Optimal Choice Between FIFO and LIFO

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1. Introduction

The decision to select one of the two accounting methods for inventory valuation—first-in, first-out (FIFO)[1] and last-in, first-out (LIFO)—involves consideration of several factors of which potential tax effect of the accounting method is often the most important.[2] While the general relationships of inflation, inventory levels, rate of taxation, etc., to the potential tax savings have been well understood for a long time, realistic decision models which quantify the impact of alternative accounting procedures on the economic value of the firm are conspicuously absent. It is difficult, then, for a manager who is weighing the choice of the two accounting methods explicitly to determine the impact of such a decision on the value of the firm.

The purpose of this study is to present procedures to estimate the difference between the net present value of tax payments under the two inventory valuation methods. Sunder [1976] developed a simple estimation procedure under conditions of certainty and level year-end inventories. Three extensions are presented here by relaxing various assumptions. First, the assumption of level year-end inventories is relaxed to compute the present value of cash-flow differences with changing inventory level within a deterministic framework. In a second extension, deterministic

* University of Chicago. I wish to thank John Bryant, Nick Dopuch, George Foster, Bob Kaplan, Bob Magee and Dov Pekelman for their help with revisions of earlier drafts of this paper.

[1] The term FIFO as used here includes variations such as the average cost method. In making a choice between LIFO and FIFO and its variations, the economic differences within the latter class are of little consequence.

[2] Butters [1949] is an early and comprehensive survey of various considerations that go into the choice of an accounting method for inventory valuation. For a recent survey of the factors considered by corporate officers in making the choice of an inventory valuation method, see Copeland and Wojdak [1970].

rates of price changes are replaced by a stochastic process to compute the *expected* present value of cash-flow differences. Finally, the deterministic inventory level is replaced by a stochastic process in the third extension allowing us to compute the expected present value of cash-flow differences for a given sequence of price changes. Procedures for handling simultaneous uncertainty in the price changes and inventory levels are also discussed. The stochastic models result in increased realism and relevance at the expense of only a small increase in computational costs.[3]

The models proposed here can be used by a firm to choose between LIFO and FIFO on the basis of the difference between the expected net present value of cash flows associated with the two methods. Once this amount has been determined, it can be adjusted for the difference in operating costs[4] of the two accounting methods. The expected value of net cash flows depends on the future marginal tax rates, anticipated rates of change in the price of inventories, cost of capital of the firm, pattern of changes in the year-end inventories, and the number of years for which the accounting change is to remain effective. Stochastic variation of year-end inventories permits realistic estimates of tax effects when there is some probability that the basic inventory may be liquidated before the firm's decision horizon. The decision model also provides information on the consequences of subsidiary decisions regarding the identification of particular inventories which may be included in the LIFO system and the size and number of inventory pools.

2. *Deterministic Model*

Consider a firm which is deliberating the adoption of LIFO for the whole or a part of its inventory. The physical quantity of the inventory involved at the beginning of the year of projected adoption of LIFO (year 1) is X_0. If the inventory is homogeneous, the physical quantity X_0 can be given in units, otherwise it is best expressed as its current acquisition cost. Let p_0 be the current price of inventory per unit.[5] Then the total value of the inventory at the beginning of year 1 is:

$$Y_0 = X_0 \cdot p_0 \tag{1}$$

[3] Beyond the costs of collecting modest amounts of data required by the model, the computational costs are really negligible. The model has already been implemented on a time-sharing computer in an interactive environment.

[4] The cost of an accounting change has two parts, a one-time set-up cost of change and a periodic cost of operating the new accounting system. The set-up cost of making the change is the major factor in most cases because the operating costs of LIFO and FIFO accounting systems do not usually differ by significant amounts.

[5] If the inventory is expressed in dollars, $p_0 = 1$ is simply the price index at the beginning of period 1.

irrespective of whether the firm continues to use FIFO or switches to LIFO.[4]

Elsewhere (Sunder [1976]), I have shown that if the quantity of inventory remains constant and other parameters are known with certainty, the present value of cash-flow differences between FIFO and LIFO is given by:

$$PV = X_0 \cdot p_0 \left[\sum_{t=1}^{K-1} \frac{I_t M_t \prod_{i=1}^{t-1} (1 + I_i)}{\prod_{i=1}^{t} (1 + d_i)} - \frac{M_K}{(1 + d_K)} \prod_{i=1}^{K-1} \left(\frac{1 + I_i}{1 + d_i} \right) + \frac{M_K}{\prod_{i=1}^{K} (1 + d_i)} \right] \tag{2}$$

where

X_0 = quantity of basic inventory
p_0 = per unit cost of basic inventory
I_t = rate of price change in period t
M_t = marginal tax rate in period t
d_t = cost of capital in period t
K = decision horizon, i.e., number of periods LIFO remains in use.

When $p_0 = 1$, $I_t = I$, $M_t = M$, and $d_t = d$ for all t, expression (2) can be simplified to:

$$PV = X_0 \cdot M \left[\frac{I}{d - I} \left(1 - \left(\frac{1 + I}{1 + d} \right)^{K-1} \right) - \frac{(1 + I)^{K-1}}{(1 + d)^K} + \frac{1}{(1 + d)^K} \right]. \tag{3}$$

The limit of PV when the decision horizon stretches indefinitely into the future ($K \to \infty$) is:

$$\lim_{K \to \infty} PV = \frac{X_0 \cdot M}{\frac{d}{I} - 1} \quad \text{for} \quad d > I. \tag{4}$$

I now develop a model where all parameters are known with certainty but level of inventory is no longer constant. Generalizations of the model

[4] I am assuming that the FIFO cost is sufficiently close to the current cost of the inventory at the beginning of year 1. If the firm is using FIFO with "lower of cost or market" rule, the law requires the firm to restate its inventory on a strict cost basis before switching to LIFO and to pay taxes on the increased value in year zero. Such taxes, if any, must be subtracted from the difference between FIFO and LIFO taxes derived later in this section.

to uncertain rates of price changes and uncertain inventory levels are presented in sections 3 and 4 respectively.

Since the LIFO method is applicable to the end-of-period inventories, let $\mathbf{X}' = \{X_0, X_1, X_2, \cdots, X_t, \cdots, X_K\}$ be the vector of the anticipated sequence of year-end inventory of the firm at base prices. In other words, X_t is the physical quantity of inventory at the end of year t. Assuming that (1) changes in inventory occur at the end of the year, (2) price change occurs earlier in the year, (3) under FIFO, all year-end inventory is carried at year-end prices, (4) taxes are paid at the end of each year, and (5) $p_0 = 1$, the present value of tax payments on realized inventory profits under FIFO is:

$$PV\,(\text{FIFO}) = \sum_{t=1}^{K} \frac{M_t \cdot I_t \cdot X_{t-1} \prod_{i=1}^{t-1} (1 + I_i)}{\prod_{i=1}^{t} (1 + d_i)} . \tag{5}$$

To determine the present value of tax payments under LIFO, we transform vector $\mathbf{X}' = \{X_0, X_1, \cdots, X_K\}$ into a $K \times K$ upper diagonal matrix $Z = \| z_{ij} \|$ so that z_{ij} is the net quantity of inventory (at base prices) deposited in year $i = 0, 1, 2, \cdots, K - 1$ and used up[7] in year $j = 1, 2, \cdots, K$. Elements of Z must satisfy the following relationships with the elements of $\mathbf{X}$.

