

Econometrics of Fair Values

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SYNOPSIS: The properties of many important valuation rules can be quantified, examined, and compared in a unified framework to assist policy decisions. Valuation rules can be viewed as econometric estimators of unobserved values of aggregates. Which valuation rule has minimum mean squared error (relative to the unobserved value of bundles of resources) is a matter of econometrics, not of theory or principle; it depends in a known fashion on the relative magnitudes of the parameters—price volatility and measurement errors—of the economy, industry, or firm. In general, no valuation rule, fair or not, dominates the others. Given the parameters of an environment, this framework can help identify efficient valuation rules.

INTRODUCTION

This article summarizes a framework and results developed in the past four decades of research to characterize various valuation rules as alternative econometric estimators of economic value.¹ Two key determinants of the properties of these estimators are the degree of price instability and the magnitude of price measurement errors. The framework can help choose valuation rules or estimators on the basis of their objective properties in the relevant economic environments, not opinions.

In accounting, few topics generate more impassioned debate than rules of valuation. They directly affect accounting numbers used in investment decisions, stewardship, management of enterprise resources, and contract enforcement. Reliability, relevance, bias, timeliness, and representational faithfulness are some of the oft-mentioned qualitative criteria for the evaluation and comparison of valuation rules.

There is little agreement, even among experts, about the qualitative properties of valuation rules (see Joyce et al. 1982), and there is no systematic way of assessing or reconciling them. Without a framework for quantified comparison, valuation debates remain largely unresolved, sometimes leading to misguided recommendations.

¹ Ijiri (1968); Tritschler (1969); Sunder (1978); Hall (1982); Sunder and Waymire (1983, 1984); Shriver (1986, 1987); Shih and Sunder (1987); Tippett (1987); Lim and Sunder (1990); Hall and Shriver (1990); Lim and Sunder (1991); and Jamal and Sunder (1995). See the Appendix for a capsule overview of this literature.

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An econometric approach can help us analyze valuation rules by considering each rule as a member of a larger class (called exchange valuation rules). Under this approach, valuation methods are viewed as estimators of the unobserved values of asset bundles of interest. Econometrics can help transform what has been essentially a qualitative debate into quantitative analysis, allowing researchers to contribute constructively to social policy by adducing evidence on falsifiable propositions.

In September, 2006, the Financial Accounting Standards Board (FASB) issued Statement of Financial Accounting Standards (SFAS) No. 157, *Fair Value Measurements*, to take effect in 2007. Fair value is defined therein as the “price that would be received to sell an asset or paid to transfer the liability at the measurement date (an exit price).” Furthermore, “a fair value measurement assumes that the asset or liability is exchanged in an orderly transaction ...; it is not a forced transaction (for example, a forced liquidation or distress sale) ... [it] is a hypothetical transaction ... considered from the perspective of a market participant that holds the asset or owes the liability.” (FASB 2006, para. 7, 3). Fair values are to be determined from the perspective of a market participant using the best-use framework, and without using any entity-specific assumptions (even if the acquirer has different plans).

Labels Matter

Before addressing the econometrics of fair values, a few words on semantics seem appropriate. Labels matter because language can do harm. What, for example, is common to the following three proposals?

- Unified Budget Act (Lyndon B. Johnson, 1964)
- Patriot Act (George W. Bush, 2002)
- Fair Values (FASB, 2006)

President Johnson wanted to use the Social Security Trust Fund surpluses to finance increased spending on Great Society programs and the Vietnam War. He sent legislation labeled Unified Budget Act to Congress, forcing his opponents to have to argue for a nonunified budget.

After the 9/11 attacks, President Bush wanted to place limits on certain civil liberties to fight the War on Terror. He sent legislation labeled the Patriot Act to Congress, forcing those worried about civil liberties to appear to be arguing against patriotism.

Now, the FASB has decided that financial reports should use current values. They have chosen the exit value (as opposed to entry value) version of this valuation rule; both have been analyzed and debated over the past century in some detail. Paton (1922), Sweeny (1936), MacNeal (1939), Alexander et al. (1950), Chambers (1966), Edwards and Bell (1961), and Sterling (1971) are but a small sampling of distinguished contributions to this literature. Yet, the FASB has decided that this old bottle of wine needs a new label—*fair values*.

Fairness is a personal judgment, not a valuation rule. Affixing a new, loaded label to a well-researched and well-discussed method of valuation may amount to playing the old game of policy rhetoric: using clever labels to put the opponents of your proposal on the defensive before the debate even starts. Who would want to defend the use of “unfair” values in accounting? It is perhaps best to put the “fair” aside and discuss current values, about which generations of accountants and researchers have thought and written. Econometrics can help us bring an element of quantified rationality to the debate about valuation rules.

ECONOMETRICS OF VALUATION

The achievements of econometrics have arisen from our ability and willingness to (1) postulate an underlying structure and unknown parameters of the problem at hand, (2) characterize the properties of alternative estimators (e.g., ordinary least squares) as a function of the underlying environment, (3) choose an estimator appropriate to the postulated environment, (4) use data to estimate the unknown parameters, holding the structure constant, (5) examine propositions about the underlying parameter on the basis of estimates, and (6) use alternative datasets to examine the propriety of the assumed structure. When the assumed structure is found not to be appropriate, we assume a different structure.

