

Previous research has suggested that households and individuals may possess multiple preferences for a given product category. These multiple preferences may be the result of multiple individuals in a household, different uses and usage occasions, and/or variety seeking. As such, single ideal point models that assume a single invariant ideal point may be operating from a false and misleading assumption. We propose a multiple ideal point model to capture these multiple preference effects. The basic premise of the model is that consumers may possess a set of ideal points, each of which represents a distinct preference. At any given purchase occasion, one of these points is "activated" with some probability, and choices are made with respect to its characteristics. In this article, the authors assess the multiple ideal point model and an associated estimation procedure with respect to the model's ability to recover a true choice structure. The authors then empirically test the model on Information Resources Inc. panel data from the powdered soft drink category. The authors discuss the results and introduce directions for further research.

A Multiple Ideal Point Model: Capturing Multiple Preference Effects from Within an Ideal Point Framework

Joint-space maps have become an important graphical tool in product design and positioning decisions by identifying consumer preferences and the products that appeal to consumers (Green 1975; Johnson 1971). In particular, ideal point models enable managers to examine how close each product is to a consumer's ideal point. Although their use is common, these models are limited in that they generally assume that consumers have a single, well-defined preference. This assumption may be violated if the household is the unit of analysis, because households often consist of multiple consumers who have their own distinct preferences (Gupta and Steckel 1993; Kahn, Morrison, and Wright 1986). In addition, situational preferences may be generated by differing uses and usage occasions (Belk 1979; Laurent 1978), as well as by variety seeking (Bass, Pessemier, and Lehmann 1972; McAlister and Pessemier 1982). In each case, an individual's or a household's preferences may vary across purchase occasions. Under these conditions, the use of a model that presumes a single ideal product may be misleading.

We present a multiple ideal point model (MIPM) for both the internal and external analysis of household preferences. The MIPM uses a product-switching matrix as its input and allows for the existence of more than one well-defined preference (ideal point) per household or individual. The identification of these multiple preferences provides a clearer picture of the forces that drive the choice process when the single preference assumption is inappropriate. In addition, the use of product-switching data enables us to use standard scanner-panel data, which is often readily available in consumer packaged goods markets.

Maps generated by the MIPM can provide managers with important insights related to positioning and segmentation. Household-specific sets of ideal points can be clustered into segments with relatively similar preference structures. Correlations between these ideal point clusters and various demographic variables (e.g., number of members in the household) may be useful in determining optimal strategies for altering product features or in identifying new product or bundling opportunities when targeting specific market segments.

The basic premise of the MIPM is that a household may have a set of ideal points, which originate from multiple individuals, different usage occasions, and/or variety seeking. We are not specifically concerned with the precise sources here. We only assume that decisions at each choice occasion are based on one of these ideal points, which is "activated" with

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some probability. The remaining "inactive" ideal points and their corresponding preferences are then irrelevant for that particular choice occasion. In a sense then, the MIPM is a type of mixture or within-individual latent class model in which the ideal points define the latent classes.

We begin our discussion by providing a brief review of the relevant literature. In the next section, we develop the proposed MIPM in detail and suggest an estimation procedure. We then discuss the results of exploratory simulations that were designed to assess the ability of the estimation procedure to recover the true parameters of the model. Next, we present an application of the MIPM to powdered soft drinks. We conclude by discussing the results, implications, potential limitations, and directions for further research.

A REVIEW OF EXISTING MODELS AND RELATED APPROACHES

The essential feature of the MIPM is the possible existence of multiple preferences as reflected by more than a single ideal point. As such, the MIPM draws its motivation from previous work on ideal point models and research streams that imply multiple or varying preferences. Furthermore, the concept of activated ideal points also demands that we address the literature on latent preference structures.

Sources of Multiple Ideal Points at the Household Level

Several factors may conceal a household's true underlying preference structure. For example, Kahn, Morrison, and Wright (1986) conclude that under certain standard assumptions, the aggregate behavior of a household always seems more zero order than the behavior of a typical individual in that household. Therefore, the aggregation of individual preferences to the household level may lead to erroneous conclusions. Our proposed model attempts to correct for this by identifying each of the distinct preferences that drive the revealed household choices.

Gupta and Steckel (1993) attempt to recover information about individual preferences from household data. By focusing on the distribution of run lengths within a purchase string, Gupta and Steckel seek to identify individual preferences and the process by which they are combined. However, the feasibility of their method is constrained by the number and the need for a priori specification of potential aggregation mechanisms and of individual preference structures. Nevertheless, their empirical work demonstrates that information regarding individuals can be inferred from household-level panel data.

Multiple preferences may also be generated by various uses and usage occasions (Dubow 1992; Laurent 1978). A product class can have many uses (e.g., wine for drinking and cooking), and a consumer may have different preferred alternatives for each. Acknowledging the need to account for these variations, researchers have both developed taxonomies of usage situations (Belk 1979; Srivastava, Shocker, and Day 1978; Stefflre 1979) and considered the possibility of situation-specific ideal points (DeSarbo and Carroll 1985; Hagerty 1980; Holbrook 1984). Unfortunately, the usefulness of these situation-specific models is restricted by the need to determine specific usage situations *a priori* (DeSarbo and Carroll 1985; Holbrook 1984). Bucklin and Srinivasan (1991) have developed a survey-based procedure for measuring interbrand substitutability that allows for the possibility of multiple usage situations (as

well as multiple users) and does not require such a predetermined specification. These "substitutabilities" can be treated as similarity measures and can then be used as input to a similarity scaling procedure.

A final potential source of multiple ideal points is variety seeking. Variety seeking implies that people may have different ideal points (preferences) at different times. Some authors (e.g., Kahn, Kalwani, and Morrison 1986) model preference shifts as random (Markov) processes. Others (e.g., Bawa 1990; Lattin 1987; McAlister 1982) suggest that preference shifts arise deterministically from discrepancies between actual and ideal inventory levels of certain attributes. The multiattribute perspectives of these approaches make them suitable for modeling reactions to specific product offerings. However, because attribute inventory levels are time dependent and based on prior consumption, these models may not be as effective for the *a priori* design and positioning of products. In addition, they do not lend themselves to the graphical maps that managers find so appealing.

In contrast to many of the approaches described in this section, our model does not require the *a priori* specification of usage contexts. Furthermore, it can graphically provide useful product strategy insights that do not depend on determining personal inventory levels.

Ideal Point Models

In the past 50 years, various ideal point models, at both the individual and aggregate levels, have been introduced into the psychology and marketing literature (e.g., Coombs 1950; Elrod 1988; Srinivasan and Shocker 1973). Coombs (1950) has been credited with introducing the individual-level deterministic ideal point model. However, as we have discussed, the single preference assumption may be unduly restrictive. Given their familiarity and widespread use, we do not go into detail about multidimensional scaling (MDS) techniques used for estimating ideal point models. We refer the reader to Green, Carmone, and Smith (1989) for a review. Here, we simply allude to data collection issues and the modeling of "nonunique" ideal points.

Various forms of data may be used for estimating ideal point models and joint-space maps, including dominance data (e.g., preference rankings/ratings) and choice data (e.g., pick-any/J, paired comparisons). Recent efforts using various types of individual choice and scanner-panel data have also emerged (Chintagunta 1994; DeSarbo and Hoffman 1987; Elrod 1988; Elrod and Keane 1995; Erdem 1996). Consistent with this trend, our approach uses preference data and relies on switching matrices derived from household panel data.

