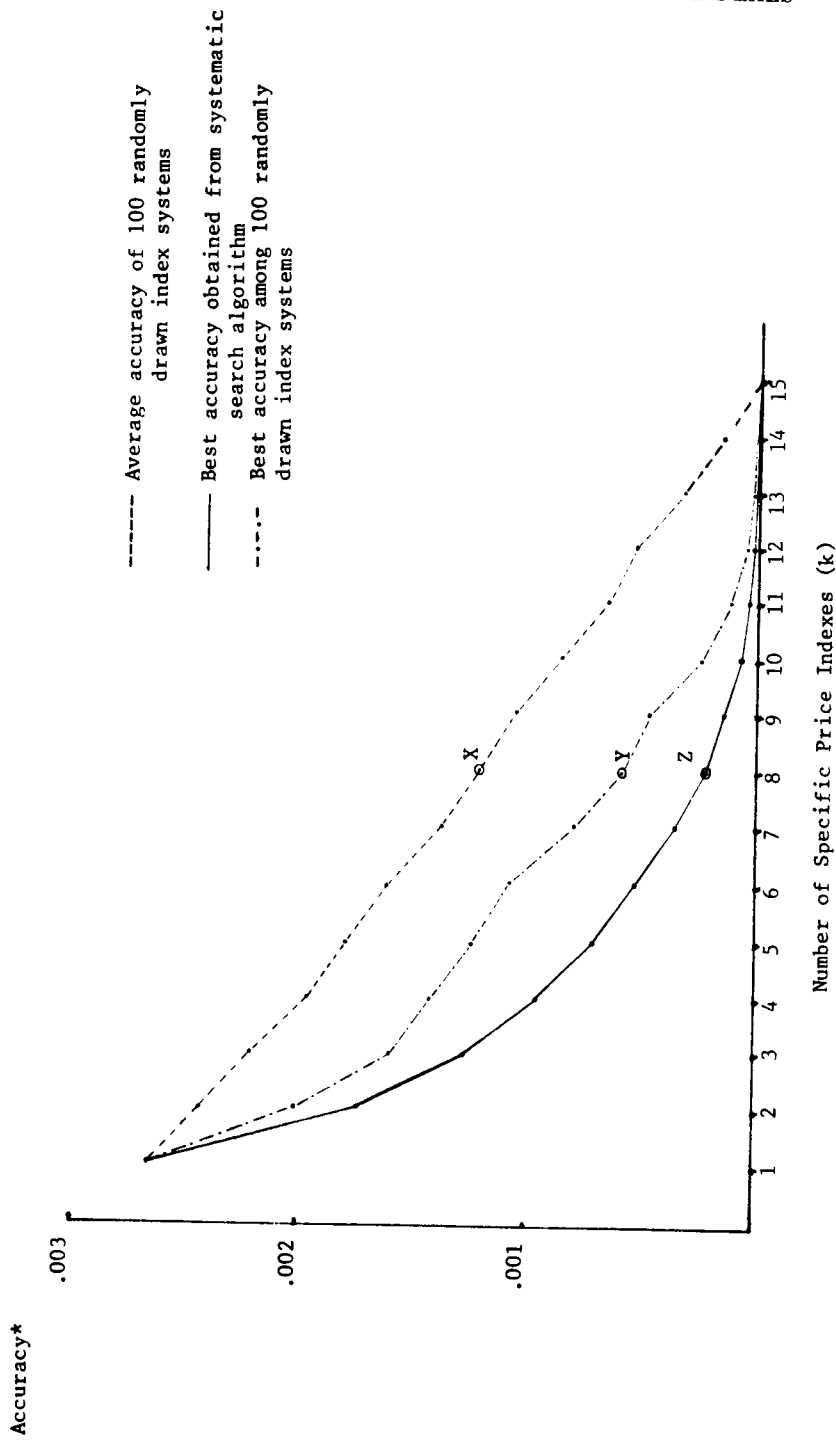


FIG. 2.—Accuracy of specific price indexes in a 15-asset economy. \*Note that mean-squared error is plotted as the inverse measure of accuracy.

the  $X$ ,  $Y$ , and  $Z$  curves merge when  $k = 1$  or  $15$  because there is only one possible way each of combining  $15$  goods into a single or into  $15$  price indexes respectively. Figure 2 shows values of  $X$ ,  $Y$ , and  $Z$  for all possible values of  $k$  from  $1$  to  $15$ .

