Abstract
Consumer goods manufacturers usually sell their brands to consumers through common independent retailers. Theoretical research on such channel structures has analyzed the optimal behavior of channel members under alternative assumptions of manufacturer-retailer interaction (Vertical Strategic Interaction). Research in Empirical Industrial Organization has focused on analyzing the competitive interactions between manufacturers (Horizontal Strategic Interaction). Decision support systems have made various assumptions about retailer-pricing rules (e.g., constant markup, category-profit-maximization). The appropriateness of such assumptions about strategic behavior for any specific market, however, is an empirical question. This paper therefore empirically infers (1) the Vertical Strategic Interaction (VSI) between manufacturers and retailer; (2) the Horizontal Strategic Interaction (HSI) between manufacturers simultaneously with the VSI, and (3) the pricing rule used by a retailer.

The approach is particularly appealing because it can be used with widely available scanner data, where there is no information on wholesale prices. Researchers usually have no access to wholesale prices. Even manufacturers, who have access to their own wholesale prices, usually have limited information on competitors’ wholesale prices. In the absence of wholesale prices, we derive formulae for wholesale prices using game-theoretic solution techniques under the specific assumptions of vertical and horizontal strategic interaction and retailer-pricing rules. We then embed the formulae for wholesale prices into the estimation equations. While our empirical illustration is using scanner data without wholesale prices, the model itself can be applied when wholesale prices are available.

Early research on the inference of HSI among manufacturers in setting wholesale prices using scanner data (e.g., Kadiyali et al. 1996, 1999) made the simplifying assumption that retailers charge a constant margin. This assumption enabled them to infer wholesale prices and analyze competitive interactions between manufacturers. In this paper, we show that this model is econometrically identical to a model that measures retail-price coordination across brands. Hence, the inferred cooperation among manufacturers could be exaggerated by the coordinated pricing (category management) done by the retailer. We find empirical support for this argument. This highlights the need to properly model and infer VSI simultaneously to accurately estimate the HSI when using data at the retail level.

Functional forms of demand have been evaluated in terms of the fit of the model to sales data. But recent theoretical research on channels (Lee and Staelin 1997, Tyagi 1999) has shown that the functional form has serious implications for strategic behavior such as retail passsthrough. While the logit and linear model implies equilibrium passthrough of less than 100% (Lee and Staelin call this Vertical Strategic Substitute (VSS)), the multiplicative model implies optimal passthrough of greater than 100% (Vertical Strategic Complement (VSC)). Because passthrough rates on promotions have been found to be below or above 100% (Chevalier and Curhan 1976, Armstrong 1991), we empirically test the appropriateness of the logit (VSS) and the multiplicative (VSC) functional form for the data.

We perform our analysis in the yogurt and peanut butter categories for the two biggest stores in a local market. We found that the VSS implications of the logit fit the data better than the multiplicative model. We also find that for both categories, the best-fitting model is one in which (1) the retailer maximizes category profits, (2) the VSI is Manufacturer-Stackelberg, and (3) manufacturer pricing (HSI) is tacitly collusive. The fact that the retailer maximizes category profits is consistent with theoretical expectations. The inference that the VSI is Manufacturer-Stackelberg reflects the institutional reality of the timing of the game. Retailers set their retail prices after manufacturers set their wholesale prices. Note that in the stores and product categories that we analyze, the two manufacturers own the dominant brands with combined market shares of about 82% in the yogurt market and 65% in the peanut butter market. The result is also consistent with a balance of power argument in the literature. The finding that manufacturer pricing is tacitly collusive is consistent with the argument that firms involved in long-term competition in concentrated markets can achieve tacit collusion.

Managers use decision support systems for promotion planning that routinely make assumptions about VSI, HSI, and the functional form. The results from our analysis are of substantive import in judging the appropriateness of assumptions made in such decision support systems.

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1. Introduction

Manufacturers of most consumer goods sell their brands to consumers through common independent retailers (e.g., supermarkets and convenience stores for grocery products, department stores, specialty stores for consumer electronics, sporting goods, etc.). There has been substantial theoretical research on such channel structures (e.g., McGuire and Staelin 1983, Choi 1991, Lee and Staelin 1997). This stream of research has analyzed the optimal behavior of channel members under alternative assumptions about vertical strategic interactions between manufacturers and retailers. Researchers of the new empirical industrial organization (e.g., Kadiyali et al. 1996, 1999) have studied the competitive interactions between manufacturers (horizontal strategic interaction). Decision support systems for promotion planning make assumptions about the pricing rules used by retailers (e.g., Silva-Risso et al. 1999 assume constant margin, Tellis and Zufryden 1995 assume category-profit maximization). The appropriateness of such assumptions for any specific market is an empirical question. Our goal in this paper is to empirically infer (1) the vertical strategic interaction (VSI) between manufacturers and retailer, (2) the horizontal strategic interaction (HSI) between manufacturers simultaneously with the VSI, and (3) the pricing rule used by a retailer.

The approach in this paper is particularly appealing because it can be used with widely available scanner data, where there is no information on wholesale prices. Researchers usually have no access to wholesale prices. Manufacturers have access to their own wholesale prices, but usually have only limited information on competitors’ wholesale prices. In the absence of wholesale prices, we derive formulae for wholesale prices using game-theoretic solution techniques under the specific assumptions of vertical and horizontal strategic interaction, demand functional form, and retailer pricing rules. We then embed the formulae for wholesale prices into the estimation equations. While our empirical illustration is using scanner data without wholesale prices, the model itself is easily applicable to the situation where wholesale prices are available by using the derived formulae for wholesale prices as additional estimation equations in the model.

Relationship with Previous Research

For a quick overview of the relative contribution of this paper, we compare related papers in the literature in Table 1.

Several papers in the new empirical industrial organization (NEIO) tradition have attempted to infer competitive interactions underlying pricing behavior among manufacturers (HSI) using national-level data (for example, see Roy et al. 1994 and Kadiyali 1996). In these papers, retailer behavior is not modeled. One justification for this is that the focus is on competition at the national level. Furthermore, any particular retailer’s strategic behavior will be lost in the data aggregation across retailers, and hence, is irrelevant for the level at which the analysis is done.

Because manufacturers do tailor their marketing mix to local market conditions (Manning et al. 1998, Nichols 1987), NEIO researchers have recently shifted their focus to analyzing HSI among manufacturers in local markets using scanner data. But at this local level, retailer strategic behavior can have a significant influence on a manufacturer’s price-setting behavior. Therefore, retailer’s interaction with manufacturers (VSI) needs to be accounted for. A further complication when using scanner data is that wholesale prices are unavailable to the researcher. As a result, to analyze manufacturer wholesale-pricing behavior, the wholesale prices need to be inferred from the data.

Kadiyali et al. (1996, 1999) infer wholesale prices by making a simplifying assumption about the VSI. They assume that the retailer is nonstrategic and charges an exogenous constant margin. This assump-
Table 1  Positioning Against Related Research

<table>
<thead>
<tr>
<th>Manufacturer-Manufacturer Interaction</th>
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<tr>
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Structural Analysis of Manufacturer Pricing

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In this paper, we investigate competitive behavior among manufacturers by allowing for other plausible manufacturer-retailer interaction and infer the interaction that best fits the data. We allow for two types of VSI between the manufacturers and the retailer: (1) manufacturers as Stackelberg leaders and (2) vertical Nash interaction. We also allow for alternative retailer pricing rules: (1) category profit maximization, (2) brand profit maximization, and (3) constant margins. Using game-theoretic analysis, we infer wholesale prices conditional on observed retail prices, sales, and other exogenous variables under each of the above assumptions. This enables us to analyze competition among manufacturers (HSI) under alternative assumptions about the VSI.

Besanko et al. (1998) (hereafter referred to as BGJ) focus on the importance of accounting for the endogeneity of pricing decisions by manufacturers and retailers to obtain unbiased estimates for a logit model using aggregate data. In contrast to our emphasis on inferring the strategic behavior of market participants, they assume (1) that manufacturers and the retailer make simultaneous pricing decisions, i.e., they assume that VSI is vertical Nash, (2) the retailer does category management by maximizing total profits from the category, and (3) that the HSI among manufacturers is Bertrand competition.

Other contemporaneous research has addressed some of the issues that we discuss in this paper. Kadiyali et al. (2000) measure the balance of power between manufacturers and retailer by measuring the share of channel profits. Although they model both
the HSI between manufacturers and the VSI between manufacturers and retailer, their econometric model does not permit one to identify the HSI between manufacturers (an issue of key interest in this paper). Another key difference is that their technique is applicable only when researchers have access to wholesale price data.

