to pay for the product (otherwise, they would not have purchased from it), relative to its competitor's customers. That is, customers' past purchases reveal their relative preference for each firm. Furthermore, if consumers recognize the possibility that they will be penalized in the future, they can alter their behavior to reduce the ability of firms to infer their true preferences. Such strategic behavior by consumers and increased competition to poach others' customers may make BBP unprofitable. Fudenberg and Villas-Boas's (2006, p. 378) succinct summary of existing literature on BBP notes that "the seller may be better off if it can commit to ignore information about buyer's past decisions ... more information will lead to more intense competition between firms." The overall lesson is that BBP cannot be profitable if both consumers and firms are rational and forward-looking; furthermore, it is never optimal to reward one's own customers.

In this paper, we resolve this discrepancy between theoretical predictions and practitioner intuitions by incorporating two simple but important features of customer behavior into our analytical model. Our results nest these two viewpoints and identify conditions in which either practitioner intuition or current theoretical results hold, even when both consumers and firms are strategic and forward-looking. We now describe the two key features that we add.

1.1. Customer Value Heterogeneity

Not all customers are equally valuable to firms. Some purchase more than others or contribute more to a firm's profits. Widespread empirical support in various categories confirms the 80/20 rule—that is, the idea that a small proportion of customers contributes to most of the purchases and profit in a category (Schmittlein et al. 1993). Such customer heterogeneity is critical for capturing the practitioner notion of a "best" customer—a feature that generally does not appear in current analytical models.

Modeling customer heterogeneity in purchase quantity also allows firms to gain a new dimension of information about consumers from their purchase history data. In addition to the usual "who they bought from" data, which provide *horizontal* information about the relative preferences of consumers, data pertaining to "how much they bought" provide *vertical* information about the relative importance of consumers to the firm.

Vertical information about quantity not only provides additional information to firms but also generates an information asymmetry among otherwise symmetric firms. With these data, a firm can identify the "best customers" only among its own customers, but not among its competitor's customers. Although the firm knows there is a mix of high- and low-volume customers among its competitor's customers, it cannot know the exact identity of the customer. Existing models assume that all customers buy just one unit of the product and that the market is fully covered, which implies that if a consumer does not buy from one firm, it must have bought from the other, and therefore, there is no information advantage about one's own customers. That is, existing models abstract away from a major reason that firms invest in customer relationship management (CRM) in practice: to obtain an information advantage over competitors about existing customers. This information asymmetry emerges as critical for obtaining more complete insights into BBP.

1.2. Stochastic Preferences

We also recognize that consumers' preferences can be stochastic. Consumer preference for a product may change across purchase occasions, independent of the marketing mix or pricing, because their needs or wants depend on the specific purchase situation, which changes over time (Wernerfelt 1994). This is relevant in many choice contexts. For example, for store choice, consumers' preferred geographic locations likely vary across time; a customer may generally prefer Lowe's for purchasing home improvement products because a store is closer to her home and offers superior quality offerings. However, she may visit the Home Depot store that is on her way home from work. A similar logic may apply to consumers' choices of hotels; even if a customer prefers Marriott in general, he may find that a Sheraton satisfies his needs better on a particular trip because of its proximity to a conference venue.[2]

Previous research in BBP (e.g., Caminal and Matutes 1990, Fudenberg and Tirole 2000) recognizes the possibility of stochastic preference, but only for the extreme case in which customers' preferences are completely independent over time. Although the independence assumption makes the analysis tractable, it is not innocuous in a customer management context. With completely independent preferences, customers' past purchases are of no use in predicting future purchases. For past purchase information to be valuable to firms in future price-setting efforts, those preferences must be correlated across time. Our formulation of stochastic preferences will accommodate such correlation in preferences across time, a characteristic of most real-world markets.

With these two features included, we identify conditions in which (1) behavior-based pricing is profitable in a competitive market, even when firms and

[2] It is important to note that we are not assuming that consumer *choice* changes over time. Consumers' purchase choice is still an endogenous decision. For example, even if the Sheraton is preferable because of its proximity to a conference venue, a consumer may still stay in the Marriott if it offers a special deal or some other incentive.

consumers are strategic and forward-looking; and (2) firms should offer lower prices to their own best customers or to their competitor's customers. We find that *either* sufficiently high heterogeneity in customer value *or* stochasticity in preference is sufficient for BBP to increase firm profits. However, *both* sufficient heterogeneity in customer value *and* stochastic preference are required for firms to reward their own best customers; if both elements do not exist, they should reward their competitor's customers.

The rest of this article is organized as follows: Section 2 describes the related literature. Sections 3 and 4 describe the model and the analysis, respectively. Section 5 concludes our paper.

2. Literature Review

Our research connects several interrelated areas of marketing and economics. Our primary contribution applies to BBP (for a review, see Fudenberg and Villas-Boas 2006). In terms of model setup, our work comes closest to Fudenberg and Tirole's (2000) analysis of a two-period duopoly model with a Hotelling line and in which customers and firms are forward-looking. Villas-Boas (1999) extends Fudenberg and Tirole (2000) to an infinite period model with overlapping generations of customers. In both studies, BBP leads to a prisoner's dilemma that induces lower profits than the case in which firms credibly commit not to use past purchase information, and it is never optimal to reward the firm's own customers. Pazgal and Soberman (2008) replicate these results but show that profits can increase if only one of the firms practices BBP, although it is still not profitable to reward own customers.[3]

Other related literature pertains to switching costs. Firms can price discriminate between their own locked-in customers and customers locked-in to a rival through switching costs. Chen (1997) analyzes a two-period duopoly model with heterogeneous switching costs among customers. Similar to BBP, firms always charge the lower price to their competitor's customers, and the total profits are lower than if they could not discriminate. Taylor (2003) extends Chen's model to competition between many firms and continues to find that firms charge the lowest prices to new customers. Shaffer and Zhang (2000) also consider a static game, similar to the second period of Fudenberg and Tirole's (2000) two-period model, but they allow switching costs to be asymmetric across the consumers of the two firms. With symmetric switching costs, firms always charge a lower price to their rival's consumers, whereas with asymmetry, the firm with lower switching costs may offer lower prices to own customers, although it is never optimal for *both* firms to charge lower prices to their own customers. In a comprehensive review, Farrell and Klemperer (2007, p. 1993) thus conclude that switching-cost literature finds it hard to explain discrimination in favor of own customers.

Our work also aligns closely with the theoretical and empirical literature on targeted pricing.[4] Although targeted pricing by a monopolist always leads to greater profits, the competitive implications in oligopoly markets are subtle. Thisse and Vives (1988) and Shaffer and Zhang (1995) show that price discrimination effects get overwhelmed by competition effects in targeted pricing, leading to a prisoner's dilemma. Chen et al. (2001) further note that targeting accuracy can moderate the profitability of firms. They recognize that consumer information is noisy; hence, targeting is imperfect. At low levels of accuracy, the positive effect of price discrimination on profit is stronger, whereas at high levels, the negative effect of competition on profit is stronger. Overall, profits are greatest at moderate levels of accuracy.