Beyond these data issues, research has also produced "map-based" models that allow for choice behavior with varying preferences. In addition to the (predefined) situation-specific ideal point models described previously (DeSarbo and Carroll 1985; Hagerty 1980; Holbrook 1984), Carroll (1980), DeSoete and Carroll (1983), and DeSoete, Carroll, and DeSarbo (1986) have all proposed models in which preferences are sampled from a multivariate normal distribution. This sampling process allows preferences to change or "wander" across choice occasions. However, the unimodal formulation still may not be suitable for capturing distinct multiple preferences.

Latent Class Preference Models

Latent class methodologies (e.g., Lazarsfeld and Henry 1968) represent one manner in which researchers have

accounted for heterogeneous preferences at the market level (e.g., Grover and Srinivasan 1987; Kamakura and Russell 1989). The basic view is that a market is composed of S distinct segments, and each segment is characterized by its own set of parameters. Consumers/households are then assumed to belong to these S segments according to a probability distribution. Probabilistic segment membership allows each household to belong to various segments with a relative frequency. This notion is reflected in our MIPM by a probabilistic process that governs ideal point activation.

Latent class approaches have also been applied to MDS (LCMDS), in which members of each particular segment are assumed to share the same perceptions and/or preferences. Researchers have estimated LCMDS models using various forms of data, including proximity data (e.g., Winsberg and De Soete 1993), dominance data (e.g., DeSarbo, Howard, and Jedidi 1991), and choice data (e.g., DeSoete 1990). For a detailed discussion of LCMDS, we refer the reader to the review by DeSarbo, Manrai, and Manrai (1994).

These LCMDS analyses produce maps on which market segments (i.e., the latent classes) rather than individuals are represented, and each individual has some probability or "degree of membership" of being in a given segment. The MIPM differs, because it is estimated for the individual or household. The latent classes are then defined by which ideal point governs a given purchase occasion rather than to which segment an individual or household belongs. In the MIPM, the individual- or household-level results are then aggregated to form market-level conclusions. As such, the MIPM provides additional household-level insights (e.g., the multiple within-household ideal points), while maintaining a similar ability to draw segment-level inferences.

Two approaches that follow the general philosophy of a preference structure being evoked on each choice occasion with a certain probability are Wedel and Steenkamp's (1989, 1991) fuzzy clusterwise regression (FCR) and Poulsen's (1990) latent Markov model. In Wedel and Steenkamp's work, the probability of evoking a preference structure is modeled as a fuzzy partition, in which preferences are a linear function of the attribute values. Poulsen's (1990) model conceptualizes the probability as a Markov process and preference structures as vectors of choice probabilities. In this article, we also model the probability as a Markov process, but here the preferences are represented by ideal point models rather than by choice probabilities.

Unfortunately, neither Wedel and Steenkamp's (1989, 1991) nor Poulsen's (1990) approach produces a visual map. Both also require that preference structures exist at the segment level and do not allow for individual and household variation. Furthermore, Wedel and Steenkamp's FCR uses survey data, and Poulsen's model examines the binary buy/no buy decision. In contrast, our approach produces a joint space map based on panel data that allows for varying individual- and household-level preferences.

THE MIPM

We propose an individual- and household-level model that provides a flexible framework to identify the number of distinct preferences (ideal points), their locations, and the probabilities with which they are activated. By allowing for more than one ideal point, the MIPM accounts for the possibility of multiple preference contexts.

A Collection of Single Ideal Point Models

For each choice occasion, there is some probability that a given ideal point may be active. When the ideal point is activated, the subsequent choice is based solely on the relative spatial locations of the alternatives with respect to the active ideal point. As such, we posit a formulation for choice probabilities to be

$$(1) \quad P[i|IP(k)] = \frac{\left\{ \sum_a [i_a - IP(k)_a]^2 \right\}^{-1/2}}{\sum_j \left\{ \sum_a [j_a - IP(k)_a]^2 \right\}^{-1/2}},$$

where

$P[i|IP(k)]$ = the probability of choosing alternative i given that ideal point k is activated,

i_a, j_a = alternative i 's, j 's perceived locations on dimension a , and

$IP(k)_a$ = ideal point k 's location on dimension a .

If the reciprocal of the Euclidean metric distance between the ideal point and the product is used as a measure of utility, smaller distances (i.e., greater similarity) result in greater probabilities of choice. Alternative metrics (e.g., distance density; DeSarbo and Manrai 1992) could be used here if so desired. The main point is that the likelihood of choosing an alternative (i.e., preference) will differ depending on which ideal point is activated. If the estimated ideal points are virtually indistinguishable, the MIPM collapses into a single ideal point as a special case.

Modeling Transitions Between Ideal Points

Following Poulsen (1990), we capture the process of switches between active ideal points in a stochastic manner using a simple first-order Markov matrix, which allows for asymmetric patterns of switching among products. The general form of this transition matrix is given by

	IP(1)	IP(2)	IP(3)	...	IP(L)
IP(1)	p_{11}	p_{12}	p_{13}	...	p_{1L}
IP(2)	p_{21}	p_{22}	p_{23}	...	p_{2L}
IP(3)	p_{31}	p_{32}	p_{33}	...	p_{3L}
:	:	:	:	..	:
IP(L)	p_{L1}	p_{L2}	p_{L3}	...	p_{LL}

where $IP(k)$ = ideal point k , p_{kl} = probability of switching to ideal point l given that ideal point k was activated last, and

$$(2) \quad \sum_l p_{kl} = 1, \text{ for all } k \leq L \text{ and } 0 \leq p_{kl} \leq 1.$$

In the case in which $p_{11} = p_{22} = \dots = p_{LL}$ for all l and L represents the total number of ideal points, the Markov chain becomes zero order. Thus, the zero-order model is a restricted version of the MIPM.

This basic framework is consistent with many of the variety-seeking and reinforcement explanations of switching behavior. In general, if a consumer switches to a new ideal point, that consumer may be more likely to make a different selection. Thus, off-diagonal probabilities (p_{kl}) that are greater than the on-diagonals (p_{kk}) are consistent with variety-seeking behavior. In contrast, when the p_{kk} are greater than p_{kl} , that is consistent with loyal behavior, mean-

ing that the consumer rarely switches. A third possibility is that one of the ideal points (say, l) is primarily used over all others ($p_{kl} > p_{k'l}$, where $l \neq l'$ and for all k). Because the consumer is more likely to operate from a particular ideal point, the others may represent a sort of variety-seeking behavior.

Probabilities of Switching

We first motivate the formula for the probability of choosing alternative i and then j on successive occasions, $P(ij)$. Consider the probability that a consumer selects alternative i from a given ideal point k , represented by $P[i|IP(k)]$. If we multiply this by the probability of subsequently making choice j after accounting for all possible ideal point transitions (as is represented by the underlined portion of Equation 3), then we obtain the probability of a switch from i to j given that the consumer starts at ideal point k . If this is multiplied by the probability of originally being in ideal point k , given by its steady state probability and represented by $P[IP(k)]$, and is then summed over all possible ideal points, we arrive at $P(ij)$:

$$(3) \quad P(ij) = \sum_k P[IP(k)] \\ \times \left(P[i|IP(k)] \times \left\{ \sum_l p_{kl} \times P[j|IP(l)] \right\} \right).$$

The steady state probability of being in ideal point k is defined as follows:

$$(4) \quad P[IP(k)] = \sum_l p_{lk} \times P[IP(l)]$$

and

$$(5) \quad \sum_{k=1}^L P[IP(k)] = 1, \text{ for all } k \leq L \text{ and } 0 \leq P[IP(k)] \leq 1.$$

The Estimation Procedure

We make the standard assumption that consumers share homogeneous perceptions of the market but have heterogeneous preferences (Green and Wind 1973). This view facilitates market-level inferences by placing all consumers into the same product space. Furthermore, data collection for generating product locations may be carried out independently of the scanner panel if it is desired and/or necessary (i.e., an external analysis of preferences).

