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METHODOLOGICAL ISSUES IN THE USE OF FINANCIAL RATIOS

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It appears that the extensive use of financial ratios by both practitioners and researchers is often motivated by tradition and convenience rather than resulting from theoretical considerations or from a careful statistical analysis. Basic questions, such as: Is the control for firm size, a major objective of the ratio form, called for by the theory examined; what is the structural relationship between the examined variables and size; and what is the optimal way to control for industry-wide factors, are rarely addressed by users of financial ratios. The major purpose of this study is to discuss the conditions under which conventional tools, such as financial ratios and measures of industry central tendency, achieve the intended objectives of analysis (e.g., size control). Various issues related to financial analysis, such as spurious correlation due to a common denominator, the choice of an optimal size variable, and the treatment of outlier observations, are also examined.

1. Introduction

Ratios of variables derived from financial statements are used extensively by both practitioners and researchers. Practitioners are generally concerned with the evaluation of corporate performance through various 'financial ratio analysis' methods, such as the study of time-series of ratios and the comparison of a given firm's ratio with industry standards. Researchers in accounting, finance, and economics use financial ratios in the examination of various issues, such as the relationship between financial data and common stock characteristics [Nerlove (1968), and Beaver et al. (1970)], the prediction of bankruptcy and bond ratings [Altman (1968), and Horrigan (1966)], the relationship between return on equity and competition [Stigler (1963)], the structures of costs and output (economies of scale) in various industries [Griliches (1972)], and the relationship between intangibles (advertising expenditures, research and development) and corporate values [Ben-Zion (1978)].

A major reason for using financial variables in the form of ratios is to control for the systematic effect of size on the variables under examination.

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The use of ratios by practitioners and researchers is necessarily based on a hypothesis (either explicitly specified or implicitly assumed) about the relationship between the numerator variable (e.g., income) and the denominator size variable (e.g., equity). Obviously, whether the use of the ratio form provides adequate control for size depends on the nature of the relationship, which can be derived from theory and/or empirical evidence. Often, however, a well-specified and empirically verified theory of this relationship is not available, yet the practitioner or researcher expects the two variables to be related. In essence, an implicit hypothesis is posited to exist. In this situation, evidence must be gathered by the researcher to determine if the use of the ratio in that context conforms to his *a priori* expectations, thereby avoiding unwarranted inferences from using the ratios.

It is shown below that control for size by the ratio form is adequate only under very restrictive conditions. When such conditions are not met by the data, perfect size control is not achieved by the use of ratios, and the resulting estimates are subject to various biases that may affect the investigator's inference. Surprisingly, despite the extensive use of ratios by practitioners and researchers, the conditions under which such use is appropriate and the consequences of using ratios when these conditions are not met are never thoroughly discussed in the accounting and finance literature. A major objective of this study is to begin to fill the void by outlining the conditions under which the ratio form adequately controls for size and the biases resulting from the use of ratios when these conditions are not met by the data. This is done in the second section of the paper.

In addition to their use in controlling for size, ratios are also used to control for (hold constant) additional factors (e.g., technology) which affect all firms within a homogeneous group such as an industry. This is generally done by incorporating in the analysis industry – and/or economy – wide standards, such as the industry mean or median ratios, intended to enable the investigator to focus on the *firm-specific* component of the ratio after the common effects of various factors on all firms within the group have been accounted for. The frequently used financial analysis technique of comparing the ratios of individual firms to an industry standard and drawing inferences from the sign and size of the differences provides one example of control for the effect of factors common to all firms within the specified group. Another example is provided by the use of index models, in which ratios of firms are regressed on economy and industry indexes to yield residuals which reflect the effect of firm-specific factors [Imel and Helmberger (1971)]. Inferences about management performance, as opposed to corporate performance, can then be drawn from these firm-specific residuals.[1]

[1] Another objective of using index models is to abstract from cross-sectional dependence in the raw observations due to common effects. Such cross-sectional dependence might adversely affect the results of statistical tests which require independence of observations.

The above mentioned use of standards or indexes raises various questions, such as which measure of location should be used as a standard and how outlier observations should be handled. As with the case of control for size, these methodological questions are not thoroughly examined in the accounting literature. Accordingly, the third section of this paper contains a discussion of various issues arising from the use of industry standards and indexes.

The general objective of this paper is thus to encourage the user of financial ratios to seriously consider the following basic questions *prior* to the use of ratios:

(1) Should the firm size (or industry-wide variables such as technology) be controlled for in the investigation? For example, in the bankruptcy prediction studies, Lev (1974b, ch. 9), size of firms was generally controlled for,[2] and consequently no inferences regarding the effect of firm size on the rate of bankruptcy could be drawn from these studies.
(2) If the hypothesis examined calls for a control of size, is the ratio form the optimal control? State differently, what is the structural relationship between the investigated variable (e.g., profit) and the size variable?
(3) Can the relationship between the examined variable and size be derived from theory (e.g., production theory – economies of scale), and what can one learn from the data about this relationship?

The indiscriminate use of ratios and industry-wide standards by many practitioners and researchers should be replaced by a careful analysis of the hypothesis tested and the data used. The following discussion provides a framework for such an analysis.

2. Control for size by ratios

The major objective of using financial variables in the form of ratios is to control for the systematic effect of size on the examined variables. Such size control underlies the most elementary use of ratios in corporate financial analysis, such as the evaluation of a given firm's profitability over time by the return on equity ratio, which controls for the changing size of equity over time. Control for size is also the major objective of using ratios in financial models, such as in Altman's (1968) bankruptcy prediction model and, in Horrigan's (1966) bond rating model, where the independent variables were various financial items (e.g., working capital and sales) scaled by total assets.[3]

[2]Size control is obtained by using ratios, including size as an independent variable in the regression, or by using samples matched by firm size.

[3]Control for size is, of course, not the only reason for using ratios. Ratios, such as return on equity, are sometimes dictated by the hypothesis examined. More on this later.