Empirical literature on this topic (Rossi et al. 1996, Besanko et al. 2003, Pancras and Sudhir 2007) indicates that firms can improve profits through targeted pricing if they use customers' past purchase histories to infer preferences. Interestingly, the probit and logit empirical models in these papers that support the profit improvement achieved through targeted pricing include the two key consumer features that we model, namely, customer heterogeneity in category usage (through the brand intercept terms relative to the outside good) and changing customer preferences (through the normal and extreme value distributions in consumer utility).

We also find connections with our work in adverse selection models in financial markets (Sharpe 1990, Pagano and Jappelli 1993, Villas-Boas and Schmidt-Mohr 1999). In contrast with the focus on a customer's relative preference (horizontal preference information) in BBP, this literature stream models vertical information about own customer types (ability to repay loans), arguing that firms use this information asymmetry to determine future loans to customers.[5] Our model combines two aspects of information

[3] Fudenberg and Tirole (1998) also identify conditions in which a firm selling successive generations of durable goods should reward current consumers or new customers in a monopoly setting.

[4] Unlike BBP models, targeting models are static, and firms discriminate among consumers on the basis of perfect or noisy information about their underlying preferences. In BBP, one explicitly models how past purchase behavior provides firms with information about preferences, which are then used to determine discriminatory prices in the future. In contrast, targeting literature does not model how firms obtain preference information.

[5] Several related theoretical papers in marketing accommodate customer heterogeneity in purchase quantities (Kim et al. 2001,

about customers' past behavior; that is, (1) horizontal preference information, which is the focus in BBP and (2) vertical information, which represents the focus of adverse selection literature in economics and finance.

Finally, our work relates to CRM literature in marketing (for a general review, see Boulding et al. 2005), which investigates whether firms should use acquisition or retention strategies (e.g., Syam and Hess 2006, Musalem and Joshi 2009). A large body of empirical research also uses data about customers' purchase history to estimate and show significant heterogeneity in customer lifetime value (Blattberg and Deighton 1996, Jain and Singh 2002, Venkatesan and Kumar 2003, Gupta et al. 2004).

3. Model

We follow Fudenberg and Tirole (2000) in considering a two-period standard Hotelling model with two retailers geographically located on the two ends of a unit line. We denote the retailer located at point 0 as retailer A and the retailer at point 1 as retailer B. The retailers sell an identical nondurable good (e.g., ice cream, gasoline), from which the consumer receives a gross utility of v. We assume that v is large enough that in equilibrium, all consumers purchase the product from one of the retailers.[6] We also assume the retailer's marginal cost for the product is constant and normalize it to zero without loss of generality. The market contains two periods, and consumers make purchase decisions in both. We denote the prices charged by retailers A and B in period t as p_t^A and p_t^B, respectively.

We adapt the Hotelling model to capture our two new features—customer value heterogeneity and unstable preference. First, to model *customer heterogeneity* parsimoniously in terms of value to the firm (i.e., the 80/20 rule), we assume there are two types of consumers in the market ($j \in \{L, H\}$): a high-type segment (H) that purchases q units of the good in each period and a low-type segment (L) that purchases only one unit.[7] The proportions of H and L types in the market are $\alpha(>0)$ and $1-\alpha$, respectively. Both H- and L-type consumers' geographic locations or preferences (denoted θ) are uniformly distributed along the Hotelling line, $\theta \sim U[0,1]$. We normalize the size of the market to one, such that a j-type ($j \in \{L, H\}$) consumer located at θ receives the following utilities from purchasing the product:

$$U^j(p_t^A, p_t^B \mid \theta) = \begin{cases} Q^j(v - p_t^A) - \theta & \text{if purchase from retailer } A, \\ Q^j(v - p_t^B) - (1-\theta) & \text{if purchase from retailer } B, \end{cases}$$
$$\text{where } Q^L = 1,\ Q^H = q.$$

We treat q as exogenous; some consumers need more of the product than others for reasons exogenous to the model. For example, a household with two people will need two cones of ice cream rather than the one cone demanded by a single-person household; a person who lives farther from work will buy gasoline more frequently than will someone who lives close to work.

Second, to capture the idea of *stochastic preference*, we allow the preference for the retailer to change across periods. As we discussed previously, this change in preference may occur as a result of a change in the consumer's geographic location (e.g., shopping trip starts from home or office). Similar to Klemperer (1987), we allow the customer location in the second period to change with probability β but remain the same as in the first period with probability $1-\beta$. When the location changes in the second period, the new location comes from a uniform distribution, $\omega \sim U[0,1]$. Let θ_1 be the first-period location. Then the second-period location $\theta_2 = \omega$ with probability β, or $\theta_2 = \theta_1$ with probability $1-\beta$. Therefore, customer locations correlate over the two periods, and the expected location of the second period for a customer is $E(\theta_2) = \beta\omega + (1-\beta)\theta_1$. If preferences are stable ($\beta = 0$), $\theta_2 = \theta_1$ all the time. In contrast, when $\beta = 1$, there is maximum instability, because consumer locations are completely independent.[8]

In the first period, the retailers A and B each offer a single price, p_1^A and p_1^B, respectively, to all

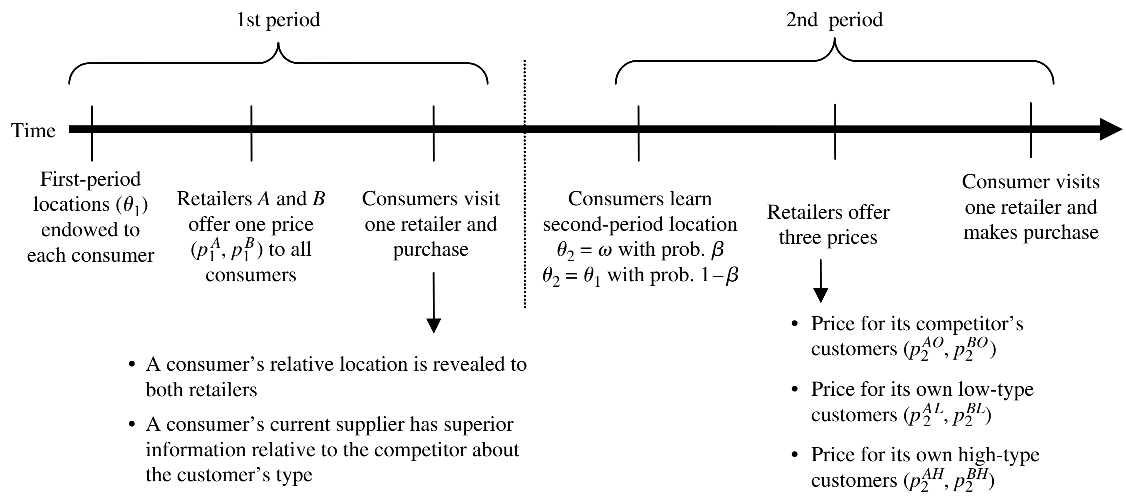
Kumar and Rao 2006) and costs to serve (Shin 2005). However, these studies do not address behavior-based pricing—that is, offering different prices to own versus competitor's customers.

[6] The assumptions of exogenous locations at the ends of the Hotelling line and full-market coverage are standard (e.g., Villas-Boas 1999, Fudenberg and Tirole 2000); they ensure no discontinuities in the demand function, which is a general problem in Hotelling-type models in which the location choices are endogenous (d'Aspremont et al. 1979).