Cotterill and Putsis (2001) also test for the manufacturer Stackelberg and vertical Nash VSI between manufacturers and retailers without wholesale price data. They also test for constant margin and category-profit-maximizing behavior by the retailer. They analyze a number of product categories and find support for both vertical Nash and manufacturer Stackelberg interaction. However, they generally reject the constant margin assumption. In contrast to our paper, they assume a linear model of demand, assume that HSI between manufacturers is Bertrand, and do not test for brand-profit-maximizing behavior by the retailer.3

In summary, this is the first paper to simultaneously infer both HSI and VSI in a channel. The technique can be applied even in the absence of wholesale price data. Methodologically, a key difference with respect to Cotterill and Putsis (2001) is that they derive their supply-side pricing equations similarly to theoretical researchers, by solving for optimal wholesale and retail prices in terms of a deterministic demand model.4 In this paper, we recognize that observed market shares include a demand-side econometric error that is known to both firms and consumers, and therefore will affect retail and wholesale passthrough differently under different types of VSI and HSI. By explicitly deriving the equations in terms of observed market shares, we account for the impact of demand-side econometric error on retail and wholesale passthrough in a manner consistent with the theoretical structure of the specific model being tested, leading to a more complete structural specification of the econometric model.

Appropriateness of Demand Functional Form

While functional forms of demand have been evaluated in terms of the fit of the model to sales data, recent game-theoretic research on channels (Lee and Staelin 1997, Tyagi 1999) has shown that the functional form of demand has serious implications for strategic behavior such as retail passthrough. While the logit and linear model implies equilibrium passthrough of less than 100% (Lee and Staelin call this vertical strategic substitute (VSS)), the multiplicative model implies optimal passthrough of greater than 100% (vertical strategic complement (VSC)). Since passthrough rates on promotions have been found to be below or above 100% (Chevalier and Curhan 1976, Armstrong 1991), a functional form with the VSC property may be appropriate for categories that have retail passthrough greater than 100%, while a functional form with the VSS property may be appropriate if the passthrough is less than 100%. In this paper, we therefore treat the appropriateness of functional form of demand as an empirical issue and choose from the logit (VSS) or multiplicative (VSC) demand models.5

4We note that Kadiyali et al. (2000) estimate conjectural variations (CVs) for VSI and, therefore, do not impose the specific restriction of leader-follower behavior. Estimating CVs, however, does not necessarily make the model more flexible, because the CV measures reactions as a constant across all periods, while the derived optimal reaction in a leader-follower model (that we use here) is flexible to accommodate changes from period to period based on changes in demand characteristics (a realistic assumption). Further research is needed on the appropriateness of the two methods. They also test specific models such as manufacturer Stackelberg, vertical Nash, etc., but, unlike the author of this paper, they assume that the HSI is Bertrand when they test those models. We also note that in the absence of wholesale price data, a CV cannot be estimated for the VSI. The specific assumption about VSI (leader-follower, vertical Nash) is necessary to estimate the model without wholesale price data.

5They also estimate an LA-AIDS model, but without the structural pricing equations. Hence it is not possible to choose among the different VSI using the LA-AIDS model.
In summary, we choose from the following strategic interactions and demand models:

1. Vertical strategic interaction: manufacturer Stackelberg and vertical Nash;
2. Horizontal strategic interaction: Bertrand, tacit collusion. We also estimate a continuous conjectural variation (CV) parameter;
3. Retailer pricing rule: category-profit-maximization, brand-profit-maximization, constant markup;

Section 2 describes the model. Section 3 describes the data, the estimation procedure, and the results. Section 4 concludes with a discussion of limitations of the model and suggestions for future research.

2. Model

Figure 1 represents the schematic model of the market that we analyze: Two manufacturers sell through a common retailer to consumers. Consumers make their choices after observing the retail prices. In the manufacturer Stackelberg models, the manufacturers choose wholesale prices first; the retailer chooses retail prices after observing the wholesale prices. In the vertical Nash models, the manufacturers and the retailer choose their prices simultaneously. Manufacturers choose their wholesale prices simultaneously. In the figure, one-sided arrows represent sequential decisions and double-sided arrows indicate simultaneous decisions. The two main assumptions embedded in the figure are that (1) there are two manufacturers in the market and (2) that manufacturers sell through one retailer. We justify both of these assumptions below.

Assumption 1. There are two manufacturers in the market.

We focus our empirical analysis on the two largest brands in the yogurt and peanut butter categories. This is reasonable for our purposes because the two largest brands have a very large market share (82% for yogurt and 65% for peanut butter). Other brands have relatively low market share of less than 10% in this market. Private labels have a minuscule market share in these categories in these stores. Hence, the use of private labels for strategic purposes is not a serious issue as in the studies by Kadiyali et al. (2000) and Putsis and Dhar (1999). While it is possible to extend model development to additional brands, solving for the optimal vertical strategic reactions becomes computationally cumbersome.

Assumption 2. Manufacturers sell through one retailer.

The primary effect of this assumption is that we do not model any effect of retail competition. This may
be a serious shortcoming when analyzing categories where retail competition is severe. Slade (1995) and Walters and McKenzie (1988), however, have shown evidence of limited retail competition in some categories. Slade reported, based on her interviews of grocery chain managers, that the vast majority of households (over 90%) do not engage in comparison shopping on a week-to-week basis. Most consumers visit the same store each week, and hence most competition seems to take place among brands in the store rather than across stores. Slade also tested this interview-based evidence statistically on the saltine cracker market and found that demand at one store was unaffected by prices in other chains. Walters and Mackenzie examined the impact of loss-leader pricing and other promotions on store sales and profits and concluded that price specials and double-coupon promotions had no significant effect on store traffic. They found that only one of eight loss-leader categories had a significant effect on store traffic. In the structural modeling literature, while analyzing the yogurt category (which is analyzed in this paper), Besanko et al. (1998) and Vilcassim et al. (1999) have also made the assumption of no cross-store competition.

To address the issue of whether the assumption is appropriate for our analysis, we follow Slade in checking whether a store's sales are affected by prices at a competing store. For our analysis we used data from two Every Day Low Price (EDLP) stores, which happen to be the biggest stores in the market. See Section 3 for discussion of the data. We ran a simultaneous equations regression with two linear demand equations for each store. As the two biggest EDLP stores may also face competition from the promotion-al (High-Low pricing) stores, we also included data from a High-Low store. Together these three stores accounted for 90% of sales in our market. Hence, we had six equations in our estimation. Each brand’s sales in a store were regressed against prices of both brands at the same store and other stores. While same store price coefficients were significant and of the right sign, other store prices were insignificant in all six equations. Hence, the assumption seems reasonable for the data that we analyze. Essentially, this implies that yogurt or peanut butter prices at competing stores have only a very minor effect on the consumer’s decision to visit a particular store. However, it is important to recognize that previous research has shown that detecting store competition requires fairly subtle modeling (Bucklin and Lattin 1992). For our purposes, it appears that retail competition is a second-order effect that is not likely to impact our results.

Demand
As discussed earlier, we consider two functional forms: the logit demand model with the VSS property and the multiplicative demand model with VSC property. These functional forms have been used widely in the marketing literature: The logit demand model has been used both for modeling household level data (for example, Guadagni and Little 1983) and aggregate market share data (for example, Allenby 1989). Recently, the logit model has been shown to provide similar substantive implications, whether one uses the model at either the aggregate (store) or disaggregate (household) level (Allenby and Rossi 1991, Gupta et al. 1996) under reasonably realistic conditions. The multiplicative demand model is a fairly popular demand model in the literature (for an example, see the SCAN*PRO-based model of Christen et al. 1997).

The Logit Model. The utility for brand \(i\) in period \(t\) for consumer \(j\) is given by
\[
U_{ijt} = \beta_{ol} + \beta_{f_{jt}} + \beta_{d_{jt}} + \beta_{dp_{jt}} + \beta_{f_{jt}d_{jt}p_{jt}} + \beta_{p_{jt}}p_{jt} + \xi_{jt} + \epsilon_{ijt}
\]
\[= U_{it} + \epsilon_{ijt} \quad i = 1, 2, \]
where \(f_{jt}\), \(d_{jt}\), and \(p_{jt}\) are the features, displays, and retail price associated with brand \(i\) in period \(t\). \(\beta_{ol}\) is

\[\text{In addition to prices of our own and competing stores, we used displays at the same stores and features at our own and competing stores (to see if they affected store traffic). We also tested other functional forms of demand, such as multiplicative and loglinear.}\]
the intrinsic attraction of brand $i$, and $\xi_i$ is the unobservable (to the econometrician, but observable to the firm and the consumer) component of utility. $U_{it}$ is the average utility across consumers for brand $i$ in period $t$. We assume that $\epsilon_{it}$ follows an i.i.d. Type-I extreme value distribution, leading to the logit model.

To allow for the market share of the two inside brands to expand and contract over different periods with the choice of the marketing mix, we allow for nonpurchases (through an outside good) denoted by $i = 0$. We normalize the utility of the outside good to zero across periods. That is, we assume $U_{i0} = 0$. The market share for each brand is given by

$$s_{it} = \frac{\exp(U_{it})}{1 + \sum_{k=1}^{2} \exp(U_{kt})}, \quad i = 0, 1, 2. \quad (2)$$

Therefore, when both brands reduce prices, the total market share of the two inside brands increases vis-à-vis the outside good and vice versa. Note that when the marketing-mix coefficients are the same for both brands the above model reduces to a logit model.