Size control by ratios is also used to satisfy the assumptions underlying the statistical techniques employed in the analysis. For example, Miller and Modigliani (1966) in their cost of capital study divided the values of the explanatory variables – equity, earnings, and asset growth – by total assets in order to remove heteroscedasticity (which is the effect of size on error variance) in the regression residuals. The same objective motivated the use of cost and output ratios to total miles-of-road (a size variable) in the research on the economies of scale in the railroad industry, Griliches (1972). Thus, ratios are used in financial analysis and in research to control for the effect of size on the investigated variables as well as on the disturbance terms in the analytic models. These two motivations may sometimes coincide.

In this section we shall consider four methodological problems associated with the use of ratios to control for size. First we consider the conditions that are necessary for ratios to serve as an adequate control for size. Second, we examine the criteria for the choice of the size variable. Third, the problem of 'spurious correlation' in ratio analysis is examined, and finally we briefly discuss the difficulties arising from the use of negative numbers in financial ratios.

2.1. Conditions for adequate size control

The ratio is an adequate instrument for size control when the variable under examination, y, is strictly proportional to the size of operation, x. Usually, the underlying relationship between y and x is suggested by theory. When a formal, well-specified theory is not readily available, the researcher should hypothesize the nature of this relationship and verify it empirically before proceeding with the use of the ratio to control for size. If the researcher's theory implies strict proportionality, $y = \beta x$ $(x \neq 0)$, the ratio $y/x = \beta$, a constant, has a straightforward interpretation: it indicates both the marginal and the average effects of a change in x on y. Thus, for example, if y represents variable costs and x represents output, the ratio y/x measures marginal cost (and, in this case, also average variable cost per unit). A time-series and/or a cross-sectional comparison of ratios is in this case meaningful, since it will indicate changes over time or interfirm differences in the values of the numerator after the systematic effect of the scale of operation has been fully accounted for. The objective of size control has in this case been fully achieved.

The case of strict proportionality between ratio components provides a point of comparison for other cases where the assumption of strict proportionality does not hold. Three such types of deviations from strict proportionality will be discussed below:

(A) An error term in the relationship.

(B) The presence of an intercept term.
(C) The dependence of y on variables other than size, and non-linearity of the relationship.

(A) The presence of error. When the relationship between y and x includes an additive error term, $y=\beta x+e$, the deviation of the ratio from β, which indicates the norm for the behavior of the numerator variable as size changes, still depends on size, $y/x-\beta=e/x$. In this case, control for size by the ratio is likely to be inadequate. The extent of deviation from perfect size control depends on the properties of the error term and its relation to the size variable, x.

If the error is homoscedastic, the ratio y/x for large firms will be closer to the norm, slope β, than the ratio of smaller firms. Thus, in the presence of homoscedastic errors, the formation of ratios and their comparison to other (time-series or cross-sectional) ratios does not provide an adequate means of control for size, since the deviations from β are small for large firms and large for small firms. Furthermore, homoscedasticity of errors in y leads to heteroscedasticity of errors in y/x and all values of y/x are not drawn from the same distribution. The larger variance of the ratio y/x for smaller firms limits the comparability of such ratios.

On the other hand, heteroscedasticity in y *might* lead to homoscedasticity of y/x. Specifically, if the error term, e, has variance proportional to x^2, the ratio y/x will have the same variance irrespective of size. It is in this heteroscedastic case that comparison of ratios to control for size finds its natural interpretation; each ratio not only has the same expectation but also the same variance.

If the degree of heteroscedasticity in y is not given by $\operatorname{Var} e \propto x^2$, the problem of heteroscedasticity of ratios remains. In formal estimation of β (discussed in the following section on industry standards), the problem can be overcome by weighted regression. But in the use of ratios for individual firms as size-controlled measures for the purpose of comparisons, a word of caution is not misplaced.

The case of multiplicative error, $y=\beta xe$, where $y/x=\beta\cdot e$, and where the disturbance term e is non-negative, is less problematic than the additive error case, since the deviation of the ratio $\beta\cdot e$ from β does not depend on the size variable x. Accordingly, comparisons among ratios, y/x, can be carried out, to obtain an adequate control for size by the ratio form.

(B) The presence of an intercept. Consider the case where the relationship between the two ratio variables includes an intercept term (i.e., a non-homogeneous or non-proportional relationship): $y=\alpha+\beta x$ $(x\neq 0)$. In this case, the ratio, $y/x=\alpha/x+\beta$, still reflects the average effect of x on y (not necessarily in a causal sense), but it is a biased measure of the marginal effect

of a change in x on y (i.e., β), where the amount of bias is α/x. Note that this bias depends on x; it will be relatively larger for small firms than for large firms. Thus, when the underlying relationship is of the form $y=\alpha+\beta x$, the ratio y/x will still be dependent on size and, therefore, will not provide a satisfactory control for size.

Ratios are frequently used to draw inferences about productivity or efficiency of firms. Consider, for example, the 'gross margin ratio' (the ratio of gross profit to sales) which is often proposed in the ratio analysis literature as an indicator of operational efficiency. The relationship between gross profit (y) and sales (x) probably contains a positive constant term, given the frequent existence of a significant fixed cost component. Accordingly, observed differences in gross margin ratios (over time or across firms) will reflect the confounding effects of differences in efficiency, reflected by β, differences in the level of fixed costs, α, and differences in sales volume, x. The latter two will be reflected by α/x. The complexity of interpreting observed cross-sectional differences and time-series changes in ratios casts doubt on the usefulness of ratios in comparing operational efficiency of firms. The same difficulty, rooted in the presence of an intercept term, is probably encountered in the evaluation of all profitability ratios, such as earnings to total assets or earnings to equity. The deviation from proportionality due to an intercept term is probably the most frequently encountered problem with financial data.

A researcher interested in investigating the existence and implications of an intercept term may explore empirically the specifications of various relationships, such as

$$y=\alpha+\beta x+u, \tag{1}$$

run on both time-series and cross-section. The statistical significance of α, as well as the stability of coefficients over time should be considered. A time-series regression for each firm may well indicate the existence of a significant intercept, α, for many firms. However, across firms, the fixed cost component is likely to vary with sales and there may be no intercept involved.[4]

(C) Dependence on other variables and non-linearity. When the variable under examination, y, depends not only on size but also on some additional variables, as is the case in most situations, deflation by size does not necessarily provide an adequate means of control for size since the other variables are also affected by the deflation. For example, consider

$$y=\alpha+\beta x+\gamma z, \qquad x\neq 0. \tag{2}$$

[4]An example of such an empirical specification test is given in Griliches (1972).