[7] Ideally, we would model customer lifetime value through multiple-period purchases. However, in a two-period framework, we abstract and capture the spirit of the 80/20 rule parsimoniously through different purchase quantities (q) and keep the model analytically tractable. In reality, q may arise from multiple purchases across each period.

[8] We consider an alternative specification for preference correlation: the second-period location, θ_2, is the weighted average of the first-period location and the external situational shock ($\theta_2 = (1-\beta)\cdot\theta_1 + \beta\omega$). The advantage of this approach is that we can capture the local movement of customers (i.e., the second location is a function of the first location). However, this specification leads to an abundance of corner solutions, which makes the analysis extremely tedious without adding insight. Nevertheless, we do find a parameter space in which the "reward own customers" and "increased profit" results hold even in this specification, which suggests that our key results are robust to alternative specifications. An alternative formulation more amenable to empirical work would be a normal distribution that allows for serial correlation across time.

Figure 1 Sequence of Events



consumers. Consumers purchase from either retailer, depending on which choice is optimal for them. Note that we allow consumers to be strategic and forward-looking; they can correctly anticipate how their purchase behavior will affect the prices they will have to pay in the future, and therefore, they may modify from whom they buy in the first period to avoid being taken advantage of by the retailers in the second period.

In the second period, the retailer can distinguish among three types of customers: (1) customers who bought q units from it, (2) customers who bought one unit from it, and (3) customers who bought no units (and therefore must have bought from the competitor). Based on the customer's purchase behavior in the first period, retailers can infer two types of information about customers: horizontal information about their relative location proximity to A or B, and vertical information about the type of customer, based on the quantity purchased. Although the horizontal information is symmetric to both retailers, because the market is covered, the vertical information is asymmetric, in that a firm knows this information only about its own customers. Therefore, customer heterogeneity in purchase quantity confers an endogenous information advantage on retailers about their current customers for the second period.[9] This point represents a key departure from existing models that exclude heterogeneity in purchase quantity.

The retailer knows that consumers who purchased from it in the first period preferred it but also recognizes that no guarantee ensures they will continue to prefer it in the second period, because consumer preferences are not fixed. However, as we show in §4.1, a consumer who purchases from a retailer in the first period is probabilistically more likely to prefer the same retailer in the second. Thus, retailers possess useful probabilistic information about the relative preferences of customers in the second period, given purchase information in the first period. We formally analyze relative location more precisely in §4.1, but for now, it suffices to say that first-period choice reveals information about relative location, even when consumer locations are stochastic.

In the second period, on the basis of its information set, each retailer offers three different prices to the three groups of customers: (1) a poaching price to the competitor's customers (p_2^{AO}, p_2^{BO}), (2) a price for its own L-type customers (p_2^{AL}, p_2^{BL}), and (3) a price for its own H-type customers (p_2^{AH}, p_2^{BH}). Consumers decide where to purchase in the second period after observing all these prices. Figure 1 summarizes the outline of the game, and Table 1 summarizes the notation we use herein.

4. Analysis

We solve the proposed game using backward induction, solving first for the second-period equilibrium strategies and then for the first-period strategies. Before we solve the game, we define retailer turf and formally demonstrate the relevance of customer purchase history information, even in the presence of unstable customer preference.

4.1. Retailer Turf and the Value of Purchase History Information

For any pair of first-period prices (all consumers purchase and both retailers have positive sales), there

[9] Regarding the idea that firms have different amounts of information about their own customers and noncustomers, Villas-Boas (1999) incorporates information asymmetry by assuming customers can be either new or competitor's customers. In contrast, our information asymmetry comes from the customer type (quantity). Also, Villas-Boas and Schmidt-Mohr (1999) consider the impact of asymmetric information between firms and consumers.

Table 1 Terminology

Q^j	Purchase quantity of customer type $j \in \{L, H\}$, where $Q^L = 1$, $Q^H = q$.
p_1^i	Retailer $i \in \{A, B\}$'s first-period price to all consumers.
p_2^{iL}	Retailer $i \in \{A, B\}$'s second-period price to its L-type customers.
p_2^{iH}	Retailer $i \in \{A, B\}$'s second-period price to its H-type customers.
p_2^{iO}	Retailer $i \in \{A, B\}$'s second-period price to its *competitor's* customers.
β	Probability that consumer location changes in period 2, $\beta \in [0, 1]$.
ω	If consumer location changes, consumer location in period 2, $\omega \sim U[0, 1]$.
θ_t	Consumer's preference or geographical location at period $t \in \{1, 2\}$. In particular, $\theta_2 = \varepsilon$ with probability β and $\theta_2 = \theta_1$ with probability $1 - \beta$.
$\tilde{\theta}_1^j$	First-period threshold for customer type $j \in \{L, H\}$, such that all consumers of type j with $\theta \leq \tilde{\theta}_1^j$ purchase from retailer A and the rest purchase from retailer B.
$\tilde{\theta}_2^{Aj}$	Second-period threshold for customer type $j \in \{L, H\}$, who was on retailer A's turf in the first period, such that all consumers of type j with $\theta \leq \tilde{\theta}_2^{Aj}$ repeat purchase from retailer A and the rest switch to B.
$\tilde{\theta}_2^{Bj}$	Second-period threshold for customer type $j \in \{L, H\}$, who was on retailer B's turf in the first period, such that all consumers of type j with $\theta \geq \tilde{\theta}_2^{Bj}$ repeat purchase from retailer B and the rest switch to A.
$\Pr^{Aj}$	Probability of repeat purchasing in second period from retailer A for customer type $j \in \{L, H\}$ who was on A's turf in the first period.
$\Pr^{Bj}$	Probability of repeat purchasing in second period from retailer B for customer type $j \in \{L, H\}$ who was on B's turf in the first period.
Π_t^i	Profit of retailer $i \in \{A, B\}$ in period t. Total profit for retailer i is $\Pi^i = \Pi_1^i + \delta\Pi_2^i$.
δ	Discount rate.
$E^A[U_2^j \mid \theta_1 = \tilde{\theta}_1^j]$	Expected second-period utility that marginal customer of type $j \in \{L, H\}$ gets when he or she purchases from retailer A. A marginal customer is indifferent between purchasing a product from A or B in the first period.
$E^B[U_2^j \mid \theta_1 = \tilde{\theta}_1^j]$	Expected second-period utility that marginal customer gets when he or she purchases from retailer B.

is a cutoff $\tilde{\theta}_1^j$ ($j \in \{L, H\}$), such that all consumers of type j whose $\theta \leq \tilde{\theta}_1^j$ decide to purchase from retailer A, and the rest purchase from retailer B. Following Fudenberg and Tirole (2000), we say that consumers to the left of $\tilde{\theta}_1^j$ lie in retailer A's turf and those on the right lie in retailer B's turf to emphasize consumers' relative locations on the Hotelling line.