It is easy to see that

$$\ln(s_{it}/s_{0t}) = U_{it} = \beta_{it} + \beta_{i} f_{it} + \beta_d d_{it} + \beta_m M_{it}$$

$$+ \beta_{i} p_{it} + \beta_{i} p_{it} + \xi_{it}, \quad i = 1, 2. \quad (3)$$

Equation (3) is the demand-side estimation equation. The term $\xi_{it}$ serves as the error term in the estimation equation. As discussed in Besanko et al. (1998) and Villas-Boas and Winer (1999), these error terms can capture the effects of manufacturer advertising and consumer promotions that we do not explicitly model in this paper.

The first derivatives of market share with respect to prices are

$$\frac{\partial s_{it}}{\partial p_{i}} = \left( \frac{\partial s_{1t}}{\partial p_{1t}}, \frac{\partial s_{2t}}{\partial p_{2t}} \right)$$

$$= \left( \alpha_1 s_{1t}(1 - s_{1t}) - \alpha_2 s_{1t} s_{2t}, \quad -\alpha_1 s_{1t} s_{2t} \right) \alpha_2 s_{2t}(1 - s_{2t}) \right), \quad (4)$$

where $\alpha_d = \beta_d d_{it} + \beta_m f_{it} + \beta_{i} p_{it}$.

The Multiplicative Model. The multiplicative demand model was

$$q_{it} = \alpha_{it} p_{it}^{\alpha_{1i}} p_{2t}^{\alpha_{2i}} f_{it}^{\alpha_{fi}} f_{2t}^{\alpha_{fi}} d_{it}^{\alpha_{di}} d_{2t}^{\alpha_{di}}.$$ 

For estimation, we used the log-log model, by taking the logs of the above equation on both sides and adding a normal additive error. The derivatives with respect to price for the multiplicative model are well known.

Retailer Model

We derive the pricing equations for the retailer under different types of VSI for the logit model. For the multiplicative model, the equations can be derived similarly.

For the category-profit-maximizing retailer the objective is to maximize category profits in period $t$. Since consumers who buy other than the two brands that we analyze may contribute to retailer profits, we assume that the outside good gives a constant contribution to retailer profit. Denoting the margin from the outside good as $m_{op}$, the retailer objective is given by

$$\Pi_{it} = [(p_{it} - w_{it})s_{it} + (p_{2t} - w_{2t})s_{2t} + m_{op} s_{0t}]M_{it}, \quad (5)$$

where $p_{it}$ and $p_{2t}$ are the retail prices of Products 1 and 2, $w_{it}$ and $w_{2t}$ are the wholesale prices of Products 1 and 2 set by the manufacturers, and $s_{it}$ and $s_{2t}$ are the shares of Products 1 and 2 defined in the demand model (note that $s_{0t} = 1 - s_{1t} - s_{2t}$ is the share of the outside good), and $M_{it}$ is the size of the market. The subscript $t$ refers to the period $t$.

For the retailer maximizing brand profits, the objective is to maximize each brand’s profits in period $t$ without considering the impact of the chosen price
on the demand and profits from the other brand. The objective is given by

$$\Pi_i^R = [(p_i - w_i)s_i]M_i, \quad i = 1, 2. \quad (6)$$

Because the retailer is a follower in the manufacturer Stackelberg games, the retailer does not incorporate manufacturer reactions when choosing retail prices. In the vertical Nash game also, the retailer does not incorporate manufacturer reactions, due to the simultaneous nature of the game. Hence, the first-order conditions and the pricing equations are identical for both the manufacturer Stackelberg models and vertical Nash models.

For the retailer maximizing category profits, the first-order conditions imply

$$\frac{\partial \Pi_i^R}{\partial p_{it}} = 0 \Rightarrow s_{it} + (p_{it} - w_{it} - m_0) \left[ \frac{\partial s_{it}}{\partial p_{it}} \right] + (p_{it} - w_{2t} - m_0) \left[ \frac{\partial s_{2t}}{\partial p_{it}} \right] = 0,$$

$$i = 1, 2. \quad (7)$$

Substituting the price derivatives from (4) and solving the first-order conditions give us the retail-pricing equation

$$\begin{pmatrix} p_{1t} \\ p_{2t} \end{pmatrix} = \begin{pmatrix} w_{1t} \\ w_{2t} \end{pmatrix} + \begin{pmatrix} 1 & s_{1t} & s_{2t} \\ \alpha_1 & \alpha_{12} & \alpha_{12} \alpha_1 \end{pmatrix} \begin{pmatrix} \frac{\partial s_{1t}}{\partial p_{1t}} \\ \frac{\partial s_{2t}}{\partial p_{1t}} \end{pmatrix} + m_0. \quad (8)$$

For brand-profit-maximizing case, the first-order conditions for the retailer are

$$\frac{\partial \Pi_i^R}{\partial p_{it}} = 0 \Rightarrow s_{it} + (p_{it} - w_{it}) \left[ \frac{\partial s_{it}}{\partial p_{it}} \right] = 0, \quad i = 1, 2. \quad (9)$$

Substituting the price derivatives from (4) and solving the first-order conditions give us the pricing equation

$$\begin{pmatrix} p_{1t} \\ p_{2t} \end{pmatrix} = \begin{pmatrix} w_{1t} \\ w_{2t} \end{pmatrix} + \begin{pmatrix} 1 & s_{1t} & s_{2t} \\ \alpha_1 & \alpha_{12} & \alpha_{12} \alpha_1 \end{pmatrix} \begin{pmatrix} \frac{\partial s_{1t}}{\partial p_{1t}} \\ \frac{\partial s_{2t}}{\partial p_{1t}} \end{pmatrix} + m_0. \quad (10)$$

For the constant retailer margin model, the pricing equation is simply

$$\begin{pmatrix} p_{1t} \\ p_{2t} \end{pmatrix} = \begin{pmatrix} w_{1t} \\ w_{2t} \end{pmatrix} (1 + m) = \begin{pmatrix} w_{1t} \\ w_{2t} \end{pmatrix} \begin{pmatrix} \frac{\partial s_{1t}}{\partial p_{1t}} \\ \frac{\partial s_{2t}}{\partial p_{1t}} \end{pmatrix} + m_0. \quad (11)$$

In Pricing Equations (8), (10), and (11) the first term represents the input cost (wholesale price) to the retailer and the second term represents the retailer margin.

For the manufacturer Stackelberg model, the retailer’s reactions to manufacturers’ wholesaler prices are obtained by taking the derivatives of the retail prices in (8) and (10). It can be shown that (proof in Appendix) for the manufacturer Stackelberg model with the retailer maximizing category profits, the reactions are given by

$$\begin{pmatrix} \frac{\partial p_{1t}}{\partial w_{1t}} \\ \frac{\partial p_{1t}}{\partial w_{2t}} \\ \frac{\partial p_{2t}}{\partial w_{1t}} \\ \frac{\partial p_{2t}}{\partial w_{2t}} \end{pmatrix} = \begin{pmatrix} 1 - s_{1t} \\ -s_{2t} \\ 1 - s_{1t} \\ 1 - s_{1t} \end{pmatrix} \quad (12)$$

For the manufacturer Stackelberg model, with the retailer maximizing brand profits, the reactions are given by

$$\frac{\partial p_{1t}}{\partial w_{1t}} = \begin{pmatrix} 1 - s_{1t}^2(1 - s_{2t}) \\ \alpha_1(1 - s_{1t})(1 - s_{2t}) - (s_{1t}s_{2t})^2 \\ \alpha_1(1 - s_{1t})(1 - s_{2t}) - (s_{1t}s_{2t})^2 \\ \alpha_2(1 - s_{1t})(1 - s_{2t}) - (s_{1t}s_{2t})^2 \end{pmatrix} \quad (13)$$

As we stated earlier, the logit demand model leads to vertical strategic substitutability (VSS) since $\partial p_i/\partial w_i < 1.$
For the vertical Nash model, both the retailer and the manufacturers take their decisions simultaneously. Hence, only the direct effect of the change in wholesale price on retail price through the retailer input cost (wholesale price) is accounted for but not the indirect effect through its impact on the choice of retail margin. Hence, for both the category-profit-maximizing case and the brand-profit-maximizing case the impact on retail price for a change in wholesale price is given by

$$\frac{dp_t}{dw_t} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (14)$$

For the constant margin model, the retailer reaction to a change in wholesale price is

$$\frac{dp_t}{dw_t} = \begin{pmatrix} \frac{dp_t}{dw_{it}} & \frac{dp_t}{dw_{it}} \\ \frac{dp_t}{dw_{2t}} & \frac{dp_t}{dw_{2t}} \end{pmatrix} = \begin{pmatrix} 1 + m & 0 \\ 0 & 1 + m \end{pmatrix}. \quad (15)$$

**Manufacturer Interactions**

We now develop the model of manufacturer interactions (HSI) and the supply-side estimation equations. We allow for two alternative manufacturer interactions: (1) tacit collusion and (2) Bertrand competition.\(^{11}\) The objective function of manufacturer \(i\) selling brand \(i\) in period \(t\) is given by

$$\Pi_{it}^M = (w_{it} - c_{it})s_{iti}M_{it} + \theta(w_{it} - c_{it})s_{iti}M_{it} - F_{it},$$

$$i = 1, 2; \quad j \neq i, \quad (16)$$

where \(w_{it}\) is the wholesale price for brand \(i\) that the manufacturer charges the retailer and \(c_{it}\) is the margin cost of brand \(i\). \(F_{it}\) is the fixed cost to the manufacturer (it can include costs that are not related to the marginal sales of the brand; e.g., slotting allowances). Note that \(\theta = 1\) for the case of tacit collusion and \(\theta = 0\) for the case of Bertrand competition. We define the marginal cost of brand \(i\) as \(c_{it} = \gamma_i + \omega_{it}\), where \(\gamma_i\) is the brand-specific marginal cost, and \(\omega_{it}\) is the brand-specific unobservable marginal cost at time \(t\). Note that \(\omega_{it}\) is unobservable to the researcher, but observable to the manufacturers.