We can apply this strategy to examine the properties of valuation rules in various environments. This strategy will not get rid of judgments entirely, but will help move the debates over valuation rules from the domain of opinion to the empirical arena. To begin analyzing valuation rules as econometric estimators, let us postulate a structure, subject to later revisions on the basis of data and observations.

Postulated Structure

The resources in the economy constitute a finite set, and are represented by a vector of relative weights based on their value in the economy as a whole at the beginning of the time interval of interest. Each firm is represented as a bundle of these resources drawn randomly (with multinomial distribution) from the resource bundle of the whole economy, and is represented by a vector of relative weights of those resource values. Current prices of the resources in the economy are subject to change over the time interval of interest. Their relative (percentage) changes over this interval have a given vector of expectations and a given matrix of their covariances. In general, observations of these relative price changes are subject to an (unbiased) measurement error that has a given covariance matrix. The beginning-of-the-interval (historical) prices of resources in the bundles are known.²

In the presence of measurement error in the current prices of individual resources, the measured current values of bundles of resources also deviate from their respective true but unobserved current values. Econometric analysis can be used to derive the properties of valuation rules as alternative estimators of the true value of any bundle of resources, statistical proximity of the estimated to the true values being the property of most interest.³

Two Sources of Error in Valuation

The difference between the valuation of a bundle of resources (estimate) and its unobserved true value is the valuation error. It can be broken into two parts. First, prices may change over time but the valuation rules may either ignore these changes or incorporate them less than perfectly. Errors of valuation arising from this source can be labeled price movement errors. Second, the current prices used to revalue the resource bundles are prone to errors because of the imperfection and incompleteness of markets from which current prices are gathered. These can be labeled price measurement errors.

Metric and Magnitude of the Errors of Valuation Rules

The actual valuation error for a given firm depends on the realized price changes and on the composition of the bundle of resources it controls. Following standard econometric

² This is a nontechnical summary. For further specification of the technical details of the postulated structure, see Sunder (1978) and Lim and Sunder (1991).

³ This is analogous to the standard econometric practice of postulating a true but unobserved set of parameters of the model, defining alternative estimators of the parameters as functions of the observed data, and analyzing the properties of the estimators in terms of the postulated characteristics of the data.

practice, we can take the expectation of this error (to get the bias) and the expectation of the squared error (to get the mean squared error) with respect to the postulated probability distributions of price changes and the compositions of resource bundles. Let us focus on the mean squared error (MSE) as the metric for assessing how well various valuation rules capture the true unobserved value of bundles of assets. This metric is frequently used in econometrics, and has the advantage of allowing us to break down the two components of the error term mentioned above.

The magnitude of MSE associated with various valuation rules depends on the structural parameters postulated above: the vector of relative weights of various resources in the economy, the vector of expectations of price changes for individual resources in the economy, the covariance matrix of price changes for individual resources in the economy, and the covariance matrix of measurement errors in price changes for individual resources in the economy.

Valuation rules differ in how each rule adjusts historical to current prices. The space of valuation rules, even their linear subset, is huge. For the sake of simplicity, we limit the present discussion to three—two polar and one intermediate—elements of the linear subset of valuation rules: historical, general price level, and current valuation. The numerical example in the following section illustrates a slightly more general case.

The three panels of schematic Figure 1 show how the two kinds of errors and their sums might vary from one valuation rule to another. Each of the three valuation rules can be described by the number of price indexes used to adjust the historical numbers. The historical (0-price index) valuation rule is to the left; the general price level (1-index) valuation rule is in the middle, and the current (N-index) valuation rule is to the right.

Historical valuation ignores price changes from the time of resource acquisition to the time of valuation and therefore suffers from price movement errors. However, since it does not depend on potentially error-prone current values, this valuation is free of the second kind of error that arises from measurement. The magnitude of the MSE depends on the parameters of the economy: the mean of the vector of relative price changes and the covariance matrix of the vector of relative price changes. The greater the “magnitude” of these two parameters, the greater the price movement error associated with historical valuation.

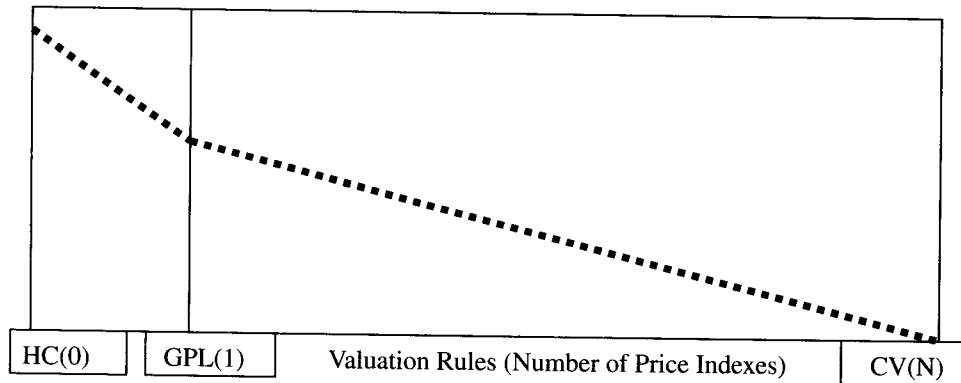
Current valuation at the right end of the panels of Figure 1 takes into account changes in the prices of each resource individually and is therefore free of price movement errors. It does have price measurement errors arising from the assessment of current values, and its MSE depends on the parameters of the economy. If we assume that the relative changes in current prices are measured without bias, the MSE arising from the mean of measurement errors is 0, and covariances of measurement errors are the only remaining source of error. The greater the “magnitude” of this covariance matrix, the greater the measurement error associated with current valuation.