For consumers who purchased from retailer A in the first period, retailer A offers the prices p_2^{AH}, p_2^{AL} to H- and L-type customers, respectively, whereas retailer B offers the price p_2^{BO} to both types at the beginning of the second period. Similarly, among consumers who purchased from retailer B in the first period, retailer B charges p_2^{BH}, p_2^{BL}, whereas retailer A charges p_2^{AO}.[10]

Given preference correlation in the second period ($E(\theta_2) = \beta\omega + (1 - \beta)\theta_1$, where $\beta \in [0, 1]$ and $\omega \sim U[0, 1]$), the conditional probability that consumers will locate in a certain range of the Hotelling line, given their first-period purchase choice of A or B, is as follows:

$$\Pr[\theta_2 \leq x \mid \theta_1 \leq \tilde{\theta}_1] = \begin{cases} (1-\beta) + \beta x & \text{if } \tilde{\theta} \leq x, \\ (1-\beta)\dfrac{x}{\tilde{\theta}_1} + \beta x & \text{if } \tilde{\theta}_1 > x; \end{cases} \tag{1}$$

$$\Pr[\theta_2 > x \mid \theta_1 > \tilde{\theta}_1] = \begin{cases} (1-\beta) + \beta(1-x) & \text{if } \tilde{\theta}_1 \geq x, \\ (1-\beta)\dfrac{1-x}{1-\tilde{\theta}_1} + \beta(1-x) & \text{if } \tilde{\theta}_1 < x. \end{cases} \tag{2}$$

Note that Equations (1) and (2) both indicate the general conditional probabilities of the second-period locations, assuming that a consumer purchases from retailer A or B in the first period. Therefore, we offer the following lemma:

Lemma 1 (Value of Purchase History Information). *When stochasticity in preference is not extreme ($\beta \leq 1/(2(1-z))$, where z is the first-period market share), a consumer who purchases from retailer j ($j \in \{A, B\}$) in period 1 is more likely to stay in the same retailer's turf rather than move to the competitor's turf.*

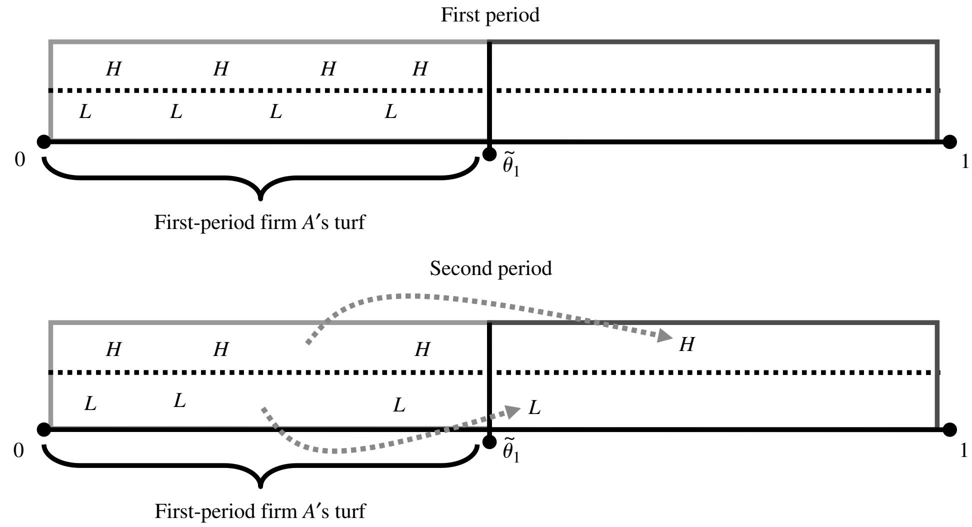
Proof. See the appendix.

This lemma shows that the customer's first-period purchase is *probabilistically* informative of second-period preference, because she or he is more likely to remain on the same turf unless preference is extremely stochastic. For example, consider the specific case of $\tilde{\theta}_1 = \frac{1}{2}$ and $\beta = \frac{1}{2}$. A consumer on A's turf relocates to the same turf with a probability of $\frac{3}{4}$ and relocates to B's turf with only a probability of $\frac{1}{4}$.[11] Figure 2 illustrates the redistribution of customer locations in the second period for four H-type and L-type first-period customers of A. Each letter represents one H- or L-type consumer. Probabilistically, three H- and L-type consumers remain relatively close to retailer

[10] When we analyze the symmetric case, "no poaching" does not arise in equilibrium, but in the asymmetric case, when $\tilde{\theta}_1 < 1/4$, A's turf is very small and consists only of consumers with a strong preference for A, and retailer A can charge a monopoly price but not lose any sales to retailer B.

[11] Lemma 1 focuses on the horizontal information about the customer's first-period relative location, as observed from past purchases. Past purchases also reveal vertical information about the customer quantities, which is always valuable, irrespective of the degree of preference stochasticity.

Figure 2 Redistribution of Consumer Locations for Consumers on Retailer *A*'s Turf



A, and one of each type changes his or her location toward retailer B in the second period, as indicated by the arrows in Figure 2.

Note that retailers do not know the exact locations of their own customers in the first period, but they know their relative location, in that these customers must be somewhere on their turf. In the second period, retailers still do not know the exact locations of their first-period customers, but they can assess the average probability of the relative proximity (closer to A or B) of their first-period customers.

When the turf is symmetric ($\tilde{\theta}_1 = \frac{1}{2}$), a customer is always more likely to stay on the same turf for all $\beta \in [0, 1)$. When $\beta = 1$ (complete independence in preferences), not surprisingly, they are equally likely to appear on either turf in the second period. Thus, past purchases are informative of future preferences for all $\beta \in [0, 1)$.

4.2. Second Period

A consumer on retailer A's turf purchases again from retailer A in the second period if and only if $Q^j v - Q^j p_2^{Aj} - \theta_2 \geq Q_v^j - Q^j p_2^{BO} - (1 - \theta_2) \Leftrightarrow \theta_2 \leq (1 + (Q^j p_2^{BO} - Q^j p_2^{Aj}))/2 \equiv \tilde{\theta}_2^{Aj}$, where $j \in \{L, H\}$ and $Q^L = 1$, $Q^H = q$. Otherwise, the consumer switches to retailer B. We denote the repeat purchase probability of the high and low types for retailer A as $\Pr^{AH}$ and $\Pr^{AL}$, respectively. Thus, $\Pr^{AH} = \Pr[\theta_2 \leq [1 + q(p_2^{BO} - p_2^{AH})]/2 \mid \theta_1 \leq \tilde{\theta}_1^H]$ and $\Pr^{AL} = \Pr[\theta_2 \leq [1 + (p_2^{BO} - p_2^{AL})]/2 \mid \theta_1 \leq \tilde{\theta}_1^L]$, where $\tilde{\theta}_1^j$ is the first-period cut-off for type $j \in \{L, H\}$. Similarly, the repeat purchase probabilities for retailer B are $\Pr^{BH} = \Pr[\theta_2 > [1 + q(p_2^{BH} - p_2^{AO})]/2 \mid \theta_1 > \tilde{\theta}_1^H]$ and $\Pr^{BL} = \Pr[\theta_2 > [1 + p_2^{BL} - p_2^{AO}]/2 \mid \theta_1 > \tilde{\theta}_1^L]$.