The first-order conditions for the manufacturer are given by

$$\frac{\partial \Pi_{it}^M}{\partial w_{it}} = s_{it} + (w_{it} - c_{it}) \left[ \begin{pmatrix} \frac{\partial s_{iti}}{\partial p_{iti}} & \frac{\partial s_{iti}}{\partial w_{iti}} + \frac{\partial s_{iti}}{\partial w_{2iti}} \\ \frac{\partial s_{iti}}{\partial w_{1iti}} + \frac{\partial s_{iti}}{\partial w_{2iti}} \end{pmatrix} \right] \left(1 + \frac{\partial p_t}{\partial w_{iti}} + \frac{\partial p_t}{\partial w_{2iti}} \right)$$

$$+ \theta(w_{it} - c_{it}) \left[ \begin{pmatrix} \frac{\partial s_{iti}}{\partial p_{iti}} & \frac{\partial s_{iti}}{\partial w_{iti}} + \frac{\partial s_{iti}}{\partial w_{2iti}} \\ \frac{\partial s_{iti}}{\partial w_{1iti}} + \frac{\partial s_{iti}}{\partial w_{2iti}} \end{pmatrix} \right] = 0,$$

$$i = 1, 2; \quad j \neq i,$$

$$s_{it} + \left( \begin{pmatrix} \frac{\partial p_t}{\partial w_{iti}} & \frac{\partial s_{iti}}{\partial p_{iti}} \end{pmatrix} \cdot \Theta \right) (w_{it} - c_{it}) = 0, \quad (17)$$

where

$$\Theta = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

for tacit collusion and

$$\Theta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

for Bertrand competition. The \(\cdot \ast \) operator denotes element-by-element multiplication of a matrix.

We can thus solve for the wholesale prices as

$$w_{it} = c_{it} + \left( - \frac{\partial p_t}{\partial w_{iti}} \frac{\partial s_{iti}}{\partial p_{iti}} \right) \ast \Theta^{-1} s_{it}, \quad (18)$$

where the term

$$\left( - \frac{\partial p_t}{\partial w_{iti}} \frac{\partial s_{iti}}{\partial p_{iti}} \right) \ast \Theta^{-1} s_{it}$$

is the vector of margins that manufacturers choose for their brands.

Note that Equation (18) is purely in terms of observable data and model parameters that we will estimate. In the absence of data on wholesale prices, we can substitute out the expression for wholesale prices in (18) into the Retail-Pricing Equation (8), (10), and (11) to get the appropriate supply-side estimation equation that accounts for manufacturer and retailer behavior.

\(^{11}\)After choosing the best-fitting model using Vuong’s (1989) test for these two extreme cases, we also estimate the CVs later. For the CV estimates, refer to the section "Implications of the Identification Problem."
The actual estimation equations for the five models can be obtained by appropriately substituting into the above model the expression of \( r_t \) and \( \partial p_t / \partial w_t \). We summarize these in Table 2 and Table 3, respectively, for quick reference. Because these expressions are in terms of observed market shares, which include the demand-side econometric error, we have incorporated demand-side econometric errors into the supply-side estimation equations in a manner consistent with the theoretical structure of the model. This is a key difference with respect to Cotterill and Putsis (2001), who use deterministic demand models in deriving their supply-side estimation equations, and therefore do not model the structural effects of demand-side econometric error in their supply-side model.

Specifically, denoting the \( i, j \) element of matrices \( \partial p_t / \partial w_t, \partial s_t / \partial p_t, \) and \( \Theta \) as \( pw_{ij}, sp_{ij}, \) and \( \theta_{ij} \), we can write the two pricing-estimation equations by doing the appropriate matrix algebra as follows:

\[
p_t = c_t + \left( -\frac{\partial p_t}{\partial w_t} \frac{\partial s_t}{\partial p_t} \right) \Theta^{-1} s_t + m_t \quad (19)
\]

**Table 2 Retail Margins**

<table>
<thead>
<tr>
<th>Model</th>
<th>Retailer Margin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer Stackelberg</td>
<td></td>
</tr>
<tr>
<td>Category profit maximization</td>
<td></td>
</tr>
</tbody>
</table>
| \[
\begin{pmatrix}
\frac{1}{\alpha_{12}} + \frac{s_{1i}}{\alpha_{21}S_{i2}} + \frac{s_{2i}}{\alpha_{22}S_{i2}} \\
\frac{1}{\alpha_{21}} + \frac{s_{1i}}{\alpha_{12}S_{i2}} + \frac{s_{2i}}{\alpha_{22}S_{i2}}
\end{pmatrix}
\] + \( m_t \) |
| Vertical Nash                  |                 |
| Category profit maximization   |                 |
| \[
\begin{pmatrix}
\frac{1}{\alpha_{1i}(1 - s_{1i})} \\
\frac{1}{\alpha_{2i}(1 - s_{2i})}
\end{pmatrix}
\] |
| Vertical Nash                  |                 |
| Brand profit maximization      |                 |
| \[
\begin{pmatrix}
\frac{1}{\alpha_{1i}(1 - s_{1i})} \\
\frac{1}{\alpha_{2i}(1 - s_{2i})}
\end{pmatrix}
\] |
| Constant margin retailer       | \( (w_{t1} w_{t2})^m \) |

**Table 3 Retailer Reactions**

<table>
<thead>
<tr>
<th>Model</th>
<th>Retailer Reaction ( \frac{\partial p_t}{\partial w_t} )</th>
</tr>
</thead>
</table>
| Manufacturer Stackelberg       | \( \begin{pmatrix}
1 - s_{1i} & -s_{1i} \\
-s_{2i} & 1 - s_{2i}
\end{pmatrix} \) |
| Category profit maximization   | \( \begin{pmatrix}
\frac{(1 - s_{1i})(1 - s_{2i})}{(1 - s_{1i})(1 - s_{2i}) - s_{1i}s_{2i}} & \frac{s_{1i}(1 - s_{1i})(1 - s_{2i})}{\alpha_{1i}(1 - s_{1i})(1 - s_{2i}) - s_{1i}s_{2i}} \\
\frac{s_{2i}(1 - s_{1i})(1 - s_{2i})}{\alpha_{1i}(1 - s_{1i})(1 - s_{2i}) - s_{1i}s_{2i}} & \frac{(1 - s_{1i})(1 - s_{2i})}{(1 - s_{1i})(1 - s_{2i}) - s_{1i}s_{2i}}
\end{pmatrix} \) |
| Brand profit maximization      | \( \begin{pmatrix}
(1 - s_{1i})(1 - s_{2i}) & 0 \\
0 & 1
\end{pmatrix} \) |
| Vertical Nash                  | \( \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} \) |
| Category profit maximization   | \( \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} \) |
| Brand profit maximization      | \( \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} \) |
| Constant margin retailer       | \( \begin{pmatrix}
1 + m & 0 \\
0 & 1 + m
\end{pmatrix} \) |
\[ p_{it} = \gamma_1 + \left\{ \theta_{12} s_{it} (pw_{11i} + pw_{12i}) - \theta_{22} s_{it} (pw_{11i} + pw_{12i}) \right\} / \text{Det}_i \]
\[ + r_{1i} + \omega_{1i}, \]
\[ p_{2i} = \gamma_2 + \left\{ \theta_{21} s_{it} (pw_{21i} + pw_{22i}) - \theta_{11} s_{it} (pw_{21i} + pw_{22i}) \right\} / \text{Det}_i \]
\[ + r_{2i} + \omega_{2i}, \]
where
\[ \text{Det}_i = \theta_{11} \theta_{22} (pw_{11i} + pw_{12i}) \]
\[ \times (pw_{21i} + pw_{22i}) - \theta_{21} \theta_{21} (pw_{11i} + pw_{12i}) \]
\[ \times (pw_{21i} + pw_{22i}). \]  

Therefore, our estimation equations for the logit model are the demand equations in (3) and the pricing equations in (20). Note, however, that in the category-profit-maximization case the quantity \( m_0 \) in retailer margin cannot be separately identified from the cost intercept (\( \gamma_Y \)). Hence, the intercept estimate in the pricing equation in the category-profit-maximization model is the sum of the brand-specific cost and the per-unit profit from the outside good.