General price level valuation (GPL) uses a single price index to adjust the historical prices toward current prices (see the intermediate point in the three panels of Figure 1). The single price index decreases the price movement error associated with the historical estimator but does not eliminate it.⁴ The use of a single price index also introduces some measurement error, although not as large as the error associated with the current value

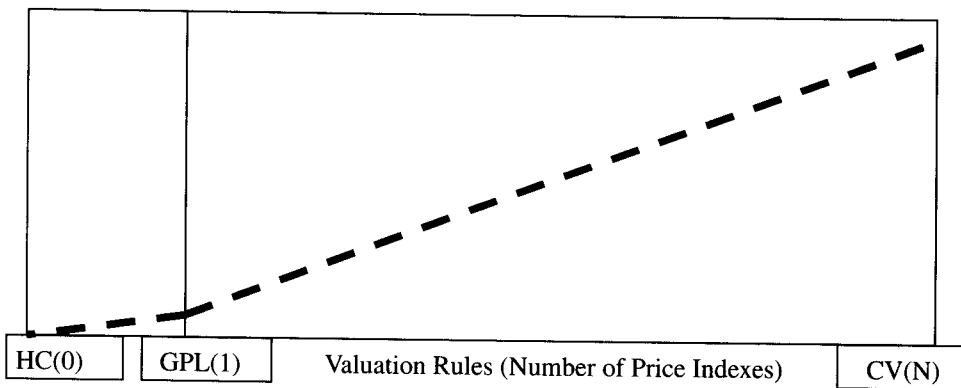
⁴ See the last column of Table 1 and Equations (A.22) and (A.23) in Lim and Sunder (1991) for comparison of price movement errors associated with historical and GPL valuation rules; the former is strictly greater than the latter.

FIGURE 1
Schematic Diagram of Valuation Errors (Not to Scale)

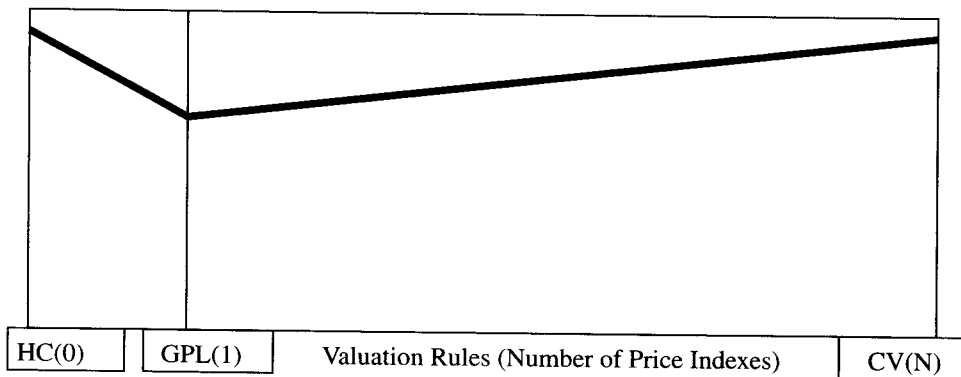
Panel A: Price Movement Error (MSE) of Three Valuation Rules



Panel B: Price Measurement Error (MSE) of Three Valuation Rules



Panel C: Total Error (MSE) of Three Valuation Rules



estimator.⁵ The magnitude of these two kinds of errors, and their sum associated with the GPL estimator, depends on the values of the mean, relative weights, and covariance parameters discussed earlier.

Panel A shows that the price movement error is the highest for historical valuation, 0 for current valuation, and has an intermediate value for GPL. The actual values depend on parameters (mean and covariance of relative price changes, and relative weights of various resources) in the economy.

Panel B shows the behavior of price measurement error, which is 0 for historical valuation, highest for current valuation, and has an intermediate value for GPL. The actual magnitudes of the current and GPL measurement errors depend on the parameters (covariance of price measurement errors and relative weights of various resources) in the economy.

Panel C shows the behavior of the total valuation error, which is the sum of the two components described above. The total error for GPL valuation is shown to be the lowest of the three valuation rules. However, this is not true in general; depending on the values of the parameters of the economy, the lowest MSE could be associated with any of the three estimators or valuation rules.

If price volatility is high and measurement errors are small, the MSE of the current value estimator could be the lowest. With low price volatility and high measurement errors, GPL, or even the historical estimator, could have the lowest MSE. In general, we should not expect that the MSE-minimizing estimator will be any one of the three explicitly considered above. Instead, it is likely that the minimum MSE estimator would be one of the very large numbers of estimators that use an intermediate number (between 1 and N, the number of resources in the economy) and configuration of specific price indexes to adjust historical to current values.

The results summarized above for three valuation rules (historical, GPL, and current) are special cases of the general results for the properties of valuation rules when the number of price indexes used takes any integer value from 0 (for historical valuation) to 1 (for GPL valuation), 2, 3, 4, ... (for specific price index valuation) and N (for current valuation in an N-good economy). In general, as valuation rules use a more disaggregated set of price indexes, their price movement error tends to decline at a decreasing rate, and their price measurement error tends to rise at an increasing rate. In the following section we provide numerical examples.