The second-period profits of retailers A and B then are

$$\begin{aligned}
\Pi_2^A &= p_2^{AH} \alpha q \tilde{\theta}_1^H \Pr^{AH} + p_2^{AL}(1-\alpha)\tilde{\theta}_1^L \Pr^{AL} \\
&\quad + p_2^{AO}\big\{\alpha q(1-\tilde{\theta}_1^H)(1-\Pr^{BH}) \\
&\qquad + (1-\alpha)(1-\tilde{\theta}_1^L)(1-\Pr^{BL})\big\}, \qquad (3) \\
\Pi_2^B &= p_2^{BH} \alpha q(1-\tilde{\theta}_1^H)\Pr^{BH} + p_2^{BL}(1-\alpha)(1-\tilde{\theta}_1^L)\Pr^{BL} \\
&\quad + p_2^{BO}\big\{\alpha q \tilde{\theta}_1^H(1-\Pr^{AH}) + (1-\alpha)\tilde{\theta}_1^L(1-\Pr^{AL})\big\}.
\end{aligned}$$

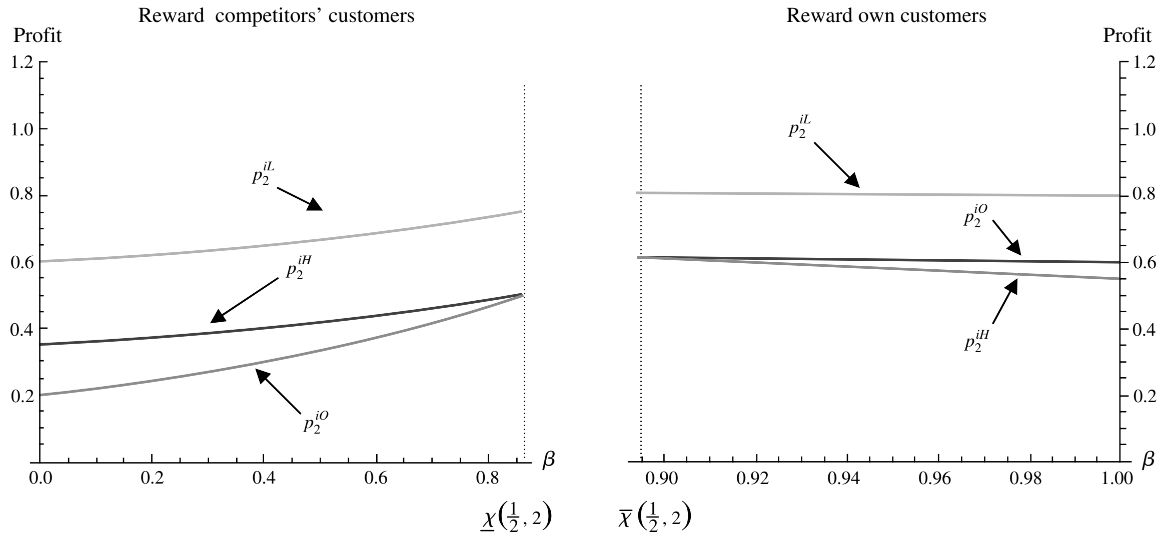
Each retailer's second-period demand consists of three parts. For example, retailer A enjoys demand from (1) its own previous H-type customers ($\tilde{\theta}_1^H$), who continue to be on the retailer's turf in the second period (with probability $\Pr^{AH}$) and pay a price p_2^{AH}; (2) its own previous L-type customers ($\tilde{\theta}_1^L$), who continue to be on the retailer's turf in the second period (with probability $\Pr^{AL}$) and pay a price p_2^{AL}; and (3) a mix of the competitor's previous high- and low-type customers ($(1 - \tilde{\theta}_1^H) + (1 - \tilde{\theta}_1^L)$) who have shifted to the retailer's turf (with probability $1 - \Pr^{Bj}$, where $j \in \{L, H\}$) and pay a price p_2^{AO}.

Using Equations (1) and (2), we can obtain the second-period prices by solving the retailers' first-order conditions. We only look for the pure strategy equilibrium in a symmetric game. In the first-period analysis, we show that the symmetric outcome, in which both firms charge the same price in the first period, is indeed the equilibrium solution.

Proposition 1. *Suppose there exists a nonzero α portion of H-type customers in the market* ($0 < \alpha < 1$).

(a) Reward Competitor's Customers: *When preference stochasticity is low* $\big(\beta < \underline{\chi}(\alpha, q)\big)$, *there exists*

Figure 3 **Prices When $\alpha = \frac{1}{2}$ and $q = 2$**



a symmetric pure strategy equilibrium in second-period prices, such that retailers charge

$$p_2^{AH} = p_2^{BH} = \frac{1}{2q} + \frac{(2+\beta)(1+(q-1)\alpha)}{6(2-\beta)(1+(q^2-1)\alpha)},$$

$$p_2^{AL} = p_2^{BL} = \frac{1}{2} + \frac{(2+\beta)(1+(q-1)\alpha)}{6(2-\beta)(1+(q^2-1)\alpha)}, \quad \textit{and}$$

$$p_2^{AO} = p_2^{BO} = \frac{(2+\beta)(1+(q-1)\alpha)}{3(2-\beta)(1+(q^2-1)\alpha)}.$$

Prices follow an ordinal relationship, $p_2^{iO} \le p_2^{iH} \le p_2^{iL}$, where $i \in \{A, B\}$; that is, the competitor's customers receive the lowest price.

(b) Reward One's Own Best Customers: *When preference stochasticity is sufficiently high ($\beta \ge \bar{\chi}(\alpha, q)$)* and *consumer heterogeneity in purchase quantity is sufficiently high ($q > q^*$), there exists a symmetric pure-strategy equilibrium in second-period prices, such that retailers charge*

$$p_2^{AH} = p_2^{BH} = \frac{2-\beta}{2q\beta} + \frac{(2+\beta)(1+(q-1)\alpha)}{6((2-\beta)(1-\alpha)+q^2\alpha\beta)},$$

$$p_2^{AL} = p_2^{BL} = \frac{1}{2} + \frac{(2+\beta)(1+(q-1)\alpha)}{6((2-\beta)(1-\alpha)+q^2\alpha\beta)}, \quad \textit{and}$$

$$p_2^{AO} = p_2^{BO} = \frac{(2+\beta)(1+(q-1)\alpha)}{3((2-\beta)(1-\alpha)+q^2\alpha\beta)}.$$

Prices follow an ordinal relationship, $p_2^{iH} \le p_2^{iO} \le p_2^{iL}$, where $i \in \{A, B\}$; that is, the retailer's own high-type customers receive the lowest price.[12]

[12] The exact expressions for $\underline{\chi}(\alpha, q)$, $\bar{\chi}(\alpha, q)$, and q^* are provided in the appendix. For any given $0 < \alpha < 1$, in a small range ($\underline{\chi}(\alpha, q) < \beta < \bar{\chi}(\alpha, q)$) of β, the pure-strategy equilibrium does not exist. We focus our analysis only on the pure-strategy equilibrium area.

Proof. See the appendix.

Proposition 1 thus highlights the importance of the new features that we incorporate in our model. Both customer heterogeneity in quantity and sufficiently high levels of preference stochasticity are necessary to justify rewarding the firm's own customers. Otherwise, it is optimal to reward the competitor's customers (see Figure 3). Consistent with existing literature (Fudenberg and Tirole 2000), when there is no heterogeneity ($q = 1$) and preferences are stable ($\beta = 0$), it is optimal to reward the competitor's customers.

