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## ASYMMETRIC INFORMATION AND NEWS DISCLOSURE RULES

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**Abstract**

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When the imminence of news announcements is not public knowledge, many traders will lack information on both the mean and variance of private information. Our analysis of such a setting in both single and multi-security contexts implies that disclosure of impending information events by firms can bound variance uncertainty and thereby improve investor welfare by mitigating the market breakdown problem. We also find that the equilibrium pricing functions are non-linear; specifically, convex for small trades and concave for larger ones. In addition, we predict that large transactions will be followed by large levels of volatility.

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# 1 Introduction

Trading suspensions in advance of material news disclosures are a common phenomenon on visible stock exchanges such as the New York Stock Exchange (NYSE) and Nasdaq. These exchanges require firms to disclose significant news to the exchange prior to public disclosure of that news, whereupon trading in a given stock is suspended until the news becomes public knowledge.<sup>1</sup> The existence of such disclosure rules and impediments to trade remains a puzzle from a theoretical standpoint. In this paper, we present a model that provides a rationale for disclosure rules and trading suspensions, and analyze their effects on investor welfare and the liquidity of financial markets. Our starting point is the observation that when market makers are unaware that a significant news announcement is imminent, they will lack enough information to accurately compute the payoff variance (which proxies for the severity of adverse selection in the market). Our analysis of the equilibrium in such a setting indicates that uncertainty over the variance can lead to very illiquid markets unless the exchange implements a policy which involves firms reporting to the exchange whenever the variance of private information is high, whereupon the exchange may choose to suspend trade.

In fact, we find that disclosure rules may be necessary for markets to open at all. This is illustrated by considering a situation in which, without a disclosure rule, the degree of asymmetric information is so high that the only equilibrium involves no trade. When a disclosure rule is introduced, however, trading does occur under some states. Although the market closes whenever the firm reports to the exchange that the amount of private information is high, it remains open when there is no such report; the *absence* of a report conveys information to the market maker that the variance of private information is low. In this scenario, it is evident that disclosure rules are welfare-enhancing. More generally, in instituting an optimal disclosure policy, an exchange faces a trade-off. On the down side, a stricter policy increases the probability that traders will be forced to maintain unhedged positions (‘unbalanced portfolios’), since the market may be closed more often because of severe asymmetric information. Positively, however, a stricter policy also increases liquidity whenever the market is open, thereby providing better terms of trade. We demonstrate that the optimal reporting cutoff balances these two effects, and that, under reasonable

parameter values, there exists a cutoff that increases ex ante investor welfare.

Our paper, by analyzing an equilibrium in which the informed agent has private information about both the mean and variance of a normally distributed asset payoff, also contributes to market microstructure methodology. Indeed, in our setting we find that the linear equilibria extensively considered in the literature do not exist. Furthermore, the derived function is convex for small trades and concave for larger ones. Intuitively, this result arises from the way market maker's update their beliefs. For very small trades the price does not move very much, since the market maker takes these trades to imply that the variance of the investor's information is small. However, for the larger trades, the market maker revises his beliefs about the variance upward, and the curve steepens. The concavity in our equilibrium price schedule is consistent with that documented by Madhavan and Smidt (1991) and Hasbrouck (1991) for NYSE stocks. We also find that the price variance following a trade depends upon the size of the transaction, or equivalently, the absolute price change in response to a trade. In our model, the result obtains because a high trader demand implies that the private information likely obtains from a high variance event. The conditional variance of the price based on the trader demand thus increases in trade size.

Previous papers on the feasibility of trade (e.g., Bhattacharya and Spiegel, 1991; and Glosten, 1989) assume that the informed investor draws his private information from a normal distribution with a known mean and variance, and find that if the variance of the informational variable exceeds a critical value, the only feasible equilibrium involves no trade. We relax the common knowledge assumption on the distribution's variance, and find that markets cannot open if the market making agent attaches *any positive probability* to the event that the variance of the informed investors' information exceeds an endogenously calculated upper bound. This is a stronger result than what has been derived in the earlier literature and indicates that market breakdowns can be far more common than what the Glosten (1989) and Bhattacharya and Spiegel (1991) papers suggest. For this reason, our setting is an ideal one to rationalize the relatively frequent trading suspensions on NYSE and Nasdaq, and why these visible and successful markets have news disclosure policies in place. The same intuition may also help to explain why events in one market (such as the default by Russia on its bond) can trigger a liquidity crisis in other markets. If an event

in one market makes people sufficiently suspicious about the variance of information held by others in another market, trade may simply cease.

In principle, one might consider a rule that requires firms to report any information that will move the stock price by more than some amount. However, while it is easy to discuss such a rule, it would be very difficult to apply. For the most part agents do not know the exact amount by which a particular news story will impact a stock's price. Even worse, sometimes it is not clear if a news event will move a stock's price up or down. Take for example, the news that Intel came to a settlement with the government regarding its anti-trust litigation. Prior to the announcement it was impossible to tell if this would be viewed as good or bad news by the market or if the impact would be large or small. The actual price move depends upon market expectations and this is difficult to know ahead of time. However, while it may be difficult to write an enforceable rule that tells a firm to disclose any information that will have a large price impact (e.g. cause the stock price to move by more than 20%), it is easy to write a rule that simply requires the disclosure of particular types of information (such as quarterly earnings). Our model suggests that such rules may be sufficient to ensure market liquidity, so long as the rules require the disclosure of "high variance" types of information. Thus, when crafting a disclosure rule neither the firm nor the exchange needs to know in advance the exact impact of a piece of information on the stock's price. All that is necessary is the knowledge that the information will on average cause a large price swing one way or the other.

We consider an extension of the model to allow for multiple securities and exchanges. This part of the analysis yields the cross-sectional implication that a high variance in one security should be followed by a high variance in another security whose private information variance is positively correlated with that of the first (e.g., a stock in the same industry). The analysis also suggests that if such related securities are traded on different competing exchanges, the resulting news disclosure policies will usually be socially inferior to those in a consolidated exchange structure. We begin with the reasonable premise that an exchange's objective is to maximize the ex ante expected utility of the investors trading on its floor. Since such exchanges do not consider the influence of their rules on investors trading in other markets, they may set a disclosure policy that is suboptimal. Our analysis suggests that this result depends upon the degree to which the adverse selection in market

A is correlated with that in market B. Intuitively, if the degree of correlation is high, the knowledge that market A has not closed implies to the market maker in market B that they probably face a low degree of adverse selection. In turn, this increases the liquidity in market B when exchange A is open. A consolidated exchange can take these cross-effects into account and can therefore implement a superior policy. This indicates that empirical evidence comparing the leniency of one exchange’s policy with another cannot provide *prima facie* evidence as to the efficacy of the news disclosure policies. It is possible for the exchange with the tighter rules to provide a superior trading environment from the perspective of social welfare.

Several other papers have departed from the standard assumption of normally distributed private information, though in different contexts. Foster and Viswanathan (1993) consider elliptical distributions; Kumar and Seppi (1994) examine a model of “arbitrage gaps” between an index futures contract and the underlying stocks, in which the *precision* of the heterogeneous signals received by market makers at different geographic locations is private information; Forster and George (1992) and Lindsey (1991) analyze models in which market makers possess superior information about the variance of “noise trading” in the market. In contrast, we examine how the inference problem of the market makers is affected when only the *informed investor* knows the ex ante variance of his signal.

This paper is organized as follows. The next section discusses the basic model and characterizes its equilibrium. Section 3 applies the analysis to news disclosure policies and trading suspensions. Section 4 analyzes the case of multiple securities. Section 5 concludes. The proofs of all propositions, unless otherwise stated, appear in the Appendix.

## 2 The Basic Model

### 2.1 The Economic Setting

The model of this section uses a structure which resembles that of Bhattacharya and Spiegel (1991), Bhattacharya *et al.* (1995), and Glosten (1989). A single representative investor with negative exponential utility and a risk aversion coefficient  $A$  trades a risky security and a riskless bond over a single period. Trading takes place in period 1, and in period 2, the risky security pays  $\epsilon + \eta$ . The variable  $\eta$  is normally distributed with a mean of zero

and a variance of  $\sigma_\eta^2$ . This variable allows the model to capture the idea that information flows which cannot be anticipated by any agent in the model arrive continually. The term  $\epsilon$  represents private information known only to the investor. Conditional on knowing its variance  $\sigma_\epsilon^2$ , the value of  $\epsilon$  follows a normal distribution with mean zero. After period 1,  $\epsilon$  becomes public knowledge. Thus, one can conceptualize  $\epsilon$  as, for example, a new product announcement about which the trader has advance knowledge relative to the market. Period 1 represents trading prior to the announcement, and period 2 the closing out of positions after the announcement.

As discussed in the introduction, a central premise of the analysis is the observation that when market makers are unaware that significant firm-specific announcements are imminent, they will lack enough information to accurately compute the degree of adverse selection in the market (proxied by the variance of  $\epsilon$ ). The model therefore departs from the standard rational expectations framework by assuming that, from the perspective of the market maker, the ex ante variance of  $\epsilon$  is random with a density function  $g(\sigma_\epsilon^2)$ , whose support is bounded above by a number  $\sigma_u^2$ . Thus, at the start of the game, the market maker does not know either the realization or the variance of  $\epsilon$ .

Note that in this paper, the uncertainty about second moments derives from the unknown variance of the private information variable  $\epsilon$ . In this respect, the paper differs from Kumar and Seppi (1994) and Forster and George (1992), in which informed traders possess superior information about the variance of the *noise* in the private information signal, and the variance of noise trading, respectively.

To motivate trade in the presence of asymmetric information, we assume that the trader has a hedging motive as well as an informational motive for trade. Some papers (e.g., Diamond and Verrecchia, 1981) model this motive by assuming that informed traders have an unhedged endowment of the risky asset. Here we make the mathematically equivalent assumption that the trader holds  $s$  units of a non-tradeable asset (human capital, for instance), which has a payoff that is correlated with that of the tradeable risky asset (our approach is similar to that adopted by Bhattacharya and Spiegel, 1991, and Bhattacharya *et al.*, 1995). Since the correlation between the traded and nontraded asset is not the focus of this paper, we analyze the case in which the payoff on the non-traded asset is perfectly correlated with the tradeable risky asset, though our results are robust to the imperfect

correlation case as well. From the perspective of all other participants in the market, the investor's stock of the untraded asset,  $s$ , is normally distributed with variance  $\sigma_s^2$ . To avoid notational clutter, the model also assumes that the investor begins with zero units of the tradeable risky asset and the bond. None of the model's primary conclusions are changed if one relaxes either of these supply-related assumptions.

Trading takes place in a market populated by competitive, risk neutral market makers. These market makers stand ready to execute orders as they arrive at the exchange. Upon seeing an order they each submit a price at which they are willing to partake in a position opposite that of the order. The set of potential prices include the entire extended real line, so quotes of plus or minus infinity for particular quantities are possible. Allowing the market makers to quote infinite prices allows them the strategic option of refusing to trade if they so desire. After seeing the price offered by the market makers the trader can then either go through with the trade or cancel it. In the event that the trader decides to cancel the order he simply retains his initial endowment.

Since the market makers are all identical, the model assumes that if they submit identical price quotes in response to any given trade, then they split the order evenly. Altering either assumption will not have any impact on the analysis so long as one assumes that no individual market maker will be assigned enough of the order flow to negate the assumption of competitive behavior. In period 1, the aggregate pricing function maps the order submitted by the investor  $X$  into a price  $P(X)$ . The model does not restrict this price function in any way, so it may be discontinuous.

## 2.2 Equilibrium

The investor's terminal (post-round 2) wealth equals

$$V = (\epsilon + \eta)(X + s) - XP.$$

Since the conditional wealth is normally distributed, the investor solves

$$\max_X \epsilon(X + s) - XP - \frac{A}{2}\sigma_\eta^2(X + s)^2. \tag{1}$$



For the moment, let us assume that  $P$  is differentiable. Then, one obtains the first-order condition

$$\epsilon - A\sigma_\eta^2 s = P + XP' + A\sigma_\eta^2 X. \quad (2)$$

In a standard model, one can argue that the market makers observe a noisy transformation of the information  $\tau = \epsilon - w$ , where for notational convenience  $w \equiv A\sigma_\eta^2 s$ . The market makers then apply standard statistical updating techniques to obtain the linear pricing function. However, since the market maker does not know the realization of  $\sigma_\epsilon^2$ , the derivation of the equilibrium must be modified.

Relaxing the assumption that the market makers submit a differentiable aggregate demand function presents a number of difficulties. To overcome them, the model now requires a formal definition of what constitutes an equilibrium. Rewrite (1) as

$$\max_X X(\tau - P) - \frac{A}{2}\sigma_\eta^2 X^2 + \tau s + \frac{A}{2}\sigma_\eta^2 s^2. \quad (3)$$

Notice that in (3) above,  $X$  only occurs in the first two terms, which depend on  $\epsilon$  and  $s$  only through  $\tau$ . Thus, the investor's maximization problem depends upon  $\epsilon$  and  $s$  only to the extent that they influence  $\tau$ . In other words, any two informed investors with the same value of  $\tau$  should select the same equilibrium demand. One can therefore think of  $\tau$  as representing the investor's type and the demand  $X$  as a signal that reveals, at least to some extent and perhaps perfectly, the realized value of  $\tau$ . The model thus falls under the category of a signalling game and one can therefore employ the standard tools of that literature to describe the sets of behavior which constitute equilibrium strategies. As with most signalling games, an important issue is the beliefs held by players when they observe an out-of-equilibrium move. Unless one is willing to impose the *ad hoc* assumption that the aggregate demand function is differentiable, the issue arises as to how to address demands that should never occur in equilibrium. To restrict the market makers' beliefs, the model follows Bhattacharya *et al.* (1995) in applying a variant of the commonly used Cho and Kreps (1987) D1 criterion; the details of this application appear in the Appendix.<sup>2</sup>

### 2.3 Supply Schedule

This section of the paper explores several aspects of the equilibrium supply schedule. First, we show that there always exists a “no trade” equilibrium in which all of the players simply

hold their endowments. In this equilibrium, the market makers quote a price  $+\infty$  for any strictly positive demand and  $-\infty$  for any strictly negative demand by the investor. Second, if an equilibrium with trade exists, the supply schedule is both continuous and differentiable for all possible demands. Using these results, the paper develops an ordinary differential equation that describes the equilibrium supply schedule, and subsequently explores its properties. We begin with the following proposition.

- Proposition 1**    1. *The no-trade equilibrium always exists. Under this equilibrium the market makers quote a price of  $+\infty$  for strictly positive demands  $X > 0$  and  $-\infty$  for strictly negative demands  $X < 0$ . In this equilibrium, the investor always demands a zero quantity.*
2. *If an equilibrium with trade exists, the equilibrium supply function is continuous and differentiable. In an equilibrium with trade there does not exist a largest or smallest demand above or below which no trade can take place.*

The idea that there exists an equilibrium without trade is not just an artifact of the model but is an example of a standard coordination problem in which one ends up at an undesirable equilibrium. In financial market settings, it is worth noting that certain assets are either totally illiquid or can only be traded after a great deal of scrutiny by both sides. For example, very small firms find it difficult if not impossible to sell equity in part because nobody believes there will be a market for their security. Similarly bonds that are “on the run” are very liquid in part because everybody believes that a liquid market exists and so traders concentrate their trading activities in such bond markets. However, once a bond is no longer on the run it becomes much more difficult to buy or sell it. Basically, if an agent does not believe anybody else will trade in a particular market it is in that agent’s best interest to also desist from attempting to trade in that market.