**Equivalence of the Constant-Margin Retailer Model and Retailer Coordination Model.** In this section we demonstrate that the constant-margin retailer model that measures HSI using the CV approach (that has been used in previous studies) is econometrically identical to an alternative model that ignores manufacturers and just measures the degree of coordination in pricing across brands by the retailer using the CV approach. To do this, we first show that the margin parameter \( (m) \) in the estimation equation for the constant-margin retailer is not identified, and we therefore derive the actual estimation equation that is fit on the data. Then we derive the estimation equation for the retailer coordination model and show that the two equations are equivalent.

Substituting the retailer reaction to wholesale prices for the constant-margin model in Equation (15), the estimation equation becomes
\[ p_t = \left\{ c_0 + \frac{1}{1 + m} \left\{ -\Theta \frac{\partial s_t}{\partial \rho_{p_t}} \right\} * I \right\} s_t (1 + m). \]

Hence,
\[ p_t = c_0 (1 + m) \left\{ -\Theta \frac{\partial s_t}{\partial \rho_{p_t}} \right\} * I s_t. \]  

Therefore, the final estimation equation in this case is
\[ p_t = \gamma_0 + \omega_t + \left\{ -\Theta \frac{\partial s_t}{\partial \rho_{p_t}} \right\} * I s_t. \]  

where \( \gamma' = \gamma(1 + m) \) and \( \omega' = \omega(1 + m) \). Of course, \( m \) cannot be identified from this estimation equation. This estimation equation generalizes the equation derived in Vilcassim et al. (1999) for their linear demand model to a nonlinear demand model.

We now show that the supply-side estimation equation for a model measuring the degree of coordination in pricing across brands using the CV approach is econometrically identical to (22). Here we parameterize the level of coordination in retail pricing across brands.

The objective function of a retailer maximizing profits from brand \( i \) in period \( t \) is given by
\[ \Pi_{it}^R = (p_{it} - w_i) s_{it} M_{it}, \quad i = 1, 2. \]  

The first-order conditions are given by
\[ \frac{\partial \Pi_{it}}{\partial p_{it}} = 0 \Rightarrow s_{it} + (p_{it} - w_i) \left\{ \frac{\partial s_{it}}{\partial \rho_{p_{it}}} + \frac{\partial s_{it}}{\partial \rho_{p_{jt}}} \right\} = 0, \]
\[ i = 1, 2; \quad j \neq i. \]

Denoting \( \theta_{21} = \rho_{p_{11}} / \rho_{p_{21}} \) and \( \theta_{12} = \rho_{p_{21}} / \rho_{p_{11}} \) for all \( t \), we can write the above equation in matrix form as
\[ s_t + \left( \Theta \frac{\partial s_t}{\partial \rho_{p_t}} \right) * I (p_t - w_t) = 0, \]
where
Structural Analysis of Manufacturer Pricing

and the \( \cdot \ast \) operator denotes element-by-element multiplication of a matrix and \( I \) is the identity matrix. We can interpret the \( \theta_{ij} \) terms as the measure of coordination in pricing across brands by the retailer. Therefore,

\[
p_{ij} = w_{ij} + \left( -\Theta \frac{\partial s_{ij}}{\partial p_{ij}} \ast I \right)^{-1} s_{ij},
\]

If we express \( w_{ij} \) in terms of factor prices and an error term, then the estimation Equation (25) is econometrically identical to (22). Hence, the ambiguity about what the estimates of the \( \theta_{ij} \) mean, i.e., whether they measure the extent of retail-pricing coordination or the degree of cooperation among manufacturers when retailers are nonstrategic and charge a constant margin.

3. Empirical Analysis

Data

We do our analysis on the yogurt and peanut butter categories on two EDLP stores from two different regional chains in a suburban market. These stores happen to be the two largest in this market. The data are for a period of 104 weeks during 1991–1993. In the yogurt market, we analyze competition between the two largest brands, Dannon and Yoplait. Together, they constitute about 82% of sales in this category in this market. In the peanut butter market, we analyze competition between the two largest brands in the market: Skippy and Jif. Together they constitute about 66% of sales in this category. The next largest national brand has less than 10% of market share in both categories in these stores. Private labels have a very small market share in these stores.

Because store-level data do not provide any information on the share of the “no purchase” alternative, we follow BGJ in using information from the household-level data to compute the share of “no purchase.” Assuming that the panel is representative of the universe of consumers, BGJ take the share of “no purchase” as the proportion of store visits by the panelists in the data that did not lead to a purchase in the category.

Estimation

Our estimation equations are the demand equations in (3) and the pricing equations in (20). As discussed earlier, details such as consumer promotions and advertising are not available in scanner data and are unobserved by the researcher and therefore captured by the demand error terms \( \xi_{it} \). However, manufacturers, retailers, and consumers have information on these, and therefore they affect the manufacturer’s and retailer’s choice of prices and the consumer’s decision to buy. Villas-Boas and Winer (1999) and BGJ offer evidence that this is indeed true. We do the same variation of the Hausman (1978) test that BGJ use and find price is endogenous. We use lagged price as an instrument in conducting this test.

Because features and displays are also strategic decisions made by the retailer, theoretically they should be considered endogenous. Putsis and Dhar (1999) found that features and displays are indeed endogenous. We test for endogeneity of features and display in two ways. First we follow Villas-Boas and Winer (1999) and look at the correlation between estimated residuals (using both three-stage least square (3SLS) and full-information maximum likelihood (FIML), where only price was treated as endogenous and feature and display were treated as exogenous) and feature and display. The correlations were not significant (\( p \) values > .2), indicating that there would be no endogeneity bias if feature and display were treated as exogenous. One may note that in our demand model, we allow for interaction effects between price and feature and between price and display. This in-

\[ \Theta = \begin{pmatrix} 1 & \theta_{12} \\ \theta_{21} & 1 \end{pmatrix} \]
interaction variable was crucial in eliminating the endogeneity bias problem.\(^\text{14}\) The second test we do is a variation of the Hausman test. We estimate two models with price treated as endogenous using 3SLS: (i) with feature and display as endogenous and using instruments (lagged feature and display) and (ii) with feature and display as exogenous. If feature and display were indeed exogenous, then both models would be consistent, but the first model’s estimates would have a higher variance because it is an instrumental variable method. However if feature and display were endogenous, only the first model would be consistent. If $q$ denotes the vector of parameters to be estimated, then the test statistic $(\hat{q}_1 - \hat{q}_2)'(\hat{V}_1 - \hat{V}_2)^{-1}(\hat{q}_1 - \hat{q}_2)$, where $\hat{V}$ is the estimated covariance matrix, is distributed as $\psi^2(\#q)$ distribution, where $\#q$ is the number of parameters in the vector $q$. We find that the null hypothesis of exogeneity cannot be rejected ($p = .17$).

This endogeneity of prices, however, implies that we still need to use an instrumental variables estimation procedure. Furthermore, errors across the supply and estimation equations are correlated due to price. We therefore need to use a simultaneous equation instrumental variables estimation procedure such as FIML or 3SLS. The major advantage of FIML estimation is that we do not need to provide instruments for prices, because the optimal instruments are generated within the estimation procedure. To compute the optimal instruments in FIML, however, we need to make the normal distribution assumption on the error terms. That is, we assume that $\xi_{1t}$, $\xi_{2t}$, $\omega_{1t}$, $\omega_{2t}$ are assumed multivariate normal. In 3SLS there is no need to make the multivariate normal assumption, but we need to provide instruments. We prefer FIML in this paper, because formal tests for nonnested models (Vuong 1989) are available using likelihood estimates. However, to check whether our results are robust to the choice of estimation procedure, we also estimated the model using nonlinear 3SLS, using lagged prices as instruments for prices. The results that we report are consistent with both types of estimation procedures. We, however, report only FIML-based estimates, because we use these to do our tests of model selection.

**Results**

**Model Selection: Choice of Functional Form.** We formally test whether the difference in log-likelihoods significantly favors one model using the Vuong’s (1989) test of model selection for nonnested models. The best-fitting multiplicative model in the yogurt category had a log-likelihood of $-343.09$ for Store 1 and $-423.9$ for Store 2. For the peanut butter category, the best-fitting multiplicative model had a log-likelihood of $-320.14$ for Store 1 and $-421.39$ for Store 2. Clearly, the logit demand model performed much better in terms of fit compared with the multiplicative demand model. This is especially true considering that there were three extra parameters in the multiplicative model compared with the logit model. We performed the Vuong’s test for this case, and reject the multiplicative model at $p < 0.001$. It therefore appears that VSS implication of the logit model fits the data better than VSC implication of the multiplicative demand model.