The intuition for why both customer heterogeneity and sufficient preference stochasticity are needed to reward the firm's own customers is as follows: as Lemma 1 shows, in a symmetric market, customers are more likely to stay with the current retailer than they are to switch to a rival for all $\beta \in [0, 1)$. If customers are all equal in value (i.e., no customer heterogeneity in quantity), retailers take advantage of customers' revealed preferences from the previous purchase and charge a higher price to their own customers while they expend more effort to acquire the competitor's customers through low prices. Although retailers understand that some existing customers switch in the second period (they cannot identify exactly who; they only know the aggregate probability of customers changing turfs), they find it optimal to offer a lower price to their current customers because both own and competitor's customers are all equal in value.

However, if customers are heterogeneous, retailers must assess whether selective retention (rewarding their own best customers) is profitable. The value of the firm's own high-type customers is greater than the expected average value of the competitor's customers.

Figure 4(a) **Customer Switching When** $\beta \le \underline{\chi}(\alpha, q)$: $\tilde{\theta}_1^L > \tilde{\theta}_2^{AL}$ **and** $\tilde{\theta}_1^H > \tilde{\theta}_2^{AH}$

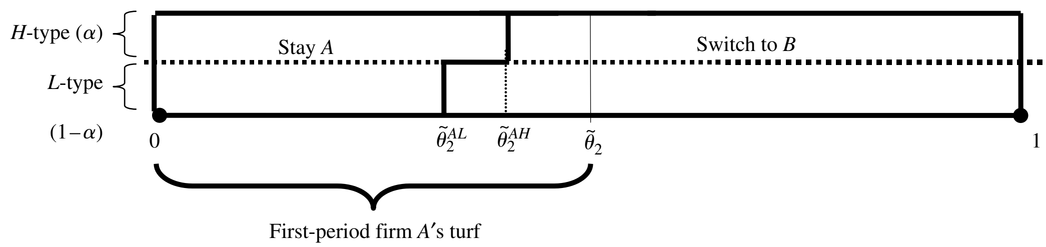
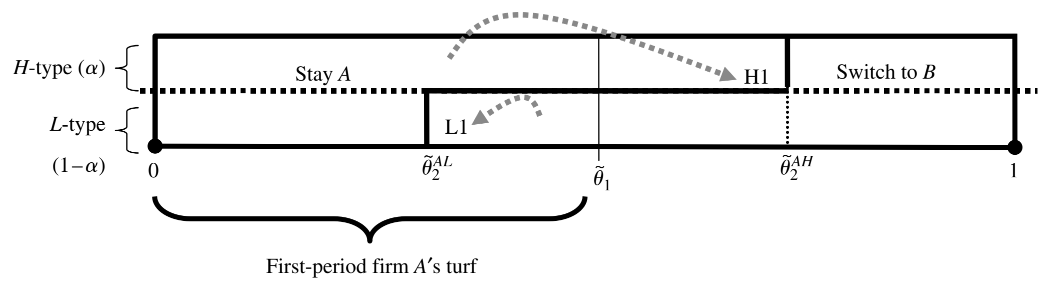


Figure 4(b) **Customer Switching When** $\beta \ge \bar{\chi}(\alpha, q)$: $\tilde{\theta}_1^L > \tilde{\theta}_2^{AL}$ **and** $\tilde{\theta}_1^H \le \tilde{\theta}_2^{AH}$



Furthermore, as preference stochasticity increases, the monopoly power that the retailer enjoys from horizontal preference weakens. Beyond a certain threshold of preference stochasticity, the marginal gain in profit from retaining high-type customers through lowered prices becomes greater than the marginal benefit of poaching a mix of high- and low-type competitor's customers. Therefore, both heterogeneity in quantities and high levels of preference stochasticity are necessary to make rewarding the firm's own high-type customers an effective strategy.

Finally, it is never optimal to offer a better value to own low-type customers, whose value is less than the expected average value of the competitor's customers. They always receive the highest price offer. This result echoes practitioners' assertion that the firm must reward its own *best* customers (not all of its customers!).[13]

We illustrate the relationship between preference stochasticity and actual customer switching behavior in the second period in Figure 4. Specifically, Figure 4(a) illustrates the case in which β is low, so retailers reward the competitor's customers. The second-period cutoff thresholds for both high and low types ($\tilde{\theta}_2^{AH}$ and $\tilde{\theta}_2^{AL}$) shift to the left of the first-period threshold, though the shift is greater for low-type customers who pay the highest price. The observed level of switching is greater for both high and low types relative to their intrinsic preference stochasticity levels.

In Figure 4(b), we present the case when β is high, so retailers reward their own best customers. An H-type customer (H_1) who bought from retailer A in the first period but gets a shock in the second period that moves him closer to retailer B still remains with retailer A. An L-type customer (L_1) who bought from retailer A and receives a shock that makes her even closer to retailer A instead may switch in the second period. In effect, the observed level of switching among H-types is lower relative to their preference stochasticity level, because they still repurchase from the same retailer even though they are much closer to the alternative. In contrast, the observed level of switching among L-type customers is greater than their preference stochasticity level. Even if the level of preference stochasticity is identical for the two types (i.e., no exogenous relationship between quantity and loyalty), firms' BBP practice creates an endogenous tendency for the most valuable (H-type) customers to stay more loyal than do L-type customers.[14]

[13] Interestingly, we also find a condition in which the average prices, $p_2^{i\_avg} = (p_2^{iH} + p_2^{iL})/2$, offered to one's own customers can be lower than the price for the competitor's customers when both heterogeneity in quantities and high levels of preference stochasticity exist (i.e., the price for the H-type is sufficiently low) and the portion of H-type customers is sufficiently small; thus the poaching price (p_2^{iO}) increases (note that the poaching price increases as α decreases). For example, when $\beta = 1$ and $q = 2$, we can show that if $\alpha < 0.2$, then $p_2^{i\_avg} < p_2^{iO}$. (See details in the electronic companion to this paper, available as part of the online version that can be found at http://mktsci.pubs.informs.org/.) When $\beta = 1$, $q = 2$, and $\alpha = 0.1$, $p_2^{i\_avg} = 0.798 < p_2^{iO} = 0.846$. We thank a reviewer and the area editor for suggesting that we compare the average prices offered to the firm's own and competitor's customers.

[14] We thank the editor for suggesting an analysis of the interplay between quantity and customer loyalty. To this end, we consider the possibility of different preference stochasticity levels across two

4.3. First Period

In the first period, the forward-looking consumer solves a dynamic program that takes into account the probabilities of second-period location and the prices offered by both retailers, as we show in Figure 5. Because these prices are the outcome of a dynamic strategic game played by the retailers, solving for the equilibrium retailer and consumer strategies requires embedding the consumer's dynamic programming problem within a dynamic strategic game that involves one price for each retailer in the first period and three prices (own high and low types, and competitor's customers) for each retailer in the second period.