The only data requirements for the MIPM are individual- and household-level switching matrices. We then use these data to determine the product locations and the MIPM parameters for each household by maximizing Equation 6, which we derive by summing the log-likelihoods of all possible product transitions for each household, over all households:

$$(6) \quad LL(\theta_N, \theta_M) = \sum_{n=1}^N \sum_{i=1}^M \sum_{j=1}^M R(ij)_n \\ \times \log[P(ij)_n],$$

where

θ_N = the vectors of parameter estimates for all households (i.e., ideal point locations $[IP(k)_a]$ and transition probabilities $[p_{kl}]$),

θ_M = the vector of estimates for the market parameters (i.e., product locations $[i_a]$),

n indexes the N households included in the analysis, i, j indexes the M available products or alternatives in the market,

$R(ij)_n$ = total number of observed transitions from product i to j for household n , and

$P(ij)_n$ = probability of switching from i to j for household n as defined in Equations 1 to 5.

Our assumption of homogeneous perceptions enables us to include all households in the estimation. However, because of the indeterminacies associated with translation, scale, and rotation of the joint space, it is necessary to fix some of the product location parameters of the model. For an n -dimensional space, we need to fix n parameters for translational invariance (this is often done by either fixing the origin to be the centroid of the spatial configuration or fixing one brand at the origin). We also fix $n(n - 1)/2$ parameters for rotational invariance. In addition, we fix one parameter for scale invariance (i.e., to fix the scale of the axes).¹ Therefore, the number of parameters that must be fixed for a one-, two-, and three-dimensional map are 1, 4, and 7, respectively.

We propose the following optimization heuristic for estimating the model:

Initialization. Fix an initial value of θ_M .

Stage 1. Household MIPM parameters, one household at a time.

1. Max $LL(\theta_n|n, l, A, \theta_M)$ for $n = 1, 2, \dots, N$ and $l = 1, 2, \dots, L$.
2. Determine θ_n on the basis of model comparison tests using results from Step 1.

Stage 2. Market parameters

3. Max $LL(\theta_M|A, \theta_N)$.
4. Repeat Steps 1–3 until θ_M converges.

The process consists of two iterative stages, in which we alternately maximize Equation 6 with respect to the free parameters in the model, θ_N and θ_M . In the first stage, we estimate the household-level parameters for each household given the product locations. We accomplish this by maximizing the household likelihood function for varying numbers of ideal points (e.g., 1, 2, and 3). We can then determine the optimal number of ideal points, their locations, and transition probabilities for each household through a series of model comparison tests. The second stage involves estimating the product locations by maximizing the market-level likelihood function (Equation 6) given the individual household parameters just calculated. We repeat this iterative process of alternately estimating household and product parameters until the difference in the estimates is less than some predetermined threshold.

In the model comparison tests of Stage 1, it is necessary to balance the improvement in fit (provided by adding more ideal points) against the increasing number of parameters in the model. We suggest using the consistent Akaike's (1974) information criterion (CAIC) for this decision. Researchers (Bozdogan 1987; Bozdogan and Sclove 1984) have suggested this method for latent structure models. Moreover, it accounts for the natural tension between fit and parsimony.

¹Note that when the origin and rotational invariance restrictions are imposed, fixing the scale of one axis fixes the scales of the other axes as well.

To reduce the computational burden, we recommend that the dimensionality of the product map (A) be determined a priori. Prior knowledge could point to an appropriate value of A . In practice, it also makes computational sense for the researcher to decide on the maximum number of ideal points (L), because the estimation time increases exponentially as the number of ideal points increases. This increase occurs because the number of parameters increases in the square of the number of ideal points as a result of the transition matrix. Also, in practice, the CAIC, which penalizes models with large numbers of parameters, rejects models with too many ideal points.² Furthermore, if there are too many ideal points relative to the alternatives, then the program would simply drive the ideal points to the product addresses. Finally, as is often the case in these types of models, researchers need to be concerned that the estimates may represent a local, rather than global, maximum. As such, it may be necessary to use multiple starting values. It is also helpful to use any prior knowledge about product locations in initializing the model.

For the first stage of the first iteration, we need to provide starting values for the product locations in Equation 1. These values can be set arbitrarily if we do not have product coordinates and need to obtain them through an internal analysis of preference. In contrast, if we have a separately created perceptual map of product addresses (e.g., choice map; Elrod 1988), then we only need to estimate the ideal points, and consequently, the second stage of the procedure is unnecessary. In that case, we are left with an external analysis of preference.

The internal analysis simplifies data collection by deriving the product addresses from the actual choice data. However, this iterative estimation process can be lengthy and computationally cumbersome. In comparison, an external analysis may involve collecting additional data, but it greatly simplifies and accelerates the estimation procedure. One other potential advantage is that an independently determined perceptual map will not be distorted by panels that exhibit heavy variety-seeking or reinforcing behavior (Erdem 1996). The decision of which approach to take largely depends on the nature of the application and is ultimately left to the discretion of the researcher. For the purposes of our study, we illustrate the full internal analysis estimation procedure, realizing that the external approach is embedded within its framework.

EXPLORATORY SIMULATIONS

The finite sample properties of the maximum likelihood estimator we propose are unknown. Because there is a large number of household-level parameters that must be estimated from limited purchase string data, it is important that we use simulations to assess the performance of the estimation procedure in recovering the model parameters. We varied both household variables (length of purchase string, number of ideal points, their locations, and transition probabilities) and "market" variables (N , A , M , and product locations) using a full-factorial design. The factors we varied are

²We illustrate the increase in parameters for a two-dimensional product map ($A = 2$). For a given number of ideal points L , the corresponding number of household-level parameters is shown in parentheses: 1 (2), 2 (6), 3 (12), 4 (20), 5 (30). Thus, as we increase the number of ideal points from 3 to 4, the number of parameters for each household increases from 12 to 20.

(levels in parentheses) as follows: number of consumers (60, 200), number of dimensions (1, 2), number of products (6, 9), length of purchase string (50, 100), and number of ideal points (1, 2, 3). We drew the ideal points and product locations from a uniform distribution $(-10, 10)$ on each dimension, and the transition matrices were also randomly drawn, subject to regularity conditions. We measured the algorithm performance using the Euclidean distances of the estimates to their actual values.³

The analysis of variance (ANOVA) tables for the errors in product locations and ideal points are shown in Table 1, Parts A and B, respectively. The corresponding main effects plots are shown in Figure 1, Panels A and B, respectively. The number of products did not have a significant effect on the estimates of product locations. But as the number of products increased, the consumer ideal points could be estimated more accurately because of the increased variance in choices. An increase in the length of the purchase string improved the accuracy of the estimates of both product locations and consumer ideal points. This is not surprising because we have more data from which to infer the product locations and ideal points. As the number of consumers in the simulation increased, we estimated the product locations more accurately. As should be expected, there was no significant improvement in the accuracy of each consumer's ideal points as we increased the number of consumers. An increase in the number of product dimensions reduced the accuracy of the estimated product locations and consumer ideal points, because the potential locations increase dramatically with the increase in the number of dimensions. As the number of consumer ideal points increases, we find that the product locations can be estimated more accurately. This is because an increase in the number of ideal points bounded the brand locations and reduced the range of possible errors for brand locations.⁴ But the impact on consumer ideal points was more complicated. When the number of ideal points increased from one to two, the estimation errors for the ideal points increased. However, they reduced significantly when the number of ideal points increased from two to three. A possible explanation is that as the number of ideal points increases, the range of possible values that the ideal point estimates can take is bounded by the other ideal point estimates, thus reducing the overall range of possible errors.