However, an alternative explanation for the poorer fit of the multiplicative model could be that it also fits the demand equations rather poorly, compared with the logit model. We therefore estimated only the demand models using both functional forms using 3SLS.\(^\text{15}\) The logit model had a higher sum of squared errors than the multiplicative model, indicating that the multiplicative model with a greater number of parameters fits the data better if we use purely the demand equations. Hence, the reduction in fit when we include the pricing equations can be attributed to the explanation that the supply-side implications of the logit model (VSS) fit the data better than the supply-side implications of the multiplicative model (VSC).

\(^\text{14}\)We briefly explain the intuition for why modeling the interaction can help in eliminating the endogeneity bias problem. Let $y_t = x_t \beta_1 + x_t \beta_2 + x_t \beta_3 + \epsilon$ be the true model. Suppose $x_t$ is endogenous, but $x_2$ is not. Instrumenting $x_1$ will eliminate the endogeneity bias, but suppose we only fit the model $y_t = x_t \beta_1 + x_t \beta_2 + \epsilon_t$, then $\epsilon_t = \epsilon + x_t \beta_3$. In this case we will find that $\text{cov}(\epsilon_t, x_t) \neq 0$ and $\text{cov}(\epsilon_t, x_2) \neq 0$. Therefore, it will seem that both variables are endogenous if we do not include the interaction term.

\(^\text{15}\)Note that we cannot use FIML if we do not have the same number of endogenous parameters and equations.
### Table 4  Model Likelihoods and the Vuong Test Statistics

<table>
<thead>
<tr>
<th>Manufacturer Interaction</th>
<th>Retailer Objective/Manufacturer-Retailer Interaction</th>
<th>Likelyhoods for Yogurt (Vuong Test Statistic)</th>
<th>Likelyhoods for Peanut Butter (Vuong Test Statistic)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Store 1</td>
<td>Store 2</td>
</tr>
<tr>
<td>Tacit collusion</td>
<td>Category profit maximization</td>
<td>-196.90</td>
<td>-247.27</td>
</tr>
<tr>
<td></td>
<td>Manufacturer Stackelberg</td>
<td>(—)</td>
<td>(—)</td>
</tr>
<tr>
<td></td>
<td>Category profit maximization</td>
<td>-206.19</td>
<td>-298.29</td>
</tr>
<tr>
<td></td>
<td>Vertical Nash</td>
<td>(2.269)</td>
<td>(2.101)</td>
</tr>
<tr>
<td></td>
<td>Brand profit maximization</td>
<td>-202.38</td>
<td>-292.19</td>
</tr>
<tr>
<td></td>
<td>Manufacturer Stackelberg</td>
<td>(2.028)</td>
<td>(1.972)</td>
</tr>
<tr>
<td></td>
<td>Brand profit maximization</td>
<td>-208.25</td>
<td>-321.17</td>
</tr>
<tr>
<td></td>
<td>Vertical Nash</td>
<td>(2.643)</td>
<td>(2.529)</td>
</tr>
<tr>
<td></td>
<td>Constant margin</td>
<td>-207.23</td>
<td>-309.35</td>
</tr>
<tr>
<td></td>
<td>Manufacturer Stackelberg</td>
<td>(2.529)</td>
<td>(2.318)</td>
</tr>
<tr>
<td>Bertrand competition</td>
<td>Category profit maximization</td>
<td>-203.66</td>
<td>-249.9</td>
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<td>(2.040)</td>
<td>(2.030)</td>
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<td>Category profit maximization</td>
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<td></td>
<td>Vertical Nash</td>
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<td>Brand profit maximization</td>
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<td>Manufacturer Stackelberg</td>
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<td>Vertical Nash</td>
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<td>(2.347)</td>
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<td>Constant margin</td>
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<td>-325.85</td>
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<tr>
<td></td>
<td>Manufacturer Stackelberg</td>
<td>(2.949)</td>
<td>(2.608)</td>
</tr>
</tbody>
</table>

Note: The Vuong test statistic is with respect to the best-fitting model (with the highest log-likelihood).

The VSS implication that retail passthrough is less than 100% appears to be intuitive for the two categories that we analyze, since yogurt or peanut butter are not usually used as loss-leaders to drive store traffic. This implication is particularly valid for this data set, because our earlier analysis revealed that yogurt and peanut butter prices had a very limited impact on competitors' sales. Hence, these categories are unlikely to have been used to build store traffic.

**Model Selection: Results for the Logit Model.** The likelihoods for the different models of the yogurt and peanut butter markets for the logit functional form are reported in Table 4. Based on the log-likelihoods it seems obvious that, for both stores, the manufacturer Stackelberg model where manufacturers are involved in tacit collusion and the retailer maximizes category profits fits best in both categories. The Vuong statistics for the different models are computed with respect to this best-fitting model. Since the Vuong statistic is distributed as a standard normal, the critical value for rejecting a model at a p value of 0.05 is 1.645. Given that all of the Vuong statistics are greater than 1.645, we can reject the other models in favor of the best-fitting model.

**Retailer Objective.** The stores that we analyze were part of large regional chains. Category management was becoming an increasingly popular concept at the beginning of the 1990s. In a survey of retailers, McLaughlin and Hawkes (1994) found that many large retailers that had the technological infrastructure to collect and analyze data had adopted the category management concept. Hence the result that retailer-maximized category profits has face validity. Our results also support the assumption widely made in the theoretical literature that retailers maximize category profits (Choi 1991, Lee and Staelin 1997).

**Manufacturer-Retailer Interaction (VSI).** The result that the manufacturer Stackelberg model best fits the data is consistent with the institutional practice.
in which manufacturers announce their wholesale prices and the retailers choose their retail prices as a response to these wholesale prices (Quelch and Farris 1983). This sequential approach has also been used in theoretical models of manufacturer-retailer pricing interactions (McGuire and Staelin 1983, Agrawal 1996).

Some theoretical research (e.g., Choi 1991) has put forth the argument that leadership in the manufacturer-retailer interaction may be a result of power balance between manufacturers and retailers. Using this argument, BGJ argue that vertical Nash is an appropriate assumption because increasingly there is balance of power between manufacturers and retailers. Our result that manufacturer Stackelberg leadership may be a reflection of the fact that the balance of power in these categories is in favor of manufacturers. Given the dominance of the two leading brands in these categories, our result also has substantial face validity on the basis of the power argument. In an interview by Manning et al. (1998), one executive says it tellingly: “We can’t walk away from the kind of shares that they have in some of the categories, so we have to work with them and we can’t afford not to promote these products.”

Table 5a Estimates for Best-Fitting Model in Yogurt Category

<table>
<thead>
<tr>
<th>Store 1</th>
<th>Store 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>SE</td>
</tr>
<tr>
<td>Intercept (Dannon®)</td>
<td>2.740</td>
</tr>
<tr>
<td>Intercept (Yoplait)</td>
<td>2.920</td>
</tr>
<tr>
<td>Feature</td>
<td>2.580</td>
</tr>
<tr>
<td>Display</td>
<td>2.122</td>
</tr>
<tr>
<td>Display + Price</td>
<td>-0.800</td>
</tr>
<tr>
<td>Price</td>
<td>-6.364</td>
</tr>
<tr>
<td>Marginal cost (Dannon®) + outside good retail margin</td>
<td>0.337</td>
</tr>
<tr>
<td>Marginal cost (Yoplait) + outside good retail margin</td>
<td>0.457</td>
</tr>
</tbody>
</table>

Table 5b Estimates for Best-Fitting Model in Peanut Butter Category

<table>
<thead>
<tr>
<th>Store 1</th>
<th>Store 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>SE</td>
</tr>
<tr>
<td>Intercept (Skippy®)</td>
<td>10.414</td>
</tr>
<tr>
<td>Intercept (Jif®)</td>
<td>10.523</td>
</tr>
<tr>
<td>Feature</td>
<td>3.470</td>
</tr>
<tr>
<td>Display</td>
<td>14.400</td>
</tr>
<tr>
<td>Feature + Price</td>
<td>-4.942</td>
</tr>
<tr>
<td>Display + Price</td>
<td>-12.544</td>
</tr>
<tr>
<td>Price</td>
<td>-10.539</td>
</tr>
<tr>
<td>Marginal cost (Skippy®) + outside good retail margin</td>
<td>0.718</td>
</tr>
<tr>
<td>Marginal cost (Jif®) + outside good retail margin</td>
<td>0.714</td>
</tr>
</tbody>
</table>

16The author notes that Lee and Staelin (1997) show that Stackelberg leadership leads to greater profitability (and therefore implies greater power) only in the case of demand models characterized by VSS. The power balance argument the author makes holds only because
Manufacturer-Manufacturer Interaction (HSI). A cooperative outcome (relative to Bertrand competition) can be achieved in a noncooperative game by using appropriate punishment strategies when firms deviate from cooperative behavior. Such cooperative behavior is easier to sustain under certain market conditions than others. Besanko et al. (1996) provide an excellent discussion on how such coordination can be achieved in a noncooperative setting. A highly concentrated market facilitates cooperative behavior. This is because coordination is easier among a small number of firms. Further, detection of deviations from cooperative behavior is relatively easy, making the threat of punishment strategies more credible, thus enforcing cooperation. Because Dannon® and Yoplait have a high concentration in the yogurt market (82% in our data), it is not surprising that these firms behave cooperatively relative to Bertrand competition. The same arguments hold for Skippy® and Jif®, who collectively have over 66% market share in the stores that we analyzed.