The  $Z$ -curve, which shows the accuracy of the most accurate  $k$ -index system that we could create from these  $15$  assets, is of primary interest. Two points are worth noting. First, the  $Z$ -curve is convex. The marginal gain in accuracy from increasing  $k$ , the number of price indexes in this economy, keeps declining. More than half the gain in accuracy can be realized from only three price indexes, and there is hardly any gain beyond eight price indexes. Second, the systematic search algorithm yields substantially more accurate price index systems than the random search procedure.  $Z$  is an upward biased estimate of the accuracy function because further search with this algorithm (and more computer time) and with more efficient algorithms can be expected to yield an even more accurate  $k$ -index system for various values of  $k$ .

The second approximation of the U.S. economy is assumed to consist of  $68$  assets, each of which is represented by one of the  $68$  three-digit indexes in the  $PPI$  data base with appropriate relative weights.<sup>4</sup> In this economy, use of  $68$  price indexes ( $k = 68$ ), one for each asset, automatically yields perfectly accurate current valuation for every firm, while a single price index ( $k = 1$ , the  $PPI$  itself) yields the least accurate approximation of current value of firms. Figure 3 shows  $X$ ,  $Y$ , and  $Z$  curves for all values of  $k$  between  $1$  and  $68$ . In this approximate economy, the accuracy function,  $Z$ , appears even more convex than in the first approximate economy. Furthermore, the systematic search algorithm yields even better results.

The third and final approximation of the U.S. economy is assumed to consist of  $199$  assets, each corresponding to one of the  $199$  four-digit price indexes in the  $PPI$  data bases with appropriate relative weights. The results shown in figure 4 are similar to those for the two other approximate economies.

In figure 5, we exploit the results in Sunder and Waymire [forthcoming] on the additivity of valuation rule accuracy and plot to a common scale the three accuracy functions estimated using systematic search. The points labeled  $A(\Pi_{15}^{PPI})$  and  $A(\Pi_{68}^{PPI})$  give the accuracy of the two-digit  $PPI$  ( $\Pi_{15}^{PPI}$ ) and the three-digit  $PPI$  ( $\Pi_{68}^{PPI}$ ), respectively, with respect to the current value given by the four-digit  $PPI$  ( $\Pi_{199}^{PPI}$ ). Two results of interest are apparent in figure 5. First, note the "envelope curve" appearance of the graph. That is, the estimated accuracy function shows further improvements in accuracy for a given  $k$  as the size ( $n$ ) of the partition defining the principal aggregation is increased. This enables us to draw some inferences about the convexity properties of the accuracy function for the much larger number of actual commodities in the U.S.

<sup>4</sup> See section 3 for a more detailed description of these  $68$  price indexes.