Numerical Examples

To illustrate the main point of this article, we use two numerical examples from Lim and Sunder (1991). Consider an economy with four goods. Consider a firm whose bundle of assets has these four goods in relative proportions (0.1, 0.35, 0.42, 0.13). Suppose that over a given interval, the true relative price change realized for these four goods is known to be (0.12, 0.15, 0.55, 0.45). The relative change in the true value of this asset bundle is: $0.354 (= 0.1 \times 0.12 + 0.35 \times 0.15 + 0.42 \times 0.55 + 0.13 \times 0.45)$ or 35.4 percent.

Suppose that the realized values of relative price changes are measured with error, and these measured changes over the relevant interval are (0.09, 0.19, 0.44, 0.47). Applying these measured price changes to the firm's asset bundle, its estimated current value changes by 32.14 percent ($= 0.1 \times 0.09 + 0.35 \times 0.19 + 0.42 \times 0.44 + 0.13 \times 0.47$). Because of measurement errors in relative price changes, the current valuation of the bundle has an error of -3.26 percent ($32.14 - 35.4$).

⁵ See Lim and Sunder (1991, Theorem 1, 675).

Suppose the relative abundance of the four goods in the economy as a whole is given by (0.2, 0.3, 0.4, 0.1), and these weights are used for constructing price indexes. The general price index for this period would change by 29.8 percent ($= 0.2 \times 0.09 + 0.3 \times 0.19 + 0.4 \times 0.44 + 0.1 \times 0.47$). Applying this single index to all assets of the firm, the GPL valuation of this firm's bundle of assets has an error of -5.6 percent ($= 29.8 - 35.4$).

If we were to use economy-wide weights to construct two price indexes (the first consisting of goods 1 and 2 and the second consisting of goods 3 and 4), the first price index would change by 15 percent $= (0.2 \times 0.09 + 0.3 \times 0.19)/(0.2 + 0.3)$, and the second by 44.6 percent $= (0.4 \times 0.44 + 0.1 \times 0.47)/(0.4 + 0.1)$. Applying these two price indexes to the actual proportions of the asset bundle of the firm in our example, this two-index valuation rule yields an estimate of 31.28 percent ($= 0.15 \times (0.1 + 0.35) + .446 \times (0.42 + 0.13)$) which has an error of -4.12 percent ($31.28 - 35.4$).

Finally, because the historical valuation rule ignores all price changes, it yields 0 percent price changes, and therefore has an error of -35.4 percent in our example.

So far we have calculated the actual realized error in valuation of a given firm over a given interval using current valuation, one of several possible two-index valuation rules, GPL, and historical cost by assuming that we know the true price changes. In practice we cannot know the true price changes, but we can choose valuation rules on the basis of mean squared errors, assuming that we know the relevant probability distributions. This is what we do in the remaining part of the example.

Suppose the expectation (mean) of the relative price changes of the four goods in the economy are (0.1, 0.2, 0.6, 0.4). The variances of the relative price changes and of the measurement errors in the relative price changes of the four goods are (1, 3, 5, 2) and (3, 4, 1, 2) respectively, and all covariances are 0.⁶

Table 1 (abstracted from Table 1 from Lim and Sunder 1991, 673) lists the mean squared errors (price movement errors in Column 5, price measurement errors in Column 6, and their sum total error in Column 4) associated with all 16 possible valuation rules based on all possible price index configurations (listed in Column 3 of the table).

The first row of Table 1 for historical valuation (using $k = 0$ price indexes) shows that the mean squared error arising from price movements is $(1.260 + 2.214/\rho)$ and the error arising from price measurements is 0. The total error of this valuation rule therefore is the same as the price movement error.

The second row of Table 1 for general price level valuation (using $k = 1$ price indexes) shows that its movement error $(2.214/\rho)$ is lower; the measurement error is higher (0.66), and the total error $(0.66 + 2.214/\rho)$ is lower than historical valuation.

The third to ninth rows show the calculations for seven possible two-index valuation rules (see Column 3 for the index configurations). The third of these seven rules with price index configuration (c,abd) has the lowest price movement error $(0.756/\rho)$ and the lowest total error $(0.66 + 1.329/\rho)$, while the fourth (d,abc) has the lowest price measurement error $(0.66 + 0.251/\rho)$. Note that the total error associated with the best of the seven two-index valuation rules is lower than the GPL error.

Rows 10 to 15 show the calculations for six possible three-index valuation rules. The third of these six rules with price index configuration (ad,b,c) has the lowest price movement error $(0.0297/\rho)$; the fourth with price index configuration (a,bc,d) has the lowest price

⁶ The actual values of MSEs of various valuation rules also depend on the degree of diversification of assets belonging to individual firms in the economy. Fortunately, it is not necessary to assume any specific value for the asset diversification parameter (ρ) to compare the accuracy of various valuation rules.