$$\begin{aligned} X_i, z_{ij} &\geq 0 \quad \text{for all } i, j \\ z_{ij} &= 0 \quad \text{for } j \leq i \\ z_{ij} &= 0 \quad \text{for } j > i, \quad \text{if } (X_i - X_{i-1}) \leq 0 \end{aligned} \tag{6}$$

$$X_i - X_{i-1} = \sum_{j=i+1}^{K} z_{ij} \quad \text{if} \quad (X_i - X_{i-1}) \geq 0 \quad \text{for} \quad i = 0, \cdots, K-1$$

$$X_{j-1} - X_j = \sum_{i=0}^{j-1} z_{ij} \quad \text{if} \quad (X_{j-1} - X_j) \geq 0 \quad \text{for} \quad j = 1, \cdots, K.$$

Then, the present value of tax payments on inventory holding gains under LIFO is given by:

$$PV\,(\text{LIFO}) = \sum_{i=0}^{K-1} \sum_{j=i+1}^{K} \frac{z_{ij} \cdot M_j \left\{ \prod_{m=1}^{j} (1 + I_m) - \prod_{m=1}^{i} (1 + I_m) \right\}}{\prod_{m=1}^{j} (1 + d_m)} , \tag{7}$$

[7] z_{iK}, $i = 0, \cdots, K - 1$ is the amount of inventory deposited in year i and remaining unused at the end of the decision horizon. Income taxes on accumulated holding gains or this inventory are paid when LIFO is abandoned in year K. Since the timing of tax payments is the only aspect of the problem we are concerned with, I treat z_{iK} the same as other z_{ij}.

and the economic value of accounting change from FIFO to LIFO is the difference of expressions (5) and (7). A special case of changing but deterministic inventory occurs when inventory level is expected to grow at a constant rate g each period, that is:

$$\frac{X_t}{X_{t-1}} = (1 + g) \quad \text{for} \quad t = 1, 2, \cdots, K. \tag{8}$$

Under constant marginal tax rate (M) and discount rate (d), the present value of tax differences is:

$$PV = X_0 \cdot M \left\{ \frac{I}{d - I'} \left(1 - \left(\frac{1 + I'}{1 + d} \right)^{K-1} \right) - \frac{I}{I'(1 + d)^K} ((1 + I')^{K-1} - 1) \right\} \tag{9}$$

and

$$\lim_{K \to \infty} PV = \frac{X_0 \cdot M \cdot I}{d - I'} \quad \text{for} \quad d > I', \quad \text{where} \quad (1 + I') = (1 + I)(1 + g). \tag{10}$$

The effect of an increase in growth rate g is positive when $K \to \infty$. For a finite decision horizon, however, it is not necessarily so.

3. *Uncertainty in Price Changes*

I now consider the inventory valuation decision of the firm analyzed above when the future rates of price changes (I_t) are uncertain but their probability distribution is known. The continuously compounded rate of price changes per year for the firm's inventory is normally distributed with mean μ and variance σ^2. The serial correlation coefficient of the successive price changes is ρ.[8]

$$\tilde{I}_t \sim N(\mu, \sigma^2) \tag{11}$$

and

$$E(\tilde{I}_t - \mu)(\tilde{I}_{t-1} - \mu) = \rho\sigma^2. \tag{12}$$

Since $\tilde{I}_t$ is the continuously compounded rate, the realized rate of price change for year t is $\exp(\tilde{I}_t) - 1$, which corresponds to a compounding factor of $\exp(\tilde{I}_t)$. The present value of tax differences for the firm under FIFO and LIFO can be obtained by substituting the random compounding factor $\exp(\tilde{I}_t)$ for its deterministic analog $(1 + I_t)$ in expressions (5)

[8] Substantial empirical evidence is available to support the view that periodic rates of change in economic quantities follow the lognormal law. As a consequence, log transformations of periodic rates follow the normal law. If it is felt that more complicated serial dependence structure is desirable, it can be used in the model just as easily. Here, I shall use only first-order serial dependence case.

and (7) to obtain:

$$\widetilde{PV}\,(\text{FIFO}) = \sum_{t=1}^{K} \frac{M_t \cdot X_{t-1}}{\prod_{i=1}^{t}(1+d_i)} \left\{\exp\left(\sum_{i=1}^{t} \tilde{I}_i\right) - \exp\left(\sum_{i=1}^{t-1} \tilde{I}_i\right)\right\} \tag{13}$$

and

$$\widetilde{PV}\,(\text{LIFO}) = \sum_{i=0}^{K-1} \sum_{j=i+1}^{K} \left[\frac{M_j \cdot z_{ij}}{\prod_{m=1}^{j}(1+d_m)} \left\{\exp\left(\sum_{m=1}^{j} \tilde{I}_m\right) - \exp\left(\sum_{m=1}^{i} \tilde{I}_m\right)\right\}\right]. \tag{14}$$

From the normality of $\tilde{I}_t$, $\sum_{i=1}^{t} \tilde{I}_i$ also is normally distributed with expectation and variance given by:

$$E\left(\sum_{i=1}^{t} \tilde{I}_i\right) = t \cdot \mu \tag{15}$$

and

$$\operatorname{Var}\left(\sum_{i=1}^{t} \tilde{I}_i\right) = \mathbf{e}' V_t \mathbf{e}, \tag{16}$$

where V_t is the covariance matrix of $\tilde{I}_1, \tilde{I}_2, \cdots, \tilde{I}_t$ and $\mathbf{e}$ is a vector of 1's. V can have any form in general; for the first-order serial dependence structure I have assumed here:

$$V_t = \| v_{ij} \|, \qquad v_{ij} = \sigma^2 \rho^{|i-j|} \quad \text{for} \quad i, j = 1, 2, \cdots, t \tag{17}$$

and

$$\mathbf{e}' V_t \mathbf{e} = \sigma^2 \left\{ t + \frac{2\rho(t-1)}{1-\rho} - \frac{2\rho^2(1-\rho^{t-1})}{(1-\rho)^2} \right\}. \tag{18}$$

It also follows that $\exp\left(\sum_{i=1}^{t} \tilde{I}_i\right)$ is log normally distributed with:

$$E \exp\left(\sum_{i=1}^{t} \tilde{I}_i\right) = \exp\left(t\mu + \tfrac{1}{2}\mathbf{e}' V_t \mathbf{e}\right) \tag{19}$$

and

$$\operatorname{Var} \exp\left(\sum_{i=1}^{t} \tilde{I}_i\right) = \exp\left(2t\mu + 2\mathbf{e}' V_t \mathbf{e}\right) - \exp\left(2t\mu + \mathbf{e}' V_t \mathbf{e}\right). \tag{20}$$

Therefore, expected cash flows $E(\widetilde{PV})$ can be written as:

$$E\,\widetilde{PV}\,(\text{FIFO}) = \sum_{t=1}^{K} \frac{M_t X_{t-1}}{\prod_{i=1}^{t}(1+d_i)} \left\{\exp\left(t\mu + \tfrac{1}{2}\mathbf{e}' V_t \mathbf{e}\right) - \exp\left((t-1)\mu + \tfrac{1}{2}\mathbf{e}' V_{t-1} \mathbf{e}\right)\right\} \tag{21}$$

$$E\,\widetilde{PV}\,(\text{LIFO}) = \sum_{i=0}^{K-1} \sum_{j=i+1}^{K} \frac{M_j z_{ij}}{\prod_{m=1}^{j}(1+d_m)} \left\{\exp\left(j\mu + \tfrac{1}{2}\mathbf{e}' V_j \mathbf{e}\right) - \exp\left(i\mu + \tfrac{1}{2}\mathbf{e}' V_i \mathbf{e}\right)\right\}. \tag{22}$$