Let retailer A's first-period price be p_1^A and retailer B's first-period price be p_1^B. If the first-period prices lead to a cut-off $\tilde{\theta}_1^j$, the marginal consumer of type j, where $j \in \{L, H\}$ with location $\tilde{\theta}_1^j$, must be indifferent between buying a product from A or B in period 1. In other words, the consumer compares the utility of purchasing from either retailer A or retailer B, recognizing that his or her location may change because of preference stochasticity. Consumers are forward-looking, so they rationally anticipate the consequence of their first-period choice in terms of the prices they will receive in the second period. Thus, the following equation holds for the marginal consumer:

$$\begin{aligned} &Q^j v - Q^j p_1^A - \tilde{\theta}_1^j + \delta\{E^A[U_2^j \mid \theta_1 = \tilde{\theta}_1^j]\} \\ &\quad = Q^j v - Q^j p_1^B - (1-\tilde{\theta}_1^j) + \delta\{E^B[U_2^j \mid \theta_1 = \tilde{\theta}_1^j]\}, \quad (4) \end{aligned}$$

where $\delta < 1$ is the discount factor, and $E^A[U_2^j|\theta_1=\tilde{\theta}_1^j] = E_2^{Aj}$, $E^B[U_2^j|\theta_1=\tilde{\theta}_1^j]=E_2^{Bj}$ represents the expected second-period utility that this "marginal" consumer derives from purchasing from retailer A or B in the first period, respectively. Thus,

$$\begin{aligned} E_2^{Aj} &= E^A[U_2^j \mid \theta_1 = \tilde{\theta}_1^j] \\ &= \int_{\theta_2=0}^{\tilde{\theta}_2^{Aj}} \Pr(\theta_2 \mid \theta_1 = \tilde{\theta}_1^j) \times (Q^j v - Q^j p_2^{Aj} - \theta_2)\, d\theta_2 \\ &\quad + \int_{\theta_2=\tilde{\theta}_2^{Aj}}^{1} \Pr(\theta_2 \mid \theta_1 = \tilde{\theta}_1^j) \\ &\qquad\qquad \times (Q^j v - Q^j p_2^{BO} - (1-\theta_2))\, d\theta_2, \quad (5) \end{aligned}$$

$$\begin{aligned} E_2^{Bj} &= E^B[U_2^j \mid \theta_1 = \tilde{\theta}_1^j] \\ &= \int_{\theta_2=0}^{\tilde{\theta}_2^{Bj}} \Pr(\theta_2 \mid \theta_1 = \tilde{\theta}_1^j) \times (Q^j v - Q^j p_2^{AO} - \theta_2)\, d\theta_2 \\ &\quad + \int_{\theta_2=\tilde{\theta}_2^{Bj}}^{1} \Pr(\theta_2 \mid \theta_1 = \tilde{\theta}_1^j) \\ &\qquad\qquad \times (Q^j v - Q^j p_2^{Bj} - (1-\theta_2))\, d\theta_2, \quad (6) \end{aligned}$$

where $\tilde{\theta}_2^{Aj}$ and $\tilde{\theta}_2^{Bj}$ represent the second-period cut-off locations of j-type customers who purchase from retailer A and retailer B in the first period, respectively.

The expected second-period utility in Equations (5) and (6) contains two components in each case. The first term in Equation (5),

$$\int_{\theta_2=0}^{\tilde{\theta}_2^{Aj}} \Pr(\theta_2 \mid \theta_1 = \tilde{\theta}_1^j) \times (Q^j v - Q^j p_2^{Aj} - \theta_2)\, d\theta_2,$$

represents the case in which a second-period location (θ_2) is such that the first-period marginal consumer decides to repurchase from retailer A at its repeat purchase price. The second term,

$$\int_{\theta_2=\tilde{\theta}_2^{Aj}}^{1} \Pr(\theta_2 \mid \theta_1 = \tilde{\theta}_1^j) \times (Q^j v - Q^j p_2^{BO} - (1-\theta_2))\, d\theta_2,$$

represents the case in which the second-period location is such that the first-period marginal consumer has decided to purchase from retailer B with its poaching price. Both the repeat purchase case and poaching case similarly constitute Equation (6).

From Equation (4), it follows that the marginal first-period customer of type j is

$$\tilde{\theta}_1^j = \frac{\{1 + Q^j p_1^B - Q^j p_1^A + \delta(E^A[U_2^j \mid \theta_1 = \tilde{\theta}_1^j] - E^B[U_2^j \mid \theta_1 = \tilde{\theta}_1^j])\}}{2}. \quad (7)$$

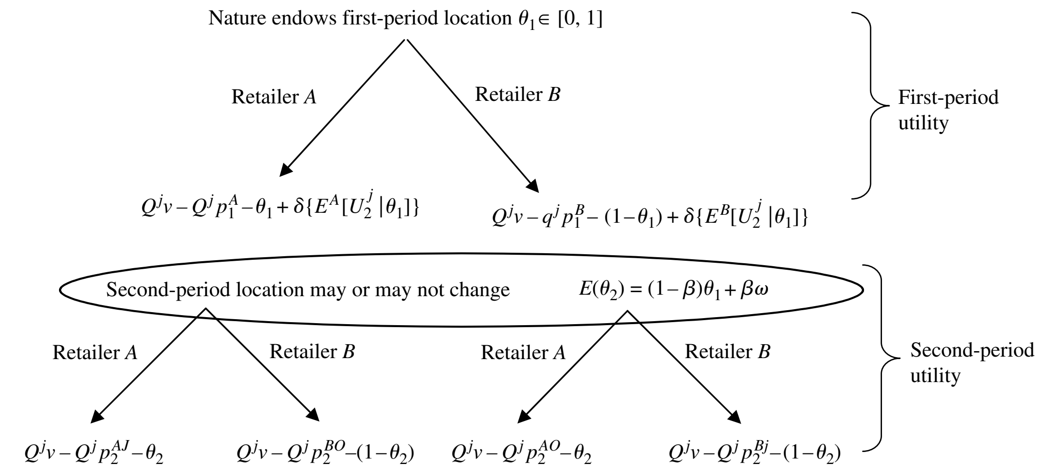
Added complexity arises because p_2^{AO}, p_2^{BO}, p_2^{Aj}, and p_2^{Bj}, which are embedded in E_2^{Aj} and E_2^{Bj}, are all functions of both $\tilde{\theta}_1^L$ and $\tilde{\theta}_1^H$. Hence, E_2^{AL}, E_2^{BL}, E_2^{AH}, and E_2^{BH} are again functions of both $\tilde{\theta}_1^L$ and $\tilde{\theta}_1^H$.

We solve for the first-period equilibrium prices and overall profits.[15] The overall net present values of

segments, say β for L-type and $\kappa\beta$ for H-type. Our main result—pertaining to rewarding the firm's own customers by offering them a lower price beyond a certain threshold level of β—remains robust as long as $\kappa\beta$ is higher than a threshold level, that is, if the preference stochasticity of the high types is not much lower than that of the low types. However, if $\kappa\beta$ for H-type customers is extremely low (i.e., little preference stochasticity), the firm has no incentive to reward its own high-type customers, who are very likely to stay with the firm. See the electronic companion for details.