If trade does take place, questions then arise about general attributes of the equilibrium supply schedule. One can imagine a situation where the supply schedule has kinks, and perhaps jumps. Part 2 of Proposition 1 shows that such poorly behaved functions cannot arise in equilibrium. Thus, the equilibrium supply function is both continuous and differentiable everywhere except possibly at  $X = 0$ .<sup>3</sup>

Having addressed issues relating to continuity and differentiability, one can return to the original first order conditions for the investor presented in equation (2). After observing  $\tau$ , the market maker updates his priors on both  $\epsilon$  and  $\sigma_\epsilon^2$ . This leads to the familiar formula,

$$f(\epsilon, \sigma_\epsilon^2 | \tau) = \frac{f(\tau | \epsilon, \sigma_\epsilon^2) g(\epsilon, \sigma_\epsilon^2)}{\int_{\Omega_1} \int_{\Omega_2} f(\tau | \epsilon, \sigma_\epsilon^2) g(\epsilon, \sigma_\epsilon^2) d\epsilon d\sigma_\epsilon^2}, \quad (4)$$

for the updating rule, where  $g(\epsilon, \sigma_\epsilon^2)$  is the joint distribution of  $\epsilon$  and  $\sigma_\epsilon^2$ ,  $f$  is a conditional distribution function over the relevant variables, and  $\Omega_1$  and  $\Omega_2$  are the supports of the distributions of  $\epsilon$  and  $\sigma_\epsilon^2$ , respectively. Since  $\tau$  is normally distributed given  $\epsilon$  and  $\sigma_\epsilon^2$ , and the prior distribution of  $\epsilon$  is also normal, one can write  $f(\tau | \epsilon, \sigma_\epsilon^2) g(\epsilon, \sigma_\epsilon^2)$  as

$$f(\tau | \epsilon, \sigma_\epsilon^2) g(\epsilon, \sigma_\epsilon^2) = \frac{1}{2\pi\sigma_w\sigma_\epsilon} g(\sigma_\epsilon^2) \exp \left[ -\frac{1}{2} \left( \frac{(\tau - \epsilon)^2}{\sigma_w^2} + \frac{\tau^2}{\sigma_\epsilon^2} \right) \right]. \quad (5)$$

By using Equation (5) in Equation (4), we obtain a solution for the posterior on  $\epsilon$  and  $\sigma_\epsilon^2$ . Multiplying the resulting expression by  $\epsilon$  and integrating over both  $\epsilon$  and  $\sigma_\epsilon^2$  yields the expected value of  $\epsilon$  given  $\tau$ . An extensive amount of algebra shows that it has the form

$$E(\epsilon | \tau) = \frac{\int_{\Omega_2} \frac{\sigma_\epsilon^2 \tau}{(\sigma_\epsilon^2 + \sigma_w^2) \sqrt{2\pi(\sigma_w^2 + \sigma_\epsilon^2)}} \exp \left\{ -\frac{\tau^2 \sigma_w^2}{2(\sigma_\epsilon^2 + \sigma_w^2)} \right\} g(\sigma_\epsilon^2) d\sigma_\epsilon^2}{\int_{\Omega_2} \frac{1}{\sqrt{2\pi(\sigma_w^2 + \sigma_\epsilon^2)}} \exp \left\{ -\frac{\tau^2 \sigma_w^2}{2(\sigma_\epsilon^2 + \sigma_w^2)} \right\} g(\sigma_\epsilon^2) d\sigma_\epsilon^2}. \quad (6)$$

In a model where  $\sigma_\epsilon^2$  is not stochastic the term  $\sigma_\epsilon^2 / (\sigma_\epsilon^2 + \sigma_w^2)$  represents the updating rule on  $\epsilon$  given  $\tau$ . That is, with a known  $\sigma_\epsilon^2$ , the market maker sets  $E(\epsilon | \tau) = \tau \sigma_\epsilon^2 / (\sigma_\epsilon^2 + \sigma_w^2)$  (the standard regression formula). Define  $\hat{\alpha} \equiv \sigma_\epsilon^2 / (\sigma_\epsilon^2 + \sigma_w^2)$ . Note that the market maker can recover  $\tau$  from the expression on the right-hand side in (2). Thus, one can equivalently write (6) as  $E(\epsilon | \tau) = \tau E(\hat{\alpha} | \tau)$ . To conserve on notation, we further define  $\alpha \equiv E(\hat{\alpha} | \tau)$ , and thus write (6) as simply

$$E(\epsilon | \tau) = \tau \alpha. \quad (7)$$

Upon observing the investor's demand, the market maker infers  $\tau$ , and forms a posterior based on (7). Given that the market maker is competitive and risk neutral, the schedule faced by the investor is  $P = \tau \alpha$ . Despite the fact that we will often write  $P = \tau \alpha$ ,  $P$  is still only a function of  $X$ . The quantity  $\alpha$  represents the market maker's beliefs given  $X$ , and we require that in equilibrium these beliefs correctly forecast  $\epsilon$ . Therefore, one can

now use the relationship  $P = \alpha\tau$  to eliminate  $\tau$  from Equation (2) and thereby produce the ordinary differential equation,

$$P(\alpha - 1) + \alpha(XP' + A\sigma_\eta^2 X) = 0, \quad (8)$$

after recalling that  $\tau = \epsilon - A\sigma_\eta^2 s$ . For a price function to form an equilibrium it must satisfy (8), and additional restrictions must be placed on the system to ensure that the investor's second order conditions also hold.

The next proposition shows that if the upper bound on the support of distribution for  $\sigma_\epsilon^2$  equals or exceeds the variance of  $w$  (i.e., if  $\sigma_u^2 \geq \sigma_w^2$ ) then the only equilibrium involves no trade. The proof of the proposition shows that this condition is equivalent to the condition that  $\alpha$  exceeds 0.5 for some realization of  $\tau$ .

**Proposition 2** *If the upper bound of the support of the private information variance,  $\sigma_u^2$ , exceeds the variance of the liquidity component of demand,  $\sigma_w^2$ , the only equilibrium involves no trade.*

To obtain some intuition on the above proposition, first consider the market maker's problem with known variance. The market maker makes zero expected profits conditional on each realization of the order flow (or  $\tau$ ). Since  $\alpha = \hat{\alpha}\tau$ , and  $\hat{\alpha} = \sigma_\epsilon^2 / (\sigma_\epsilon^2 + \sigma_w^2)$ , if the market maker believes almost surely that  $\sigma_\epsilon^2 \geq \sigma_w^2$ , this, in a sense, amounts to believing that "over half" of the investor's desire to trade comes from his desire to exploit an informational advantage rather than any desire to hedge. In this case, the market makers do not want to take the opposite side of the trade, market clearing cannot occur, so that the only possible equilibrium is the no-trade equilibrium.

Now, with a random variance, as  $\tau$  becomes very large, the market makers believe that the information shock was almost surely drawn from the top of the distribution. Thus, if at the top of the distribution the market makers believe that over half of the investor's demand derives from an informational motive the market cannot clear for those demands. Since market makers are unable to quote break-even prices for these quantities, this then unravels the entire supply function, and thereby closes down trade.<sup>4</sup>

The above arguments suggest that for a market to function, uninformed participants need some "assurance" that the level of asymmetric information can never exceed some

bound. Section 3 analyzes this observation in greater depth. At this point, it is worth comparing our intuition with that of Glosten (1989) and Bhattacharya and Spiegel (1991), which show that if the fixed value of  $\sigma_\epsilon^2$  exceeds some bound, then only the no trade solution exists. Based on this result, one may conjecture that if  $\sigma_\epsilon^2$  is random, then the no-trade result would require that  $\sigma_\epsilon^2$ 's *expected* value fall below some level. Our results do not support this intuition; rather they suggest that the *maximum* value  $\sigma_\epsilon^2$  can take on must fall below the critical bound.

We next show that if  $\sigma_{\bar{a}}^2 < \sigma_w^2$ , then an equilibrium with trade always exists. A characterization of this equilibrium is provided in the following proposition.

- Proposition 3** *1. If the upper bound of the support of the private information variance,  $\sigma_{\bar{a}}^2$  is smaller than the variance of the liquidity component of demand,  $\sigma_w^2$ , then an equilibrium with trade exists. Given the realization of the noisy transformation of private information revealed by the investor's demand,  $\tau$ , the price schedule  $P$  takes the form  $\tau\alpha$ . Now, let  $p_L(X, \bar{a})$  denote the linear pricing schedule obtained by setting  $\alpha$  equal to the value of  $\alpha$  when the variance of private information is known. Then a candidate price schedule  $P$  forms an equilibrium supply function if and only if it meets the following condition: For any  $\delta > 0$ , there must exist a level of investor demand  $X = x_1$  such that  $|P(x_1) - p_L(x_1, \bar{a})| < \delta$ , where  $\bar{a} = \lim_{\tau \rightarrow \infty} \alpha$ . Less formally, an equilibrium supply function must cross or come arbitrarily close to the linear supply function formed when  $\alpha$  is a constant equal to  $\bar{a}$ .*
- 2. If trade can take place, the equilibrium supply function is convex at the origin. Further, the equilibrium supply function with the greatest liquidity has at least one inflection point, and, for sufficiently large levels of investor demand,  $X$ , is concave.*

When the uninformed agents know the variance of the investor's information, the equilibrium with the greatest liquidity is linear. In the present case, however, we know that a linear equilibrium does not exist because the conditional expectation of  $\epsilon$  on  $\tau$  is not linear in  $\tau$ . It is evident that the exact shape of the equilibrium supply curve will depend on the distribution of  $\sigma_\epsilon^2$ , though some general characteristics can be derived. Thus, as part 2 of the above proposition shows, the supply function must be somewhat "S" shaped.<sup>5</sup>

To help develop some intuition about the model’s predictions, Figure 1 contains the results of several simulations.

**Please insert Figure 1 here.**

From Proposition 3, an equilibrium supply function must either cross  $P_L(X, \bar{a})$  or come arbitrarily close as  $X$  becomes large. Thus, to obtain numerical solutions one may proceed as follows. First, calculate the value of  $\alpha$  when  $\sigma_\epsilon^2$  equals its maximum value. Second, begin with a large value of  $X$ , and set  $P$  close to the value corresponding to the linear ( $p_L$ ) schedule as defined in Proposition 2. Third, use a standard numerical algorithm to trace out the function.

The equilibrium supply functions graphed in Figure 1 are based on the common assumptions that  $\sigma_\eta^2 = \sigma_w^2 = 1$  and that  $\sigma_\epsilon^2 \in [0, 0.5]$ . The line labeled Constant High represents the linear equilibrium under the assumption that all traders know that  $\sigma_\epsilon^2 = 0.5$ . The “Uniform” line displays the equilibrium supply function when the market maker believes that  $\sigma_\epsilon^2$  has a uniform distribution between 0 and 0.5. The “Gamma” lines are drawn assuming that the market maker believes that  $\sigma_\epsilon^2$  has a truncated gamma distribution. Let  $G(x)$  equal the cumulative density function of the gamma distribution from 0 to 0.5, i.e.  $\int_0^{0.5} \exp(-t)t^{x-1}/\Gamma(x)dt$  where  $x$  is an arbitrary parameter and  $t$  is an integration variable. Then in the case labeled Gamma 1, the distribution function is  $\exp(-\sigma_\epsilon^2)\sigma_\epsilon^2/\Gamma(1)G(1)$ , and in the Gamma 2 case  $\exp(-\sigma_\epsilon^2)\sigma_\epsilon^2/\Gamma(2)G(2)$ . Notice that these curves are initially convex and then concave. For very small trades the price does not move very much, since the market maker takes these trades to imply that  $\sigma_\epsilon^2$  is small. For larger trades, the market maker revises his beliefs about  $\sigma_\epsilon^2$  upward, and the curve steepens. Since the market maker knows that the market will close if  $\sigma_\epsilon^2$  exceeds some boundary, his belief about the value of  $\sigma_\epsilon^2$  is itself bounded. As trades cause him to approach that bound, his revisions become smaller, so that the supply curve takes on a concave form as the trade size becomes large. The concavity of the supply schedule in Figure 1 is consistent with that documented for NYSE stocks by Madhavan and Smidt (1991) and Hasbrouck (1991).

Notice that in each example in Figure 1, as  $X$  becomes large, the equilibrium price function approaches the linear equilibrium curve  $p_L$  which corresponds to  $\sigma_\epsilon^2 = 0.5$ , the upper bound of the support of  $\sigma_\epsilon^2$ ’s distribution. The next proposition generalizes this

observation by stating that the equilibrium supply function approaches the  $p_L$  line obtained from the assumption that  $\sigma_\epsilon^2$  equals the top of the support of its distribution.

Observe that the market makers learn  $\tau$  from the submitted trade and use this to update their beliefs, which in turn, implies that the post-trade variance of the stock given all public information equals the residual variance of  $\epsilon$  plus  $\sigma_\eta^2$ . Since  $\sigma_\eta^2$  is common knowledge, changes in the conditional variance of the final payoff due to the trade can occur only through the variance of  $\epsilon$  given  $\tau$ . As the next proposition shows, the latter quantity increases with the absolute value of  $\tau$  and thus in the size of the trade.

**Proposition 4** *An increase in the absolute value of  $\tau$ , the noisy transformation of private information revealed by the investor's demand, increases the variance of the asset value conditional on the investor's demand.*

It is worth noting that the variance of the price following a trade varies positively with  $|\tau|$ , or equivalently,  $\text{var}(\tau)$  despite the fact that the residual variance faced by the informed investor does not depend upon his information set.<sup>6</sup>

A question arises as to whether our nonlinear supply schedules would obtain in a dynamic Kyle (1985) setting, wherein informed investors split up their trades over time. The setting of impending public announcements considered in this paper corresponds to situations where private information is short-lived and perishable, in which case the number of trading opportunities is likely to be small, so that dynamic issues will not be very relevant. Further, the analysis of Meulbroek (1992) suggests that repeated trades by insiders prior to a public announcement are an important trigger for the initiation of an investigation by Securities and Exchange Commission; this would also deter multiple trades. In general, however, extending our model to a dynamic setting is an important area for future research.

### 3 News Disclosure Policies and Liquidity Crises

Existing models such as Glosten (1989) and Bhattacharya and Spiegel (1991) require for a market breakdown that the variance of private information exceed a certain bound. However, as mentioned earlier, Proposition 2 implies that one only needs the upper bound

of the support of  $\sigma_\epsilon^2$  to exceed a threshold for the market to break down, a result which is of considerable significance. Our model implies that liquidity crises can be far more common than conventional models would predict. When the volatility of future prices is uncertain, we only require an increase in the upper bound of the uncertainty for a market breakdown to occur.