Then the ratio form is

$$y/x = \alpha/x + \beta + \gamma z/x. \tag{3}$$

The ratio, y/x, is affected in this case by the values of other variables involved in the relationship, z/x, which now become functions of size. Consider, for example, Longbrake's (1973) study on the cost function of banks' demand deposit operations. The operating cost of demand deposits was found to be significantly associated with variables such as the average number of accounts per banking office, average dollar size of account, number of offices operated by the bank, the average wage per demand deposit employee, etc.[5] Given this complex relationship, how should one interpret observed differences in the widely used ratio of operating (demand deposits) costs to the number of demand deposit accounts (i.e., the average cost per account)? Obviously, no economic interpretation is possible unless all other associated variables are considered. However, once this is done there seems to be no reason for using the ratio as a shortcut for a more complete analysis.[6]

The interpretation of ratios becomes even more difficult when the underlying relationship is non-linear. Consider, for example, the case where the well-known EOQ, or 'square root formula' adequately describes the inventory management policy of a firm. The square root formula implies that the inventory turnover ratio (sales to average inventory) will be a function of the *square root* of sales. Accordingly, comparison of the inventory turnover ratio of a given firm with the industry mean (or median) ratio – a standard tool of financial ratio analysis – is incomplete since sales level is not explicitly accounted for.[7] Similar problems arise in the interpretation of a time series of the inventory turnover ratio, when changes in the level of operations are not considered explicitly.

Summarizing, the ratio form adequately controls for size only under highly restrictive conditions. When these conditions are not met, size is not adequately controlled for and, more seriously, the amount of bias varies with size; it is large for small firms and relatively small for large firms. Given these problems, the use of ratios in financial analysis and research should be

[5] A linear regression function on the logarithms of these and other variables was used for estimation.

[6] Ratios are often computed for predictive purposes. Suppose for example, a bank wishes to predict the average cost per demand deposit account in a new office (branch). Obviously, past ratios for existing branches are of little use in this case since the average cost will depend on variables such as the number of accounts in the new office, average size of those accounts, etc., which have to be considered explicitly.

[7] For example, suppose the inventory turnover ratio of a relatively small form is found to be smaller than the industry mean. This finding is obviously insufficient to judge the firm's inventory management as inefficient.

accompanied by a theoretical justification and an empirical analysis of the degree to which the data meet the ratio assumptions. In the likely case of deviations from these assumptions, an analysis of the sensitivity of findings to these deviations should be conducted. An example of such a careful analysis of the consequences of using ratios by researchers can be found in Griliches (1972). Standard econometric texts such as Maddala (1977) discuss the appropriate procedures for testing the validity of the assumptions.

2.2. *Choice of the size variable*

The conditions under which the size effect can be controlled by a ratio were discussed in the preceding section. We turn now to the second major question arising from the control for size – the choice of a size variable. Specifically, given the availability of several alternative measures of size, the choice of the appropriate measure in an application is obviously important for the successful control of the size effect. Unfortunately, in most empirical applications the choice of a specific size measure is made in an ad hoc manner,[8] often leading to ambiguities in the interpretation and generalization of empirical findings. For example, Beaver et al. (1970) and Ball and Brown (1969) reported a statistically significant and strong relationship between stock market-based estimates of systematic risk of common stocks and accounting-based estimates of systematic risk ('accounting betas').[9] Gonedes (1973a), on the other hand, found only a week relationship between these two estimates of systematic risk, arguing that the difference between the findings may be due to differences in the size measure used for deflation: while Beaver et al., and Ball and Brown used the market value of the firm to deflate income, Gonedes used total 'book value' of assets as the deflator.[10] Such controversy might well be avoided by a more careful choice of the most appropriate size deflator.

There are at least three major considerations, one theoretical and two empirical, in the choice of a size variable: (a) identification of the relevant set of reference, or 'market', (b) relationship of the size variables to the variable under examination (e.g., earnings), and (c) error in measurement of the size variable. Stigler (1968, p. 30) discusses the theoretical problem of identifying the reference market, as follows:

'The *purpose* of a measure of concentration is to predict the extent of

[8]For example, Nerlove (1968) deflated observations on retained earnings and dividends by total assets. However, no economic arguments or empirical evidence were provided as to the reasons underlying the choice of this specific size measure.

[9]The latter were derived from time-series regression of corporate income numbers on an economy-wide index of income.

[10]This deflation controversy was pursued in Beaver and Manegold (1975) and Gonedes (1975).

the departure of price (or, alternatively, of rate of return) from the competitive rate of return. This purpose supplies an answer to our earlier question of how to measure the size of firm: two firms are equal in a market if they sell or buy equal quantities in that market. Hence measure a firm's size by sales, in a product market; by employees, in a labor market; by materials, in a material market; by assets, in a capital market.'

For example, when employee productivity is of interest (the labor market), then total output relative to the number of employees or to total man-hours may be the appropriate measures of size. When the productivity of the capital employed is of interest (the capital market), then earnings as a function of equity, the latter measured by book or market values, is appropriate.

When the economic phenomenon under examination affects the size measure drawn from the appropriate market differently for different firms, the market criteria may have to be abandoned. For example, Deakin (1979) in his study of accounting methods in the oil and gas industry avoids using income, equity or market value as the measure of size since these variables are affected or alleged to be affected by the accounting method used for exploration costs (full costs or successful efforts). More generally, if the size variable itself is affected by the phenomenon being analyzed, particular care is needed in using this variable as a size measure.

While the 'market' criterion narrows down the choice of size variable, it is in general insufficient for the choice of a unique measure of size. For example, total assets (to be used according to Stigler in capital markets) can be measured by historical cost, replacement cost, or the total market value (of stock and bonds) of the firm. Which one should be used? Note that two issues are involved in this choice: (i) the relevant size *variable* based on a priori theoretical considerations (e.g., total sales or total number of employees in the preceding quotation), and (ii) the *estimator* with the most appropriate statistical properties (e.g., correlation, bias, consistency) for each size variable, such as replacement cost or historical costs for total asset size.