The expressions can be simplified if we assume that rates of price changes are serially independent and the rate of discount and marginal tax rate are constant at d and M respectively.

$$
\begin{aligned}
E(\widetilde{PV}) &= E\,\widetilde{PV}(\text{FIFO}) - E\,\widetilde{PV}(\text{LIFO}) \\
&= M\Bigg[\sum_{t=1}^{K}\frac{X_{t-1}}{(1+d)^t}\{\exp t(\mu+\sigma^2/2) - \exp(t-1)(\mu+\sigma^2/2)\} \\
&\quad - \sum_{i=0}^{K-1}\sum_{j=i+1}^{K}\frac{z_{ij}}{(1+d)^j}\{\exp j(\mu+\sigma^2/2) - \exp i(\mu+\sigma^2/2)\}\Bigg] \qquad (23)\\
&= M\Bigg[\sum_{t=1}^{K}\frac{X_{t-1}}{(1+d)^t}\exp\{(t-1)(\mu+\sigma^2/2)\}\cdot\{\exp(\mu+\sigma^2/2) - 1\} \\
&\quad - \sum_{i=0}^{K-1}\sum_{j=i+1}^{K}\frac{z_{ij}}{(1+d)^j}\exp\{i(\mu+\sigma^2/2)\} \\
&\quad \cdot\{\exp((j-1)(\mu+\sigma^2/2)) - 1\}\Bigg].
\end{aligned}
$$

Again, a special case of deterministic inventory levels occurs when inventories grow at a positive continuously compounded rate of g per year:

$$X_t = X_0 \exp(gt) \qquad (24)$$

$$
z_{ij} = \begin{cases} X_0 \exp(g(i-1))\cdot(\exp(g) - 1) & \text{for } i = 1, \cdots, K-1, j = K \\ X_0 & \text{for } i = 0, \ j = K \\ 0 & \text{otherwise.} \end{cases} \qquad (25)
$$

Then the net present value of expected cash-flow differences between FIFO and LIFO is given by:

$$
\begin{aligned}
E(\widetilde{PV}) &= MX_0\Bigg[\sum_{t=1}^{K}\frac{\exp(g(t-1))}{(1+d)^t} \\
&\quad \cdot\{\exp(t(\mu+\sigma^2/2)) - \exp((t-1)(\mu+\sigma^2/2))\} \\
&\quad - \frac{X_0(\exp(K(\mu+\sigma^2/2)) - 1)}{(1+d)^K} \qquad (26)\\
&\quad - \sum_{t=1}^{K-1}\frac{(\exp(gt) - \exp(g(t-1)))}{(1+d)^K} \\
&\quad \cdot\{\exp(K(\mu+\sigma^2/2)) - \exp(t(\mu+\sigma^2/2))\}\Bigg].
\end{aligned}
$$

4. *Uncertainty in Inventory Levels*

Let us now consider optimal choice between LIFO and FIFO methods of inventory valuation under conditions of uncertainty about the level of inventories. In considering stochastic inventory levels, we assume first that the quantity of inventory in the future will remain unaffected by whether management decides to stay with FIFO or to switch to LIFO. The possible effects of inventory valuation policy on the level of inventories will be considered in section 6.

Also assume that the physical quantity of inventory changes from year-end to year-end in discrete steps of x units, α being the probability of a positive change and $(1 - \alpha)$ the probability of a negative change.[9] Thus, given X_t, the physical quantity of inventory at the end of year t, the physical quantity at the end of year $t + 1$ has a Bernoulli distribution:

$$\begin{aligned} Pr\{X_{t+1} &= a + x \mid X_t = a\} = \alpha \\ Pr\{X_{t+1} &= a - x \mid X_t = a\} = 1 - \alpha. \end{aligned} \tag{27}$$

In other words, the physical quantity of inventory follows a random walk.[10] Values of α greater than, equal to, or less than 0.5 imply a random walk with positive trend, no trend, and negative trend, respectively. Since the results in this paper depend, in important respects, on the properties of this random walk, I will first establish a few propositions about the properties of this random walk that are necessary in the subsequent analysis.

Without loss of generality, let the step change in physical quantity of inventory be 1. Thus, each year, inventory increases or decreases by 1 with probability α and $(1 - \alpha)$, respectively. Let b, a positive integer, be the initial inventory at the beginning of period 1. Then the physical quantity of inventory at the end of any period t is:

$$X_t = b + \epsilon_1 + \epsilon_2 + \cdots + \epsilon_t . \tag{28}$$

Where ϵ_i, $i = 1, \cdots, t$ is the Bernoulli random variable:

$$\begin{aligned} Pr(\epsilon_i &= 1) = \alpha \\ Pr(\epsilon_i &= -1) = 1 - \alpha. \end{aligned} \tag{29}$$

An exception is made to this rule when the physical quantity X_t at the end of period t becomes zero. In order to avoid negative inventory, the Bernoulli process is modified as follows:

$$\begin{aligned} Pr(\epsilon_i &= 1/X_{i-1} = 0) = \alpha \\ Pr(\epsilon_i &= 0/X_{i-1} = 0) = 1 - \alpha. \end{aligned} \tag{30}$$

The physical quantity of inventory at any time t can be represented by

[9] I have selected this simple distribution to represent the behavior of inventory level because it is amenable to the formal analysis which follows and it provides excellent approximation of expected value of cash-flow differences. Evidence of how good this approximation is, is provided in section 6 of the paper. I also explain later why a probability distribution that does not even allow for the possibility of zero change provides such good approximations.

[10] Dov Pekelman has pointed out to me that a random walk assumption is in accordance with the "order up to" inventory reordering policy—a method frequently used in the industry.

the number of heads in excess of tails in a coin-tossing game after t trials in which the probability of heads is α and the game starts with an excess b of heads over tails. The state of the coin-tossing game, and therefore of the inventory at any time, can be described by an ordered pair (n, s_n), where n is the serial number of trial or period and s_n is the cumulative excess of heads over tails. Therefore, at the beginning of period 1, the state of the system is $(0, b)$ and at the end of period 1 it will be $(1, b + 1)$ with probability α and $(1, b - 1)$ with probability $(1 - \alpha)$.

Definition 1. A first passage through a point $r > 0$ is said to have taken place in period n if

$$s_0 = b, \quad s_1 > b - r, \quad s_2 > b - r, \cdots, s_{n-1} > b - r, \quad s_n = b - r. \quad (31)$$

Since the value of s_{n-1} and s_n differs by unity, it must be the case that $s_{n-1} = b - r + 1$. In the context of LIFO inventories, the first passage through $r = 1$ implies the consumption of the top layer of the base inventory X_0. Since in this case, the initial inventory has b layers, the first passage through $r(1 \leq r \leq b)$ in period t implies that the rth layer from the top is consumed in period t.

Proposition 1. The probability that the first passage through r occurs in period t when the probability of the associated Bernoulli process is α is given by:

$$\phi_{\alpha,r,t} = \frac{r}{t}\binom{t}{\frac{t-r}{2}}\alpha^{(t-r/2)}(1 - \alpha)^{(t+r/2)}. \quad (32)$$

Proposition 1 is a generalization of Theorem III.7.2 given by Feller [1968] for $\alpha = 0.5$. It is easily shown that $\phi_{\alpha,r,t} = 0$ unless both r and t are either even or odd. Equation (32) gives the probability of first passage for $(t = 1, 3, 5, \cdots, r = 1, 3, \cdots, t)$ and $(t = 2, 4, 6, \cdots, r = 2, 4, \cdots, t)$ because the rth layer from the top cannot be used up before period r.

Proposition 1 allows us to compute the probability of any given layer in the basic inventory, existing at the time of adoption of LIFO, being consumed in any future period.