TABLE 1
Properties of Estimators of Current Value in a Four-Good Economy: Example 1

1	2	3	4	5	6
			Economy-Wide Mean Squared Errors		
Number of Price Indexes	Serial Number	Valuation Rule (Partition)	Total	From Price Movement	From Price Measurement
0	1	Historical	$1.260 + 2.214/\rho^*$	$1.260 + 2.214/\rho^*$	0^*
1	1	GPL(abcd)	$0.66 + 2.214/\rho^*$	$2.214/\rho^*$	0.66^*
2	1	(a,bcd)	$0.66 + 2.380/\rho$	$1.765/\rho$	$0.66 + 0.615/\rho$
	2	(b,acd)	$0.66 + 2.174/\rho$	$1.205/\rho$	$0.66 + 0.969/\rho$
	3	(c,abd)	$0.66 + 1.329/\rho^*$	$0.756/\rho^*$	$0.66 + 0.573/\rho$
	4	(d,abc)	$0.66 + 2.162/\rho$	$1.911/\rho$	$0.66 + 0.251/\rho^*$
	5	(ab,cd)	$0.66 + 1.704/\rho$	$1.044/\rho$	$0.66 + 0.660/\rho$
	6	(ac,bd)	$0.66 + 1.968/\rho$	$1.211/\rho$	$0.66 + 0.757/\rho$
	7	(ad,bc)	$0.66 + 2.155/\rho$	$1.605/\rho$	$0.66 + 0.550/\rho$
3	1	(ab,c,d)	$0.66 + 1.381/\rho^*$	$0.481/\rho$	$0.66 + 0.900/\rho$
	2	(ac,b,d)	$0.66 + 2.040/\rho$	$0.833/\rho$	$0.66 + 1.207/\rho$
	3	(ad,b,c)	$0.66 + 1.704/\rho$	$0.030/\rho^*$	$0.66 + 1.407/\rho$
	4	(a,bc,d)	$0.66 + 2.282/\rho$	$1.399/\rho$	$0.66 + 0.883/\rho^*$
	5	(a,bd,c)	$0.66 + 1.668/\rho$	$0.378/\rho$	$0.66 + 1.290/\rho$
	6	(a,b,cd)	$0.66 + 2.063/\rho$	$0.563/\rho$	$0.66 + 1.500/\rho$
4	1	Current Valuation (a,b,c,d)	$0.66 + 1.740/\rho^*$	0^*	$0.66 + 1.740/\rho^*$

* Denotes members of the efficient set and efficient frontier.

ρ is a measure of the degree of diversification of assets belonging to individual firms in the economy. It can take any value greater than 1.

measurement error ($0.66 + 0.883/\rho$); and the first (ab,c,d) has the lowest total error ($0.66 + 1.381/\rho$). Note that the total error associated with the best of the six three-index valuation rules is higher than the error associated with the best of the seven two-index valuation rules.

The final row of Table 1 corresponds to current valuation, in which each good has an "index" of its own yielding configuration (a,b,c,d). Current valuation has no price movement error but relatively high price measurement error ($0.66 + 1.740/\rho$), which is also the total error of current valuation. Note that this error is lower than the historical and GPL valuations but higher than what is achievable with two- and three-index valuation rules.

Thus, for the parameters chosen for this numerical example, there exist two- and three-index valuation rules that dominate historical, GPL, and current valuation. If different parameters were chosen, it is possible to show that any one of the three (historical, GPL, or current valuation rule) could dominate the others. For example, suppose we increase the variances of errors associated with the measurement of price relatives by a factor of 2.6, so instead of (3, 4, 1, 2), the variances are now (7.8, 10.4, 2.6, 5.2), though all other parameters of Example 1 remain unchanged.

Table 2 (abstracted from Table 2 in Lim and Sunder 1991, 684) shows the calculations for the 16 possible valuation rules in the four-good economy for Example 2. The price movement errors (in Column 5) remain unchanged from Example 1, but the price

TABLE 2
Properties of Estimators of Current Value in a Four-Good Economy: Example 2

1	2	3	Economy-Wide Mean Squared Errors		
			4	5	6
Number of Price Indexes	Serial Number	Valuation Rule (Partition)	Total	From Price Movement	From Price Measurement
0	1	Historical	$1.260 + 2.214/\rho^*$	$1.260 + 2.214/\rho^*$	0*
1	1	GPL(abcd)	$1.716 + 2.214/\rho^*$	$2.214/\rho^*$	1.716^*
2	1	(a,bcd)	$1.716 + 3.364/\rho$	$1.765/\rho$	$1.716 + 1.599/\rho$
	2	(b,acd)	$1.716 + 3.724/\rho$	$1.205/\rho$	$1.716 + 2.519/\rho$
	3	(c,abd)	$1.716 + 2.246/\rho^*$	$0.756/\rho^*$	$1.716 + 1.490/\rho$
	4	(d,abc)	$1.716 + 2.564/\rho$	$1.911/\rho$	$1.716 + 0.653/\rho^*$
	5	(ab,cd)	$1.716 + 2.760/\rho$	$1.044/\rho$	$1.716 + 1.716/\rho$
	6	(ac,bd)	$1.716 + 3.179/\rho$	$1.211/\rho$	$1.716 + 1.968/\rho$
	7	(ad,bc)	$1.716 + 3.035/\rho$	$1.605/\rho$	$1.716 + 1.430/\rho$
3	1	(ab,c,d)	$1.716 + 2.821/\rho^*$	$0.481/\rho$	$1.716 + 2.340/\rho$
	2	(ac,b,d)	$1.716 + 3.971/\rho$	$0.833/\rho$	$1.716 + 3.138/\rho$
	3	(ad,b,c)	$1.716 + 3.955/\rho$	$0.297/\rho^*$	$1.716 + 3.658/\rho$
	4	(a,bc,d)	$1.716 + 3.695/\rho$	$1.399/\rho$	$1.716 + 2.296/\rho^*$
	5	(a,bd,c)	$1.716 + 3.732/\rho$	$0.378/\rho$	$1.716 + 3.354/\rho$
	6	(a,b,cd)	$1.716 + 4.463/\rho$	$0.563/\rho$	$1.716 + 3.900/\rho$
4	1	Current Valuation (a,b,c,d)	$1.716 + 4.524/\rho^*$	0*	$1.716 + 4.524/\rho^*$

* Denotes members of the efficient set and efficient frontier.