Demand and Cost Estimates for the Best-Fitting Model. We report the results of our estimation for the best-fitting models in the yogurt and peanut butter category in Table 5a and 5b (see p. 258), respectively. All of the coefficients have the expected signs, giving the analysis substantial face validity.

As expected, features and display have a positive effect on sales in both categories and stores. Price has a negative effect on sales. In the final estimations we did not estimate brand-specific price coefficients, because it turned out in the estimation that they were not significantly different from each other when we incorporate the price-feature and price-display interaction effects. It is well known that a decrease in price when accompanied by a feature or display has a greater impact on sales than their separate effects. This was confirmed in that the interaction terms between price and display and between price and feature are negative and significant.

The demand intercept for Yoplait is marginally higher than for Dannon®. However, the estimate of manufacturing cost plus retailer margin from outside good for Dannon® is about 30% lower than that of Yoplait. This implies that Dannon® is the low-cost leader. These results are similar to those of BGJ, even though our estimates are on a very different data set. However, the differences in the estimates of manufacturing cost plus retailer margin from the outside good indicates that Store 1 gets a greater margin from the outside good in the yogurt category.

In the case of the peanut butter category, the demand intercept for Skippy® is marginally lower than for Jif®. The estimate of manufacturing cost plus retailer margin from outside good for Skippy® is marginally lower than that of Jif®. However, the differences in the estimates of manufacturing cost plus retailer margin from the outside good indicate that Store 2 gets a greater margin from the outside good in the peanut butter category.

We report estimates of elasticities for the two categories in Table 6a and 6b. Across both stores, we find that Yoplait has greater price elasticity than Dannon®. Further, Yoplait has greater cross-elasticity with respect to Dannon®’s prices than vice versa. This indicates a very strong market position for Dannon®, and it is not surprising when we consider that Dannon® is usually regarded as “synonymous with yogurt.”

In the peanut butter category, across both stores, Skippy® has greater price elasticity than Jif®. Further, Skippy® has greater cross-elasticity with respect to Jif®’s prices than vice versa. This indicates a stronger market position for Jif® than for Skippy®. Our results in the peanut butter market are consistent

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17According to the teaching case YoplaitUSA in the marketing text by Berkowitz et al. (1999, p. 607), some serious concerns for YoplaitUSA in 1993 (the period of our analysis) were “(i) Retail Prices: Yoplait’s prices for a six ounce cup was higher on some lines than competitor’s eight ounce cups. For example, the prices on Yoplait’s 4 pack are about 20% higher per cup than Dannon®’s and Kraft’s 6 pack. (ii) Low gross margins: Margins have declined, at least partly because of high production and overhead costs.” The concern about the relatively high production costs and its impact on prices give face validity to our estimates.

structural analysis of manufacturer pricing

Table 6a Elasticity Estimates for Yogurt Category

<table>
<thead>
<tr>
<th></th>
<th>Store 1</th>
<th>Store 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of Dannon w.r.t. Dannon® price</td>
<td>-4.293</td>
<td>-4.601</td>
</tr>
<tr>
<td>Elasticity of Yoplait w.r.t. Dannon® price</td>
<td>0.286</td>
<td>0.537</td>
</tr>
<tr>
<td>Elasticity of Dannon w.r.t. Yoplait price</td>
<td>-5.420</td>
<td>-6.462</td>
</tr>
<tr>
<td>Elasticity of Yoplait w.r.t. Yoplait price</td>
<td>0.640</td>
<td>1.289</td>
</tr>
</tbody>
</table>

Table 6b Elasticity Estimates for Peanut Butter Category

<table>
<thead>
<tr>
<th></th>
<th>Store 1</th>
<th>Store 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of Skippy w.r.t. Skippy® price</td>
<td>-11.026</td>
<td>-8.755</td>
</tr>
<tr>
<td>Elasticity of Jif w.r.t. Skippy® price</td>
<td>2.328</td>
<td>2.118</td>
</tr>
<tr>
<td>Elasticity of Skippy w.r.t. Jif® price</td>
<td>-10.505</td>
<td>2.583</td>
</tr>
<tr>
<td>Elasticity of Jif w.r.t. Jif® price</td>
<td>2.488</td>
<td>-8.370</td>
</tr>
</tbody>
</table>

Table 7 CV Estimates for the Logit Model: Category-Profit-Maximizing Retailer and Constant-Margin Retailer

<table>
<thead>
<tr>
<th></th>
<th>Yogurt</th>
<th>Peanut Butter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category profit maximizing retailer $\theta_1$</td>
<td>0.122</td>
<td>0.151</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>0.154</td>
<td>0.399</td>
</tr>
<tr>
<td>Constant margin retailer $\theta_3$</td>
<td>0.232</td>
<td>0.297</td>
</tr>
<tr>
<td>$\theta_4$</td>
<td>0.252</td>
<td>0.454</td>
</tr>
</tbody>
</table>

4. Conclusion

In this paper, we empirically inferred the VSI between manufacturers and retailers and the HSI between manufacturers simultaneously. The approach is particularly appealing because it can be used in the common situation in which the wholesale price information of all competitors is not available. We showed analytically that the simplifying assumption that a retailer charges a constant margin (used in earlier research) may misinterpret category management behavior by the retailer to be cooperative behavior by manufacturers. Consistent with the intuition from the analytical result, we find that the estimated cooperation among manufacturers is exaggerated for the constant-margin model, highlighting the need to simultaneously model and infer the VSI when analyzing the HSI.

Implications of the Identification Problem. We have shown that the constant-margin retailer model is econometrically identical to a model measuring the degree of retail coordination. Hence, intuitively we should expect that CVs estimated for the HSI might be exaggerated under the assumption of a constant-margin retailer compared with CV estimates when the retailer strategically maximized category profits with the manufacturer as the Stackelberg leader. We test this intuition by comparing the CV estimates for the two models. The CV estimates for the two models in both categories and stores are reported in Table 7. CV estimates are consistently higher, exaggerating the degree of cooperation among manufacturers for the constant-margin retailer model. This is consistent with the intuition we gain from the analytical results. The empirical result further underscores the need to properly model the VSI in inferring the HSI.

4. Conclusion

In this paper, we empirically inferred the VSI between manufacturers and retailers and the HSI between manufacturers simultaneously. The approach is particularly appealing because it can be used in the common situation in which the wholesale price information of all competitors is not available. We showed analytically that the simplifying assumption that a retailer charges a constant margin (used in earlier research) may misinterpret category management behavior by the retailer to be cooperative behavior by manufacturers. Consistent with the intuition from the analytical result, we find that the estimated cooperation among manufacturers is exaggerated for the constant-margin model, highlighting the need to simultaneously model and infer the VSI when analyzing the HSI.

Since packaged goods manufacturers spend a sizable share of their marketing-mix budget on trade promotions (Bucklin and Gupta 1999), several decision support systems (DSSs) have been developed to aid manufacturers in their promotion planning (e.g., Neslin et al. 1995, Tellis and Zufryden 1995, Midgley et al. 1997, Silva-Risso et al. 1999). Our results are of substantive import in guiding the assumptions that go into such DSSs. For example,

1. DSSs usually assume Bertrand behavior among manufacturers (e.g., Midgley et al. 1997). By explicitly testing for Bertrand behavior as well as cooperative behavior among manufacturers, we found that manufacturers in these markets tend to be more cooperative. A DSS that assumes Bertrand pricing behavior would have led to more aggressive pricing than would be warranted by the actual behavior of the market participants.

2. Many DSSs assume manufacturer Stackelberg behavior between manufacturers and retailers (e.g.,

3Since FIML does not converge for the CV models, we use 3SLS for estimation.
Tellis and Zufryden (1995). However, the theoretical literature does not provide clear guidance on whether such an assumption is appropriate; it shows that a change in assumption can have substantive implications for pricing behavior of the channel member. Given the apparent shift in power to retailers, this question becomes even more important to managers. We found that for the categories and markets that we analyzed, the manufacturer Stackelberg assumption is appropriate.

3. DSSs have used different assumptions about retailer-pricing rules. Silva-Risso et al. (1999) assume that retailers follow a simple pass-through rule and manufacturers incorporate that assumption when deciding their trade deal. Tellis and Zufryden (1995) assume that the retailer’s objective is to maximize category profits. By testing both these specific assumptions, we found that both retailers’ objective is to maximize category profits.

4. DSSs also make assumptions about the functional form of demand. While functional forms of demand have been evaluated in terms of the fit of the model to sales data, its ability to accommodate the behavioral implications of the supply side have not been evaluated. We found that the logit model performed better than the multiplicative model in its ability to accommodate the strategic behavior of firms for the categories that we analyze.