Proposition 2. If a system is in state (t, a) at the end of period t, the probability that the first passage through 1 will occur in period $t + 2k + 1$ is given by:

$$\theta_{\alpha,2k+1} = \frac{1}{(2k + 1)}\binom{2k + 1}{k}\alpha^k(1 - \alpha)^{k+1}, \quad k = 0, 1, 2, \cdots, \quad (33)$$

and is independent of t and a.

Proposition 2 is obtained from Proposition 1 by substituting $r = 1$, $t = 2k + 1$ and recognizing that the properties of a random walk do not depend on its past history. I return to these propositions later when computing the expected value of tax savings under LIFO.

Let p_t be the price of inventory per unit at the end of period t; then the rate of price change during period t, I_t, is:

$$I_t = p_t/p_{t-1} - 1, \qquad t = 1, 2, \cdots. \tag{34}$$

According to this scheme, the price per unit of inventory at the beginning of period 1, the time of possible adoption of LIFO, is p_0. Under the FIFO system of inventory valuation, the entire physical amount of inventory at the end of period t, X_t, is valued at rate p_t.[11] Under LIFO, on the other hand, the price of the inventory depends on the vintage of individual layers which constitute X_t. This difference in valuation of inventory induces differences in tax liability of the firm at different points in time. The time differential in taxes payable under the two systems of valuation creates differences in the net present value of tax liability of the firm under the two valuation systems for given rates of inflation, discount and taxation, and levels of inventory. In the following analysis, I determine the expected present value of the tax difference for prespecified rates of inflation, discount and taxation, and an uncertain inventory level which follows a random walk. The tax difference is analyzed in two parts, with the first dealing with the inventory X_0 which is on hand at the time LIFO is adopted, and the second with the additional layers of inventory that might accumulate from time to time in future years while the LIFO method remains in use.

X_0 is the physical quantity of inventory on hand at the beginning of period 1. We can view this quantity as consisting of b layers of equal size x and price $x \cdot p_0$. Thus $b = X_0/x$ is a positive integer.[12] Let these b layers be indexed ($r = 1, 2, 3, \cdots, b$) from the top. Consider the net present value of the tax difference between the two accounting methods for the rth layer.

Under the FIFO system, the rth layer will be valued at $p_0 \cdot \prod_{i=1}^{t} (1 + I_i)$ at the end of any period t, where I_t is the rate of price change anticipated for period t as given by equation (34). The appreciation in the value of this layer in period t is:

$$p_0 x \left\{ \prod_{i=1}^{t} (1 + I_i) - \prod_{i=1}^{t-1} (1 + I_i) \right\} = p_0 x I_t \prod_{i=1}^{t-1} (1 + I_i). \tag{35}$$

Under the FIFO system, this inventory holding gain will be subject to a tax payment:

$$M_t p_0 x I_t \prod_{i=1}^{t-1} (1 + I_i) \tag{36}$$

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[11] This is a reasonable approximation. For example, an inventory turnover of six implies that the average FIFO price of ending inventory will be the price observed one month before the year end. If this difference of a month in the multiyear discounted model is considered nontrivial, adjustments to the subsequent expression can easily be made.

[12] Later in section 6, I describe an appropriate selection of b, the number of layers.

at the end of period t. The present value of the tax payment due on the holding gain on the rth layer of the basic inventory in period t under FIFO is:

$$M_t p_0 x I_t \prod_{i=1}^{t-1} (1 + I_i) \Big/ \prod_{i=1}^{t} (1 + d_i), \tag{37}$$

where d_i is the discount rate or time value of money to the firm in period i.[13]

The present value of all taxes paid on inventory holding gains from period 1 to t on any one basic layer under FIFO is:

$$T_t^{FB} = \sum_{r=1}^{t} M_r \cdot p_0 \cdot x \cdot I_r \prod_{i=1}^{r-1} (1 + I_i) \Big/ \prod_{i=1}^{r} (1 + d_i). \tag{38}$$

Now consider the alternative when the firm switches to the LIFO method of inventory valuation for K periods, $t = 1, 2, \cdots, K$. If a layer of the basic inventory gets used up in period t, the total amount of inventory holding gain from period 1 to t will be taxed at rate M_t in period t, $(t = 1, 2, \cdots, K)$.[14] If this inventory layer is not liquidated before period K, taxes on the holding gain over the K periods will have to be paid at the end of period K when the firm switches back to FIFO. If any layer of the basic inventory is liquidated in period t, the present value of tax on inventory holding gains under LIFO is:

$$T_t^{LB} = M_t p_0 \cdot x \left\{ \prod_{i=1}^{t} (1 + I_i) - 1 \right\} \Big/ \prod_{i=1}^{t} (1 + d_i). \tag{39}$$

The difference between the present value of tax payments under FIFO and LIFO conditional on a layer of base inventory being liquidated in period t is:

$$T_t^{FB} - T_t^{LB}, \qquad t = 1, 2, \cdots, K. \tag{40}$$

The probability of the rth layer of basic inventory being liquidated in period t is given by proposition 1 and is equal to:

$$\phi_{\alpha,r,t} = \frac{r}{t} \binom{t}{\frac{t-r}{2}} \alpha^{t-r/2} (1 - \alpha)^{t+r/2}. \tag{41}$$

Therefore, the expected present value of the tax difference for the rth

[13] Selection of the appropriate discount rate is discussed in section 6.

[14] The tax law does, at present, allow spreading this payment over a number of years. To the extent that this delay is not considered here, the results will have a small bias in favor of FIFO under inflation and in favor of LIFO under deflation. In my judgment, however, the difference is small. The model can be altered to take into account increases where this effect is regarded as nontrivial.

layer of basic inventory under LIFO system adopted for $K-1$ periods is:

$$\sum_{t=1}^{K-1}(T_t^{FB}-T_t^{LB})\phi_{\alpha,r,t}. \tag{42}$$

Since the probability of the rth layer being liquidated in the first $K-1$ period is $\sum_{t=1}^{K-1}\phi_{\alpha,r,t}$, the probability that the LIFO taxes on inventory holding gains on the rth layer are paid in the Kth period when the firm reverts back to FIFO is $(1-\sum_{t=1}^{K-1}\phi_{\alpha,r,t})$. (Note that by allowing K to become arbitrarily large, we can calculate the effects of LIFO used for an arbitrarily long period of time.) The net present value of the tax difference if layer r is not liquidated within the first $(K-1)$ period is $(T_K^{FB}-T_K^{LB})$, so the total expected tax difference for the rth layer is:

$$\sum_{t=1}^{K-1}(T_t^{FB}-T_t^{LB})\phi_{\alpha,r,t}+(T_K^{FB}-T_K^{LB})\left(1-\sum_{t=1}^{K-1}\phi_{\alpha,r,t}\right). \tag{43}$$

The expected value of the tax difference for all b layers of the basic inventory will then be:

$$\begin{aligned}T^B=\sum_{r=1}^{b}\Bigg\{&\sum_{t=1}^{K-1}(T_t^{FB}-T_t^{LB})\phi_{\alpha,r,t}\\&+(T_K^{FB}-T_K^{LB})\left(1-\sum_{t=1}^{K-1}\phi_{\alpha,r,t}\right)\Bigg\}.\end{aligned} \tag{44}$$

Expression (44) gives the expected net present value of all tax differences between the FIFO and LIFO methods of valuation for inventory on hand at the time LIFO is adopted. Next, I determine the tax differences associated with the delay of tax payments on inventory holding gains on layers of inventory which might be deposited during any period $t=1,2,\cdots,K-1$. Under the scheme considered, a layer of inventory consisting of physical quantity x is deposited in any period t with probability α, and the top layer of the opening inventory for period t is consumed in period t with probability $(1-\alpha)$. The price of the layer deposited is $x\cdot p_0\prod_{i=1}^{t}(1+I_i)$. Following an argument similar to the one given above for the basic inventory, we can calculate the present values of tax payments due on a layer deposited in period t and liquidated in period $t+r\le K$ under each system of inventory valuation:[15]

For FIFO:

$$T_{t,r}^{FS}=\sum_{j=t+1}^{t+r}M_jp_0xI_j\prod_{i=1}^{j-1}(1+I_i)\Big/\prod_{i=1}^{j}(1+d_i),\qquad t+r\le K,\quad r\text{ odd}. \tag{45}$$

[15] Note that r must be an odd number because a layer deposited in period t can only be consumed in periods $t+1, t+3, \cdots, t+2k+1$, etc.