Along similar lines, a common informal intuition is that markets for many assets fail to exist because of “insufficient liquidity.” Our analysis formalizes this notion precisely in a specific context, in that it shows that the *support* of the private information variance’s distribution needs to be bounded above for markets to open. One way to accomplish this is to impose disclosure requirements on security issuers such as firms and governments (henceforth, we refer to security issuers as “firms” for convenience). In this section, we discuss how such reporting rules can help resolve the market breakdown problem in our setting.

It is well-known from event studies in finance (see, e.g., the seminal study of Fama *et al.*, 1969; Rendleman *et al.*, 1982; and, more recently, Ikenberry *et al.*, 1995) that there is a price drift in the direction of the event-date price reaction during the period preceding the event. This suggests that private information about the event leaks out to agents, who may either be corporate employees or individuals outside of the firm who have been tipped off. In large, modern corporations it would be difficult for the firm to control trading on such inside information by numerous employees and “tippees” when important announcements are pending. Indeed, the empirical literature suggests that firms generally are unsuccessful in this endeavor.

Based upon the model any disclosure rule that helps to keep the upper bound of the distribution governing the asymmetric information’s variance below the critical bound will help markets to function. This is a useful result from a regulatory point of view. However, note that disclosure would also open a Glosten (1989) market without variance uncertainty if the information about the mean could be disclosed directly; all traders would then be symmetrically informed. There are two issues to consider in this regard.

First, we show that market breakdowns can be far more frequent than the analysis of Glosten (1989) would suggest, since we demonstrate that with uncertain variance, if there is any positive probability that the variance exceeds the upper bound, the market cannot



open. This implies that the need for disclosure is far greater than would appear based on previous literature. Second, in general, we believe it is easier for a regulatory agency to write and enforce a “variance-based” rule than a “mean-based” rule. A “variance-based” rule requires firms to report specific types of information that are known to have (on average) a large price impact. In fact, such news disclosure policies are common and often take the form of accounting requirements. In the U.S., for example, exchange listed firms are required to release very specific information every quarter. Additionally, the government requires firms to file particular documents when they engage in unusual activities (e.g. takeovers) that also have a large average price impact. Note, these rules do not state that the firm must report any news that will move the price by a great deal (which would be a “mean-based” rule). Instead the rules require the firm to report specific types of information, and they must do so regardless of their beliefs about its market impact. The reason for this (in part) is that a rule based upon a projected price impact is impractical. Often firms themselves do not know how the market will react to a particular bit of news, and thus will not know if it should be reported under such a rule.<sup>7</sup> However, the model suggests that mean based rules are in fact unnecessary. It suffices instead to simply require firms to disclose information that is known to have a high variance price impact, regardless of the news’ current value.

The July 1999 issue of the *Exchange* magazine published by NYSE supports this view of rules requiring disclosure of impending informational events. To quote (p. 3):

Regulatory halts are designed to alleviate the instantaneous volatility that occurs when significant news is released, to give investors a chance to interpret the news and make informed investment decisions, and to price the stock in a calm environment. If a drilling company comes out with a technical release, such as an announcement that it has discovered 300,000 barrels of oil per day out of a 3/4-inch choke, many individual investors and brokers might not know what it means to the firm...A company contacting the Exchange before releasing the news would let us know the impact is big. We would then recommend a trading halt, research analysts would interpret that news and release their recommendations, and brokers would get on the phone with their clients.

Similarly, Christie *et al.* (1999, p. 7) state that news-pending trading suspensions on Nasdaq are not uncommon and are governed by the following disclosure policy:

...Traditionally, companies are required to provide [Nasdaq's] StockWatch with a press release no later than 15 minutes prior to the public disclosure of information contained in the release. Analysts at Nasdaq evaluate the information and determine whether the news would materially affect stock prices. In the event the news is deemed material, StockWatch notifies the news wires and all markets where the Nasdaq issue and its derivative securities are traded. The issuing company must also be notified of the impending halt before it could be implemented.

Note that our analysis also applies to market-wide disclosures about informational events. For example, the Federal Reserve usually times important announcements to coincide with the end of the week (see, for example, Ederington and Lee, 1993). This strategy does not preclude investors trading on leaked information towards the end of the week and potentially increasing adverse selection. Our analysis suggests that investor welfare could be higher if the Fed implemented a policy that involved reporting to the exchange whenever a sufficiently important announcement is imminent.

### **3.1 News Disclosure Policies and Investor Welfare**

If a regulatory agency can require the disclosure of high variance events prior to trade then clearly this will enhance welfare so long as the disclosure costs are not too high. However, in many cases firms may be aware that tippees have important information but do not yet want it released to the general public. In this case it may be useful to have the firm report that it has important information without revealing what the information is. For a firm this rule has obvious advantages. However, from the perspective of trade it is a second best solution. By requiring such announcements, trade is facilitated when an announcement does not occur, but ceases when it does. Note, that this rule is still better than no rule. Absent such a rule trade may not take place at all if otherwise the upper bound on the variance of insider's information exceeds the critical bound. With the disclosure bound it will at least function whenever the firm does not notify the authorities of a high variance

event.<sup>8</sup>

Propositions 2 and 3 shed light on the manner in which rules requiring disclosure of impending information events can be welfare-enhancing. Specifically, suppose that without an information disclosure rule (i.e., with  $c = \infty$ ), the upper bound of the support of  $\sigma_\epsilon^2$ 's distribution,  $\sigma_u^2$ , exceeds  $\sigma_w^2$ , so that  $\alpha$  exceeds 0.5 for some realization of  $\tau$ . Then, as Proposition 2 indicates, the only equilibrium involves no trade in every state of nature. Now consider what happens as  $c$  is reduced. Initially,  $\alpha$  will continue to exceed 0.5 for some  $\tau$  even when  $\sigma_\epsilon^2$  is below  $c$  (i.e., when no report is provided), and the no-trade equilibrium will continue to prevail in the absence of a report. However, as  $c$  is further reduced, eventually  $\alpha$  (for  $\sigma_\epsilon^2 < c$ ) will fall below 0.5 for all  $\tau$ . At this point, if firms do not provide a report (i.e., if  $\sigma_\epsilon^2 < c$ ), trade will actually take place. On the other hand, since the market is unable to open without a disclosure rule, it is evident that the no-trade equilibrium will prevail when a report is provided (i.e., when  $\sigma_\epsilon^2$  is greater than  $c$ ).

**Proposition 5** *Suppose that the upper bound of the support for the private information variance,  $\sigma_u^2$ , exceeds the variance of the liquidity component of the investor's demand,  $\sigma_w^2$ , and that the probability of the private information variance falling below  $\sigma_w^2$  is non-zero. Then there exists a disclosure rule, wherein the firm reports to the exchange if the private information variance is greater than a positive cutoff  $c < \sigma_w^2$ , thereby causing the market to close upon release of the report, such that the representative investor's ex ante welfare is strictly higher in the presence of the rule than in its absence.*

Note that with the rule in place, the representative investor is able to trade under at least *some* parameter realizations, whereas without the rule, he is not able to trade at all. Since trade is voluntary, it must occur when the welfare of the investor is increased by trading. This leads to the inference that the welfare of the investor is higher with the rule than without it. Further, since the rule leaves the zero-expected-profit earning market makers no worse off, it results in a Pareto improvement. The above proposition thus demonstrates the welfare-enhancing potential of such rules.

A natural issue that arises here is the manner in which the disclosure cutoff  $c$  is determined. While this issue is difficult to address analytically, the trade-offs involved in the choice of  $c$  by the exchange can be described as follows. Differentiating the investor's

expected utility with respect to  $c$  and setting the resulting expression to zero yields

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ 1 - \exp \left[ -\frac{A}{2}(x(\tau - P) - \frac{A}{2}\sigma_{\eta}^2 x^2) \right] \right\} \exp \left[ -(A/2)(\tau s + (A/2)\sigma_{\eta}^2 s^2) \right] h(\epsilon, w, c) d\epsilon ds - \int_0^c \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial P}{\partial c} x \exp \left[ -(A/2)(x(\tau - P) - (A/2)\sigma_{\eta}^2 x^2 + \tau s + (A/2)\sigma_{\eta}^2 s^2) \right] h(\epsilon, w, c) d\epsilon ds d\sigma_{\epsilon}^2 = 0, \quad (9)$$

where  $h$  represents the joint density function for  $\epsilon$ ,  $w$ , and  $x$  the equilibrium value of  $X$  selected by the insider conditional on his type and the disclosure rule. The following proposition then obtains.

**Proposition 6** *The optimal value of the cutoff on the private information variance  $c$ , i.e., the value of the variance of private information above which firms are required to report the variance to the exchange, solves equation (9). Both integrals are in this equation at least weakly positive.*

Proposition 6 illustrates the tradeoffs faced by the investor if the exchange wishes to increase  $c$ .<sup>9</sup> The first integral in (9) represents the direct influence. Increasing  $c$  leaves the market open more frequently. The expression in curly braces in the first term represents the utility gain to the investor from being able to trade when  $\sigma_{\epsilon}^2 \leq c$ . Instead of not being able to trade ( $X = 0$ ), the trader instead sets  $X$  to a nonzero number  $x$ . The second integral in (9) represents the indirect effect that increasing  $c$  has on investor utility. An increase in  $c$  makes the market maker more “suspicious” that the investor has valuable information whenever trade does take place. This results in a steeper equilibrium supply curve which makes the investor worse off.

### 3.2 Implications and Alternative Specifications

Our analysis indicates that an exchange can improve investor welfare by requiring firms to notify it whenever there is a significant adverse selection problem. As we discussed earlier in this section, the disclosure model in the previous section closely mimics that used by the NYSE and Nasdaq. Under its listing rules, firms must notify exchange officials whenever important news announcements are pending. Our model suggests a distinct prediction which follows from the notion that the suspension rule uses a variance-based trigger, as

opposed to a mean-based trigger. Under a pure variance trigger, the distribution of returns during a cessation of trade should be unimodal, with the modal return equal to zero. In contrast, under a mean-based rule the return distribution around the cessation should be bimodal, with the two peaks corresponding to the positive and negative price revisions which trigger a stoppage in trade. Since the actual returns during a news-pending cessation of trade are both unimodal and centered at zero (Bhattacharya and Spiegel, 1998), the data indicate that these stoppages are triggered by a variance-based rule, as suggested by the model presented here.

Our discussion also suggests some empirical implications. The news cutoff should be stricter for firms in which the asymmetric information problem is more severe. This indicates that cessation of trade due to impending informational events should occur less often for large firms and/or firms which are followed by a relatively large number of security analysts, since these are the firms for which relatively large stocks of public information are likely to be available to investors. By an opposite argument, firms with large proportions of insider ownership should have stricter rules. In contrast, stocks with relatively diffuse ownership (as measured by the number of shareholders, for example) should have more lenient news disclosure policies (and less frequent trade cessations due to impending informational events), as uninformed hedging trades are likely to be relatively more important in such stocks.

An alternative view of the rule analyzed in the previous section can be derived from the several empirical studies that have found a positive relationship between mean returns and subsequent volatility. In practice, large price shocks tend to be followed by additional large shocks.<sup>10</sup> It seems reasonable that shocks should be correlated with potentially high variances on the informed investor's information, since this is presumably a partial source of the additional price moves. For example, if an informed investor knows about a product's quality, this information will be worth more if sales are unexpectedly high. Now suppose a firm knows the realization of a random variable correlated with the variance, and that it can communicate the value of the variable that it knows about to the exchange. Our model suggests that liquidity can be improved by requiring that firms report to the exchange whenever this variable has a mean exceeding some boundary, since the absence of a report then assures the market maker that  $\sigma_\epsilon^2$  is itself below some boundary.

It is to be noted that the reason market collapses are more frequent in a setting with uncertain variance is because market makers are unable to quote breakeven prices for large orders. This is an argument in favor of encouraging imperfect competition because if small trades can subsidize large trades then market breakdowns should become less frequent (Glosten, 1989).<sup>11</sup> Yet, it would be difficult to regulate imperfect competition because specialists face competition from other sources such as floor traders and limit order submitters. An anonymous dealer market such as Nasdaq is in a better position to encourage such imperfect competition as a means to keep markets open for larger orders. In addition, Nasdaq dealer firms often act in a dual capacity by having their brokerage/analyst divisions disclose information about the firm, and this may serve the same purpose as news disclosure policies in mitigating the market breakdown problem. This point notwithstanding, the fact that even Nasdaq uses trading halts and news disclosure requirements (see Christie *et al.*, 1999) suggests that imperfect competition and disclosure by dealer firms is unable to fully address the adverse selection problem.

Finally, it is to be noted that while disclosure of an impending informational event helps keep markets open in our model, a large literature has argued in different contexts that disclosure of trade information may also impose costs. For example, Madhavan (1996) argues that disclosing information about price-inelastic trades may increase price volatility by increasing the effects of informed trading on prices. Similarly, Madhavan (1995) argues that disclosure of trading information can increase the trading costs of large institutional traders who would like to break up their orders under adequate camouflage but are prevented from doing so by disclosure requirements. A related argument is made by Gemmill (1996) in an empirical context. Porter and Weaver (1998) present evidence that dissemination of the best bid-ask quotations and depth actually decreased liquidity on the TSE. This is ostensibly because increased transparency may decrease the depth of the limit order book as limit order submitters are reluctant to gift free options to others. This research is a counterpoint to the notion that increased disclosure is necessarily desirable. Consistent with this literature, it is not the goal of our work to argue for more disclosure in a broad sense. Instead, we argue that policies allowing for dissemination of information about impending news events, can bound variance uncertainty in the absence of such events and thereby improve overall liquidity.

## 4 Multiple Securities and Exchanges

The focus in the present section is on how the disclosure rule in one security alters the liquidity of another security when there are interdependencies in the degree of adverse selection. To illustrate the issues involved, suppose that the future cash flows of General Motors (GM) become very volatile. Traders in GM will then probably face increased levels of asymmetric information. However, if the volatility of GM's cash flows goes up, then it is likely that so will that of the cash flows of other firms in the same industry, e.g., Ford. Now consider the implications of a disclosure rule for the two stocks. If GM reports to the exchange that a significant news announcement is imminent, then the market for GM's stock closes. The market maker in Ford's stock observes that GM has closed, and therefore infers that Ford's investors are also likely to have unusually valuable information. Intuitively, one expects Ford's stock to become less liquid in this situation, a conjecture verified by the analysis provided below. Also, once one recognizes that news disclosures in one stock can influence liquidity in another stock, the regulatory environment in which securities trade becomes an issue. Thus, a competing exchange structure will fail to take cross-effects into account. The analysis below yields insights regarding the conditions under which a consolidated exchange will employ higher or lower reporting requirements (i.e., set  $c$  higher or lower) than a competing exchange structure.