We also conducted the ANOVA with all two-way interaction effects. We found that for consumer ideal points, none of the interactions was substantively significant.⁵ For product locations, some effects were substantively significant. We found that the number of consumers had no significant effect on the accuracy of product locations when there was only one product dimension. But for two product dimensions, an increase in the number of consumers helped

³We also computed the absolute deviations of the estimates, and the overall performance was similar.

⁴Note, however, that the reduction in errors was not significant when the number of ideal points increased from two to three.

⁵Although we did find that a few interactions were statistically significant, an inspection of the interaction plots revealed that the magnitude of the difference was too low to be substantively significant. Note that because there are 60 and 200 consumers in the remaining 240 conditions, there are as many as $260 \times 24 = 6240$ data points in this ANOVA. Given the large degrees of freedom, even substantively insignificant differences tend to be statistically significant.

Table 1
ESTIMATION ERROR ANOVA TABLES

<i>A: Errors in Brand Product Locations: ANOVA Table</i>						
<i>Source</i>	<i>Degrees of Freedom</i>	<i>Sequential Sum of Squares</i>	<i>Adjusted Sum of Squares</i>	<i>Adjusted Mean Square Error</i>	<i>F</i>	<i>p-Value</i>
Products	1	.0024	.0024	.0024	.02	.891
Purchase string length	1	.4394	.4394	.4394	3.4	.066
Consumers	1	1.5371	1.5371	1.5371	11.91	.001
Product dimensions	1	12.2982	12.2999	12.2999	95.27	.000
Ideal points	2	9.9233	9.9233	4.9617	38.43	.000
Error	281	36.2791	36.2791	.1291		
Total	287	60.4795				

<i>B: Errors in Consumer Ideal Points: ANOVA Table</i>						
<i>Source</i>	<i>Degrees of Freedom</i>	<i>Sequential Sum of Squares</i>	<i>Adjusted Sum of Squares</i>	<i>Adjusted Mean Square Error</i>	<i>F</i>	<i>p-Value</i>
Products	1	292.79	292.79	292.79	197.47	.000
Purchase string length	1	7.96	7.96	7.96	5.37	.020
Consumers	1	4.5	4.5	4.5	3.04	.081
Product dimensions	1	2970.2	2970.2	2970.2	2003.26	.000
Ideal points	2	975.83	975.83	487.91	329.07	.000
Error	6233	9241.56	9241.56	1.48		
Total	6239	13492.83				

improve the estimates of product locations. This indicates that we must have greater numbers of consumers in our analysis if we need to estimate higher-dimensional maps. We also found that the number of ideal points had significant interaction effects with the number of consumers, the product dimensions, and the purchase string length. The basic idea is related to the main effect we observed: An increase in the number of ideal points reduces the errors in estimated product locations. Therefore, an increase in the number of consumers and the purchase string length (which improve the estimates of product locations) has the greatest (and significant) impact on the estimates of product locations when there is only one ideal point. Similarly, a reduction in the product dimensions (which improve the estimates of product locations) has the greatest (and significant) impact on the estimates of product locations when there is only one ideal point.

Overall, we find that product locations are estimated more accurately than consumer ideal points, because they are based on the choices of all consumers. The average error for each product location parameter is .21 versus 1.44 for the ideal points. The greatest error of 2.12 occurs for consumer ideal points when we use a two-dimensional map. This is equivalent to an error of approximately 1.50 on each dimension. Given that these locations were drawn from a range of -10 to 10, the error is approximately 7.5% of the range. In summary, we find our estimates to be robust and reasonable to variation in several factors, which gives us confidence in the performance of our estimation algorithm.

AN EMPIRICAL ILLUSTRATION

We estimate the model using the iterative estimation procedure and validate the results for each household in our

study. To illustrate the benefits of accounting for multiple preferences, we compare the MIPM with a simplified single ideal point and DeSoete, Carroll, and DeSarbo's (1986) wandering ideal point (WIP) model. The WIP provides another useful benchmark in that it allows the ideal point to shift and thus can account for preferences that may vary over time. However, because the WIP uses a multivariate normal distribution (i.e., single peaked), its structure is still consistent with that of a single preference. Following these comparisons, we consider specific examples from individual households and conclude with some aggregate-level inferences.

The Data

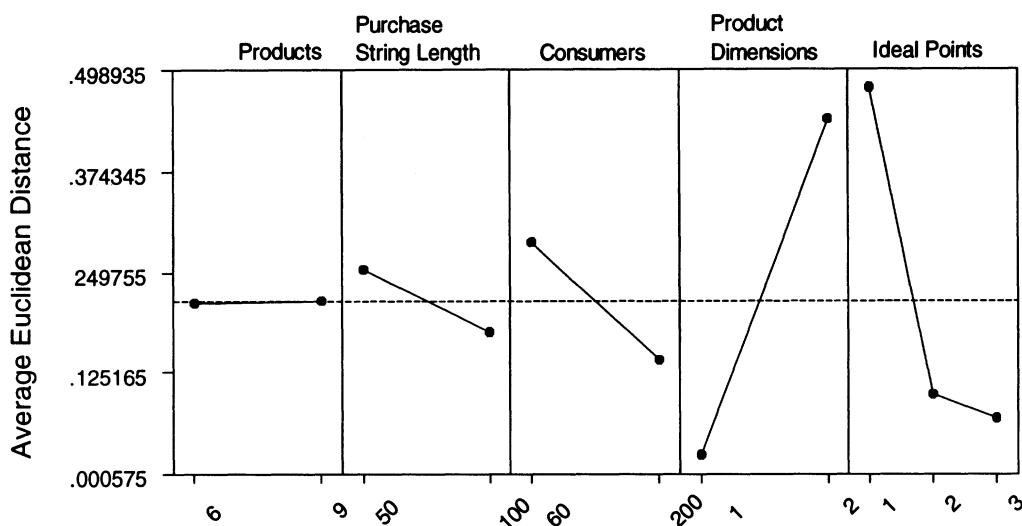
We obtained a subset of the Information Resources Inc. scanner-panel data for powdered soft drinks used by Harlam and Lodish (1995).⁶ The data set consisted of information on the purchase history (from 1985 to 1987) of 91 households from five markets: (1) Pittsfield, Mass.; (2) Marion, Ind.; (3) Eau Claire, Wis.; (4) Midland, Tex.; and (5) Rome, Ga.

Prior to our analysis, we eliminated households that made fewer than ten purchases over the two years. Households were included if they only purchased seven flavors of interest: orange, lemonade, fruit punch, cherry, grape, berry, and iced tea. Two flavors (root beer and assorted) were excluded because of infrequency of purchase. This resulted in a final sample of 59 households that accounted for 5826 choices with purchase string lengths that varied between 32 and 307. Following Steckel and Vanhonacker (1993), we used 2/3 of each purchase string for estimating the MIPM and the final 1/3 of each as a holdout sample for validation purposes. An