Choice criteria (b) and (c) above are concerned with such statistical properties and are therefore largely empirical. Criterion (b) relates to the strength of correlation and homogeneity of the examined variable to the size measure. In the ratio context, when the correlation between the numerator and the size measure is weak, little by the way of control for size is needed or accomplished by using a ratio. Yet researchers sometimes use size deflators which bear weak or no relationship to the numerator variable of interest.

For example, in the extensive research on railroad cost structures, Griliches (1972), raw cost and output data were in general deflated by the miles-of-road size measure, yielding the two ratios: cost/miles-of-road, and output/miles-of-road. However, when Griliches estimated the cost–size relationship by the following regression on the data of 97 railroads:

$$\text{cost} = \alpha(\text{miles-of-road}) + \beta(\text{output}), \tag{4}$$

he found α to be statistically insignificant, concluding that 'there is no evidence that M [miles-of-road] belongs in the equation in any form . . . Anyway, division by an irrelevant variable is uncalled for' (p. 32). Researchers on the structure of railroad cost were thus using an inappropriate size measure.[11]

Having chosen the size variable of interest, one must then determine how to make use of it. Griliches (1972) suggests that prior to using a size deflator, various alternative measures should be examined for their extent of correlation with the variables under examination (cost and output in the railroad case) and to determine whether a ratio or some other method of control is appropriate. In general, where the theory is not sufficiently well specified to determine whether control for size is necessary, and if necessary, what is the optimal measure of size, the researcher should use the data analysis approach to address these questions. For example, by running alternative regressions, such as $y = a + bx + u$ and $y/x = a/x + b + w$, one can infer from the significance of the coefficient b whether size deflation by x is adequate. The significance of coefficient a will provide evidence about the existence of an intercept. The behavior of the residuals from each regression can be investigated to determine if the errors are homescedastic in the levels or deflated form. This empirical approach to specifying the ratio can be generalized to situations in which y is expected to vary with the other variable, z, as well as with size, x.

A related issue to the choice of size measure is the substitutability among such measures. If, for example, two alternative size measures are highly correlated, they can be regarded as substitutes, and the choice will be a matter of convenience or data availability. Some empirical evidence on substitutability is provided by Smyth et al. (1975), and by Shalit and Sankar (1977, p. 297) who conclude:

> 'The inequality coefficient is very high (around 0.4) for a pair involving employment and another proxy, and in each case bias is a dominant source of inequality. Thus employment appears to be a poor substitute for any of the other four [sales, total assets, stockholders' equity, and market value of the firm] for prediction purposes. Also, employment and market value (or sales) are not good substitutes for estimation because

[11]Another example is reported by Kuh and Meyer (1955) in a review of Chenery's (1952) study in which the correlation of the size variable, capital stock, with output varied substantially across industries and the use of capital stock for deflation yielded non-comparable results and misleading conclusions.

> the correlation coefficient is relatively low. The highest correlation coefficient is between assets and stockholders' equity . . . For these reasons it is clear that assets may be interchanged with stockholders' equity as measures of size.'

Additional empirical evidence on the substitutability of alternative size measures will be of considerable help to researchers employing size deflators.

The third issue of concern in the choice of a size variable involves the extent of measurement errors in the size variable. Unfortunately, the measurement error for most financial variables is not easily determined, making it difficult to select size variables on the basis of minimum measurement error.[12] Beaver, Kettler and Scholes' (1970) use of the market value of owners' equity as the size measure might have been motivated by the assumption that the market value has a smaller measurement error than accounting variables. However, the total market value of the firm is not directly observable if some of the claims are not traded (e.g., private debt, lease commitments, etc.). Whether accounting owners' equity, total reported assets, or the market value of the equity (i.e., the product of the price of individual shares and the number of outstanding shares of stock) are closer approximations of the market value of the whole firm is as yet an untested question.

One might encounter cases where allowance for size effect will not be achieved by a conventional measure of location, such as average total assets of the firm. In these cases other properties of the size variable's distribution such as its dispersion, can serve as candidates for deflation.[13] For example, Fama (1974) deflated dividend prediction errors for each sampled firm by the standard deviation of dividend changes for that firm arguing that 'since the dispersion of the distribution of the [dividend] prediction error for any given model is likely to vary from firm to firm in constructing cross-sectional distributions, the prediction errors are measured in units of the standard deviation of the dependent variable [dividend change]' (pp. 306–307). A similar deflation by a dispersion measure was employed by Beaver (1968) in examining the reaction of stock prices to earnings announcements. Individual

[12]The problem of measurement errors in macro- and microeconomic data, including several financial variables, was extensively investigated by Morgenstern (1950). Surprisingly, this important issue has not been thoroughly investigated in the accounting literature.

[13]The general issue here is the choice between raw and standardized variables to be used in the analysis. A suggested standardization of variables in regression analysis, known as 'beta coefficients', involves the scale free variables expressed in units of their standard deviation:

$$y/\sigma_y=\alpha_1^*+\beta_1^*(x_1/\sigma_1)+\beta_2^*(x_2/\sigma_2)+\ldots, \tag{5}$$

where σ_1 and σ_2 are the standard deviations of x_1 and x_2, respectively, and α^*, β_1^*, β_2^* are the estimated regression coefficients when the variables are all defined in terms of standard deviation units. For elaboration, see Maddala (1977, p. 119).

stock price residuals (from the 'market model') were deflated by the standard deviation of these residuals. It should be noted that despite the intuitive appeal of the deflators in the above two examples, no evidence was provided to the effect that relative to other candidate deflators they had indeed achieved their objective of increasing cross-sectional homogeneity of observations.

Summarizing, when control for size is employed, researchers should be careful in the choice of size variable. When the choice of a unique measure cannot be justified on theoretical grounds, as is often the case, information should be provided on the degree of substitutability of the different measures, and the sensitivity of findings to alternative measures.

2.3. *'Spurious correlation' due to size deflation*

Correlation r_{xy} between two variables, x and y, is not, in general, equal to the correlation $\rho(x/z, y/z)$ between the ratios x/z and y/z, where z is any other variable. Whenever inference about the relationship between x and y (e.g., output and cost) is based on correlation of x/z and y/z (output/size and cost/size), there is a possibility of spurious inference. This is often, and mistakenly, referred to as the problem of spurious correlation. Though the problem was thoroughly analyzed by Pearson (1897) and Kuh and Meyer (1955), spurious inference from data and somewhat indiscriminate references to spurious correlation continue to appear in the accounting and finance literature [for example, Gonedes (1973a, p. 436)]. The mere presence of ratios with a common denominator on both sides of a regression equation does not make the results or inference from the model spurious. More careful analysis is necessary.