The structure of the model in this section is as follows. There are two securities and two investors, each with information about a particular security. Assume that investor 1 only trades in security 1 while investor 2 only trades in security 2. Aside from tractability, this case is of interest since informed investors who simultaneously trade several related stocks are likely to attract unwanted attention from the Securities and Exchange Commission. Trade occurs across two periods with the timing of events and stochastic structure identical to the one-security model. We index the quantities in the single-security model, i.e.,  $\epsilon$ ,  $\eta$ ,  $w$ , etc., by  $i$ , where  $i$  equals 1 or 2, depending on the market in question. Let  $g(\sigma_{\epsilon_1}^2, \sigma_{\epsilon_2}^2)$  denote the joint distribution of  $\epsilon_1$  and  $\epsilon_2$ . Each market maker knows  $g$  but not the realized values of  $\sigma_{\epsilon_i}^2$  prior to trade, and also knows whether the other market is open or closed at the time of trade in his own market.

We make the following assumptions about  $g$ :

1.  $g(\sigma_{\epsilon_1}^2, \sigma_{\epsilon_2}^2) = 0$  if  $\sigma_{\epsilon_1}^2$  or  $\sigma_{\epsilon_2}^2 \leq 0$ .
2. Let  $g_j^m$  be the marginal distribution of  $\sigma_{\epsilon_j}^2$ . Then  $g_j^m(\sigma_{\epsilon_j}^2 | \sigma_{\epsilon_i}^2 = a)$  first order dominates  $g_j^m(\sigma_{\epsilon_j}^2 | \sigma_{\epsilon_i}^2 = b)$  if  $a > b$  for  $i \neq j$  and  $i = 1, 2$  and  $j = 1, 2$ . This condition is also assumed to hold for all distributions of  $\sigma_{\epsilon_j}^2$  truncated from below.

The first assumption implies that negative variances occur with zero probability. The second assumption states that the variances of private information in the two stocks are positively correlated, in the sense that if the private information variance of stock  $i$  is unusually high, then, on average, so is that of stock  $j$ . From Proposition 4, the latter postulation immediately gives rise to the following result (stated without proof):

**Proposition 7** *Let  $\tau_i$  ( $i = 1, 2$ ) be the noisy transformation of private information revealed by investor demands in market  $i$ . Then, given  $\tau_i$ , an increase in the absolute value of  $\tau_j$  ( $j \neq i$ ) increases the post-trade conditional variance of the value of stock  $i$ .*

The above proposition implies a cross-sectional lead-lag relationship between intraday volatilities, in the sense that a high variance in one stock should be followed by a high variance in another stock with a positively correlated private information variance (this is most likely to be true for stocks in the same industry). A possible empirical test of this implication is to group stocks by industry and examine lead-lag patterns in intraday volatility across the stocks.

We now analyze the implications of a disclosure rule in one stock for market liquidity in another. As noted earlier, if disclosures about the exact price impact of the information signal can be made in full and in advance at no cost then the issue is moot. Such a disclosure will unambiguously aid liquidity since it removes all asymmetric information. However, as in earlier sections, here we examine the more complex issue of crafting a disclosure rule for implementation between the time when outside agents (e.g., “tippees”) first become aware of the information, and when the firm wishes to disclose the information. In this case the exchange must ask the firm to notify it when a high variance event is about to occur, at which time trade can be suspended.

Denote the reporting cutoffs on the variances in the two markets (through the respective cutoffs on the  $\delta$ 's in the markets) by  $c_1$  and  $c_2$ . If exchange  $i$  is open, the market maker



will base his schedule on his observations of the investor's demand, and whether exchange  $j$  is open. Let  $k_i = \{o, n\}$  represent market maker  $i$ 's knowledge of whether exchange  $j$  is open or not open, respectively. Essentially,  $k_i$  ( $i = 1, 2$ ) is a signal about the variance of the investor's observation in the other exchange. This information is useful because of the assumption that the variances are correlated. As in the one-period model, the market maker uses the investor's demand to infer  $\tau$ . However, now the update on the price function includes not only the inferred  $\tau_i$  but also on the observed value of  $k_i$ . Therefore, in the two-market setting, we define  $E(\hat{\alpha}_i | \tau_i, k_i) \equiv \alpha_i(\tau_i, k_i)$ .

Since we wish to focus on trading stoppages caused by news disclosures, we make the assumption that the exogenous parameter values are such that for *some* realization of  $\tau_i = \epsilon_i - w_i$ ,  $\alpha_i \geq 0.5$ , for  $i = 1, 2$  (or, equivalently, that  $\sigma_{\bar{u}_i}^2 > \sigma_{w_i}^2$ ). From Proposition 2, this implies that the two markets fail to open in the absence of a disclosure rule. We further assume that  $\alpha_i(\tau_i, k_i) \geq 0.5$  for some  $\tau_i$  (for  $i = 1, 2$ ), regardless of the realization of  $\sigma_{\epsilon_j}^2$  (or again, equivalently, that  $\sigma_{\bar{u}_i}^2 > \sigma_{w_i}^2$  regardless of the realization of  $\sigma_{\epsilon_j}^2$ ). Basically, this simply entails suitably restricting the support of the conditional distribution of  $\sigma_{\epsilon_i}^2$  on  $\sigma_{\epsilon_j}^2$ . These assumptions ensure that, regardless of whether market  $j$  is open or closed, market  $i$  always closes whenever  $\sigma_{\epsilon_i}^2$  is reported to be greater than  $c_i$ . Under the assumption, therefore, the disclosure rule is equivalent to a disclosure rule which involves a cessation of trade whenever the reported variance is greater than  $c_i$ .

Consider two market structures. In the first structure each exchange sets  $c_i$ ,  $i = 1, 2$  independently to maximize the welfare of the investor in that stock alone. Call this the competing exchange structure. In the second structure, the exchange sets  $c_1$  and  $c_2$  to jointly maximize some weighted average of the investors' utilities. Call this the consolidated exchange structure. Assume complete symmetry in the two markets (i.e.,  $\sigma_{\eta_1}^2 = \sigma_{\eta_2}^2$ ,  $\sigma_{w_1}^2 = \sigma_{w_2}^2$ , and  $g(\sigma_{\epsilon_1}^2, \sigma_{\epsilon_2}^2) = g(\sigma_{\epsilon_2}^2, \sigma_{\epsilon_1}^2)$ ). Given the symmetry assumption, we omit all subscripts except where needed for clarity. Further, unless otherwise stated, all integrations are performed over the entire support of the relevant distributions.

At the beginning of this section, we conjectured that if a market maker notices that a related stock has experienced a stoppage in trade, he will quote a less liquid supply function in his own stock. Proposition 8 formalizes this intuition.

**Proposition 8** *Suppose the market maker in stock  $i$  observes whether the market for stock  $j$  ( $i, j = 1, 2, i \neq j$ ) is open or closed. Then the illiquidity in the market for stock  $i$ , measured by the derivative of the pricing schedule in stock  $i$  with respect to quantity, is greater when the market for stock  $j$  is closed than when it is open.*

Proposition 8 demonstrates how the cessation of trade in one market affects liquidity in related markets. When one market closes, the derivative of the supply function (with respect to quantity) in the other market increases not just on average but everywhere. This provides a simple empirical prediction, namely, that during a cessation of trade in one security, stocks in the same or related industries should experience an increase in the price impact of market orders.

While a market breakdown in one security can reduce liquidity in a related security in our model, it will not induce the market in the related security to close. This is “good news” in that a cessation of trade in one market will not generate a cascade of breakdowns, leading perhaps to a widespread market failure, as explained below.

Recall that under the disclosure rule proposed in this paper, the exchange requires the firm to issue a report whenever the variance of an upcoming news event exceeds some prespecified boundary. Therefore, if a firm does not issue a report, the market makers in that stock know that the variance of the trader’s information cannot exceed the requisite boundary.

Now, a report that the adverse selection problem associated with GM has triggered a cessation of trade in security GM may shift the a market maker’s prior distribution over the variance of Ford. However, given that there has been no announcement in Ford, the news of GM’s closure cannot shift the upper boundary of that distribution past the boundary set by the reporting rule in Ford. Then, appealing to Proposition 3, an equilibrium with trade must exist in Ford’s stock absent a disclosure in Ford no matter what report has been issued by GM. One therefore concludes that a cessation of trade in one market cannot by itself lead to a cessation of trade in another market.

## 4.1 News Disclosure Policies With Multiple Securities

Proposition 8 shows that a breakdown of trading in one market can influence the supply function in a related market. The next proposition extends this result to the disclosure point  $c$  itself.

**Proposition 9** *Ceteris paribus, increasing either either the closure bound on stock 1,  $c_1$ , or on stock 2,  $c_2$ , increases the illiquidity function in stock 1,  $\alpha_1(X_1, s_1)$ , for all  $X_1$  and  $S_1$ .*

Proposition 9 implies that news disclosure policies in one security have an external effect on other securities, in the sense that increasing  $c_j$  reduces liquidity in market  $i$ . Also, it is evident that the benefit of raising  $c_j$  on  $c_i$  is through the relationship  $\alpha_i(\tau_i, o) \leq \alpha_i(\tau_i, c)$  in Proposition 8.

It is worth noting that Proposition 9 does not necessarily imply that traders in one security prefer as tight a disclosure rule in the other security as possible. A lower value of  $c_1$  does increase liquidity in market 2 conditional on the first market being open or closed. However, a lower value of  $c_1$  implies that market 1 will be closed more often, and thus traders in security 2 will face a steeper supply function when this happens. On net, the tradeoffs are similar to the case of a single security. Tightening the rule increases liquidity to some degree, but leaves traders within a less liquid exchange structure more often.

Using Propositions 8 and 9, one can prove the intuitive result that the supply function in market  $i$  when market  $j$  is closed is above that when it is open. Essentially, the market maker in market  $i$  takes the breakdown of market  $j$  as an indication that there exists an ‘above average’ amount of asymmetric information in the market.

**Proposition 10** *Let  $P_i(X_i, k_i)$  represent the equilibrium price function given the signal  $k_i$ , where  $k_i = o$  denotes the case in which market  $j$  ( $j \neq i$ ) is open and  $k_i = c$  denotes the case in which market  $j$  is closed. Then  $P_i(X_i, o) < P_i(X_i, c)$  for all  $X_i > 0$ , and  $P_i(X_i, o) > P_i(X_i, c)$  for all  $X_i < 0$ .*

So far the analysis has characterized how the disclosure rule in one market affects liquidity in a related market. The next question is whether competitive exchanges will set

the disclosure points to a higher or lower level than a consolidated exchange. It turns out that the answer to this question depends upon the properties of the distribution function  $g(\sigma_{\epsilon_1}^2, \sigma_{\epsilon_2}^2)$ .

In a competing exchange structure, the news disclosure policy will be set to maximize the utility of the investor's in that market, and cross-market influences will be ignored. Let  $Eu_i(c_1, c_2)$  represent trader  $i$ 's ( $i = 1, 2$ ) expected utility given  $c_1$  and  $c_2$ . Then market  $i$  will set  $c_i$  to maximize  $Eu_i(c_1, c_2)$ . By the envelope theorem, at the  $c$ 's corresponding to the Nash equilibrium between the two exchanges, a small change in  $c_i$  has no impact on the utility of trader  $i$ . Thus, if  $dEu_1/dc_2 \neq 0$  and  $dEu_2/dc_1 \neq 0$ , then a consolidated exchange can improve every trader's welfare. It follows that a consolidated exchange can set a set of news disclosure policies superior to those that prevail in a competing exchange structure.

With some additional notation and another assumption one can characterize the difference between the competitive and consolidated exchange. Let  $V(\epsilon, w; \alpha(X_i, k_i))$  equal the expected utility of trader  $i$ , given the realization of  $\epsilon$ ,  $w$ , and  $k_i$ . Now define

$$U(\sigma_{\epsilon_i}^2; \alpha(\cdot)) \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} V(\epsilon, w; \alpha(\cdot)) h(\epsilon, w, \sigma_{\epsilon_1}^2, \sigma_{\epsilon_2}^2) d\epsilon dw, \quad (10)$$

where  $h$  is the joint density function for  $\epsilon$ ,  $w$ ,  $\sigma_{\epsilon_1}^2$  and  $\sigma_{\epsilon_2}^2$ . Notice that both  $V$  and  $U$  are *functionals* since  $\alpha(X_i, k_i)$  is a function.

Using the above notation, one can write the expected utility of the trader in market one as

$$\int_0^{c_2} \int_0^{c_1} U_1(\sigma_{\epsilon_1}^2, \alpha_1(X_1, o)) g(\sigma_{\epsilon_1}^2, \sigma_{\epsilon_2}^2) d\sigma_{\epsilon_1}^2 d\sigma_{\epsilon_2}^2 + \int_{c_2}^{\infty} \int_0^{c_1} U_1(\sigma_{\epsilon_1}^2, \alpha_1(X_1, c)) g(\sigma_{\epsilon_1}^2, \sigma_{\epsilon_2}^2) d\sigma_{\epsilon_1}^2 d\sigma_{\epsilon_2}^2 + \int_{c_1}^{\infty} U_1(\sigma_{\epsilon_1}^2, \alpha_1^*) g_1^m(\sigma_{\epsilon_1}^2) d\sigma_{\epsilon_1}^2.$$

By the envelope theorem, the impact of a change in  $c_2$  on trader one's utility is given by

$$\begin{aligned} \frac{\partial Eu_1}{\partial c_2} &= \int_0^{c_1} U_1(\sigma_{\epsilon_1}^2, \alpha_1(X_1, o)) g(\sigma_{\epsilon_1}^2, c_2) d\sigma_{\epsilon_1}^2 + \int_0^{c_2} \int_0^{c_1} \frac{dU_1}{dc_2} g(\sigma_{\epsilon_1}^2, \sigma_{\epsilon_2}^2) d\sigma_{\epsilon_1}^2 d\sigma_{\epsilon_2}^2 \\ &\quad - \int_0^{c_1} U_1(\sigma_{\epsilon_1}^2, \alpha_1(X_1, c)) g(\sigma_{\epsilon_1}^2, c_2) d\sigma_{\epsilon_1}^2 + \int_{c_2}^{\infty} \int_0^{c_1} \frac{dU_1}{dc_2} g(\sigma_{\epsilon_1}^2, \sigma_{\epsilon_2}^2) d\sigma_{\epsilon_1}^2 d\sigma_{\epsilon_2}^2. \end{aligned} \quad (11)$$

Let us now consider the sign of the various terms in equation (11). Initially, let us consider the first and third terms. From Proposition 8,  $\alpha_1(X_1, o) \leq \alpha_1(X_1, c)$  for all  $X_1$ .

From Proposition 9, we know that the investor always faces a more illiquid schedule in market 1 when market 2 is closed than when it is open. This implies that for any given  $\tau$ , larger values of  $\alpha$  result in lower welfare, so that the first integral in (11) must be larger than the second integral.

Now consider the second and fourth terms. In the second integral, the change in  $c_2$  affects  $\alpha_1$  when the market in stock 2 is open. In contrast, the fourth integral provides the impact of  $c_1$  on  $\alpha_1$  when the second market is closed. However, in both cases  $dU_1/dc_2 < 0$  since Proposition 9 demonstrates that an increase in  $c_2$  causes  $\alpha_1$  to increase for all  $\sigma_{\epsilon_1}^2$  and  $X_1$ , which reduces the investor's expected utility.