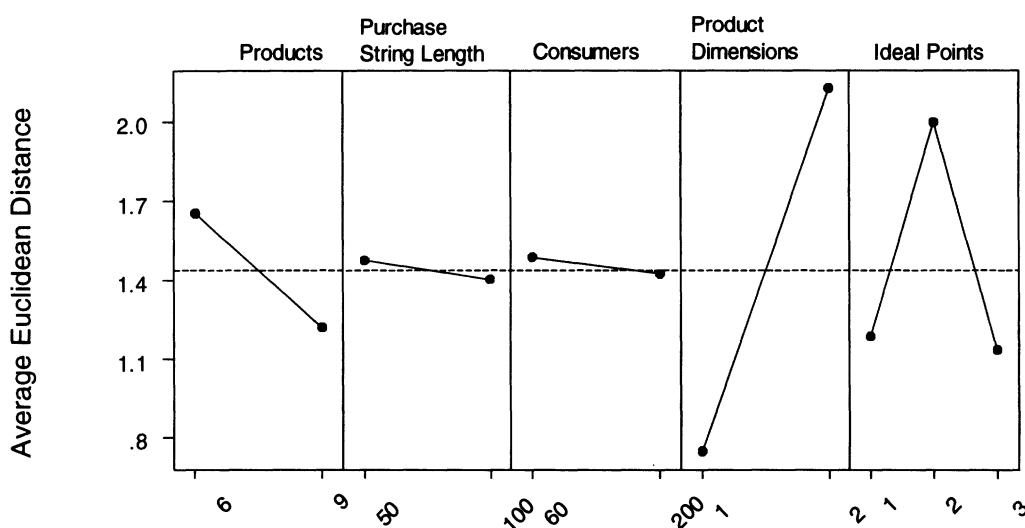
⁶We thank Bari Harlam for providing us this data.

Figure 1
MAIN EFFECTS

A: Main Effects Plot: Average Euclidean Distances Between Actual and Estimated Brand Locations



B: Main Effects Plot: Average Euclidean Distances Between Actual and Estimated Consumer Ideal Points



independent mapping exercise suggested that a two-dimensional map was appropriate.⁷

Estimation and Validation Results

In the first step of the estimation procedure, we estimated one-, two-, and three-ideal point models for each house-

hold using 2/3 of each purchase string. We did not consider models of four or more ideal points for reasons of parsimony because there are only seven products.⁸ The initial values for the product locations were based on prior knowledge of the product category from the same independent mapping exercise. On the basis of these household-level results, we then reestimated the product addresses and repeated this process until there was a minimal improvement in fit. The conservative CAIC assigned 34%, 48%, and 18% of the sample to

⁷The current application of the model involved an *a priori* specification of dimensionality. As one reviewer pointed out, there may be a natural trade-off between the number of ideal points and the number of dimensions (i.e., higher dimensionality might require fewer ideal points). After all, both dimensionality and extra ideal points supply additional parameters that can lead to improved fit. However, increasing the number of dimensions could render the results (i.e., the extra dimensions) more difficult to interpret. Concerned researchers can apply the procedure for multiple dimensions and make a selection on the basis of either fit or interpretability.

⁸In our empirical illustration with seven products, we estimated four ideal points for some randomly selected households for which the three-ideal point model fitted better than the one- and two-ideal point models. The additional fit from the four-ideal point models never outweighed the parameter penalty, and therefore the four-ideal point models were always rejected by the CAIC.

one-, two-, and three-ideal point models, respectively. Thus, although the CAIC favors parsimonious models, a substantial portion of the sample (66%) is identified as having multiple ideal points. We interpret this result as supporting our situation-dependent multiple ideal point conceptual framework.

Using these estimates and the validation sample, we then compared the overall performance of the MIPM, the single ideal point, and the WIP models. We calculated parameter estimates for the WIP using maximum likelihood and the same homogeneous perceptions/heterogeneous preferences assumption. In these comparisons, we used Rust and Schmittlein's (1985) Bayesian cross-validated likelihood (BCVL) method with the single ideal point and the WIP as "null models." In doing the BCVL test, we assigned a prior probability of .95 for both the null models. Despite this heavy favoring of the null models through the priors, the BCVL test rejected the single ideal point model and the WIP model for 63% and 66% of the households, respectively ($p < .05$). Overall, the total log-likelihoods for the MIPM, single ideal point, and the WIP were -6498.33, -6787.40, and -7161.38 in the validation sample, respectively. The MIPM therefore outperforms the two null models fairly convincingly.

Order of the Transition Matrix

For each household, we analyzed the order of the ideal point transitions by comparing the first-order switching matrix with a zero-order model. Although a majority of households (63%) were adequately modeled by the zero-order assumption, a substantial portion of the sample (37%) exhibited higher-order behavior ($p < .05$). These results suggest that households vary with respect to the order of the transition process. These findings are similar to Bass and colleagues' (1984) findings, in which approximately 74% of households exhibited zero-order behavior in several product categories.

Two Sample Households

To illustrate the benefits of identifying multiple preferences, we present the results from two households in our sample. In both cases, the multiple ideal point solution yields a significantly better fit to the observed data. More important, these results are consistent with the view that households exhibit behavior that can be captured by transitions among multiple ideal points (see Figures 2 and 3).

Household 057 was estimated on a total of 66 purchases: 11 orange, 5 lemonade, 26 fruit punch, 10 cherry, 4 grape, and 10 berry. The first-order two-ideal point solution identifies a preference for citrus drinks, $IP(1) = (9.42, 1.53)$, and one for fruitier and sweeter drinks, $IP(2) = (-3.01, 3.72)$. The steady state probabilities, $P[IP(1)] = .27$; $P[IP(2)] = .73$, indicate that most of the decisions occur from $IP(2)$, making it the more dominant preference. Further examination of the switching probabilities shows that the consumer is unlikely to make successive purchases from Ideal Point 1, perhaps indicating that this represents a preference that is quickly satiated.

In comparison, the single ideal point model (.04, 3.78) would have suggested that the household's ideal preference is medium-high sweetness and medium citrus flavor. This is consistent with the notion that a single point solution often represents the "average" ideal preference.

Both the single ideal point and WIP results suggest the possibility of a new product offering near the center of the

product space for this household. The WIP result indicates that the center of the ideal point distribution for this household is located at $WIP_{\mu} = (.85, 1.27)$, which represents the dominant preference. The shaded region approximates the area where the ideal point is most commonly found ($WIP_{\sigma_x} \approx 6.26$, $WIP_{\sigma_y} \approx 1.70$, $WIP_{\sigma_{xy}} \approx -2.83$). Given the normal distribution, the WIP implies that the household's ideal point is as likely to wander to a' and a'' , as it is to a . This kind of result makes it difficult to draw specific and meaningful conclusions from the WIP distribution.

We consider another example: Household 199, with 74 purchases: 3 orange, 18 lemonade, 21 cherry, 20 grape, and 12 berry. The MIPM arrives at a zero-order three-ideal point solution with the following locations: $IP(1) = (9.92, -1.07)$, $IP(2) = (-4.63, 4.19)$, and $IP(3) = (-7.57, 1.00)$. The results indicate that each ideal point is frequently activated [$P[IP(1)] = .30$; $P[IP(2)] = .44$; $P[IP(3)] = .26$], with a slight propensity for $IP(2)$. It also suggests that this household's multiple preferences arise independently (i.e., activation of the next ideal point is independent of the previous ones).

The simple one-ideal point model (-13.89, 4.92) suggests that this household's ideal preference is fairly high sweetness and virtually no citrus flavor. Because this solution lies outside the range of the stimuli (i.e., the product alternatives), this could be interpreted as a preference vector (as indicated in Figure 4). It is worth noting that the WIP result is also similar. Because the ideal point distribution is relatively compact and located far outside of the product space, we also represent the WIP solution as a vector in Figure 4. Although both comparison models attempt to capture the dominant preference of this household ($IP[2]$ and $IP[3]$), the vectors do not appear to account for the significant preference for citrus items ($IP[1]$).