Correlation between ratios with a common deflator may be greater or smaller than the correlation between the undeflated variables. If the coefficient of variation of the deflator z is small enough so that the third and higher order terms of the expansion could be ignored, the relationship between ratio and variable correlations was derived by Pearson (1897) to be

$$\rho\left(\frac{x}{z}, \frac{y}{z}\right) = \frac{r_{xy}v_x v_y - r_{xz}v_x v_z - r_{yz}v_y v_z + v_z^2}{(v_x^2 + v_z^2 - 2r_{xz}v_x v_z)^{\frac{1}{2}}(v_y^2 + v_z^2 - 2r_{yz}v_y v_z)^{\frac{1}{2}}}, \tag{6}$$

where r_{xy}, r_{yz}, r_{xz} are the respective coefficients of correlation, and v_x, v_y, v_z are the respective coefficients of variation.

The basic problem is best summarized by Kuh and Meyer (1955, p. 403):

> 'Correlating ratio variables and making inferences from there to the *simple* correlations between the series in the numerator is an extremely hazardous business. A possibly unexpected result is that in the context

of spurious correlation the ratio correlations may just as well be spuriously low as spuriously high. Consequently, if primary interest centers on the *simple* correlation between the numerator series, it would usually be appropriate to proceed in a straightforward manner by relating these values to one another.' (Emphasis added)

Of course, the primary interest of researchers in correlating ratios usually lies not in the simple correlation between the numerator series but in their partial correlation, when the denominator variable is controlled or held constant. What, then, are the conditions that equate the ratio correlation (ρ) to this partial correlation ($r_{xy \cdot z}$)? Kuh and Meyer showed that two conditions are necessary and sufficient for equality of ρ and $r_{xy \cdot z}$: (a) the coefficient of variation v_z be small so that series expansion yields a good approximation of ρ, and (b) the numerator variables x and y be linear homogeneous functions of the denominator z. In other words, if v_z is sufficiently small to make the probability of a zero or negative denominator very small, homogeneity of x and y in z is sufficient to eliminate the risk of spurious inference from ratio correlation, provided that the researcher is interested in the partial correlation, $r_{xy \cdot z}$, between the numerators when the denominator is held constant.

When the homogeneity condition is not met, the presence of a negative (positive) intercept renders the ratio correlation a positively (negatively) biased estimate of partial correlation. Furthermore, the magnitude of the bias for a given deviation from homogeneity depends on the strength of correlation, r_{xy}, relative to r_{xz} and r_{yz}. If the influence of size on x and y is weak, a given deviation from homogeneity results in a smaller bias than would be the case if this influence were strong. When the influence of size is strong, the need to control for size is also strong; it is only then that the lack of homogeneity leads to large biases in ratio correlation as an estimator of partial correlation $r_{xy \cdot z}$.

Note that the above discussion of spurious inference is limited to cases when inference about simple or partial correlations between numerator variables is drawn on the basis of ratio correlations. When the *ratio* correlation itself is of intrinsic interest, there can be no question of spurious correlation. Consider, for example, the Whitbeck and Kisor (1963) model where the following relationship was hypothesized:

$$\begin{aligned}\text{price/earnings} = \alpha + \beta_1 &\text{(earnings' growth rate)} + \beta_2\text{(dividends/earnings)} \\ &+ \beta_3 \text{(earnings' variability)}. \qquad (7)\end{aligned}$$

The relationship between the price/earnings ratio and the dividend/earnings (payout) ratio was hypothesized by the authors on economic grounds: 'To the extent that investors as a whole consider current dividends a desirable investment characteristic, we would expect high dividend pay-out ratios to contribute positively to the prices of common stocks' (p. 58).[14] Accordingly, given that the hypothesis was formulated in terms of *ratios*, the existence of a common denominator (earnings) does not, by itself, give rise to the problem of spurious inference.

Misconceptions about the nature of 'spurious correlation' are exemplified by the argument raised by Gonedes (1973a) with regard to the Beaver et al. (1970) results showing significant correlation between the market and accounting betas. Since Gonedes did not find the significant correlations reported by the earlier study, he reasoned that the higher correlations obtained by Beaver et al. can be 'spurious' due to the use of market value of equity as the scaling factor in that study. This use of the term 'spurious correlation' was quite different from the specific problems analyzed by Pearson and by Kuh and Meyer as we have already discussed. The mere fact that two variables whose correlations are under scrutiny are functions of a third variable is no basis to declare the observed phenomenon spurious.

The fundamental issue in using ratios with the same or correlated denominators is therefore not one of spurious correlation but rather of *model specification*. One should not carelessly deflate observations by some intuitively appealing variable without reference to the objective of investigation. Raw and deflated quantities reflect different variables (e.g., total earnings and rates of return), and a hypothesis formulated in terms of one variable cannot be directly tested using data based on another variable. It is a matter of concern that the use of financial ratios appears to be often motivated more by custom and tradition than by explicit reference to the specific hypothesis.[15] If the investigator's interest is in the numerator series (e.g., cost and output), there seems to be no reason to use ratios (e.g., cost and output per miles of road). The size effect, for example, can be often accounted for by adding a relevant variable to the regression equation. Thus, instead of being concerned with spurious correlation, one should pay careful attention to model formulation in the light of theoretical and statistical considerations at hand.

2.4. Negative numbers and control for size

The use of ratios as a means for size control presents special problems when one or both variables take negative values. The problem is rooted in the following results from calculus:

[14]The reader should note that we use this statement as an example only. The consistency of this argument with financial theories and/or its empirical validity is not at issue here.

[15]It is interesting to note in this context the reaction of W.F.R. Weldon to K. Pearson's criticism on his use of ratios in biological research: '. . . I have always expressed the size of the organs measured in terms of body length' [Pearson (1897, p. 498)]. Thus, unreflecting choice of deflators is apparently not restricted to users of financial data.