One can now determine whether a consolidated exchange improves welfare by setting the values of  $c_1$  and  $c_2$  higher or lower than the competitive exchange. The answer to this question depends on the size of the marginal impact of  $c_2$  on  $U_1$ , which, in turn depends on how changing  $c_2$  alters the available information about market 1's variance. For example, suppose that at the Nash value for  $c_2$ , a small change in  $c_2$  leaves  $g_1^m$  unchanged. Then it is evident that changing  $c_2$  a small amount leaves the supply function in market 1 unchanged, so that  $dU_1/dc_2 = 0$ . In this case, the right-hand side of equation (11) is necessarily positive (because the second and fourth terms disappear and the first term is always greater than the third term), so that the consolidated exchange sets a more lenient disclosure rule. Under this scenario, the consolidated exchange recognizes that when market 2 closes, the liquidity in market 1 declines. Thus, a small increase in  $c_2$  increases the probability that market 1 will operate in a more liquid environment.<sup>12</sup>

Conversely, if a small change in  $c_2$  causes a large change in  $g_1^m$ , the right-hand side of (11) will be negative, and the consolidated exchange will set a tighter disclosure rule. It is thus evident that the direction of the net effect depends on the properties of the distribution  $g$ .

A general policy implication of our analysis is that there is a benefit to having a single regulatory body implement disclosure policies across stocks, especially across those with interdependencies in the degree of adverse selection (e.g., in those in the same industries). This can evidently be achieved by having one consolidated exchange rather than several fragmented ones. The analysis also suggests that society suffers a net welfare loss by having different stocks in the same industry listed on multiple exchanges. Finally, while not

explicitly modeled here, our results also have implications for disclosure policies across multiple international markets for the same stock (given that the degree of adverse selection is non-trivially correlated across such markets). Specifically, if the goal of policy-makers is to jointly maximize the welfare of investors across all markets, our analysis presents a case for coordinated policies across international markets. In addition, our analysis suggests that better liquidity could be achieved across markets for securities and their contingent claims (e.g., equity markets and their underlying derivatives) by having both sets of markets regulated by the same umbrella organization, as opposed to the current fragmented structure (Commodity Futures Trading Commission for derivatives and Securities and Exchange Commission for equities).

## 5 Summary and Concluding Remarks

As Lee *et al.* (1994) and Christie *et al.* (1999) point out, trading suspensions in advance of material news disclosures are common on the New York Stock Exchange and Nasdaq. We rationalize disclosure rules and trading suspensions in a model which allows for information asymmetry about both the mean and variance of a normally distributed asset payoff. We show that markets cannot function if uninformed traders believe the upper bound on the distribution of the variance exceeds a certain critical bound. However, one can restore trade by simply requiring firms to report high variance information to the market. While trade ceases when such a report is made to the exchange, the absence of a report bounds the variance uncertainty in the absence of such a report and thereby facilitates trade. Thus, disclosure rules increase *ex ante* investor welfare.

Our analysis of the equilibrium with information asymmetry about both the first and second moments is of interest beyond the specific application to disclosure rules and trading suspensions. We find that in our framework, the familiar linear equilibria of standard settings do not exist. The non-linear equilibrium price schedules we obtain are consistent with those documented by Madhavan and Smidt (1991) and Hasbrouck (1991). We also show that market breakdowns can be far more common than what is suggested by existing models. In our model, market breakdown requires only that the upper bound of the support of the variance of private information exceeds some finite maximum. This strong

result obtains because market makers believe that large orders accompany a large variance event, which precludes the existence of a breakeven price for those orders, whereby the liquidity supply schedule unravels. The result suggests that liquidity crises and market crashes such as those which occurred in the international bond markets in 1998 would be far more common than what existing models would suggest, and would also be particularly likely to occur if there is a great deal of uncertainty surrounding the extent (volatility) of future price movements (e.g., prior to important macroeconomic announcements).

Our study of a multi-security setting indicates that if securities with correlated asymmetric information are traded on different competing exchanges, the resulting news disclosure policies will usually be socially inferior to those in a consolidated exchange structure. This is because such exchanges do not consider the influence of their rules on investors trading in other markets, so that they may set a disclosure policy that is suboptimal. This result suggests not only that equity exchanges may set better disclosure rules if they are overseen by one regulatory body, but also that the liquidity of equity markets as well as derivative markets for equities (e.g., Chicago Board Options Exchange) may be higher if they are governed by one umbrella organization. Coordination of disclosure policies is particularly important when the nature of non-systematic information flows are correlated across companies in a sector. This does appear to be the case for technology and the “dot-com” sectors where there is much commonality in news flows (e.g., operating system innovations, adoption of on-line shopping by consumers, new security technology for on-line shopping, and so on). Thus, our analysis may provide a partial explanation for the current structure in which technology firms are overwhelmingly listed on a single exchange, Nasdaq.

The model also provides further empirical implications, which are as yet untested:

- Cessation of trade due to impending public disclosures should occur less often for stocks that are followed by many analysts and are widely held, because variance uncertainty in particular and asymmetric information in general is likely to be less of a problem for such stocks.
- In a given stock, high order flows should be followed by a high volatility of price changes because a high order flow is likely to be associated with a high variance

event.

- If one accepts that disclosures by analysts associated with Nasdaq dealer firms is an alternative way to mitigate the market breakdown problem, one should see more disclosures for firms whose cash flows are relatively more uncertain.
- Our multi-security analysis indicates that a high volatility of price changes in one stock should be followed by a high volatility in another stock with a positively correlated private information variance (e.g., another stock in the same industry).
- A trading halt in one stock should decrease the liquidity of other stocks in the same industry.

Testing these implications would appear to be a reasonable topic for future research.



## Appendix

**Proof of Proposition 1:** We first describe how we adapt the Cho-Kreps D1 criterion to address out-of-equilibrium moves in our setting with an infinite type space. The adaptation follows Bhattacharya *et al.* (1995). For expositional ease, the phrase “better off” in the material to follow will be taken to apply relative to the investor’s equilibrium payoff.

For demands that correspond to a move that can occur in equilibrium one simply requires the market makers to update their beliefs via Bayes’ rule. However, for moves that should not occur in equilibrium, the market makers need to evaluate the message the investor may be trying to send. If the investor submits an out-of-equilibrium demand, the market makers need to determine which insider type or types may have sent the errant signal. Standard refinements require the market makers to ask themselves which investor types would benefit the most from such a move and then assume that an investor from that set was responsible for the action.

When the investor submits a demand, the market makers will respond with an aggregate demand function that produces a price. For an investor to be better off, the resulting price must leave him with a higher payoff relative to what he would have earned had he submitted his equilibrium demand. It is evident that for any investor type there exists a price response to an out-of-equilibrium move that can potentially make him better off. But some investor types are more likely to benefit than others. Consider some set of types  $T$  with the following property. Suppose that a price response which makes any investor outside of  $T$  better off also makes every investor in  $T$  better off, but that the converse does not hold. Under these circumstances, the D1 criterion, as well all those listed in Footnote 2, require the market makers to believe that an insider from the set  $T$  submitted the errant demand, and then react appropriately given that belief. Note that in this paper, the market maker can assign any probabilities they wish to the elements of  $T$ , but they must assign zero probability to elements outside of  $T$ .

A minor modification is necessary to handle the unbounded type space in the current model. As noted earlier, the current paper seeks, among other things, to determine when markets will or will not function. This requires one to include the possibility that market

makers will quote an infinite price for some demands, in other words, refuse to trade if a trader wishes to buy or sell a particular quantity of the security. Thus, any demand that “should” produce an infinite price must be an “out-of-equilibrium” move. Intuitively, this should occur only when the market makers cannot place either an upper or lower bound on the set of types that may have defected. To see how this can happen, suppose that any reaction by the market makers that makes an investor of type  $\tau_1$  better off will also make an investor of type  $\tau_2$  better off if  $\tau_1 < \tau_2$ .<sup>13</sup> The natural extension of the D1 criterion in this case clearly requires the market maker to believe the investor who submits an out-of-equilibrium demand is of type  $\tau_2$ , and since  $\tau_2$  is arbitrarily chosen (i.e., it can be as large as  $+\infty$ ), the market maker must believe that the investor is of type  $+\infty$ .<sup>14</sup>

Now, consider an out-of-equilibrium move in which the investor demands  $X > 0$ . Under the no-trade equilibrium, the investor’s maximization problem has a value of

$$\tau s + \frac{A}{2}\sigma_\eta^2 s^2, \quad (12)$$

while a demand of  $X$  produces a value of

$$\tau X - \frac{A}{2}\sigma_\eta^2 X^2 - XP + \tau s + \frac{A}{2}\sigma_\eta^2 s^2. \quad (13)$$

The value from Equation (13) will exceed that in (12) if

$$\tau > P - \frac{A}{2}\sigma_\eta^2 X.$$

Thus, suppose that in response to the demand  $X$  the market makers quote a finite price  $P$ . If the price  $P$  makes an investor of type  $\tau_1$  better off then it must make any investor of  $\tau_2$  better off if  $\tau_1 < \tau_2$ . Therefore, under the D1 criterion the market maker cannot believe that an insider whose type lies below  $\tau_2$  was responsible for the out-of-equilibrium move. However, the same argument holds for any  $\tau_2$  no matter how large. Thus, the market maker cannot believe the insider is of any finite type. If the market maker does not believe the investor’s type is finite, then the market maker cannot set a finite price as a best response and must set  $P$  to  $+\infty$ . Finally, from the optimization problem (3), the investor’s best response to a vertical supply function is to submit a zero demand.  $\square$

To prove part 2 of the proposition, we first state and prove the following lemmas.

**Lemma 1** *Higher types, ordered by  $\tau$ , select weakly higher demands.*

**Proof:** Consider two types  $\tau_1$  and  $\tau_2$  such that  $\tau_1 < \tau_2$  and two potential demands  $X_1$  and  $X_2$  such  $X_1 < X_2$ . Suppose that type  $\tau_1$  prefers  $X_2$  to  $X_1$ . Substituting both demands into equation (3) implies

$$\tau_1(X_2 - X_1) \geq \frac{A}{2}\sigma_\eta^2(X_2^2 - X_1^2) + X_2P_2 - X_1P_1. \quad (14)$$

Since  $X_1 < X_2$  and  $\tau_1 < \tau_2$  one has  $\tau_2(X_2 - X_1) > \tau_1(X_2 - X_1)$ . Thus, if an investor of type  $\tau_1$  prefers  $X_2$  to  $X_1$  then so does an investor of type  $\tau_2$ .  $\square$

**Lemma 2** *Suppose that there exist types such that  $X_1$  and  $X_2$  are selected as equilibrium demands. Then every demand between  $X_1$  and  $X_2$  must also be selected in equilibrium. Furthermore, if a particular demand is selected in equilibrium then it is selected by one and only one type.*

**Proof:** The proof is by contradiction. Suppose that there exist two points  $X_1$  and  $X_2$  such that in equilibrium no investor type selects any point  $x$  such that  $X_1 < x < X_2$ . Then there must exist a type  $\tau^*$  such that  $\tau^*$  earns the same utility from selecting either  $X_1$  or  $X_2$ . From Lemma 1, all of the types that select  $X_1$  must lie below  $\tau^*$ , and those selecting  $X_2$  must lie above  $\tau^*$ . Thus, when comparing the utility of type  $\tau^*$  with that of the other investors selecting either  $X_1$  or  $X_2$  one needs only conduct comparisons with  $\tau < \tau^*$  at  $X_1$  and  $\tau > \tau^*$  at  $X_2$ .

Consider an out-of-equilibrium demand  $x$  such that  $X_1 < x < X_2$  and a price response by the market maker  $P(x)$ . Suppose the price response makes some type  $\tau_1 < \tau^*$  better off. Then the following inequality must hold:

$$\tau_1(x - X_1) > \frac{A}{2}\sigma_\eta^2(x^2 - X_1^2) + xP(x) - X_1P(X_1) \quad (15)$$

Since  $x - X_1 > 0$ , and  $\tau_1 < \tau^*$ , it is evident from (15) that any price response by the market maker that makes an investor of type  $\tau_1$  better off must also make an investor of type  $\tau^*$  better off. For  $\tau_2 > \tau^*$  the relevant inequality is

$$\tau_1(x - X_2) > \frac{A}{2}\sigma_\eta^2(x^2 - X_2^2) + xP(x) - X_2P(X_2) \quad (16)$$

However, now  $x - X_2 < 0$ , and  $\tau_2 > \tau^*$  so again if an investor of type  $\tau_2 > \tau^*$  is made better off then the investor of  $\tau^*$  is made better off. Thus, under the D1 criterion if the market makers see a defection between  $X_1$  and  $X_2$ , they must believe the errant demand was submitted by an investor of type  $\tau^*$ .

Define  $P(X|\tau)$  as the price response of the market makers if they know the insider is of type  $\tau$  and the investor demands  $X$ . Recall from Lemma 1 that higher types submit weakly higher demands. Thus,  $P(X_1|\tau^*) \geq P(X_1)$  and  $P(X_2|\tau^*) \leq P(X_2)$ . Also note that since the market makers are risk neutral, if they know  $\tau$  they must set the same price for all possible demands. Therefore one must have  $P(x|\tau)$  for all  $X_1 < x < X_2$ . It is evident that if  $P(X_2|\tau^*) < P(X_2)$ , then the investor can submit a demand slightly below  $X_2$  and strictly increase his utility. Thus, one must have  $P(X_2|\tau^*) = P(X_2)$ .

If  $P$  is a constant, then the insider's objective function (equation (3)) is strictly concave in  $X$ . Thus, if  $P(X_1|\tau^*) = P(X_1)$  and  $P(X_2|\tau^*) = P(X_2)$  and the investor is indifferent between  $X_1$  and  $X_2$ , he must strictly prefer some demand between  $X_1$  and  $X_2$ . Therefore, one must have  $P(X_1|\tau^*) > P(X_1)$ . Now consider what must occur at  $X_1$ . Using the same argument as given before there cannot exist an  $X_0 < X_1$  such that no investor ever selects a demand between  $X_0$  and  $X_1$ .<sup>15</sup> However, for  $P(X_1|\tau^*) > P(X_1)$ , one must also have a set of types below  $\tau^*$  with strictly positive measure, that also select  $X_1$ . Assume that all types between  $\tau_1$  and  $\tau^*$  select  $X_1$  (recall from Lemma 1 that demands are weakly increasing in the investor's type). Now consider a demand  $X_1 - \delta$  where  $\delta$  is a small positive number. From the arguments given above, some investor type or types must select the demands  $X_1 - \delta$ . We also know the set of investor types that select this demand lie below  $\tau_1$ . Thus, at this demand, if one has  $P(X_1|\tau^*) > P(X_1 - \delta)$  for  $\delta$  arbitrarily small, then the investor of type  $\tau^*$  can be made strictly better off by selecting a demand  $X_1 - \delta$  for sufficiently small  $\delta$ . This implies that one cannot have a situation where a set of types below  $\tau^*$  with strictly positive measure also select  $X_1$ . But, as shown above, the case where only type  $\tau^*$  selects  $X_1$  has also been ruled out. Therefore, in equilibrium, there cannot exist any two demands  $X_1 < X_2$  such that no investor type selects any type  $x$  between  $X_1$  and  $X_2$ . Using the  $\delta$  argument given above, one can also rule out the case where multiple types

select the same equilibrium demands.  $\square$

**Lemma 3** *An insider of type 0 always selects a demand of zero.*

**Proof:** From Lemma 2, whatever demand the investor of type 0 selects only that investor type selects that demand. The demand selected by an insider of type 0 must therefore produce a price of 0. From equations (12) and (13), an insider of type 0 will prefer to select a zero demand over any other demand  $X$  if

$$0 > -\frac{A}{2}\sigma_\eta^2 X^2,$$

which is always true.  $\square$

**Lemma 4** *If trade can take place, then every demand is selected by one and only one type in equilibrium.*

**Proof:** From the arguments given so far, if a particular demand ( $X_1$ ) is selected in equilibrium, then the price function in the half open interval  $[0, X_1)$  must be continuous and differentiable. Thus, one can use the investor's first-order and second-order conditions to derive

$$\tau - P - P'X - A\sigma_\eta^2 X = 0 \tag{17}$$

and

$$-2P' - XP'' - A\sigma_\eta^2 \leq 0. \tag{18}$$

Since the market maker sets  $P = \alpha\tau$ , we can substitute for  $\tau$  in (17) to get

$$P(\alpha - 1) + \alpha[P'X + A\sigma_\eta^2 X] = 0. \tag{19}$$

We proceed by first proving the following two useful Lemmas.