AGGREGATE-LEVEL IMPLICATIONS

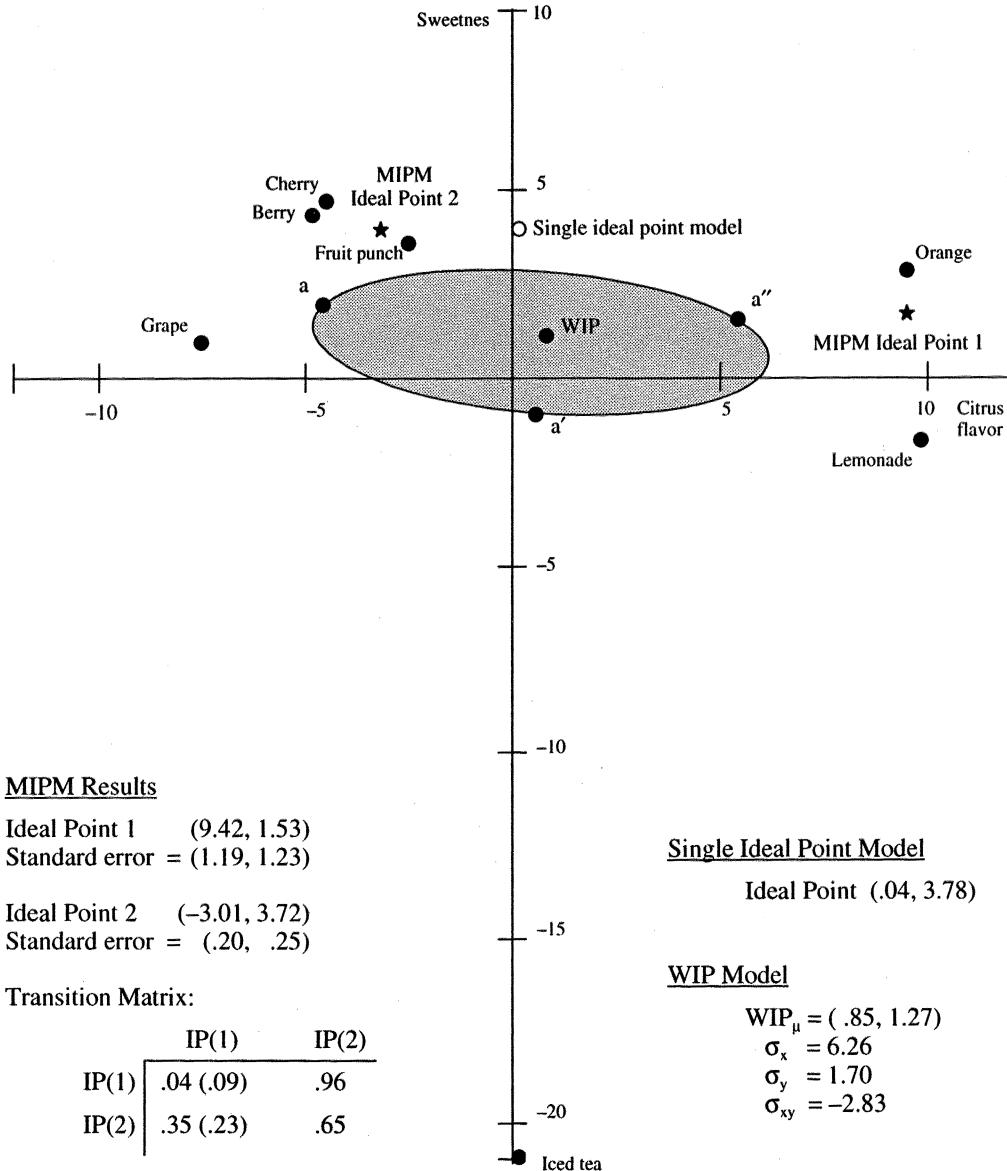
Although our discussion thus far has focused on the household, the proposed model may be used to draw market-level inferences. Therefore, in addition to providing better predictions of consumer behavior, the MIPM can provide meaningful managerial implications.

Segments of Ideal Points

As an example of the type of analyses that may be performed, we aggregated all household MIPM results. We then clustered the ideal points using an average linkage algorithm to form a market-level map (Figure 4). Although this analysis produced seven distinct clusters, there are three primary groupings of ideal points. The largest clusters are 1, 2, and 3, with 71, 47, and 12 ideal points in each, respectively. These three clusters account for 96% of all the ideal points in this group of households. This map may then be used to generate additional insights regarding multiple preference trends in the market.

The market-level cluster map is similar to the latent class model by DeSarbo and colleagues (1991). These authors develop a simultaneous unfolding and cluster analysis procedure for two-way preference data, resulting in a joint space of stimuli and clusters. A single ideal point characterizes each cluster, and a mixing proportion determines a subject's probability of cluster membership. Our model is a variation of this. We cluster the household-level results to produce the market map, and as such, membership in a spe-

Figure 2
MIPM RESULTS FOR HOUSEHOLD 057



cific cluster is not probabilistic. Comparing the differences and results of these two approaches may be worthwhile and is left for further research.

As illustrated previously at the household level, the MIPM and single ideal point model can produce different results and conclusions (see Figures 4 and 5). Figure 5 presents a similar market-level map for the more conventional single ideal point model. The results suggest a six-cluster solution, in which Clusters 1, 2, and 3 account for 85% of all the ideal points.

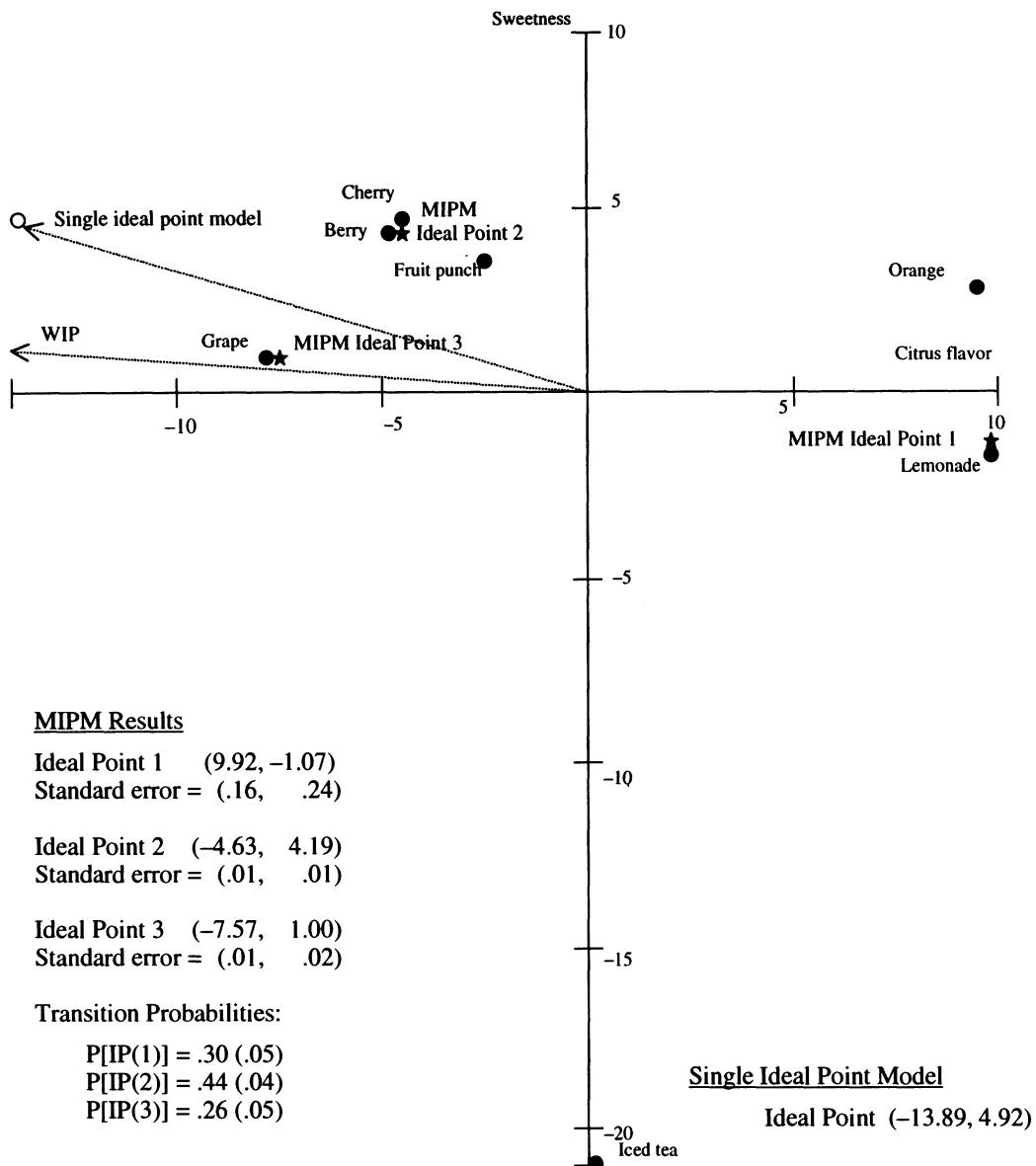
For the MIPM, ideal points in each cluster are tightly grouped and located in close proximity to the choice alternatives. In comparison, most of the ideal points in Figure 5 are clustered loosely near the center of the map (Clusters 1 and 3 represent 63% of the market). In addition, the single-point market map suggests that there are possible new product or repositioning opportunities near Clusters 1, 3, and 4.