For LIFO:

$$T_{t,\tau}^{LS} = M_{t+\tau}\cdot p_0\cdot x\cdot\left\{\prod_{i=1}^{t+\tau}(1+I_i)-\prod_{i=1}^{t}(1+I_i)\right\}\Big/\prod_{i=1}^{t+\tau}(1+d_i),\qquad t+\tau\le K,\quad \tau \text{ odd.} \tag{46}$$

The difference between the tax payments under FIFO and LIFO is:

$$T_{t,\tau}^{FS}-T_{t,\tau}^{LS},\quad t=1,2,\cdots,K-1;\quad \tau=1,3,\cdots,K-t-1. \tag{47}$$

The probability that a layer deposited in period t is consumed in period $t+\tau$ is given by Proposition 2, where $\tau = 2k+1$ is an odd number:

$$\theta_{\alpha,2k+1}=\frac{1}{2k+1}\binom{2k+1}{K}\alpha^k(1-\alpha)^{k+1},\qquad k=0,1,2,\cdots. \tag{48}$$

The probability that a layer deposited in period t would be liquidated before period K, therefore, is $\sum_{k=0}^{K-t-1/2}\theta_{\alpha,2k+1}$. When the LIFO method is abandoned in period K, there is probability $\{1-\sum_{k=0}^{K-t-1/2}\theta_{\alpha,2k+1}\}$ that a net present value of the tax difference $(T_{t,K-t}^{FS}-T_{t,K-t}^{LS})$ will be realized in that period.

Therefore, the expected tax difference on a layer deposited in period t is:

$$\sum_{k=0}^{K-t-2/2}(T_{t,2k+1}^{FS}-T_{t,2k+1}^{LS})\theta_{\alpha,2k+1}+(T_{t,K-t}^{FS}-T_{t,K-t}^{LS})\left(1-\sum_{k=0}^{K-t-2/2}\theta_{\alpha,2k+1}\right). \tag{49}$$

Since the probability that a layer will be deposited in any period t is α, the expected present value of all tax savings on layers deposited subsequent to adoption of LIFO until period K is:

$$T^S=\sum_{t=1}^{K-1}\alpha\left\{\sum_{k=0}^{K-t-2/2}(T_{t,2k+1}^{FS}-T_{t,2k+1}^{LS})\theta_{\alpha,2k+1}+(T_{t,K-t}^{FS}-T_{t,K-t}^{LS})\left(1-\sum_{k=0}^{K-t-2/2}\theta_{\alpha,2k+1}\right)\right\}. \tag{50}$$

The sum of expressions (44) and (50) gives the total net present value of tax differences between FIFO and LIFO when LIFO is adopted in year 1 and abandoned in period K:

$$T=T^B+T^S. \tag{51}$$

By allowing K to become very large, we can approximate the net present value of the tax differences from the adoption of LIFO for an infinite horizon.

The expressions developed above are very general in the sense that they can incorporate deterministic changes in inflation, discount and marginal tax rates from year to year, as well as stochastic variations in the level of physical inventory from year end to year end. Given the values of parameters α, K, I_i, d_i, M_i, x, and b, the expected present value of cash-flow differences can be explicitly computed from these expressions.

A considerable simplification is accomplished by assuming $I_t = I$, $M_t = M$, and $d_t = d$. Equations (38)–(44) for tax differences on basic layers can be rewritten as:

$$\begin{aligned} T_t^{FB} &= \frac{M \cdot p_0 \cdot x \cdot I}{1+d} \sum_{r=1}^{t} \left(\frac{1+I}{1+d}\right)^{(r-1)} \\ &= \frac{M \cdot p_0 \cdot x \cdot I}{d-I} \left\{1 - \left(\frac{1+I}{1+d}\right)^t\right\}. \end{aligned} \tag{52}$$

$$T_t^{LB} = M \cdot p_0 \cdot x \cdot \frac{(1+I)^t - 1}{(1+d)^t}. \tag{53}$$

$$T_t^{FB} - T_t^{LB} = M \cdot p_0 \cdot x \left\{\frac{I}{d-I} - \frac{d}{d-I}\left(\frac{1+I}{1+d}\right)^t + \frac{1}{(1+d)^t}\right\}. \tag{54}$$

$$\begin{aligned} T^B &= M \cdot p_0 \cdot x \sum_{r=1}^{b} \Bigg[\sum_{t=1}^{K-1} \left\{\frac{I}{d-I} - \frac{d}{d-I}\left(\frac{1+I}{1+d}\right)^t \right. \\ &\quad \left. + \left(\frac{1}{1+d}\right)^t\right\} \cdot \phi_{\alpha,r,t} + \left(1 - \sum_{t=1}^{K-1} \phi_{\alpha,r,t}\right) \\ &\quad \cdot \left\{\frac{I}{d-I} - \frac{d}{d-I}\left(\frac{1+I}{1+d}\right)^K + \left(\frac{1}{1+d}\right)^K\right\}\Bigg]. \end{aligned} \tag{55}$$

The equations (45)–(50) for expected tax differences from the layers deposited subsequent to adoption of LIFO can be rewritten as:

$$\begin{aligned} T_{t,r}^{FS} &= \frac{M \cdot p_0 \cdot x \cdot I}{1+d} \sum_{j=t+1}^{t+r} \left(\frac{1+I}{1+d}\right)^{j-1} \\ &= \frac{M \cdot p_0 \cdot x \cdot I}{d-I}\left(\frac{1+I}{1+d}\right)^t \left\{1 - \left(\frac{1+I}{1+d}\right)^r\right\}, \qquad t + r \le K,\ r \text{ odd}. \end{aligned} \tag{56}$$

$$T_{t,r}^{LS} = M \cdot p_0 \cdot x \cdot \left(\frac{1+I}{1+d}\right)^t \left\{\frac{(1+I)^r - 1}{(1+d)^r}\right\}, \qquad t + r \le K,\ r \text{ odd}. \tag{57}$$

$$\begin{aligned} T^S &= M \cdot p_0 \cdot x \cdot \alpha \sum_{t=1}^{K} \left(\frac{1+I}{1+d}\right)^t \left\{\sum_{k=0}^{K-t-2/2} \left(\frac{I}{d-I} - \frac{d}{d-I}\left(\frac{1+I}{1+d}\right)^{2k+1}\right.\right. \\ &\quad \left.\left. + \frac{1}{(1+d)^{2K+1}}\right)\right\} \cdot \theta_{\alpha,2k+1} + \left\{\left(\frac{I}{d-I} - \frac{d}{d-I}\left(\frac{1+I}{1+d}\right)^{K-t}\right.\right. \\ &\quad \left.\left. - \frac{1}{(1+d)} K - t\left(1 - \sum_{k=0}^{K-t-2/2} \theta_{\alpha,2k+1}\right)\right)\right\}. \end{aligned} \tag{58}$$

Recall that $\phi_{\alpha,r,t}$ and $\theta_{\alpha,2k+1}$ are given by:

$$\phi_{\alpha,rt,} = r \cdot \frac{(t-1)!}{(t-r/2)!(t+r/2)!}\, \alpha^{(t-r/2)}(1-\alpha)^{t+r/2} \tag{59}$$

and $\theta_{\alpha,2k+1} = \phi_{\alpha,1,2k+1}$ or:

$$\theta_{\alpha,2k+1} = \frac{(2k)!}{k!(k+1)!}\alpha^{k}(1-\alpha)^{(k+1)}. \qquad (60)$$

The sum of expressions (55) and (58) yields the expected present value of the difference between net cash flows associated with the use of FIFO and LIFO for a total of $(K - 1)$ years beginning immediately. It is not necessary that these differences be positive. In the case of an appropriately timed price decline of sufficient magnitude, the tax savings from LIFO may actually be negative.