**Lemma 5** *An equilibrium supply function must have a strictly positive derivative at all possible equilibrium demands.*

**Proof:** Using (23), the second-order condition (18) implies

$$\frac{1}{\alpha} \left[ -P' + \frac{d\alpha}{d\tau} \frac{d\tau}{dX} \tau \right] \leq 0. \quad (20)$$

Now from the investor's first-order condition, one can show that

$$\frac{d\tau}{dX} = 2P' + P''X + A\sigma_\eta^2 \geq 0. \quad (21)$$

Now, use (17) and (21) to eliminate  $\tau$  and  $d\tau/dX$  from (28). Some additional algebra shows that

$$\frac{P'}{\alpha} = (2P' + P''X + A\sigma_\eta^2) \left[ 1 + \frac{(P + P'X + A\sigma_\eta^2)}{\alpha\sigma_w^2} \text{var}(\hat{\alpha}|\tau) \right], \quad (22)$$

which implies that the investor's second-order condition will hold if and only if  $P' > 0$ .  $\square$

**Lemma 6** *Let  $p(X, a)$  represent a solution to the differential equation (19) under the assumption that  $\sigma_\epsilon^2$  is known. Suppose that  $P(X_1) = p(X_1, a)$  for some  $X_1$ , and  $a = \alpha(X_1)$  at  $X_1$ , where  $\alpha(X)$  represents the beliefs about  $\alpha$  induced by a demand of  $X$ . Then, for all  $X > X_1$ , we have  $P(X) < p(X, a)$ .<sup>16</sup>*

**Proof:** Differentiate (19) with respect to  $X$  and then rearrange to obtain

$$-2P' - XP'' - A\sigma_\eta^2 = \frac{1}{\alpha} \left\{ -P' + \frac{d\alpha}{d\tau} \frac{d\tau}{dX} [P + P'X + A\sigma_\eta^2 X] \right\}. \quad (23)$$

Consider a point  $x_1 > 0$ . Under  $p$ , the equilibrium conditions Equations (17) and (19) become

$$\tau - p - p'x_1 - A\sigma_\eta^2 x_1 = 0 \quad (24)$$

and

$$p(a - 1) + a(p'x_1 + A\sigma_\eta^2 x_1) = 0. \quad (25)$$

Set  $a = E(\alpha|\tau)$  at  $x_1$  under the true price function  $P$ . Now select a price function  $p(X)$  such that  $P(x_1) = p(x_1)$ .<sup>17</sup> Notice that if  $P(x_1) = p(x_1)$ , then Equations (17) and (24) imply that  $P' = p'$  at  $x_1$ .

From (6), one has

$$\alpha = \frac{\int_{\Omega_2} \frac{\hat{\alpha}}{\sqrt{2\pi(\sigma_w^2 + \sigma_\epsilon^2)}} \exp\left\{-\frac{\tau^2}{2} \frac{\sigma_w^2}{\sigma_\epsilon^2 + \sigma_w^2}\right\} g(\sigma_\epsilon^2) d\sigma_\epsilon^2}{\int_{\Omega_2} \frac{1}{\sqrt{2\pi(\sigma_w^2 + \sigma_\epsilon^2)}} \exp\left\{-\frac{\tau^2}{2} \frac{\sigma_w^2}{\sigma_\epsilon^2 + \sigma_w^2}\right\} g(\sigma_\epsilon^2) d\sigma_\epsilon^2}. \quad (26)$$

Differentiating (6) with respect to  $\tau$  yields

$$\frac{d\alpha}{d\tau} = \frac{\tau}{\sigma_w^2} \text{var}(\hat{\alpha}|\tau). \quad (27)$$

Next, use (27) in (23) to show that

$$x_1 P'' = P'(\alpha^{-1} - 2) - A\sigma_\eta^2 - \frac{\tau^2}{\alpha\sigma_\eta^2} \text{var}(\hat{\alpha}|\tau) \frac{d\tau}{dX}. \quad (28)$$

In comparison, differentiation of Equation (25) produces an identical equation with  $p$  replacing  $P$  everywhere, and  $\text{var}(\hat{\alpha}|\tau) = 0$ . When  $\sigma_\epsilon^2$  is unknown, then  $\text{var}(\hat{\alpha}|\tau) > 0$ . Using this fact in Equation (28), along with the earlier finding that at  $x_1$ , one has  $P' = p'$ . Thus, we find that locally  $P$  must lie below  $p$  to the right of  $x_1$ . We now show that  $P$  cannot cross  $p$  from some  $X > x_1$ . Suppose that  $P(x_2) = p(x_2)$  for some  $x_2 > x_1$ . At  $x_2$ , Equation (19) implies

$$p(a - 1) + a[p'x_2 + A\sigma_\eta^2 x_2] = P(\alpha - 1) + \alpha[P'x_2 + A\sigma_\eta^2 x_2]. \quad (29)$$

At a crossing point  $P = p$  and  $a < \alpha$  so

$$(ap' - \alpha P')x_2 = P(\alpha - a) + A\sigma_\eta^2 x_2(\alpha - a) = [P + A\sigma_\eta^2 x_2](\alpha - a). \quad (30)$$

Since the right hand side of the above equation is strictly positive, one has  $ap' - \alpha P' > 0$ , so that  $p' > \alpha P'/a > P'$ . So  $P$  cannot cross  $p$  a second time.  $\square$

From Bhattacharya and Spiegel (1991), none of the solutions to the differential equation with a fixed variance asymptote to plus infinity for  $X > 0$ . Consider some demand  $X_1$  and the value of  $\alpha$  at that point. Lemma 2 shows that any solution to the differential equation with a random variance must lie below the solution to the differential equation with a fixed variance set equal to  $\alpha$  for all demands greater than  $X_1$ . Thus, the solution to the differential equation with random variances cannot asymptote to plus infinity. Neither can it go to minus infinity without violating the investor's second order conditions.

Therefore, every demand produces a finite price response from the market makers. Since every demand produces a finite price response, and the equilibrium price function is  $C^1$ , in equilibrium every demand is selected by one and only one type. We thus obtain Lemma 4.  $\square$

Part 2 of Proposition 1 follows directly from the above lemmas and the fact that the maximization problem (3) is continuous and differentiable in  $X$ .  $\square$

**Proof of Proposition 2:** Only the proof for  $\tau > 0$  is given. The proof for  $\tau < 0$  is symmetric.

Consider a candidate supply function  $P$  and a function  $p$  (as defined in the proof of Lemma 6), where  $a$  has been selected so that at  $x_1$ , one has  $a = \alpha$ . As stated in the proposition, for some  $X = x_1$ , suppose that  $\alpha \geq 0.5$ . From Bhattacharya and Spiegel (1991), if  $a \geq 0.5$ , then the function  $p$  will fail as an equilibrium supply function, since for all  $X$  greater than some threshold,  $p' < 0$ . Since Lemma 6 shows that  $P$  must lie below  $p$ , one must also have that  $P$  will have a negative derivative for some  $X$ . From Lemma 5, if  $P' \leq 0$  for any  $X$ , then  $P$  fails as an equilibrium supply function. This proves Part 1. This proves that if  $\alpha \geq 0.5$  for any  $X$  then the only equilibrium involves no trade.

To prove the proposition one now needs to show that if  $\sigma_\epsilon^2 \geq \sigma_w^2$ , then  $\alpha \geq 0.5$  for some  $X$ . For this, consider an arbitrarily small  $\delta > 0$ . From equation (6) in the text, one can express  $\alpha$  as

$$\begin{aligned} \alpha = & \left[ \int_0^{u-\delta} \frac{\hat{\alpha}}{\sqrt{2\pi(\sigma_w^2 + \sigma_\epsilon^2)}} \exp\left\{-\frac{\tau}{2}[1 - \hat{\alpha}]\right\} g(\sigma_\epsilon^2) d\sigma_\epsilon^2 \right. \\ & \left. + \int_{\sigma_u^2 - \delta}^{\sigma_u^2} \frac{\hat{\alpha}}{\sqrt{2\pi(\sigma_w^2 + \sigma_\epsilon^2)}} \exp\left\{-\frac{\tau}{2}[1 - \hat{\alpha}]\right\} g(\sigma_\epsilon^2) d\sigma_\epsilon^2 \right] / \\ & \left[ \int_0^u \frac{\hat{\alpha}}{\sqrt{2\pi(\sigma_w^2 + \sigma_\epsilon^2)}} \exp\left\{-\frac{\tau}{2}[1 - \hat{\alpha}]\right\} g(\sigma_\epsilon^2) d\sigma_\epsilon^2 \right]. \end{aligned}$$

Now multiply the numerator and denominator by  $\exp\left\{-\frac{\tau}{2}[\hat{\alpha}(u - \delta) - 1]\right\}$  to obtain

$$\alpha = \left[ \int_0^{\sigma_u^2 - \delta} \frac{\hat{\alpha}}{\sqrt{2\pi(\sigma_w^2 + \sigma_\epsilon^2)}} \exp\left\{-\frac{\tau}{2}[\hat{\alpha}(u - \delta) - \hat{\alpha}]\right\} g(\sigma_\epsilon^2) d\sigma_\epsilon^2 \right]$$



$$\begin{aligned}
& + \int_{\sigma_u^2 - \delta}^{\sigma_u^2} \frac{\hat{\alpha}}{\sqrt{2\pi(\sigma_w^2 + \sigma_\epsilon^2)}} \exp \left\{ -\frac{\tau}{2} [\hat{\alpha}(\sigma_u^2 - \delta) - \hat{\alpha}] \right\} g(\sigma_\epsilon^2) d\sigma_\epsilon^2 \Bigg] / \\
& \left[ \int_0^{\sigma_u^2} \frac{\hat{\alpha}}{\sqrt{2\pi(\sigma_w^2 + \sigma_\epsilon^2)}} \exp \left\{ -\frac{\tau}{2} [\hat{\alpha}(\sigma_u^2 - \delta) - \hat{\alpha}] \right\} g(\sigma_\epsilon^2) d\sigma_\epsilon^2 \right]. \tag{31}
\end{aligned}$$

Since  $\hat{\alpha}$  is increasing in  $\sigma_\epsilon^2$ , we have  $\hat{\alpha}(\sigma_u^2 - \delta) - \hat{\alpha} > 0$  if  $\sigma_\epsilon^2 < \sigma_u^2 - \delta$  and  $\hat{\alpha}(\sigma_u^2 - \delta) - \hat{\alpha} < 0$  if  $\sigma_\epsilon^2 > \sigma_u^2 - \delta$ . Thus if  $\int_{\sigma_u^2 - \delta}^{\sigma_u^2} g(\sigma_\epsilon^2) d\sigma_\epsilon^2 > 0$ ,  $\lim_{\tau \rightarrow \infty} \alpha > \hat{\alpha}(\sigma_u^2 - \delta)$  for any  $\delta > 0$ , since the first term in the numerator of (31) goes to zero, while the second term goes to infinity. Therefore  $\lim_{\tau \rightarrow \infty} \alpha \rightarrow \hat{\alpha}(c)$ .

Since  $\hat{\alpha} = \sigma_\epsilon^2 / (\sigma_\epsilon^2 + \sigma_w^2)$ , one has that as  $X$  goes to infinity  $\hat{\alpha}$  approaches  $\sigma_\epsilon^2 / (\sigma_\epsilon^2 + c)$ . From the previous part of the proof, if this figure exceeds 1/2 then the only equilibrium involves no trade. Thus, only the no-trade equilibrium exists if  $\sigma_\epsilon^2 / (\sigma_\epsilon^2 + \sigma_u^2) \geq 1/2$  or upon rearrangement if  $\sigma_\epsilon^2 \geq \sigma_u^2$ .  $\square$

### Proof of Proposition 3:

*Part 1:* We first prove the “if” portion of the proposition. Consider a  $P$  that equals  $p_L(X, \bar{a})$  at some point  $x$ , where following the paper’s earlier notation,  $\bar{a} = \lim_{\tau \rightarrow \infty} \alpha$ . For  $P$  to fail as an equilibrium, there must exist an  $X$  such that  $P' \leq 0$ . At the crossing point  $x$  we know from (30) that the derivative of  $P$  is strictly greater than the derivative of  $p_L$  which is strictly greater than zero. Next divide (19) by  $\alpha$  and (25) by  $a$ . Then set the two equations equal to each other for some demand  $X$  and note that the  $A\sigma_\eta^2 X$  terms cancel with each other. This produces the following equality:

$$P' - p_L = \left(\frac{1}{\alpha} - 1\right)P - \left(\frac{1}{\bar{\alpha}} - 1\right)p_L. \tag{32}$$

By definition  $\alpha \leq \bar{\alpha}$ . At  $x$  our earlier arguments showed that  $P' > p_L$ , therefore for  $X$  just greater than  $x$  one has  $P > p_L$ . If  $P > p_L$  and  $\alpha \leq \bar{\alpha}$  at some demand  $X$  then (32) shows that  $P' > p_L$  at that demand. Therefore, for all  $X \geq x$ , the candidate equilibrium function satisfies the investor’s second-order conditions.

We now need to show that the candidate equilibrium function also satisfies the investor’s second-order conditions for all demands between 0 and  $x$ . If  $P' < 0$  for some  $X$ , then it must be that there exists an  $X = x_1$  such that  $P' = 0$  and  $P'' > 0$ . From (23), if

$P' = 0$  then

$$\alpha[P''X + A\sigma_\eta^2] + \frac{d\alpha}{d\tau} \frac{d\tau}{dX} [P + A\sigma_\eta^2 X] = 0. \quad (33)$$

From (19),  $P + A\sigma_\eta^2 X = P/\alpha$ , so  $\alpha^2[P''X + A\sigma_\eta^2] + \frac{d\alpha}{d\tau} \frac{d\tau}{dX} P = 0$ . Further  $P > 0$  from (19), since since this equation implies  $P(\alpha - 1) + \alpha A\sigma_\eta^2 X = 0$ . Therefore

$$P''X = -A\sigma_\eta^2 - \frac{1}{\alpha^2} \frac{d\alpha}{d\tau} \frac{d\tau}{dX} P < 0, \quad (34)$$

which implies that one cannot have  $P' = 0$  and  $P'' > 0$ . Thus, the pricing function must satisfy the second-order condition between 0 and an arbitrarily small  $x > 0$  as well.