The MIPM results indicate that those opportunities are unlikely and that other marketing vehicles (e.g., joint promotions, variety packs) may prove to be more worthwhile. One other notable difference is the absence of any ideal points located near orange or lemonade in the single point map (Figure 5). In contrast, the citrus cluster in the MIPM map (Cluster 2) accounts for 35% of the ideal points in the market. These types of discrepancies can have a significant impact on marketing strategies.

DISCUSSION AND CONCLUSIONS

Multiple preference contexts (i.e., multiple users, uses, usage occasions, and variety seeking) have been identified as one of the underlying reasons for switching behavior, and their existence suggests that individuals and/or households operate from multiple ideal points (Holbrook 1984;

Figure 3
MIPM RESULTS FOR HOUSEHOLD 199



Srinivasan and Shocker 1973). The MIPM we propose incorporates these ideas into a flexible framework that accounts for a wide variety of choice behaviors. The MIPM is a household-level model that can uncover a multiple ideal point structure and non-zero-order choice. It allows for the possibility that the choice process may be best characterized by simpler assumptions. In particular, the zero-order assumption and single ideal point are special cases of the MIPM.

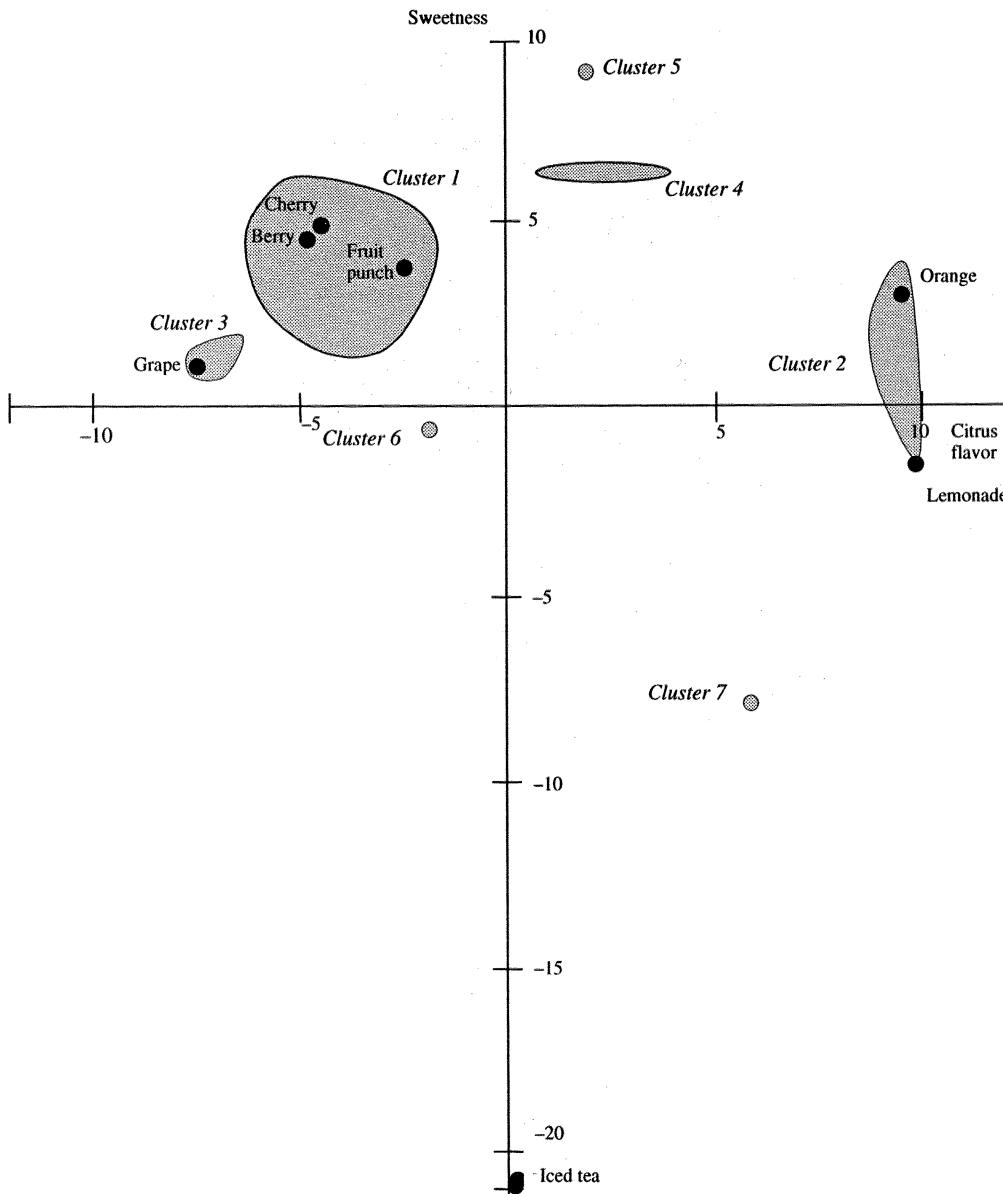
Existing single ideal point models have provided useful spatial representations of consumer preferences. Our view is simply that the single ideal point assumption may be unduly restrictive and may result in misleading conclusions. The MIPM tests for the possibility that a higher-level model may provide significantly better fits and predictions. Furthermore, multiple individuals in the household are no longer an

issue, because the MIPM allows for the existence of multiple preferences.

We have demonstrated the potential value of our model by studying the behavior of 59 households in the powdered soft drink market. Empirical results support the notion that certain consumers operate from multiple ideal points. In most cases, the MIPM not only provided a better fit to the data but also outpredicted the more conventional single ideal point and WIP models. At the individual and aggregate levels, results highlighted the potential pitfalls associated with drawing inferences from a single preference model. The small sample size, 59 households, is a limitation of our work. Further research should investigate larger applications.