5. Uncertainty in Rates of Price Changes and Inventory Level

In the preceding sections, I have presented two models of the LIFO–FIFO decision under conditions of uncertainty, one for uncertain price changes and a given sequence of inventory levels and the other for uncertain inventory levels and a given sequence of price changes. If both variables are considered uncertain, I suggest the following procedures: (1) generate several alternative paths of inventory levels and compute the expected present value of cash flow from the price-change uncertainty model for each path, or (2) generate several alternative price-change sequences and compute the expected present value of cash flow from the uncertain inventory level model for each sequence. In either case, the $E(\widetilde{PV})$ numbers can be used to compute a grand mean, assuming that each path is equally likely.

6. Estimation of Model Parameters

PHYSICAL LEVEL OF INVENTORIES

In both the deterministic (model 1) and stochastic price change (model 2) cases, it is necessary to specify the entire sequence of inventory levels from the present to the horizon. The most likely basis for specifying the sequence would be to apply a growth rate to the current inventory level. In the case of stochastic inventory levels (model 3), the physical quantity of inventories is assumed to follow an additive random walk of step size x from year end to year end, α being the probability of a positive step and $(1 - \alpha)$ the probability of a negative step. Therefore, the first stage in implementation of model 3 is to obtain estimates of x and α.

The expected value of a change in physical inventory is:

$$E(X_i - X_{i-1}) = \alpha\cdot x + (1-\alpha)(-x) = x(2\alpha - 1). \qquad (61)$$

Similarly, the variance of change in physical inventory is:

$$E\{X_i - X_{i-1} - x(2\alpha - 1)\}^2 = 4x^2\alpha(1-\alpha). \qquad (62)$$

The mean and variance of the step changes in the physical quantity of inventory can be inserted in equations (61) and (62) to estimate parameters x and α. If the mean and standard deviation are estimated to be μ and σ respectively, then:

$$\mu = x(2\alpha - 1) \tag{63}$$

$$\sigma^2 = 4x^2\alpha(1 - \alpha). \tag{64}$$

If μ is estimated to be zero, $\alpha = \frac{1}{2}$ and $x = \sigma$. If μ is not zero, α and x can be computed as:

$$\alpha = \begin{cases} 0.5 + \sqrt{0.25 - \left(\dfrac{\sigma^2}{4\mu^2 + 4\sigma^2}\right)} & \text{for} \quad \mu > 0 \\ 0.5 - \sqrt{0.25 - \left(\dfrac{\sigma^2}{4\mu^2 + 4\sigma^2}\right)} & \text{for} \quad \mu < 0 \end{cases} \tag{65}$$

and

$$x = \mu/(2\alpha - 1). \tag{66}$$

Note that when $\mu = 0$, α is one-half and the physical quantity of inventory follows an additive random walk without trend. When $\mu > 0$, α is greater than 0.5 and inventory follows a random walk with a positive trend. Similarly $\mu < 0$ implies $\alpha < 0.5$ and a random walk with a declining trend. Thus the stochastic process is capable of describing a variety of behavior of inventory levels.

The simplest way to estimate μ and σ would be to take past year-end physical inventory data and compute the sample mean and standard deviation of changes. Such estimates, however, will have to be adjusted to account for at least three factors. First, μ is the future and not the past rate of growth of the class of inventories being considered for a switch to LIFO. If the future rate of growth of the particular segment of inventory is anticipated to be different from the past, it should be reflected in the estimate of μ, which could be negative. Second, both μ and σ refer to the year-end and not the average inventory levels. If any major changes in seasonal inventory policies which might affect the year-end inventory levels are anticipated because of internal or external reasons, they should be appropriately reflected in the values of μ and σ. Third, other things being equal, the benefits of LIFO are lower when year-end inventories have higher variance. Under certain circumstances, a reduction in variance of year-end inventories will make adoption of LIFO more attractive. But lowering the variance of year-end inventories will require incurrence of extra costs in the form of forced purchases to recoup the depleted inventories. Therefore, the variance of year-end inventories is itself a decision variable from management's point of view. Model 3 allows management to estimate the tax savings from several different inventory policies specified in terms of the variance of the year-end inventories. The incremental tax benefits of lowering this variance can be compared with the corresponding incremental costs to arrive at an optimal year-end inventory variance. The input value of σ into this model, therefore, can be viewed as a tentative decision variable subject to the control of management.

At first glance, use of a Bernoulli distribution to describe the behavior of changes in the quantity of inventory appears to be an oversimplification which rules out all changes other than $+x$ and $-x$ (including a no-change condition). The distribution of inventory changes is obviously continuous and is probably described much better by a normal or lognormal distribution than by a Bernoulli distribution. For the purpose of computing expected values, however, a Bernoulli distribution does provide an excellent approximation of continuous distributions, even in the first year of the decision horizon when the difference between the underlying continuous distribution and the Bernoulli approximation of cumulative inventory changes is the greatest. In subsequent years, the Bernoulli approximation gets progressively closer to a normal distribution.

To demonstrate robustness of the proposed calculation of expected present value of cash-flow differences to the distributional assumption, I will compare the expected value of tax differences under the normal distribution with its Bernoulli approximation during the first year of decision horizon. Let inventory changes be distributed normally with $\mu = \sigma = 100{,}000$. In the first year after adoption of LIFO, taxes will have to be paid on inventory holding gain only if the change in inventory is negative. Expected value of tax payments is given by:

$$p_0 \cdot M \cdot I \cdot \left(-\int_{-\infty}^{0} \frac{y}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(y-\mu)^2}{2\sigma^2}\right) dy \right)$$

$$= p_0 \cdot M \cdot I \cdot \left(\frac{+\sigma}{\sqrt{2\pi}} \exp\left(\frac{-\mu^2}{2\sigma^2}\right) \mp \mu \operatorname{Prob}(y < 0) \right)$$

$$24197 - 15870 \quad = 8327 \cdot M \cdot I \cdot p_0$$

$$= 21{,}269 \cdot M \cdot I \cdot p_0 .$$

Under FIFO, taxes on inventory holding gain in the first year are $p_0 \cdot M \cdot I \cdot X_0$ and the difference of expected taxes is:

$$E(PV(\text{normal})) = p_0 \cdot M \cdot I(X_0 - \overset{8327}{\cancel{21{,}269}}).$$