Next, we prove the “only if” part of the proposition. For all  $X > 0$ , suppose some function  $P$  remains strictly below  $p_L(X, \bar{a})$  by an amount greater than or equal to  $\delta > 0$ . Then for some  $X = x_1$  large enough, one must have  $P(x_1) = p(x_1, a) < p_L(x_1, a)$  (where  $a = \alpha$  at the point  $x_1$  under the function  $P$ ), since Part 1 of Proposition 3 shows that  $\alpha$  goes to  $\bar{a}$  as  $X$  goes to infinity. From Bhattacharya and Spiegel (1991), any function  $p < p_L$  must fail as an equilibrium supply function since it has a negative derivative for all  $X$  above some level. Lemma 6 shows that for all  $X > x_1$ , the function  $P$  lies below  $p$ , so  $P$  must also have a negative derivative for some  $X$  above  $x_1$ . From Lemma 5, this implies that  $P$  must also fail as an equilibrium supply function.

*Part 2:* We first prove that  $\lim_{X \rightarrow 0} P'' > 0$ . Let  $\alpha'$  and  $\alpha''$  represent the first and second derivative of  $\alpha$  with respect to  $X$ , respectively. Using this notation, write Equation (23) as

$$\frac{\alpha}{1 - 2\alpha} X P'' = P' - \frac{\alpha}{1 - 2\alpha} A\sigma_\eta^2 - \frac{\alpha'}{1 - 2\alpha} [P + P'X + A\sigma_\eta^2 X]. \quad (35)$$

As  $X \rightarrow 0$ , the third term on the right hand side goes to zero. So it remains to see what happens to the first two terms. As before, define  $p_L$  as the linear equilibrium when  $\alpha$  is a constant. In this case,  $p_L = kX$ , where  $k$  is a constant. For  $\alpha$  constant and equal to some number  $a$ , Equation (19) becomes

$$kX(a - 1) + a[kX + A\sigma_\eta^2 X] = 0,$$

which implies that

$$k = \frac{aA\sigma_\eta^2}{1 - 2a}.$$

Let  $\underline{a} \equiv \lim_{\tau \rightarrow 0} \alpha$ . Lemma 6 shows that for some  $X_1$ , if  $P(X - 1)$  lies below  $p_L(X_1, a)$ , where  $a$  equals  $\alpha$  given the beliefs induced by the demand  $X_1$ , then  $P(X)$  lies below some nonlinear function  $p(X, a)$  for all  $X > X_1$ . Bhattacharya and Spiegel (1991) show that any candidate supply function  $p$  that lies below  $p_L$  fails as an equilibrium supply function, since for some  $X$  large enough, the function has a negative slope. Thus, from Proposition 3, if  $P(X_1)$  lies below  $p_L(x_1, a)$ , then  $P$  must fail as an equilibrium supply function. From Lemma 3, an insider of type 0 always selects a zero demand in equilibrium. Therefore, at  $X = 0$ , an equilibrium supply function must have the property that

$$P' \geq \frac{\underline{a}A\sigma_\eta^2}{1 - 2\underline{a}},$$

otherwise for some  $X > 0$ , the function  $P$  will lie strictly below  $p_L(\underline{a})$  since at  $X > 0$ ,  $\alpha(X) > \underline{a}$  and the function  $P$  will therefore fail as an equilibrium supply function.

If

$$\lim_{X \rightarrow 0} P' > \frac{\underline{a}A\sigma_\eta^2}{1 - 2\underline{a}},$$

then the first two terms in Equation (35) sum to a strictly positive number as  $X \rightarrow 0$ , and so one has  $P'' > 0$  as  $X \rightarrow 0$ . Now suppose that

$$\lim_{X \rightarrow 0} P' = \frac{\underline{a}A\sigma_\eta^2}{1 - 2\underline{a}},$$

and  $P$  is concave in a neighborhood around 0. Then, by the definition of concavity,  $P$  must lie below  $p_L(\underline{a})$  for some  $X > 0$ , and therefore  $P$  must fail as an equilibrium demand function. Having shown that the equilibrium supply function must be convex at the origin, the next goal is to show that the equilibrium supply function with the greatest liquidity has at least one inflection point, and, for  $X$  sufficiently large, is concave.

Define  $p_L(\bar{\alpha})$  as the pseudo-linear equilibrium supply schedule under the assumption that  $\bar{\alpha} = \lim_{\tau \rightarrow \infty} \alpha$  for all demands  $X \geq 0$ . From the proof of Proposition 3 we know that any price function that satisfies the ordinary differential equation (8) and crosses  $p_L(\bar{\alpha})$  will globally satisfy the equilibrium conditions. Consider the supply schedule(s) that cross  $p_L(\bar{\alpha})$  as the crossing point goes to infinity; these will be the schedule(s) with the greatest liquidity. No matter what equilibrium supply schedule is considered, an investor of type

$\tau = 0$  will always select a demand of 0. Thus, by continuity (recall from Lemma 4 that every demand is selected by one and only one investor type) all potential equilibrium schedules must have the property that  $P(0) = 0$ . This means the equilibrium supply function produces the same prices as  $p_L(\bar{\alpha})$  at demands of 0 and for some  $X$  very large. Also, as shown in the proof of Proposition 3 for all values below the crossing point for  $X > 0$ , the equilibrium supply schedule lies strictly below  $p_L(\bar{\alpha})$ . Thus, at  $X = 0$  one must have  $P'(0) < p'_L(0)$ . For  $P(X)$  to equal  $p_L(\bar{\alpha}, X)$  for some  $X$  large one must have at some point between a demand of 0 and  $X$  that  $P'(x) > p'_L(x)$  for some  $x$  between 0 and the crossing point. Finally, at the crossing point one has  $\alpha \approx \bar{\alpha}$ , where the approximation can be made arbitrarily accurate by selecting a large enough value for the crossing point. Equations (19) and (25) therefore imply that at the crossing point  $X$  the derivative  $P'(X) \approx p'_L(X)$ , where again the approximation can be made arbitrarily close by selecting  $X$  to be large enough, say  $X^*$ . Thus, for small demands the derivative of the true price function lies below  $p'_L$ , for some intermediate values it lies above  $p'_L$ , and for large values arbitrarily close to  $X^*$  it equals  $p'_L$ . Therefore, the equilibrium supply schedule must have at least one inflection point. Also note that since  $P(X)$  approaches  $p_L(X)$  from below and is tangent to it at  $X^*$ ,  $P(X)$  must be concave immediately to the left of  $X^*$ .  $\square$

**Proof of Proposition 4:** From equation (6), we have

$$\text{var}(\epsilon|\tau) = \frac{\int_{\Omega_2} \frac{\sigma_\epsilon^2 \sigma_w^2}{(\sigma_\epsilon^2 + \sigma_w^2) \sqrt{2\pi(\sigma_w^2 + \sigma_\epsilon^2)}} \exp\left\{-\frac{\tau^2}{2} \frac{\sigma_w^2}{\sigma_\epsilon^2 + \sigma_w^2}\right\} g(\sigma_\epsilon^2) d\sigma_\epsilon^2}{\int_{\Omega_2} \frac{1}{\sqrt{2\pi(\sigma_w^2 + \sigma_\epsilon^2)}} \exp\left\{-\frac{\tau^2}{2} \frac{\sigma_w^2}{\sigma_\epsilon^2 + \sigma_w^2}\right\} g(\sigma_\epsilon^2) d\sigma_\epsilon^2}.$$

Now, let  $X(\sigma_\epsilon^2) = \frac{\sigma_\epsilon^2 \sigma_w^2}{\sigma_\epsilon^2 + \sigma_w^2}$ . Then, we have the result that  $X$  increases monotonically in  $\sigma_\epsilon^2$ . Thus, we can determine the partial derivative of  $E(X|\tau)$  by noting if an increase in  $\tau$  causes a first order stochastic dominating shift in the distribution. To do this, differentiate

$$\frac{\int_0^m \frac{1}{\sqrt{2\pi(\sigma_w^2 + \sigma_\epsilon^2)}} \exp\left\{-\frac{\tau^2}{2} \frac{\sigma_w^2}{\sigma_\epsilon^2 + \sigma_w^2}\right\} g(\sigma_\epsilon^2) d\sigma_\epsilon^2}{\int_0^\infty \frac{1}{\sqrt{2\pi(\sigma_w^2 + \sigma_\epsilon^2)}} \exp\left\{-\frac{\tau^2}{2} \frac{\sigma_\epsilon^2 + \sigma_w^2}{\sigma_\epsilon^2 \sigma_w^2} \left[\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_w^2} - \left(\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_w^2}\right)^2\right]\right\} g(\sigma_\epsilon^2) d\sigma_\epsilon^2}$$

with respect to  $\tau$ . To simplify the notation, let  $\beta = \sigma_\epsilon^2/(\sigma_\epsilon^2 + \sigma_w^2)$  and  $S = \sigma_\epsilon^2 \sigma_w^2/(\sigma_\epsilon^2 + \sigma_w^2)$ .

Then, the derivative reduces to

$$\frac{\tau \int_0^m \frac{1}{\sqrt{2\pi(\sigma_\epsilon^2 + \sigma_w^2)}} \exp\left[-\frac{\tau^2}{2S}(\beta - \beta^2)\right] g(\sigma_\epsilon^2) d\sigma_\epsilon^2}{\int_0^\infty \frac{1}{\sqrt{2\pi(\sigma_\epsilon^2 + \sigma_w^2)}} \exp\left[-\frac{\tau^2}{2S}(\beta - \beta^2)\right] g(\sigma_\epsilon^2) d\sigma_\epsilon^2} \times \left\{ E\left[\frac{\beta - \beta^2}{S} \middle| \tau, \frac{\beta - \beta^2}{S} < m\right] - E\left[\frac{\beta - \beta^2}{S} \middle| \tau\right] \right\} < 0.$$

This implies that increasing  $\tau$  decreases the value of the cumulative distribution function at each point, and so increases  $\text{var}(\epsilon|\tau)$ .  $\square$

**Proof of Proposition 5:** Choose a disclosure cutoff  $c$  that is greater than the lower bound of the support of  $\sigma_\epsilon^2$ 's distribution but is less than  $\sigma_w^2$ . (It is possible to choose such a  $c$  so long as the probability that  $\sigma_\epsilon^2 < \sigma_w^2$  is strictly positive.) From Part 1 of Proposition 2, absent a disclosure rule no trade ever occurs. Thus, when the exchange receives a report that  $\sigma_\epsilon^2$  is greater than any positive cutoff  $c$ , the upper bound of the support of  $\sigma_\epsilon^2$ 's distribution continues to exceed  $\sigma_w^2$ . In this case, from Proposition 2 the no-trade equilibrium continues to prevail, so that the investor is no worse off with the disclosure rule than without. Now, suppose the exchange has not received a report. Then  $c$  becomes the upper bound of the support of  $\sigma_\epsilon^2$ 's distribution, and since  $c < \sigma_w^2$ , Part 1 of Proposition 3 indicates that trade will take place. From Lemma 4, only one type of a continuum of insider types will select a demand of zero when trade can take place. Thus, with probability one the insider will submit a non-zero demand. Since the investor always has the option of selecting a zero demand, if the investor trades a non-zero amount he must be strictly better off by revealed preference. Therefore, the expected utility of any investor must be greater under the disclosure rule, if  $c$  is set so that  $c < \sigma_w^2$ , and if the probability that  $\sigma_\epsilon^2 < \sigma_w^2$  is strictly positive.  $\square$

**Proof of Proposition 6:** The ex ante expected utility of the investor equals

$$\int_0^c \int_{-\infty}^\infty \int_{-\infty}^\infty \exp\left[-(A/2)(x(\tau - P) - (A/2)\sigma_\eta^2 x^2 + \tau s + (A/2)\sigma_\eta^2 s^2)\right] h(\epsilon, w, c) d\epsilon ds d\sigma_\epsilon^2 + \int_{-\infty}^\infty \int_{-\infty}^\infty \exp\left[-(A/2)(\tau s + (A/2)\sigma_\eta^2 s^2)\right] h(\epsilon, w, c) d\epsilon ds,$$

where the second integral arises because the market does not open when  $\sigma_\epsilon^2 > c$ . Differentiating the above equation, and using the envelope theorem to eliminate the effect of  $x$ , we obtain the expression in equation (9).

To sign the first integral of the expression contained in the proposition, note that the investor seeks to maximize the term in the first exponential. Setting  $X = 0$  produces a value of 0 for that term, so that the first exponential equals  $\exp(-0) = 1$  at  $X = 0$ . Therefore, the value  $X = x$  must result in a larger value in the exponent and thus  $\exp\left\{-(A/2)(x(\tau - P) - (A/2)\sigma_\eta^2 x^2)\right\} < 1$ , which signs the first integral. To sign the second integral, note from (6) that an increase in  $c$  must increase the expected value of  $\epsilon$  given  $\tau$ . Since  $P = \alpha\tau$ , this implies that  $(\partial P/\partial c)x \geq 0$ , which signs the second integral.  $\square$

**Proof of Proposition 8:** Conditional on  $\tau_i$  and  $k_i$ , the market maker in stock  $i$  sets  $\alpha_i$  via a modified version of (26):

$$\alpha(\tau_i, k_i) = \frac{\int \frac{\hat{\alpha}}{\sqrt{2\pi(\sigma_w^2 + \sigma_{\epsilon_i}^2)}} \exp\left\{-\frac{\tau^2}{2} \frac{\sigma_w^2}{\sigma_{\epsilon_i}^2 + \sigma_w^2}\right\} g(\sigma_{\epsilon_i}^2, k_i) d\sigma_{\epsilon_i}^2}{\int \frac{1}{\sqrt{2\pi(\sigma_w^2 + \sigma_{\epsilon_i}^2)}} \exp\left\{-\frac{\tau^2}{2} \frac{\sigma_w^2}{\sigma_{\epsilon_i}^2 + \sigma_w^2}\right\} g(\sigma_{\epsilon_i}^2, k_i) d\sigma_{\epsilon_i}^2}. \quad (36)$$

By assumption 1  $g_i^m(\sigma_{\epsilon_i}^2 | \sigma_{\epsilon_j}^2 = a)$  first order dominates  $g_i^m(\sigma_{\epsilon_i}^2 | \sigma_{\epsilon_j}^2 = b)$  if  $a > b$ . If  $k_i = o$  then  $\sigma_{\epsilon_j}^2 < c_j$  and if  $k_i = c$  then  $\sigma_{\epsilon_j}^2 \geq c_j$ . From Assumption 1 this implies  $g(\sigma_{\epsilon_i}^2, c)$  dominates  $g(\sigma_{\epsilon_i}^2, o)$ . Since  $\hat{\alpha}$  is increasing in  $\sigma_{\epsilon_i}^2$ , this implies  $\alpha_i(\tau_i, o) < \alpha_i(\tau_i, c)$  for all  $\tau_i$ .  $\square$

**Proof of Proposition 9:** From the market maker's updating problem one can write

$$\alpha_i(\tau_i, o) = \frac{\int_0^{c_i} \int_0^{c_j} \frac{\hat{\alpha}}{\sqrt{2\pi(\sigma_w^2 + \sigma_{\epsilon_i}^2)}} \exp\left\{-\frac{\tau^2}{2} \frac{\sigma_w^2}{\sigma_{\epsilon_i}^2 + \sigma_w^2}\right\} g(\sigma_{\epsilon_i}^2, \sigma_{\epsilon_j}^2) d\sigma_{\epsilon_i}^2 d\sigma_{\epsilon_j}^2}{\int_0^{c_i} \int_0^{c_j} \frac{1}{\sqrt{2\pi(\sigma_w^2 + \sigma_{\epsilon_i}^2)}} \exp\left\{-\frac{\tau^2}{2} \frac{\sigma_w^2}{\sigma_{\epsilon_i}^2 + \sigma_w^2}\right\} g(\sigma_{\epsilon_i}^2, \sigma_{\epsilon_j}^2) d\sigma_{\epsilon_i}^2 d\sigma_{\epsilon_j}^2}. \quad (37)$$

One can thus write  $\alpha_i(\tau_i, o) = E(\hat{\alpha} | \tau, \sigma_{\epsilon_1}^2 < c_1, \sigma_{\epsilon_2}^2 < c_2)$ . Since  $\hat{\alpha}_1$  is increasing in  $\sigma_{\epsilon_i}^2$  any increase in either  $c_i$  or  $c_j$  must increase its expected value. To prove this statement for the case where  $k_i = c$ , simply rewrite the above equation so that the limits for  $\sigma_{\epsilon_j}^2$  go from  $c_j$  to infinity. Again, any increase in either  $c_i$  or  $c_j$  must increase  $\alpha_i(X_i, c)$  for all  $X_i$ .  $\square$

**Proof of Proposition 10:** The proof only covers the case of  $X_i > 0$ ; the signs can simply be reversed for the case  $X_i < 0$ .