ρ is a measure of the degree of diversification of assets belonging to individual firms in the economy. It can take any value greater than 1.

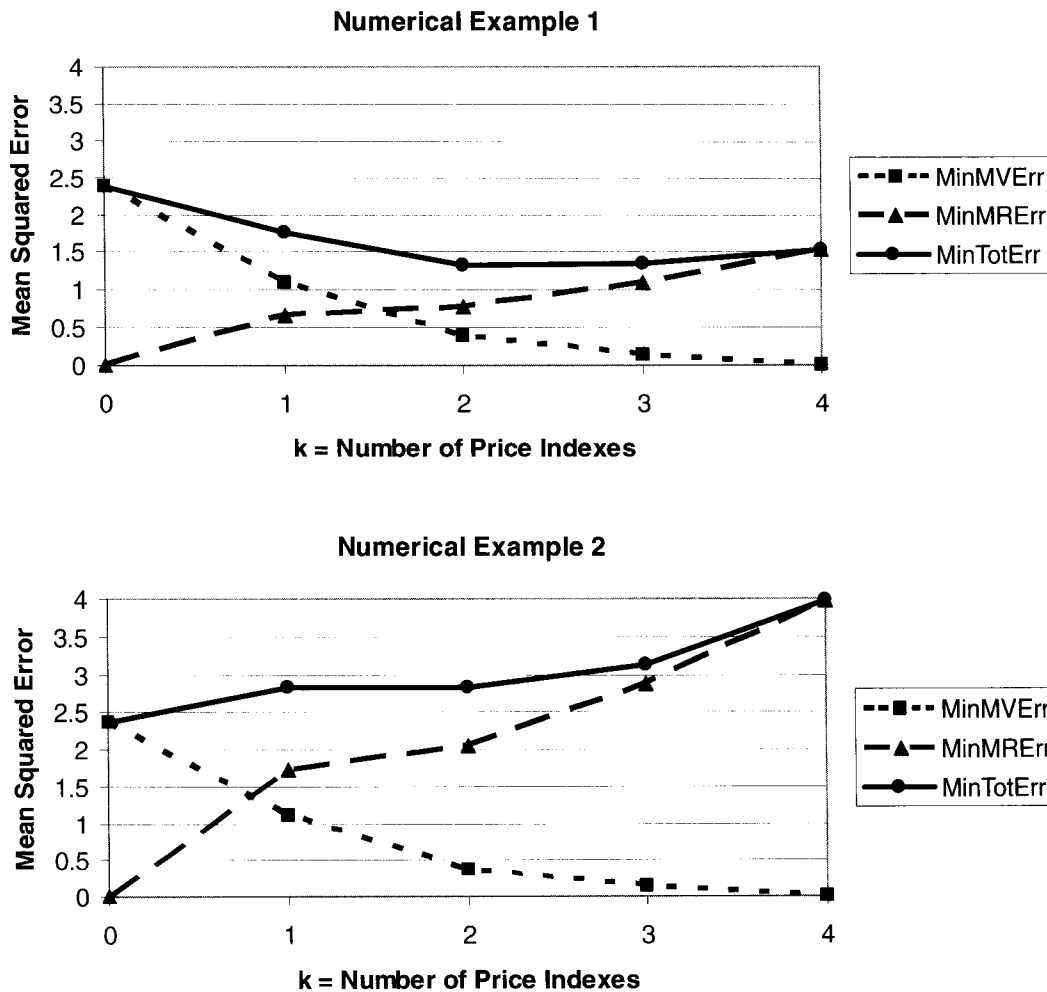
measurement errors (in Column 6) are scaled up by a factor of 2.6. We should expect that the efficient valuation rule would shift in the direction of coarser prices index systems because of larger measurement errors. Indeed, Column 4 shows that the total error of historical valuation ($1.260 + 2.214/\rho$) is not only lower than the total error of GPL ($1.716 + 2.214/\rho$) and current valuation ($1.716 + 4.524/\rho$), but also lower than the best of all the two-index ($1.716 + 2.246/\rho$ for c,abd) and three-index ($1.716 + 2.821/\rho$ for ab,c,d) valuation rules.

Figure 2 shows schematically the mean squared errors associated with the best of the k-index valuation rules (for $k = 0, 1, 2, 3$, and 4) in the two numerical examples given above. Similarly, if the variances of the measurement errors were sufficiently small relative to movement errors, a three-index or the current valuation (four-index) valuation rule could be associated with the minimum MSE.