We now discuss some of the limitations in this paper and possible avenues for future research. First, it would be useful to validate the methodology by checking how well the model predicts actual wholesale prices by estimating the model on a data set with wholesale prices. Second, the current estimation method assumes that the estimation equations can be derived in closed form. To allow for greater flexibility in the functional forms and also to extend the analysis to a larger number of brands, we need to enable estimation even when closed-form estimation equations cannot be obtained.

Clearly more research in other categories at other retail stores is needed before we can develop a body of evidence that will enable us to understand the impact of supply characteristics (for example, degree of concentration in market, manufacturer-retailer power balance, availability of private labels) and demand characteristics of category (e.g., high inertia or variety seeking, stockpiling, average price sensitivity, loss leader) on the inference of strategic interactions. For example, Cotterill and Putsis (2001) find that the Vertical Strategic Interactions are different across categories, while we found manufacturer Stackelberg behavior in both categories we analyze. Understanding the determinants of these differences should be of interest in future work.

Putsis and Dhar (1998) have also done a cross-category analysis of how competitive behavior between private labels and manufacturers changes as a function of the category and market characteristics. Sudhir (2001) develops hypotheses combining arguments from game-theoretic research and the ability-motivation paradigm (Boulding and Staelin 1995) about how competitive behavior in different segments of the auto market will differ, depending on the demand-and-supply characteristics of these markets, and then tests these hypotheses. Such an analysis will help provide deeper insights into the determinants of horizontal and vertical strategic interactions and appropriateness of functional forms of demand.

The issue of retail competition was not found to be particularly important in the yogurt and peanut butter categories. Future research into categories such as detergents and soda, which are known to have an impact on store traffic, is needed to study the role of strategic retail competition on retailer-pricing behavior. Also, retail competition may be at the basket level across a number of categories (Bell and Lattin 1998).

We have limited ourselves to the inference of manufacturer-retailer interaction only when the retailer is EDLP. However, accounting for unobserved heterogeneity among consumers in the demand model can enable us to investigate how loyalty and switching behavior, asymmetric responses to lower-price-tier brands, etc., may cause high-low pricing behavior. We are investigating how to use household-level data to infer heterogeneity distributions and how to incorporate this information into the aggregate model so that we can extend our analysis to the case of high-low retailers. Horsky and Nelson (1992) and Gold-
berg (1995) provide good starting points for this stream of research. Recently, there has been interest in recovering heterogeneity from aggregate data (Berry et al. 1995, Kim 1995, Nevo 2001, Sudhir 2001, Chintagunta 1999). In recent work, Villas-Boas and Zhao (2000) address many of the issues in modeling channel behavior with household-level data.

We have not modeled any kind of dynamics that affect demand or supply. Forward buying by consumers, habit persistence, variety seeking, advertising wearout, etc., can have intertemporal demand effects, which in turn can affect supply-side behavior. Forward buying by retailers is another supply-side intertemporal effect. While the use of efficient consumer response (ECR) has significantly reduced forward buying on the part of retailers, it is still a significant component of how a retailer reacts to a trade deal. This is an important issue for future research. For a more comprehensive set of methodological and substantive questions that await research in this area, we direct the reader to Kadiyali et al. (2001).

Acknowledgments
This paper is based on an essay from the author’s 1998 Ph.D. dissertation at Cornell University. He thanks the members of the dissertation committee, especially Vithala R. Rao and Vrinda Kadiyali, for their extensive comments and suggestions. The author also thanks Pradeep Chintagunta, Bill Puttis, and Robert Shoemaker for their comments, as well as the seminar participants at Chicago, Cornell, Dartmouth, Duke, HKUST, INSEAD, NYU, Purdue, Rochester, Stanford, SUNY Buffalo, Toronto, UCLA, and Yale. The author has benefited greatly from the review process and thanks the editor, the area editor, and the two anonymous reviewers for their input in revising this paper. The usual disclaimer applies.

Appendix

Reactions for Category-Managing Retailer
Taking the derivatives of the retail prices in Equation (8) of the paper,

\[
\frac{\partial p_{1u}}{\partial w_{1u}} = 1 + \frac{1}{s_u} \left[ \frac{1}{\alpha_2^{(1-s_1)}} \left( \frac{\partial s_{1u}}{\partial p_{1u}} \frac{\partial p_{1u}}{\partial w_{1u}} + \frac{\partial s_{2u}}{\partial p_{1u}} \frac{\partial p_{2u}}{\partial w_{1u}} \right) + \frac{1}{\alpha_2^{(1-s_2)}} \left( \frac{\partial s_{1u}}{\partial p_{2u}} \frac{\partial p_{1u}}{\partial w_{1u}} + \frac{\partial s_{2u}}{\partial p_{2u}} \frac{\partial p_{2u}}{\partial w_{1u}} \right) \right]
\]

Substituting the share derivatives with respect to prices from Equation (4), we get

\[
\frac{\partial p_{1u}}{\partial w_{1u}} = 1 - \frac{s_{1u}}{(1 - s_1 - s_2)} \frac{\partial p_{1u}}{\partial w_{1u}} - \frac{s_{2u}}{(1 - s_1 - s_2)} \frac{\partial p_{2u}}{\partial w_{1u}} \tag{A1}
\]

Similarly,

\[
\frac{\partial p_{2u}}{\partial w_{1u}} = -\frac{s_{1u}}{(1 - s_1 - s_2)} \frac{\partial p_{1u}}{\partial w_{1u}} - \frac{s_{2u}}{(1 - s_1 - s_2)} \frac{\partial p_{2u}}{\partial w_{1u}} \tag{A2}
\]

Solving (A1) and (A2) for \(\frac{\partial p_{1u}}{\partial w_{1u}}\) and \(\frac{\partial p_{2u}}{\partial w_{1u}}\) we have \(\frac{\partial p_{1u}}{\partial w_{1u}} = 1 - s_1\) and \(\frac{\partial p_{2u}}{\partial w_{1u}} = -s_2\). By symmetry, \(\frac{\partial p_{1u}}{\partial w_{2u}} = 1 - s_2\) and \(\frac{\partial p_{1u}}{\partial w_{2u}} = -s_1\).

These reactions are summarized in Equation (12) of the paper.

Reactions for Brand-Managing Retailer
Taking the derivatives of the retail prices in Equation (10) of the paper,

\[
\frac{\partial p_{1u}}{\partial w_{1u}} = 1 + \frac{1}{s_u} \left[ \frac{1}{\alpha_2^{(1-s_1)}} \left( \frac{\partial s_{1u}}{\partial p_{1u}} \frac{\partial p_{1u}}{\partial w_{1u}} + \frac{\partial s_{2u}}{\partial p_{2u}} \frac{\partial p_{2u}}{\partial w_{1u}} \right) + \frac{1}{\alpha_2^{(1-s_2)}} \left( \frac{\partial s_{1u}}{\partial p_{1u}} \frac{\partial p_{1u}}{\partial w_{1u}} + \frac{\partial s_{2u}}{\partial p_{2u}} \frac{\partial p_{2u}}{\partial w_{1u}} \right) \right]
\]

Substituting the share derivatives with respect to prices from Equation (5), we get

\[
\frac{\partial p_{1u}}{\partial w_{1u}} = 1 - \frac{s_{1u}}{(1 - s_1)} \frac{\partial p_{1u}}{\partial w_{1u}} - \frac{s_{2u}}{(1 - s_1)} \frac{\partial p_{2u}}{\partial w_{1u}} \tag{A3}
\]

\[
\frac{\partial p_{2u}}{\partial w_{1u}} = \frac{s_{1u}}{(1 - s_1)} \frac{\partial p_{1u}}{\partial w_{1u}} - \frac{s_{2u}}{(1 - s_1)} \frac{\partial p_{2u}}{\partial w_{1u}} \tag{A4}
\]

Solving (A3) and (A4) for \(\frac{\partial p_{1u}}{\partial w_{1u}}\) and \(\frac{\partial p_{2u}}{\partial w_{1u}}\), we have

\[
\frac{\partial p_{1u}}{\partial w_{1u}} = \frac{(1 - s_1)(1 - s_2)}{(1 - s_1)(1 - s_2) - (s_1 s_2)^2}
\]

\[
\frac{\partial p_{2u}}{\partial w_{1u}} = \frac{(1 - s_1)(1 - s_2)}{(1 - s_1)(1 - s_2) - (s_1 s_2)^2}
\]

By symmetry,

\[
\frac{\partial p_{1u}}{\partial w_{2u}} = \frac{(1 - s_2)(1 - s_1)}{(1 - s_1)(1 - s_2) - (s_1 s_2)^2}
\]

\[
\frac{\partial p_{2u}}{\partial w_{2u}} = \frac{(1 - s_2)(1 - s_1)}{(1 - s_1)(1 - s_2) - (s_1 s_2)^2}
\]

These reactions are summarized in Equation (13) of the paper.
References


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