From the proof of Proposition 2, in the limit as  $X \rightarrow \infty$ ,  $P_i(X_i, o) \rightarrow p_L(X_i, o)$  and

$P_i(X_i, c) \rightarrow p_L(X_i, c)$ . Since  $p_L(X_i, o) < p_L(X_i, c)$ , there exists an  $X^*$  such that for all  $X_i > X^*$ ,  $P_i(X_i, o) < P_i(X_i, c)$ .

Having shown that for large  $X_i$ , the market is more liquid when  $k_i = 0$  rather than when  $k_i = c$ , the next step is to show that this is true for all  $X_i > 0$ . To do this, we next demonstrate that the two functions  $P_i(X_i, o)$  and  $P_i(X_i, c)$  never cross, except at  $X_i = 0$ .

Rewrite Equation (19) in terms of  $P'$ :

$$P'_i = \frac{A\sigma_\eta^2 X_i - P_i}{X_i} + \frac{P_i}{\alpha_i X_i}. \quad (38)$$

If  $P_i(X_i, c)$  crosses  $P_i(X_i, o)$  at  $X^* \neq 0$ , then at  $X^*$ , one has  $P'_i(X_i, c) > P'_i(X_i^*, o)$ . If  $P'_i(X_i^*, c) > P'_i(X_i^*, o)$  then the above implies  $\alpha_i(X_i^*, c) < \alpha_i(X_i^*, o)$ . The trader's first order conditions imply that  $\tau_i = P_i + P'_i X_i + A\sigma_\eta^2 X_i$ . Since  $P'_i(X_i^*, c) > P'_i(X_i^*, o)$  one has  $\tau_i(X_i^*, c) > \tau_i(X_i^*, o)$ . But  $P_i = \tau_i \alpha_i(\tau_i, k_i)$  and if  $\tau_i(X_i^*, c) > \tau_i(X_i^*, o)$ , then  $\alpha_i(X_i^*, c) > \alpha_i(X_i^*, o)$  by the fact that  $\alpha_i$  is increasing in  $\tau_i$  and (from Lemma 8)  $\alpha_i(\tau_i, c) > \alpha_i(\tau_i, o)$  for all  $\tau_i$ . Thus  $\tau_i(X_i^*, c) \alpha_i(X_i^*, c) > \tau_i(X_i^*, o) \alpha_i(X_i^*, o)$  which implies  $P_i(X_i^*, c) > P_i(X_i^*, o)$ , a contradiction. Thus,  $P_i(X_i, o)$  cannot cross  $P_i(X_i, c)$ , and so one has  $P_i(X_i, o) > P_i(X_i, c)$  for all  $X_i > 0$ .  $\square$

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## Footnotes

1. Lee *et al.* (1994), and Christie *et al.* (1999) provide institutional details of such trading suspensions on the NYSE and Nasdaq, respectively. They find that there were more than 500 “news-pending” trading suspensions on these exchanges in the sample period of one year that they consider.
2. Since all of the following equilibrium refinements employ the D1 criterion, any of them can be substituted without altering any of the model’s results: Stability by Kohlberg and Mertens (1986), universal divinity by Banks and Sobel (1987), and never-a-weak-best-response (NWBR) by Cho and Kreps (1987).
3. Since we prove that the equilibrium supply function does not have discontinuities, and we allow for potentially infinite prices, trade in our model cannot be restored by stipulating schedules that are truncated beyond a certain order size.
4. Note that this result suggests a role for “upstairs” markets on the NYSE wherein large trades are crossed without an intermediary. However, such markets can survive only if upstairs traders cultivate a reputation for being uninformed (e.g., index funds). Otherwise such an upstairs market could not function because if large trades stem from informed agents with high variance draws, the market will not open. Our analysis applies so long as not all agents can credibly communicate that they are uninformed, which is a plausible assumption.
5. Pooling equilibria of the type mentioned by Glosten (1989, p. 221) do not arise in our setting because the investor moves first here, unlike in Glosten (1989), where investors submit orders in response to a price schedule. Indeed, note from Part 2 of Proposition 1 that our pricing function is continuous and differentiable. See also Hellwig (1992), who argues that the non-existence of an equilibrium with trade arises in Glosten because of the unboundedness of types. In addition, Bhattacharya and Spiegel (1991) derive an equivalent market breakdown condition, but in a Walrasian setting. In all these frameworks, the market breakdown occurs when the nonstochastic projection

coefficient of the market maker's linear inference problem (denoted by  $\alpha$  in our paper) exceeds 0.5. In our setting, the inference problem is nonlinear and, as Propositions 2 and 3 indicate, the condition for the existence of an equilibrium with trade is stronger because of the randomness in  $\alpha$ . We show in the next two sections that introducing updating on the variance of private information has some significant policy implications for financial markets.

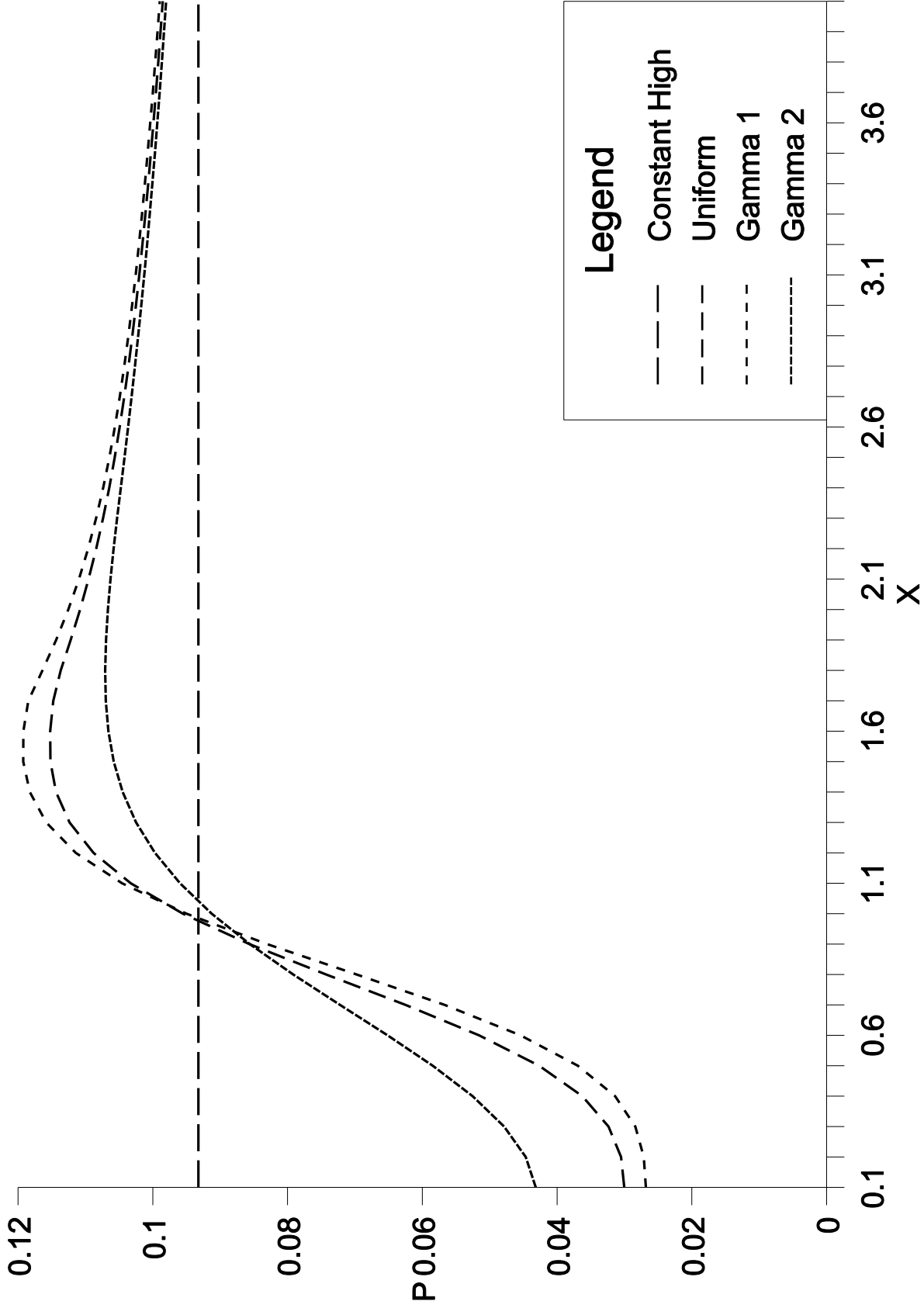
6. The above proposition thus implies that large trades will be followed by large variances and, since large trades are associated with large absolute price moves, also appears consistent with the ARCH literature, i.e., large price changes should be followed by high variances. Our analysis suggests an additional empirical test on the relation between price changes and trade size via equation (6): Once a distribution function for  $\sigma_c^2$  has been estimated, the equation provides a particular functional relationship between the trade size, post trade variance, and the shape of the supply curve.
7. It may be tempting to surmise that the rule is unnecessary since the market will itself close without the disclosure policy. However, this statement is false. Absent the disclosure policy, the only equilibrium will be the no-trade equilibrium. With the disclosure policy, other equilibria may exist. What happens is that absent a disclosure by the firm, the competitive traders in the model believe that the variance of the insider's information falls below the regulatory bound. Thus, if the firm fails to disclose when in fact the variance exceeds the bound, trade takes place since the outside investors are "fooled." Of course, if this happens investors may become suspicious of the firm's willingness to follow the regulations and will then pull out of the market. Thus, from the firm's own perspective, if it wishes to maintain liquidity in its own securities, it will follow the rule.
8. As an example, is a particular labor agreement good news or bad news? This depends upon what the market expected from the actual agreement, and thus the announcement's impact may be difficult to determine prior to seeing the market's reaction.

9. Proposition 5 shows that it is always possible to choose a  $c$  satisfying the equation in Proposition 6, and that the solution will set  $c$  less than  $\sigma_w^2$ .
10. For example, Chambers and Penman (1984) note the following: “There appears to be abnormal price variability following reports with high announcement effects for both good news and bad news reports, but little after those with relatively low effects on prices.”
11. See also Glosten (1994) and Viswanathan and Wang (1999).
12. Note that since  $dU_1/dc_2 = 0$ , the change in  $c_2$  does not alter the supply curves faced by traders in market 1.
13. In our setting, the type of an investor can be captured by the value of  $\tau$  that he possesses. Note that it is possible for multiple investors to have observed different realizations of  $\sigma_\epsilon^2$  but have the same  $\tau$ , because  $\tau$ , which equals  $\epsilon - A\sigma_\eta^2$  does not depend on  $\sigma_\epsilon^2$ . Thus, a type can be interpreted as all individuals that face the same optimization problem upto an affine transformation of their utility function. Knowing that all insiders with the same  $\tau$  but different private information variances select the same action, the market maker uses the type to update on both the mean and the variance.
14. It is perhaps useful to provide a simple analogy which illustrates why no-trade is an equilibrium in our setting. Consider a stranger who walks up to an agent with an envelope that can contain any amount of money from  $-\infty$  and  $+\infty$ , and offers to sell it to the agent. It is evident that the agent will never accept such an offer since any offer must be unprofitable to the agent. The agent thus implicitly imposes a type of  $-\infty$  to the stranger, and refuses to purchase the envelope. This is exactly the situation faced by the market maker in the discussion given above.
15. Otherwise there will exist a type  $\tau^{**} < \tau^*$  that is indifferent between  $X_0$  and  $X_1$ . Since all types between  $\tau^{**}$  and  $\tau^*$  select one must therefore have  $P(X_1|\tau^{**}) < P(X_1)$

and the investor of type  $\tau^{**}$  can strictly improve his utility by submitting a demand just below  $X_1$ .

16. For notational simplicity, we will often suppress the parameter  $a$  in the function  $p(X, a)$ , and simply write it as  $p(X)$ .
17. One can find a price function that runs through any single point by adjusting the constant associated with the solution to the ordinary differential equation describing the price function.

# Marginal Cost Per Share



## Figure Legend

**Figure 1:** This figure graphs the equilibrium supply function under the common assumptions that the variance of noise in the fundamental value,  $\sigma_\eta^2$ , and the parameter  $A\sigma_\eta^2s$ , where  $A$  is the risk aversion coefficient and  $s$  is the variance of the informed trader's endowment, both equal 1, and that the variance of private information  $\sigma_\epsilon^2 \in [0, 0.5]$ . The line labeled Constant High represents the linear equilibrium under the assumption that all traders know that  $\sigma_\epsilon^2 = 0.5$ . The "Uniform" line displays the equilibrium supply function when the market maker believes that  $\sigma_\epsilon^2$  has a uniform distribution between 0 and 0.5. The "Gamma" lines are drawn assuming that the market maker believes that  $\sigma_\epsilon^2$  has a truncated gamma distribution, where the cases labeled Gamma 1 and Gamma 2 set the parameter of this distribution to 1 and 2, respectively.