(a) When the algebraic sign of the numerator changes (e.g., from positive to negative), the sign of the partial derivative of the ratio with respect to the numerator is unchanged but the partial derivative with respect to the denominator changes sign.
(b) Similarly, when the sign of the denominator changes, the sign of the partial derivative with respect to the denominator is unchanged but the partial derivative with respect to the numerator changes sign.

Thus, whenever the sign of the numerator or denominator changes, one of the two partial derivatives of the ratio also changes sign. Accordingly, a change in a variable (e.g., earnings) which has a 'favorable' effect on the ratio before the change of sign will have an 'unfavorable' effect after the change of sign. This loss of continuity is a frequent cause of problems in interpreting ratios computed from negative numbers. Consider, for example, the case where the numerator of a ratio changes sign from one period to another, such as the earnings to equity (or total asset) ratio, where earnings were negative in one year and positive in the following year. In this case, where the firm's profitability has obviously improved, the relative (percentage) change of the earnings to equity ratio will be *negative* (assuming the denominator is positive). This basic problem renders ratios a hazardous instrument of controlling for size in the presence of negative numbers, and the researcher would be well advised to seek alternative means of exercising such control whenever feasible.

Sometimes the problems associated with certain negative variables and their ratios can be mitigated by a simple modification of the ratio form. We will discuss such a modification for the income figure which is the most susceptible of all major financial variables to taking negative values. Income, I_t, of a firm during period t can be seen as the difference between the beginning- and ending-period owners' equity, E_{t-1} and E_t, respectively, after the appropriate adjustments for capital transactions have been made to the ending-period owners' equity.[16] Therefore the ratio of E_t/E_{t-1}, which is equal to the traditional return on owners' equity plus 1, *and will in general be positive*, will mitigate the problem of discontinuity posed by the negativity of firm income.

The above-described transformation of ratios can be formulated in more general terms. Where the numerator of a ratio is a flow variable that can take negative values and the denominator is the *corresponding* stock variable, we might consider replacing this ratio by the ratio of consecutive observations on the stock variable or by a logarithmic transformation of the

[16] Adjustments to ending-period owners' equity include (a) adding the cash dividends declared and treasury stock purchased, and (b) subtracting the proceeds of new common stock issued and treasury stock sold.

latter. Of course, the observations on the stock variables must be appropriately adjusted in the manner suggested above for the owners' equity.

3. Control for industry-wide factors

In the preceding section, we have focused on the use of ratios as an instrument of control for size before making cross-sectional or time-series comparisons of financial variables. Besides size, many other factors affect the values of financial variables: For example, cross-sectional differences in corporate earnings or rates of return were found in the industrial organization literature to be partially explained by differences in risk levels and in the degree of competition among industries. Gupta and Huefner (1972) found that cross-sectional differences in many financial ratios were primarily related to industry characteristics. There are conceptual and practical difficulties in controlling for (holding constant) such factors. On the conceptual level, there is lack of adequate theoretical knowledge about the impact of many economic factors on financial variables. For example, the question as to why do firms, even within a given industry, have different financial leverages is far from being well understood. On the practical level, the number of such economic factors is large, and adequate control for them may require more observations than can be conveniently gathered.

Given that many of these economic factors, such as those arising from technological conditions, supply and demand structures, etc., are relatively uniform *within* industries, yet vary substantially *across* industries, an industry stratification provides a convenient way of controlling for such factors. Such an industry stratification is very common in the use of ratios for financial analysis. In practical ratio analysis, the ratios of an examined firm are often compared with industry location measures, such as the mean or median industry ratio, and inferences are based on the direction and extent of deviation between the examined ratio and the industry standard. In empirical research, control for industry effects is often exercised by the use of samples matched by industries.[17] Sometimes the examined ratios are divided by industry mean ratios, such as in Horrigan (1966) and Fisher and Kraft (1971). More recently, control for industry-wide factors by the use of index models has been suggested. Specifically, Lev (1974a) and Foster (1978) suggest the use of a 'residual analysis' approach to financial statement analysis, in which the time series of a given firm's ratio is regressed on the industry mean ratio (and possibly also on a corresponding economy average ratio) to yield residuals which reflect the effect of firm-specific factors. Analysis then focuses on these residuals, given that the effects of factors

[17]This sample design is very common in the bankruptcy prediction literature; see Lev (1974b, ch. 9) and Foster (1978, ch. 14).

common to the industry and to the economy have been controlled for (held constant) by the regression.

The use of industry norms to control for common factors appears, therefore, to be of importance both in practice and in research, raising the question of how to choose industry norms.[18] Here, as in the preceding case of the choice of the measure of size, there appear to be no guiding criteria in the accounting and finance literature, and consequently decisions by researchers (e.g., to employ an equally-weighted or a value-weighted industry mean ratio) appear to be ad hoc.

3.1. *Choice of industry standard*

The choice of an industry standard, to account for industry-wide factors, depends to a large extent on the cross-sectional distributional properties of financial variables.[19] Unfortunately, very little is known about these properties;[20] based on Deakin's study, about all that is known at this stage is [Deakin (1976, pp. 95–96]:

> '. . . it would appear that assumptions of normality for financial accounting ratios would not be tenable except in the case of the total debt to total assets ratio . . . However, it does appear that normality could be achieved in certain cases by transforming the data . . . there appear to be cases in which both the square roots of the data and the natural logs of the data were normally distributed. There also appeared from the study an indication that financial accounting ratios might be more normally distributed within a specific industry group.'

Given the scarcity of evidence concerning the properties of ratio distributions, we can only provide some general remarks on the appropriate industry summary measures, under the assumption that the financial variables comprising the ratios are either normally or lognormally distributed (or can be reasonably transformed to approximate such distributions).

(A) If the ratio is formed of two normally distributed variables, y and x, the distribution of the ratio, given by Fieller (1937), does not have first or any

[18] Other questions can, of course, also be raised, such as what is the most appropriate industry grouping (e.g., by product, growth rate, etc.). These questions, however, are beyond the scope of this paper.

[19] And, of course, on the objective of analysis or model used. However, available theories pertaining to the use of financial variables rarely, if ever, specify a unique industry summary measure.