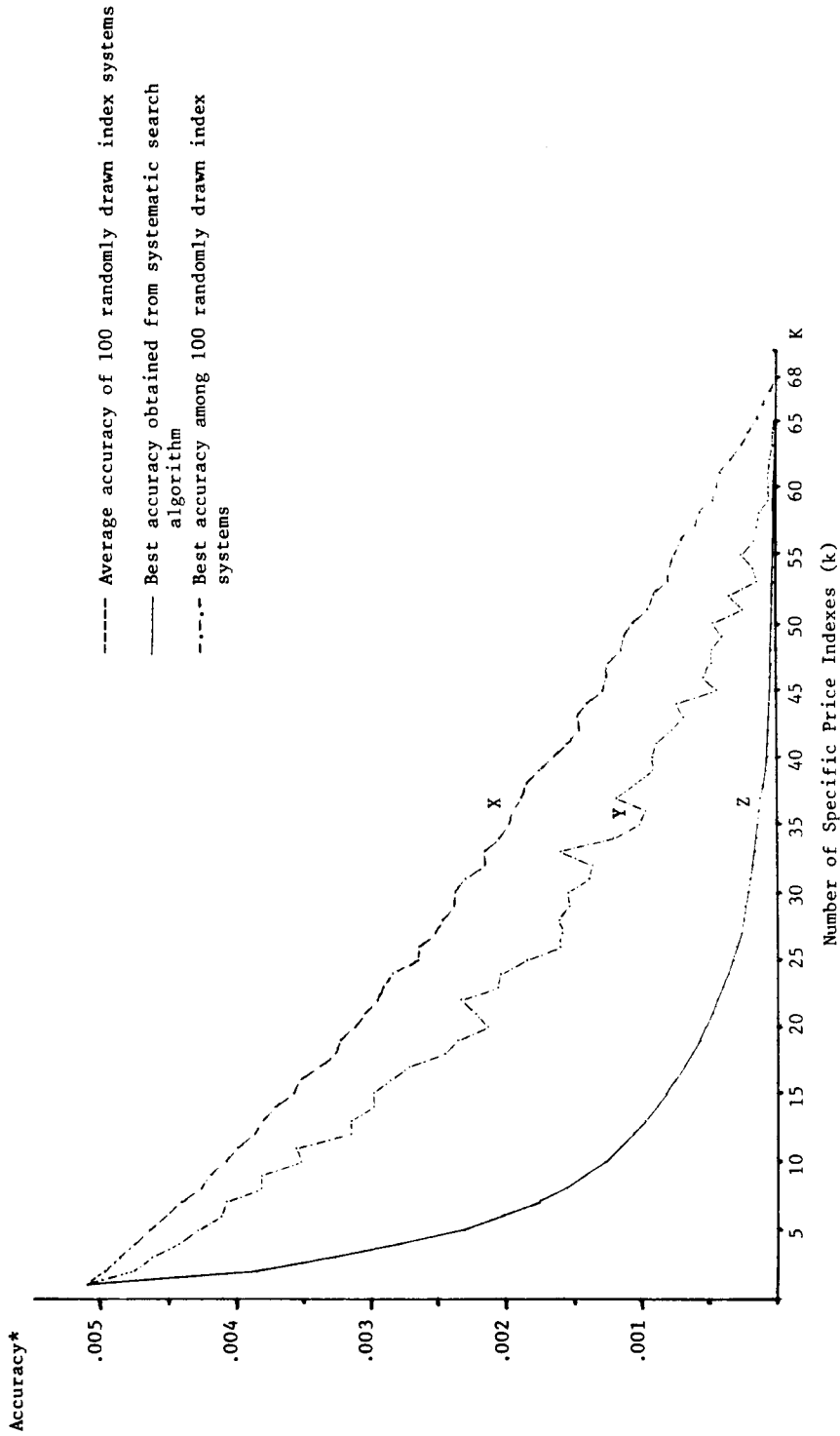


FIG. 3.—Accuracy of specific price indexes in a 68-asset economy. \*Note that mean-squared error is plotted as the inverse measure of accuracy.