In terms of managerial applications and benefits, the MIPM provides a graphical tool through which the product

Figure 4
MIPM IDEAL POINT CLUSTERS FOR THE POWDERED SOFT DRINK MARKET



manager can view the market. The proposed model also provides insights into the competitive structure of the market, possible niches, and ways consumers make choice decisions. These results can guide product managers in decisions pertaining to positioning, market entry, and overall strategy.

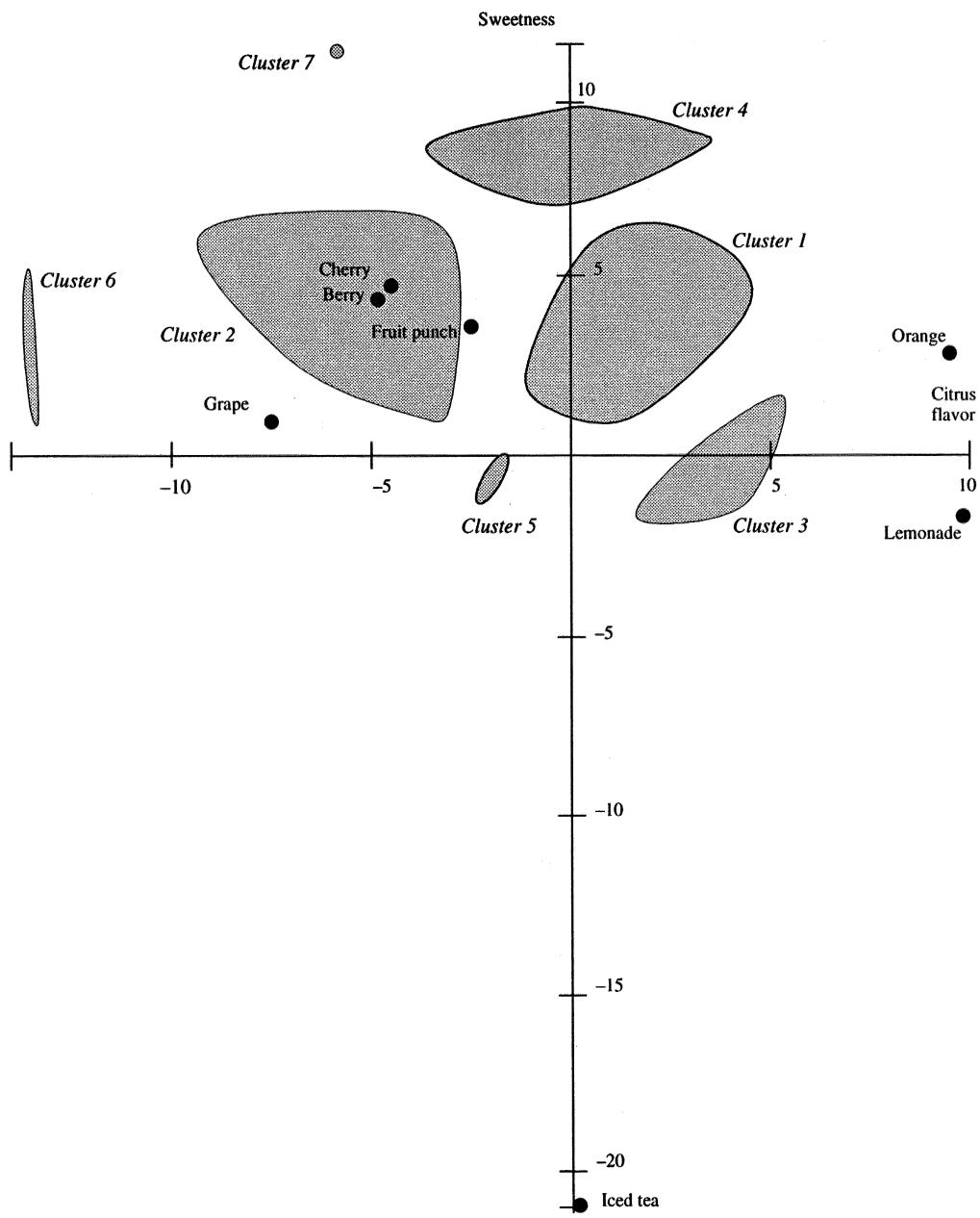
Other Potential Limitations and Extensions

For estimating the model effectively, longer purchase histories, which are readily available, are preferable. As such, studies at the household level should be focused on product categories that are frequently purchased. The requirement of a large number of observations introduces the problem of stationarity. Because these purchase histories may extend over a lengthy period of time, the researcher must ensure that significant changes in the choice problem do not occur, and

if they do, they must be accounted for. Examples of these changes include the entry or exit of products and major shifts in the perceptual locations of the products or the ideal points.

Another issue to be considered is the sensitivity of estimates to randomization effects (e.g., the order in which three products cross a scanner when bought together). We conducted a minor simulation to assess the sensitivity of the estimates to this. Using a string of 22 purchases as a base, we created six new purchase histories by permuting a block of three of the choices into all of its possible combinations and by randomizing the order of all 22 purchases once. The resulting product location and ideal point parameter estimates in all six cases were consistent with one another ($\pm .05$), and the standard deviation of the results was small ($<.01$). This provides reasonable assurance that the stability

Figure 5
SINGLE IDEAL POINT RESULTS FOR THE POWDERED SOFT DRINK MARKET



of our model's estimates will not be adversely affected by simultaneous purchases.

Despite our simulation results, multiple purchases within the same shopping trip theoretically could still present a problem for the entries of the transition matrix. Specifically, the order in which selections "cross the scanner" on a given trip is a random event. Therefore, any higher-order behavior that occurs is likely to be mitigated by this "within-trip randomization" and therefore favors a zero-order transition process. This handicap notwithstanding, we still found that 37% of the sample consisted of higher-order households.

Although the transition matrix may provide keen insights into the nature of the choice process, the essential benefit of

the MIPM is still the identification of multiple ideal points. Because only the order is affected, and not the actual selections, the randomization does not significantly influence the locations of the ideal points. Comparisons of the zero- and first-order results for the entire sample also support this conclusion. Therefore, depending on the application, the MIPM may be easily modified into a zero-order model for simpler analyses without a significant loss of accuracy. This flexibility is particularly attractive considering the mixed findings surrounding the order of choice processes (Bass et al. 1984).

One of the more promising managerial applications involves determining the specific cause of multiple ideal

points within a given household or individual. These insights would enable managers and researchers to better understand consumer wants and needs. Market segmentation and target marketing would also benefit from identifying these antecedents. Although the proposed model does not explicitly determine the sources of multiple preferences, exploratory analyses might be conducted in which the number of ideal points is linked to different demographic variables.

With the emergence of the MIPM as a potentially viable model of product choice, much research can be done. In addition to examinations of the antecedents of multiple preferences, managerially relevant and well-defined measures of market structure and segment behavior could be developed and applied to various markets. It might also be worthwhile to investigate the stability of these multiple ideal points with respect to varying market characteristics. A natural progression in the development of the MIPM would be the inclusion of explanatory variables (e.g., marketing mix) or psychological constructs (e.g., attribute satiation measures). The marginal explanatory and predictive power of these constructs remains to be seen, in particular with respect to the added computational complexity. Although the model specified here is described at the household level, market- or segment-level studies could be conducted to provide insights into how specific consumers behave (e.g., segment-level ideal points, measures of loyalty or variety seeking).

We view this work as a first step in considering the effects of multiple preferences on observed household-level behavior. As we illustrated, the assumption of a single well-defined preference may often be inappropriate and can result in misleading conclusions. Given the prevalent use of household-level data, we believe that the MIPM represents a potentially useful tool for the identification and analysis of multiple preferences.

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