Under the Bernoulli approximation, for $\mu = \sigma = 100{,}000$:

$$\alpha = 0.853,$$

$$x = 141{,}000$$

and

$$E(PV(\text{Bernoulli})) = p_0 \cdot M \cdot I(X_0 - (1-\alpha)x) = p_0 \cdot M \cdot I(X_0 - 20{,}789),$$

which is a very good approximation of normal expectation except when

TABLE 1

*Approximation of Normally Distributed Inventory Changes By Bernoulli Distribution Simulation Results**

(25 Iterations)

Year	Present value of cash flow	
	Normal distribution mean	Bernoulli distribution mean
1	27,826	26,554
2	28,027	27,188
3	28,661	28,889
4	29,451	29,266
5	29,410	29,808
6	29,014	29,993
7	29,314	30,435
8	29,198	30,243
9	28,625	30,017
10	28,514	29,636
11	28,509	29,189
12	28,174	28,324
13	28,403	28,139
14	28,218	27,761
15	27,334	27,392
16	26,807	26,964
17	26,135	26,370
18	25,186	25,695
19	24,431	25,106
20	23,712	24,494
21	22,927	23,692
22	22,233	23,091
23	21,484	22,333
24	20,693	21,716
25	20,132	21,028
26	−156,707	−158,427
Total	505,721	514,906

* PV of cash-flow differences from the model $E(PV)$ = \$517,780.

the initial inventory is very small compared to the variability of changes in earnings. The approximation works even better for subsequent years. To confirm this I conducted computer simulations of cash flows of the firm described in example 1, assuming normal and Bernoulli distribution for inventory changes, and compared the two simulation averages with the expected value of cash flows from the model. For a twenty-five-year horizon, the results are summarized in table 1. There is little doubt that for each yearly cash flow, as well as for the total cash flow over the horizon, the Bernoulli distribution provides an excellent approximation of continuous distributions.

NUMBER OF LAYERS IN THE BASIC INVENTORY (b)

In model 3, I assumed the physical quantity of inventory, X_0, at the beginning of the year when LIFO is adopted consists of b layers of quantity

x each priced at p_0 per unit. Once the value of x has been determined from equation (66), the value of b is the integer closest to (X_0/x).

MARGINAL TAX RATE (M_t)

The appropriate future marginal tax rate applicable to the differential effects of the two inventory valuation systems under consideration depends on several factors which are usually hard to predict. The first consideration is the existing tax structure and the anticipated changes in the structure. The tax structure includes such factors as the relationship of the level of income to the tax rate, existence of excess profits taxes, and averaging devices which allow high and low incomes in consecutive years to be averaged for tax purposes. The second major consideration is the level and stability of income of the firm from sources other than the inventory holding gains. Since a large degree of uncertainty is associated with both these considerations, it would be hard to specify a separate M_t for more than a few years in the immediate future. Beyond this period, practical limits may dictate the use of a uniform marginal tax rate for all future years. Note, though, that such an assumption is a matter of convenience in specifying the input parameters and is not required by the models. An implicit assumption is that the volume of inventory profits under the accounting systems under consideration does not affect the marginal tax rate. In the presence of difficulties associated with the estimation of M_t mentioned earlier, such an assumption would appear to be rather innocuous.

RATES OF PRICE CHANGE (I_t)

Price changes are the primary cause of the differences that exist between the LIFO and FIFO methods of valuation. The preference for one method over the other depends critically on the nature of price changes anticipated for the inventories under consideration. In their most general form, models 1 and 3 allow a separate rate of price change for each year up to the decision horizon of the firm. $I_t > -1$ is the only restriction on the value of I_t in these models which will accommodate monotonically rising, declining, cyclical, or random patterns of price changes. In many situations, it may be useful or convenient to specify a flat rate of price change function $I_t = I$, $t = 1, 2, \cdots$. Again, while the use of such an assumption simplifies computations, it is not required by the models.

Model 2 needs three parameters μ, σ^2, and p, the mean and variance and serial correlation of log price relatives $(ln\ I_t/I_{t-1})$. Probably the easiest approach to specification of these parameters is to estimate them from past data. These estimates can be adjusted to reflect management's assessment of the future value of the parameters, in case the latter quantities are different from the estimates made from past data.

I have deliberately avoided using the terms price-level changes or change in purchasing power of money for I_t since I_t is the rate of anticipated change in the price of the particular parts of the inventory. In some cases, changes in the general price level, often measured by the Consumer Price Index and GNP Deflator, may provide an acceptable measure of price changes for inventories in question. However, in general, this would not be the case. Each firm must make a separate estimate of the anticipated rate of price changes for various parts of its inventory for which the decision to adopt LIFO can be taken independently.

RATE OF DISCOUNT (d_t)

According to capital market theory, the appropriate rate of discount is a function of the sensitivity of the economic returns from the proposed accounting change to returns from the market portfolio. The uncertainty associated with the returns from the accounting change includes such factors as the future changes in the tax laws and accounting principles, the future inventory policies of the firm, and the supply and demand conditions in the input and the output market. I suggest that the average cost of capital of the firm be used as the discount rate until better estimates of the discount rate can be devised.

POOLS OF INVENTORY

The model presented here is applicable to each pool of inventory as well as to larger aggregations of inventory pools. An advantage of using smaller pools is that it permits exclusions from the LIFO system of those items of inventory which are likely to decline in price, thus avoiding the related tax losses. Use of larger pools, however, reduces the likelihood of the liquidation of the low-priced base inventory in the early years, a situation that reduces the tax advantage of LIFO. To determine the optimal scheme of forming inventory pools, the net present value of tax savings could be computed for each level of aggregation by adding the tax savings from each pool at that level. The level of aggregation which yields the highest net present value would then be selected for further consideration in the decision-making process.

7. *Application*

Application of models 1 and 2 is straightforward and needs little explanation. I shall present two numerical examples of application of model 3 to illustrate its use and conduct some sensitivity analysis. With respect to changes in decision horizon, average rate of change in inventory level, and tax rates, I also compare the results of model 3 with the results from the certainty model of Sunder [1976].

EXAMPLE A

Consider a firm which has a FIFO inventory worth one million dollars (X_0 = \$1,000,000). Since inventory is heterogeneous, its quality is best measured in dollars and $p_0 = 1$. The management estimates that the physical quantity of their year-end inventory will increase at a rate of \$100,000 per year at the base prices. The standard deviation of changes in the year-end inventories is \$100,000. The firm's marginal tax rate is expected to remain constant at 40 percent. Other assumptions are: a steady rate of price change of 8 percent per year for its inventories, and a cost of capital to remain at 15 percent. The firm wishes to measure the present value of tax benefits it can expect to receive if it adopts LIFO for all its inventory (1) for a period of twenty-five years and (2) forever.

Since average (μ) and standard deviation (σ) of the physical quantity of inventory are given as \$100,000 and \$100,000 respectively, we can estimate α and x from expressions (65) and (66):

$$\alpha = 0.853$$

$$x = 141{,}000.$$

The initial inventory of \$1,000,000 can be considered to be made of seven layers of \$141,000 each at base prices. Since the marginal tax rate, rate of price change, and discount rates are all given as constants, we can use simplified expressions (55) and (58) to compute the expected present value of the difference between cash flows under the two accounting procedures. The expected difference arising out of the base inventory, T^B, is \$291,500 and from layers deposited subsequently, given by T^S, is \$226,279 for a period of twenty-five years under the assumption that LIFO is abandoned in the twenty-sixth year. Thus the total expected present value of tax differences is \$517,780. The present value of the expected cash-flow difference is shown as a function of the number of years for which LIFO is used in figure 1. There is little change in *EPV* beyond a decision horizon of fifty years. A comparison of the *PV*'s of cash flow obtained from model 3 with the corresponding numbers obtained from the certainty model of Sunder [1976] is given in the last column of table 2. If LIFO is used while the inventory level is maintained constant at \$1,000,000, the certainty model would indicate a *PV* of \$300,243 for twenty-five years and \$457,143 forever. Since, in this example, the quantity of inventory is increasing at a fast rate, the certainty model understates the benefits of LIFO in spite of a sustained rise in prices. The tax difference obtained from the certainty model is only slightly greater than the tax difference due to the basic inventory in the uncertainty model. The difference is small because there is only a small chance that any part of basic inventory is ever liquidated. Since the chances of adding more layers are very high, tax differences due to other inventory (T^S) become much larger than the tax differences due to basic inventory (T^B) as the decision horizon increases.