Implications

These theoretical results about the properties of valuation rules as econometric estimators have several implications. First, current valuation should be more informative for firms and industries whose (1) assets' prices change faster on average, (2) assets' price changes exhibit greater variability, and (3) assets are traded in relatively perfect and complete markets (i.e., current prices have smaller measurement errors). In industries such as real estate, mineral deposits, films, software, and patents that are traded in less perfect

FIGURE 2
Minimum Mean Squared Errors of k-Index Valuation Rules (Movement, Measurement, and Total)



markets, current valuation should have smaller errors relative to historical valuation (Lim and Sunder 1991, 685–686).

Second, the relative informativeness of various valuation rules is not a matter of general accounting theory. Depending on the parameters of the economy, industry, and each firm involved, any valuation rule could be better than the others. In contrast, a great deal of theoretical literature in accounting theory literature tends to claim the general dominance of one valuation rule over the others.

While efficient valuation rules vary across assets, firms, and industries, empirical accounting literature on the informativeness of valuation rules tends to follow the “general theory” approach by conducting cross-sectional tests (e.g., Gheyara and Boatsman 1980;

Ro 1980; Beaver et al. 1982). Taking an econometric perspective on valuation suggests that empirical tests could benefit from greater attention to the characteristics of assets, of firms, and of industries to which valuation rules are being applied (Lim and Sunder 1991, 686).

Third, Figure 1 illustrates that the level of aggregation of price indexes used to adjust historical to current prices is a major determinant of the properties of valuation. The FASB's Fair Value Measurement standard (FASB 1979b, SFAS No. 157), following earlier proposals from the Securities and Exchange Commission (1976, ASR No. 190) and the FASB (1979a, SFAS No. 33), wisely leave this issue open. Where should the regulators look for an efficient level of aggregation? It seems reasonable to draw the following inferences from the literature on the subject (cited in footnote 1 and summarized in the Appendix):

- (1) For time periods and industries with low price instability and high measurement errors, valuation rules toward the left in Figure 1 that either use a small number of price indexes to adjust historical numbers (e.g., GPL) or use unadjusted historical costs (with depreciation) are likely to be efficient (minimize the MSE).
- (2) For time periods and industries with high price instability and small measurement errors, valuation rules toward the right in Figure 1 that use a large number of "fine" price indexes to adjust historical numbers are likely to be efficient.
- (3) Given the highly convex nature of the accuracy functions (see Hall 1982; Sunder and Waymire 1983; Shriver 1986; Tippett 1987; Jamal and Sunder 1995), most of the gains of specificity through reduced price movement errors are realized from a small number of price indexes. After a score or two of price indexes, the marginal gains become quite small. On the other hand, given a large magnitude of price measurement errors relative to price movement errors, such as for machinery and equipment data (see Hall and Shriver 1990), using a large number of price indexes subjects valuation to significant measurement errors. Therefore, in general, efficient valuation rules are likely to lie somewhere in the middle of the fineness scale. Lim and Sunder (1991) and Tippett (1987) provided analytical frameworks that can be used by standard setters to identify efficient valuation rules for specified environments.

CONCLUDING REMARKS

Traditional analyses in accounting theory and in empirical work tend to examine and compare the properties of individual valuation rules. This article is based on some four decades of theoretical and empirical literature (see footnote 2 and the Appendix) and points to the advantages of an alternative approach. Theories of valuation can be integrated into a unified framework to facilitate direct comparison of their properties in specified environments. When current prices change and are prone to measurement errors, neither current nor the general price level valuation rule is necessarily the minimum mean squared error estimator of the unobserved economic value of resources. Generally, the minimum mean squared error estimator is a specific price index rule that depends on the parameters of the economy. If the price measurement errors are sufficiently large relative to price movement errors, even historical valuation can be the minimum mean squared error estimator.

No valuation rule has minimum mean squared error in general, as a matter of principle. Instead, it is a matter of econometrics, and depends on the relative magnitudes of the parameters of the economy. Efficient (in the sense of minimum mean squared error) valuation rules vary across assets, firms, and industries. Using known methods, we can discover which rules are better in which circumstances.