[20] It is interesting to contrast the vast amount of empirical evidence relating to the distributional properties of stock prices with the scarcity of evidence on financial ratio distribution. See Foster (1978, chs. 5 and 6) for some evidence.

higher moments. In this case, the *equally-weighted* mean of the sample ratios, $(1/n)\sum_{i=1}^{n}(y_i/x_i)$, n being number of sample firms in the industry, is unstable in the sense that it does not converge as the value of n is increased.[21] Furthermore, the larger the coefficient of variation for the denominator, x, the more serious is the problem of instability. However, the weighted mean of sample ratios, with the denominator (e.g., equity, total assets, etc.) used as the weighting factor

$$\bar{y}/\bar{x} = \sum_{i=1}^{n} [(y_i/x_i)(x_i/\sum x_i)], \tag{8}$$

is the maximum likelihood estimator of the ratio of population means μ_y/μ_x.[22] Accordingly, for normally distributed variables, the use of the ratio of sample means, namely the weighted mean (8), seems more appropriate than the mean of the ratios (the equally-weighted mean) as a summary measure for the cross-sectional ratio distribution. It is interesting to note that empirical findings are consistent with this conclusion. Gonedes (1973b) found the weighted (by equity) average of firms' earnings and sales numbers to be a more representative economy-wide index (i.e., representing economy-wide factors) than the equally-weighted index.

(B) If a ratio is formed of two lognormally distributed variables, the ratio also is lognormally distributed.[23] When the cross-sectional distribution of the

[21]For example, in his study of oil industry profits, Sunder (1977) found that the equally-weighted averages of profitability ratios were very unstable and difficult to compare. He excluded all firms with a size of less than one million dollars to compute his results.

[22]Zellner (1978) has suggested a Bayesian approach to this estimation problem, termed MELO (minimum expected loss). The MELO estimate for μ_y/μ_x with diffuse prior and generalized square error loss function is given by

$$\bar{y}/\bar{x}[1+\mathrm{cov}(y,x)/\bar{y}\bar{x}]/[1+\mathrm{var}\,x/\bar{x}^2], \tag{9}$$

where $\bar{y}$ and $\bar{x}$ are posterior means, and $\mathrm{cov}(y,x)$ and $\mathrm{var}(x)$ are posterior covariance and variance, respectively. For large sample sizes the MELO estimate approaches the maximum likelihood estimator $\bar{y}/\bar{x}$, but for small samples the difference between the two estimates can be quite large. Note that for the MELO estimation the *complete* posterior distribution is available and should be used in specific applications.

[23]There is some evidence [e.g., Gibrat (1931)] that the distributions of various financial variables, such as corporate profits, bank deposits, wealth of individual persons, etc., are lognormally distributed. Deakin (1976) reports that the log-transformation of some financial ratios produced normal distributions. In general, skew distributions appear on a priori grounds to provide better approximations than symmetric distributions to financial variables and ratios, since by the multiplicative central limit theorem, the distributions of products and quotients (ratios) of positive random variables tend to lognormality, just as the sums of random variables tend to normality. Indeed it is a well-known phenomenon that the distributions of many financial ratios are skewed to the right; see Lev (1974b, pp. 63–64). Of course, the lognormal distribution is not the only non-negative, skewed to the right distribution. Other possible

ratio under examination is lognormal, there is room for disagreement about the appropriate summary measure or standard for the industry, since the arithmetic mean of the lognormal distribution is larger than the median, which in turn is larger than the modal value. Specifically, if x $(0<x<\infty)$ is distributed lognormal, $\Lambda(\mu,\sigma^2)$, then the arithmetic mean, median and mode of x are $\exp(\mu+1/2\sigma^2)$, $\exp(\mu)$, and $\exp(\mu-\sigma^2)$, respectively.[24] It should be noted that the median, $\exp(\mu)$, of the lognormal distribution is equal to its geometric mean.

When the industry distribution of a ratio is lognormal, the use of its geometric mean (median), $\exp(\mu)$, as the industry standard would seem to have several advantages. First, the logarithmic and exponential transformations that relate the normal and lognormal distributions also relate the medians of the normal and lognormal distributions. Specifically, if x is lognormal and y is the corresponding normal random variable,

$$\begin{aligned} &\log x = y, && \log \text{median}(x) = \text{median}(y), \\ &\exp(y) = x, && \exp(\text{median}(y)) = \text{median}(x). \end{aligned} \tag{10}$$

The median is thus the only parameter preserved under the transformation relating the lognormal to the normal distribution. The normal distribution is stable with respect to additive operations, and its median is equal to its arithmetic mean. Similarly, the lognormal distribution is stable with respect to multiplicative operations, and its median is equal to its geometric mean. Thus, the use of the geometric mean or median of a lognormal distribution as the industry standard is equivalent to using the arithmetic mean or median as the industry standard for a normal distribution.

Since the products and quotients of lognormal variables are also distributed lognormally, the industry standards for ratios are easily determined as follows. If x is lognormally distributed, $\Lambda(\mu,\sigma^2)$, then $1/x$ is also lognormally distributed, $\Lambda(-\mu,\sigma^2)$, with a median or geometric mean of $\exp(-\mu)$. Similarly, if x_1 and x_2 are lognormally distributed, $\Lambda(\mu_1,\sigma_{11})$ and $\Lambda(\mu_2,\sigma_{22})$, respectively, then their ratio, x_1/x_2 is also lognormally distributed, $\Lambda(\mu_1-\mu_2,\sigma_{11}+\sigma_{22}-2\sigma_{12})$, and its median (geometric mean) is $\exp(\mu_1-\mu_2)$, which can be used as the industry standard.

It is interesting to note that the median of the industry ratio distribution is often used as a standard for ratio evaluation, such as in Dun and Bradstreet's *Key Business Ratios*. The reasons given for this choice of the

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candidates for describing financial variables are the log-student t distribution (of which the lognormal is a limiting case), the inverted Gaussian distribution, various gamma and inverted-gamm distributions, etc.

[24]See Aitchison and Brown (1963), Johnson and Kotz (1970), and Zellner (1971) for a discussion of the properties of lognormal distribution and procedures for estimating its parameters.

median are generally related to its robustness to large outliers and measurement errors. However, as has been argued above, the median is also a 'natural' location parameter (standard) when the ratio distribution can be well approximated by a lognormal distribution.