EXAMPLE B

Consider the firm in example A which expects that its inventory will have an average change of zero and this standard deviation of changes will

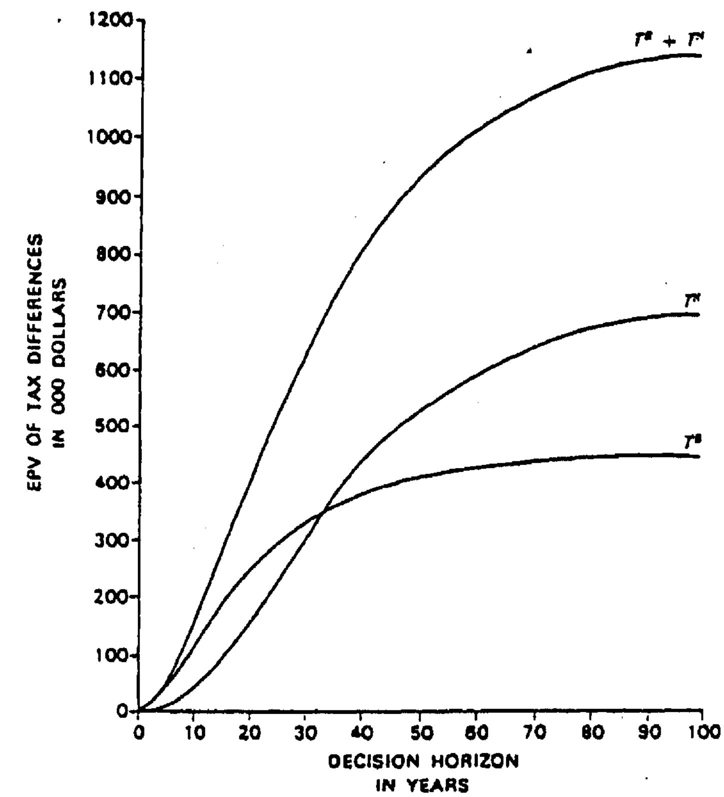

FIG. 1

TABLE 2

Expected Present Value of Tax Differences under FIFO and LIFO for Decision Horizon K-Example A

Decision horizon (years)	Uncertainty model			Certainty model
	Total difference $T^B + T^S$	Due to basic inventory T^B	Due to other inventory T^S	
1.........	3,553	3,553	0	3,630
2.........	10,315	9,960	355	10,194
3.........	19,958	18,629	1,329	19,104
5.........	46,694	40,908	5,786	42,032
7.........	81,540	67,353	14,187	69,271
10.........	144,841	110,325	34,516	113,546
15.........	267,032	180,714	86,318	186,079
20.........	395,039	241,921	153,118	249,152
25.........	517,780	291,501	226,279	300,243
50.........	936,879	410,260	526,619	422,624
75.........	1,093,031	436,733	656,298	449,904
99.........	1,139,042	442,200	696,842	455,536
∞.........				457,143

be $40,000 at base prices. Other data are the same as in example A except that its marginal tax rate is 50 percent. Table 3 and figure 2 show the difference between the net present value of tax payments under FIFO and LIFO as a function of the decision horizon. These results are compared

TABLE 3

Expected Present Value of Tax Differences under FIFO and LIFO for Decision Horizon K-Example B

Decision horizon (years)	Uncertainty model			Certainty model
	Total difference $T^B + T^d$	Due to basic inventory T^B	Due to other inventory T^d	
1........	4,446	4,446	0	4,537
2.........	12,448	12,405	43	12,743
5.........	50,901	50,281	620	52,539
10.........	136,165	133,146	3,019	141,933
25.........	354,638	339,930	14,708	375,304
50.........	495,427	466,968	28,459	528,280
99.........	533,041	498,467	34,574	569,420

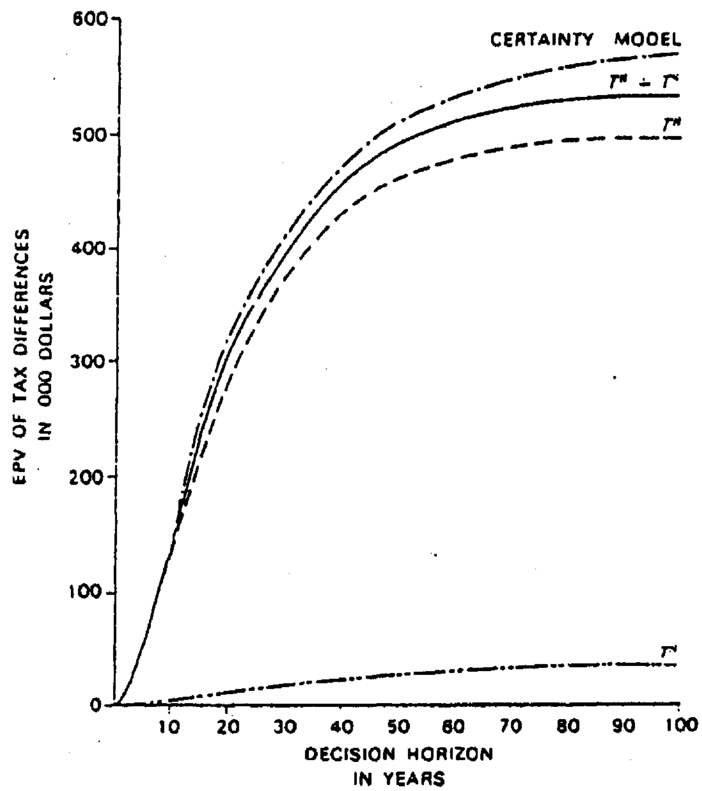

FIG. 2

with the results obtained from the certainty model under the assumption of constant inventory. Also note that the tax advantage of LIFO is much smaller when inventory is expected to remain level in spite of a higher marginal tax rate. Almost all of the tax savings arise from the basic inventory because very little other inventory is likely to accumulate.

In both examples A and B, the decision to switch to LIFO is optimal if no other costs and benefits are associated with the accounting switch. Other costs such as the cost of accounting changes, tax payments on the restatement of FIFO inventory on a price cost basis, and higher inventory management costs under LIFO to avoid liquidation of base inventory can be added to the tax effect to arrive at the optimal decision.

8. Concluding Remarks

I have presented three approaches to quantifying the cash-flow consequences of a choice between the FIFO and LIFO methods of inventory valuation. The first assumed conditions of certainty, while the other two assumed conditions of uncertainty. In all three cases, the objective was to compute the expected present value of the difference between cash flows under the two systems. The approaches can also be used to conduct analyses of sensitivity of the accounting decision to relevant parameters such as the expected value and standard deviation of year-end inventory changes, rate of price change, cost of capital and marginal tax rate, and various configurations of inventory pools. The quantified measure of the tax effects of inventory decision can be combined with other quantifiable and nonquantifiable variables to select the method of accounting which will best serve the interest of a firm.

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