APPENDIX
AN OVERVIEW OF THE ANTECEDENT LITERATURE

1. Ijiri, Yuji. 1968. The linear aggregation coefficient as the dual of the linear correlation coefficient. *Econometrica* (April): 252–259. Proved that the linear correlation coefficient and linear aggregation coefficient are two interpretations of the same parameter, depending on whether the underlying space R^n is viewed as a data or function space.
2. Ijiri, Yuji. 1967. *Foundations of Accounting Measurement*. Englewood Cliffs, N.J.: Prentice Hall. Formulated valuation as a linear aggregation of quantities and introduced the idea of statistical assessment of accounting measurement using the squared error criterion.
3. Tritschler, Charles A. 1969. Statistical criteria for asset valuation by specific price index. *The Accounting Review* 44 (1) (January): 99–123. Proposed and assessed, using squared error criterion, alternative specific price index valuation rules at varying levels of aggregation using the Wholesale Price Index (now called Producer Price Index) data for machinery group.
4. Sunder, Shyam. 1978. Accuracy of exchange valuation rules. *Journal of Accounting Research* 16 (2) (Autumn): 347–367. Defined the “exchange valuation” family of rules as estimators that use an alternative configuration of price indexes to adjust historical values toward current values, and proved that (under the given set of structural assumptions) their statistical accuracy (mean squared error) is an increasing function of the fineness (disaggregation) of the price index set.
5. Hall, T. W. 1982. An empirical test of the effect of asset aggregation on valuation accuracy. *Journal of Accounting Research* 20 (Spring): 139–151. Examined the effect of increasing the number and fineness of specific price indexes on the accuracy of valuation for 25 firms in each of four utility industries using data for five asset categories, and found that increasing the number and fineness of specific price indexes increases accuracy of valuation.
6. Sunder, S., and Gregory Waymire. 1983. Marginal gains in accuracy of valuation from increasingly specific price indexes: Empirical evidence for the U.S. economy. *Journal of Accounting Research* 21 (2) (Autumn): 565–580. Estimated the marginal gains in accuracy (mean squared error) of valuation rules from increasing specificity of price indexes used to adjust historical values using U.S. Producer Price Index data, and found that the marginal gains decline sharply as with the increase in the number of indexes. These findings are consistent with the proposition that the accuracy function is convex.
7. Sunder, S., and Gregory Waymire. 1984. Accuracy of exchange valuation rules: Additivity and unbiased estimation. *Journal of Accounting Research* 22 (1) (Spring): 396–405. Proved the additivity of accuracy of valuation rules: If valuation rules R_A , R_B , and R_C use price indexes comparable in their fineness (R_A being finer than R_B , which is finer than R_C), the mean squared difference between valuations R_A and R_C is the sum of the mean squared differences between R_A and R_B , and between R_B and R_C . Also provided an unbiased estimator of mean squared error used in Sunder and Waymire (1983).
8. Shriver, Keith A. 1986. Further evidence on the marginal gains in accuracy of alternative levels of specificity of the producer price indexes. *Journal of Accounting Research* 24 (Spring): 151–165. Used Land database for 1973, 1976, and 1980 to assess the effect of price index specificity on accuracy of valuation rules and found

- that accuracy is affected by the level of specificity; most specific indexes do not always result in improved accuracy, and Producer Price Indexes generally yield more accurate valuations for newer assets.
9. Shriver, Keith A. 1987. An empirical examination of the potential measurement error in current cost data. *The Accounting Review* 62 (January): 79–96. Examined the sign, magnitude, and sources of measurement errors in estimates of current cost generated by alternative levels of specificity of Producer Price Indexes (as compared to Land database), and found that errors tend to overstate valuation. Dominant sources of errors are product mix errors in electrical and miscellaneous equipment, pricing errors in general purpose equipment, and inadequate adjustments for quality changes in machine tools and special industry equipment.
 10. Shih, S., and S. Sunder. 1987. Design and tests of an efficient search algorithm for accurate linear valuation systems. *Contemporary Accounting Research* 4 (Fall): 16–31. Design and test of an algorithm to produce efficient valuation rules (i.e., partitions of N goods in the economy into k nonempty subsets) using information contained in the parameters of the economy (relative weights and the expected values and variance-covariance matrices of the prices changes of goods in the economy).
 11. Tippett, M. 1987. Exchange valuation rules: Optimal use of specific price indexes in asset valuation. *Accounting and Business Research* 17 (Spring): 141–154. Developed an efficient method of weighting price indexes for application to specific firms, and derived results for upper bounds on mean squared error associated with various specific index valuation rules. Examined Australian price data and found a sharp decline in marginal increase in accuracy with increasing index specificity (confirmed Hall (1982) and Sunder and Waymire (1983) results for U.S. price data).
 12. Lim, Suk S., and S. Sunder. 1991. Efficiency of asset valuation rules under price movement and measurement errors. *The Accounting Review* 66 (4) (October): 669–693. Introduced price measurement errors to the Sunder (1978) model to show that the minimum mean squared error valuation rule may be the current cost, GPL, historical cost, or any specific price index valuation rule depending on the value of parameters of the economy.
 13. Lim, Suk S., and S. Sunder. 1990. Accuracy of linear valuation rules in industry-segmented environments: Industry vs. economy-weighted indexes. *Journal of Accounting and Economics* 13 (July): 167–188. Showed that estimates of current value based on industry-weighted price indexes do not necessarily dominate the estimates based on economy-weighted price indexes; larger measurement errors favor economy-weighted indexes; and larger mean and variability of price changes favor industry-weighted indexes.
 14. Hall, T. W., and Keith A. Shriver. 1990. Econometric properties of asset valuation rules under price movement and measurement errors: An empirical test. *The Accounting Review* 65 (July): 537–562. Used Lim and Sunder (1991) framework and Land and Producer Price Index databases to examine the accuracy of valuation rules with price indexes of different specificity. Under the assumption that Land database is error-free, the study found that (1) movement error bias is zero but measurement error bias is positive, (2) both movement and measurement mean squared errors decline with finer indexes and diversification of asset portfolios, (3) both kinds of mean squared errors increase with longer asset holding periods, and (4) mean squared errors arising from price measurement are more significant than from price movement.

15. Jamal, Karim, and Shyam Sunder. 1995. Convexity of valuation accuracy function: Empirical evidence for the Canadian economy. *Contemporary Accounting Research* 11 (2) (Spring): 961–972. Examined Canadian price data to conclude that marginal gains in accuracy decline sharply as the specificity of price indexes used to estimate current values increases. These results are consistent with the results obtained for U.S. prices (Hall 1982; Sunder and Waymire 1983; Shriver 1986) and for Australian prices (Tippett 1987).

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