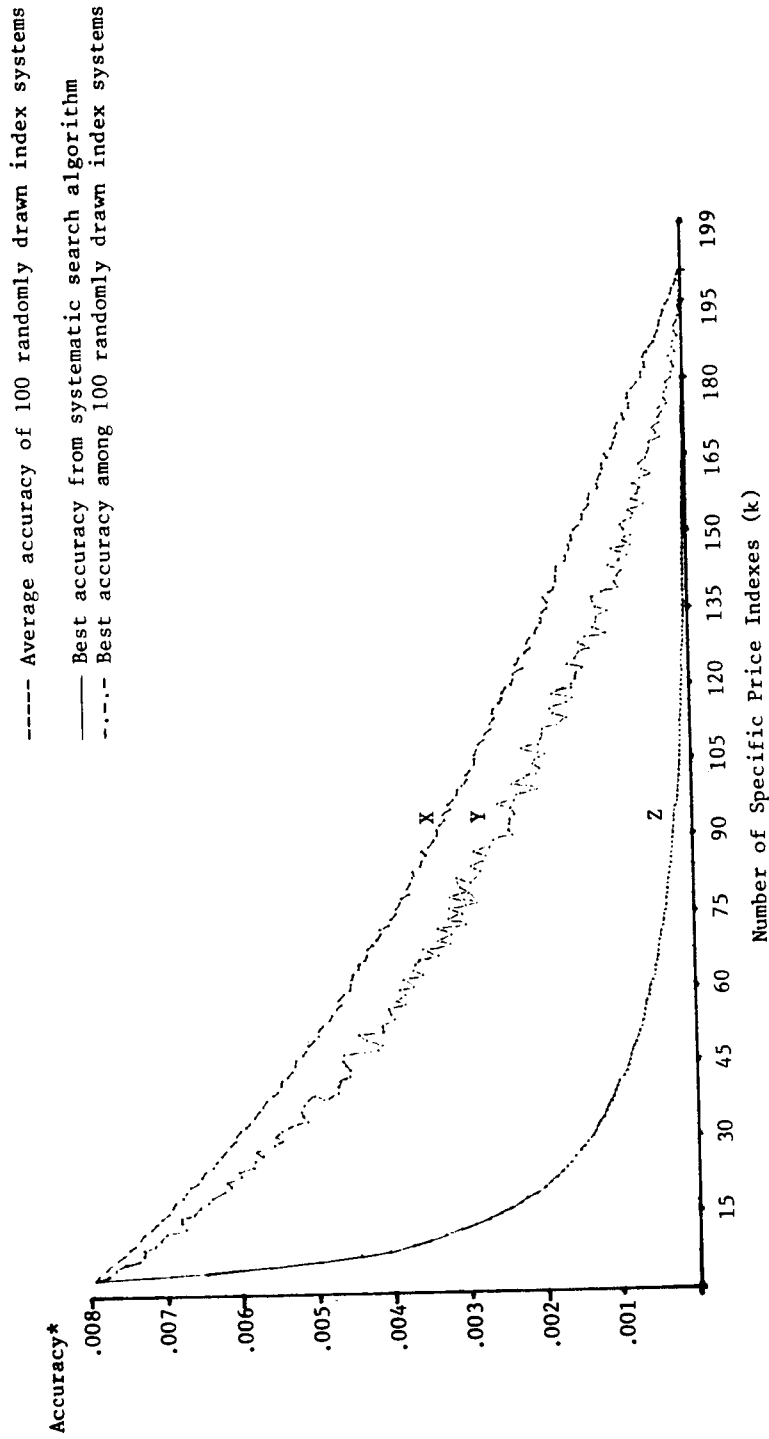


FIG. 4.—Accuracy of price indexes in a 199-asset economy. \*Note that mean-squared error is plotted as the inverse measure of accuracy.

economy. Because the estimated accuracy function shows further improvement in accuracy for a given  $k$  as  $n$  increases, the accuracy function for, say, the actual United States economy is likely to be highly convex. Second, the figure provides some evidence on the relative accuracy of the two- and three-digit *PPI* index systems,  $\Pi_{15}^{PPI}$  and  $\Pi_{68}^{PPI}$ , in estimating valuation of firms in a 199-asset compared to an alternative economy. Figure 5 shows that it is possible to construct a far more accurate set of 15 specific price indexes than the two-digit system of *PPI*. The accuracy measure of  $\Pi_{15}^{PPI}$  in the 199-asset economy is 0.00529, while the most accurate 15-index system we were able to find with only limited search had an accuracy measure of 0.00285. Similarly, the accuracy measure of  $\Pi_{68}^{PPI}$  in the 199-asset economy is 0.00285, but we were able to construct a 68-index system whose accuracy measure was about 0.00042. The *PPI*'s system of classification and aggregation may be better suited for other purposes, but for valuation purposes it does not perform very well.

5. Conclusions

Our empirical evidence strongly suggests that the accuracy function for the United States economy is highly convex. This implies that the marginal gain in accuracy of valuation declines sharply as the number and specificity of price indexes used for valuation increase. Our findings are potentially useful to those who are involved in the ongoing debate on selection of asset valuation rules—for example, auditors, practicing and academic accountants, and accounting regulators. The results are of particular relevance for selection of asset valuation rules in situations in which it is costly to employ refined measurement methods.

Before the framework employed here can be directly exploited for policy purposes, however, several research issues need to be resolved.