(C) Finally, coefficients from cross-sectional regressions can themselves be regarded as candidates for industry standards in certain cases. For example, if the relationship between y and x is approximately linear, homogeneous (i.e., zero intercept) but not exact, it can be represented as

$$y_i = \beta x_i + \varepsilon_i, \qquad i = 1, \ldots, n, \tag{11}$$

where β is the industry norm and the residual term ε_i reflects the effects of factors unique to the firm. The least squares estimator of the regression coefficient, b, is given by

$$b = \sum_i x_i y_i \Big/ \sum x_i^2 = \sum_i (y_i/x_i)\left(x_i^2 \Big/ \sum_i x_i^2\right). \tag{12}$$

Thus, by (12), if the specifications of the least squares regression model are satisfied, a weighted version of the sample ratios (the n firms' ratios are y_i/x_i; the weights are $x_i^2/\sum x_i^2$) provides an unbiased, consistent, and efficient estimate of the industry norm, β.

The weights $(x_i^2/\sum x_i^2)$ are special cases applicable when the variance of the firm specific factor ε_i is constant across firms. In general, an efficient estimator of the industry standard β is obtained by using weights $(x_i/s_i)^2/\sum(x_i/s_i)^2$ to compute the mean of individual firm ratios when the standard deviation of the firm specific factor ε_i is proportional to s_i. We shall mention two special cases: First, when $s_i = x_i^{1/2}$; the optimal estimate of β is given by weights $x_i/\sum x_i$,

$$b_1 = \sum_i \left(x_i/\sum_i x_i\right)(y_i/x_i) = \bar{y}/\bar{x}. \tag{13}$$

Second, when $s_i = x_i$, the optimal estimate of β is the equally weighted mean of firm ratios,

$$b_2 = \frac{1}{n}\sum_i (y_i/x_i).$$

Thus, different weighting schemes of the industry ratio distribution are also consistent with different remedies for heteroscedasticity.

Summarizing, the question of the appropriate industry-wide standard or index depends first on the underlying theoretical reasons as to why firms within the same industry are expected to have similar or dissimilar ratios and, second, on the distributional properties of financial variables and ratios. Lacking sufficient evidence regarding these distributional properties, we have discussed the appropriateness of various industry-wide indexes under the assumption of normality and lognormality of the variables comprising the ratios. Under these conditions, the value-weighted mean (for normally distributed variables) and the median (for lognormally distributed variables) appear to be the most representative industry-wide measures of location.

3.2. The treatment of 'outliers'

In determining industry standards, researchers are sometimes faced with the problem of 'outliers'. Outliers can be defined as observations that are so unusually small or large that the researcher feels that there is a very high probability that they are the result of an error in measurement, yet the source of error cannot be easily identified. In financial ratio analysis unusually large values often occur because the denominator of a ratio is close to zero (e.g., an earnings to equity ratio, where equity has been decimated by a streak of losses, or an inventory turnover ratio, where inventory at the end of the year happened to be very small).[25] Obviously, the equally-weighted mean of the ratio distribution will be significantly affected by an extreme observation of this type. However, value-weighted (by the denominator) means of ratios (8) or medians, which were shown above to be more appropriate than the equally-weighted mean as estimates of industry standards, will be less sensitive to unusually large ratios resulting from near-zero denominators. The use of medians and size-weighted means reduces the sensitivity of industry standards to extreme observations caused by errors of measurement and minimizes the need to devise ad hoc procedures to deal with outliers that are likely to contaminate the sample seriously.

Finally we should mention two techniques aimed at a systematic treatment of outliers, known as 'trimming' and 'winsorizing'.[26] Trimming involves the removal from the sample of an equal number of the smallest and largest observations and then proceeding as if the trimmed sample were a complete one. If the sample is drawn from a normal population, the trimming procedure results in some loss of efficiency in estimating the location parameter (e.g., the mean). However, if the distribution is longtailed, efficiency is increased by trimming. Winsorization, a technique named after

[25] A small denominator is one of the major reasons for ratio distributions to be skewed to the right (discussed above).

[26] See Tukey (1962) for elaboration on these and various other techniques suggested for data analysis.

Charles P. Winsor, does not delete outliers as in trimming; rather the outlier's value is changed to the value of the nearest observation not seriously suspect as arising from a measurement error. For normally distributed samples, winsorized means are more stable than trimmed means.[27] It should be noted, however, that the identification as well as treatment of outliers remains to some extent subjective. Recommended techniques, such as trimming and winsorizing, while capable of improving the estimation of location parameters under certain circumstances, are still somewhat ad hoc.

4. Concluding remarks

It appears that the extensive use of financial ratios by both practitioners and researchers is often motivated by tradition and convenience rather than by careful methodological analysis. Basic questions, such as does the examined hypothesis (theory) call for control of size and/or other industry-wide factors in the analysis, what is the structural relationship between the examined variable and size, and how can the control of size and other factors be best achieved, are rarely addressed by users of financial ratios. Consequently, the data analysis appears to be in many cases inconsistent with the examined theory leading to unwarranted inferences. The major objective of this study is to encourage users of financial ratios to examine carefully the adequacy of using ratios in their analysis. This was done by analyzing various major issues related to the use of financial ratios. In particular, the conditions under which ratios can fulfill their major objective – control for the size effect – were investigated. It has been shown that these conditions, are very restrictive and that deviations from these conditions result in adequate size control. It is important that the form of the relationship between the variable of interest and size be identified by theoretical considerations, empirically substantiated, before the ratio is used as an instrument of control for size.

The second major issue in the formation and use of ratios is the choice of size deflator. Economic and statistical criteria for this choice problem were discussed. The third issue in the context of size control was the so-called problem of spurious correlation of ratios having common denominators. The statistical issues arising from the existence of common denominators and the possibilities of spurious inferences (rather than spurious correlations) were discussed. Examination of the problems caused by negative numbers and suggestions for overcoming these problems closed the size control section.

In the third section of the paper we extended the scope of discussion to control for other (than size) variables common to firms within a given group

[27]For elaboration, see Sarhan and Greenberg (1958).

(e.g., industry). Criteria for the choice of adequate standards, or indexes to represent the common effects of these variables, and the treatment of outlier observations were discussed.

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