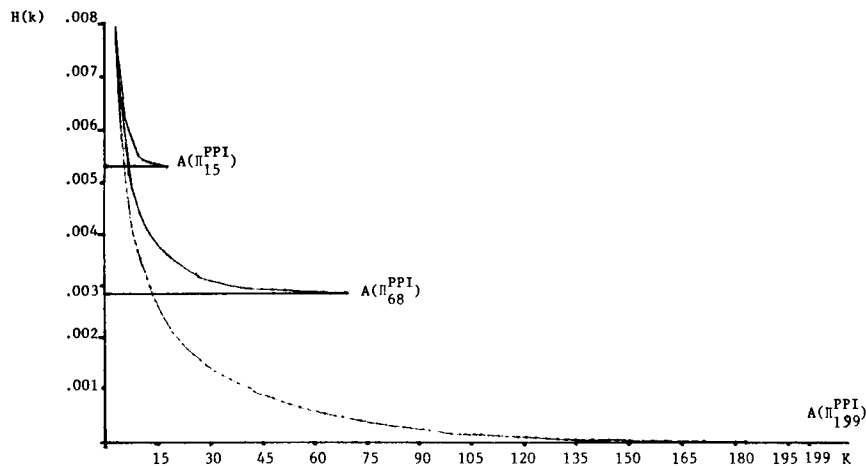


FIG. 5.—All accuracy functions estimated using systematic search plotted to a common scale.

First, it remains to be shown whether, in general, the accuracy function is convex, and if not, the conditions under which it is convex. If a given economy does not possess a convex accuracy function, then accuracy of valuation will not be substantially improved by using only a few price indexes. Second, we need to investigate the extent to which the convexity of the accuracy function is related to parameters ( $\mu$  and  $\Sigma$ ) describing the underlying process generating relative price changes. If a systematic relationship can be identified between the parameters  $\mu$  and  $\Sigma$  and convexity of the accuracy function, it would then be possible to determine the partitions along the accuracy function without employing costly search procedures for the set of valuation rules. This would allow one to exploit our framework in an ex ante manner. Finally, if it is impossible to derive the relationship between  $\mu$  and  $\Sigma$  and accuracy function convexity, we should develop superior search algorithms for the accuracy function.

## APPENDIX A

### *Search Algorithm for Accuracy Function*

(1) For each value of  $k = 2, \dots, n - 1$ , generate  $t_1$  random partitions. Identify the most accurate of these  $t_1$  partitions. Store the partition and its accuracy in the memory. Accuracy function in memory is  $H^0$ .

(2) Starting with  $k = 1$ , take the most accurate  $k$ -partition identified up to this point in the algorithm and obtain a  $(k + 1)$ -partition from it by randomly splitting one of the multiple-element subsets in the partition. If the resulting  $(k + 1)$ -partition is more accurate than the  $(k + 1)$ -partition in storage, substitute the former for the latter in the storage. Repeat  $t_2$  times.

(3) Conduct step 2 for  $k = 2, 3, \dots, n - 2$ . Accuracy function in the memory at this point is  $H^1$ .

(4) Starting with  $k = n$ , take the most accurate  $k$ -partition identified up to this point in the algorithm and obtain a  $(k - 1)$ -partition from it by combining two of its randomly chosen subsets. If the resulting  $(k - 1)$ -partition is more accurate than the  $(k - 1)$ -partition in storage, substitute the former for the latter in the storage. Repeat  $t_2$  times.

(5) Conduct step 4 for  $k = n - 1, n - 2, \dots, 3$ . Accuracy function in the memory at this point is  $H^2$ .

(6) Repeat steps 2 through 5  $t_3$  times or until the improvement in accuracy function is less than a predetermined limit.

APPENDIX B

*PPI Classification Scheme*

Two-Digit Classification	Commodity Index	Number of Three-Digit Indexes	Number of Four-Digit Indexes
01	Farm Products	9	23
02	Processed Foods	9	40
03	Textile Products and Apparel	5	14
04	Hides, Skins, Leather and Related Products	4	16
05	Fuel and Power	6	15
06	Chemicals	7	15
07	Rubber and Plastic Products	2	6
08	Lumber and Wood Products	4	9
09	Pulp and Paper Products	2	7
10	Metals	8	32
11	Machinery and Equipment	7	46
12	Furniture and Household Durables	6	24
13	Nonmetallic Mineral Products	9	19
14	Transportation Equipment	3	6
15	Miscellaneous	$\frac{6}{87}$	$\frac{20}{292}$

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