[15] Because we assume a uniform distribution of consumers, one may think that we could compute the first-order condition of Equation (8) using an explicit solution strategy, which simply plugs the primitives back into the second-period profit functions. Unfortunately, as we can see in Equations (5) and (6), this approach would require us to solve the integral with the boundary and the probability itself involving the primitives of $\tilde{\theta}_1^j$, which is again the implicit function of itself and p_1^A. It is therefore infeasible to employ this explicit solution strategy. We follow and extend the general approach, first employed by Fudenberg and Tirole (2000). Furthermore, we develop an indirect method to solve this challenge, which we show in Lemma 2 of the appendix.

Figure 5 Consumer Decision Tree



profits for retailers A and B, as viewed from period 1, respectively, are given by

$$\begin{aligned}\Pi^A &= (p_1^A)\cdot(q\tilde{\theta}_1^H + \tilde{\theta}_1^L) + \delta\Pi_2^A,\\ \Pi^B &= (p_1^B)\cdot(q(1-\tilde{\theta}_1^H) + (1-\tilde{\theta}_1^L)) + \delta\Pi_2^B,\end{aligned} \tag{8}$$

where Π_2^A, Π_2^B are as defined in Equation (3).

Taking first-order conditions with respect to prices, we find a symmetric pure-strategy equilibrium in which both retailers charge the same prices (by using the envelope theorem and indirect method to solve for the first-order conditions that we develop in Lemma 2 in the appendix, we can arrive at the closed-form solutions; see the appendix for details):

$$\begin{aligned}p_1^A &= p_1^B\\ &= \frac{((1-\alpha)+\alpha q) - 2\delta(\Omega^H(-\partial\tilde{\theta}_1^H/\partial p_1^A) + \Omega^H(-\partial\tilde{\theta}_1^L/\partial p_1^A))}{2(\alpha q(-\partial\tilde{\theta}_1^H/\partial p_1^A) + (1-\alpha)(-\partial\tilde{\theta}_1^L/\partial p_1^A))},\end{aligned} \tag{9}$$

$$\begin{aligned}\text{where }\ \Omega_L &= \frac{\partial\Pi_2^{AA}}{\partial p_2^{BO}}\frac{dp_2^{BO}}{d\tilde{\theta}_1^L} + \frac{\partial\Pi_2^{AB}}{\partial p_2^{BL}}\frac{dp_2^{BL}}{d\tilde{\theta}_1^L} + \frac{\partial\Pi_2^{AB}}{\partial p_2^{BH}}\frac{dp_2^{BH}}{d\tilde{\theta}_1^L}\\ &\quad + \frac{\partial\Pi_2^{AA}}{\partial\tilde{\theta}_1^L} + \frac{\partial\Pi_2^{AB}}{\partial\tilde{\theta}_1^L},\\ \Omega_H &= \frac{\partial\Pi_2^{AA}}{\partial p_2^{BO}}\frac{dp_2^{BO}}{d\tilde{\theta}_1^H} + \frac{\partial\Pi_2^{AB}}{\partial p_2^{BL}}\frac{dp_2^{BL}}{d\tilde{\theta}_1^H} + \frac{\partial\Pi_2^{AB}}{\partial p_2^{BH}}\frac{dp_2^{BH}}{d\tilde{\theta}_1^H}\\ &\quad + \frac{\partial\Pi_2^{AA}}{\partial\tilde{\theta}_1^H} + \frac{\partial\Pi_2^{AB}}{\partial\tilde{\theta}_1^H}.\end{aligned}$$

Therefore, the equilibrium profits are

$$\begin{aligned}\Pi^A &= \Pi^B = (p_1^A)\cdot(\alpha q\tilde{\theta}_1^H + (1-\alpha)\tilde{\theta}_2^L) + \delta\Pi_2^A\\ &= \left(\frac{((1-\alpha)+\alpha q) - 2\delta(\Omega^H(-\partial\tilde{\theta}_1^H/\partial p_1^A) + \Omega^H(-\partial\tilde{\theta}_1^L/\partial p_1^A))}{2(\alpha q(-\partial\tilde{\theta}_1^H/\partial p_1^A) + (1-\alpha)(-\partial\tilde{\theta}_1^L/\partial p_1^A))}\right)\\ &\quad\cdot\left(\frac{(1-\alpha)+\alpha q}{2}\right) + \delta\Pi_2^A.\end{aligned} \tag{10}$$

We now compare these profits obtained by BBP with a model that does not use customer purchase information. The comparison should help us understand the benefits, if any, of BBP.

The model that does not use customer purchase information reduces to a static pricing model. In each period, the two retailers maximize their current profits. In this static pricing scenario, both retailers would charge the static price (p_1^s), which maximizes the static profit functions in each period, such that

$$\begin{aligned}\Pi_1^A &= (p_1^A)\cdot\left(\alpha q\left(\frac{1-qp_1^A+qp_1^B}{2}\right)\right.\\ &\qquad\left.+(1-\alpha)\left(\frac{1-p_1^A+p_1^B}{2}\right)\right), \quad\text{and}\\ \Pi_1^B &= (p_1^B)\cdot\left(\alpha q\left(\frac{1-qp_1^B+qp_1^A}{2}\right)\right.\\ &\qquad\left.+(1-\alpha)\left(\frac{1-p_1^B+p_1^A}{2}\right)\right).\end{aligned} \tag{11}$$

Taking the first-order conditions and solving for prices, the equilibrium price in the static case is given by $p_1^{\text{No BBP}} = p_1^A = p_1^B = (1+(q-1)\alpha)/(1+(q^2-1)\alpha)$, and the per-period profit is $\Pi_1^A = \Pi_1^B = (((1+(q-1)\alpha)^2)/(2(1+(q^2-1)\alpha)))$. The discounted net present value of the total profit across two periods for both retailers is

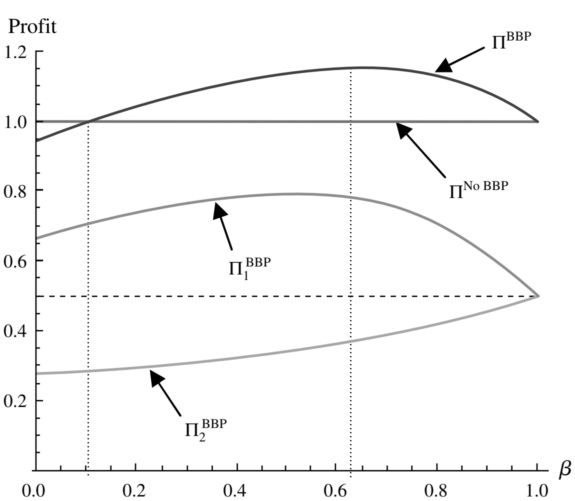
$$\Pi_1^{\text{No BBP}} = \Pi^A = \Pi^B = \left(\frac{(1+(q-1)\alpha)^2}{2(1+(q^2-1)\alpha)}\right)(1+\delta). \tag{12}$$

We begin the profit discussion by considering two special cases with (1) only preference stochasticity and (2) only customer heterogeneity in purchase quantity. These cases can be obtained by setting $q=1$ and $\beta=0$, respectively, in the general model. We isolate the true effect of these features and then describe the general profit result with both customer heterogeneity and preference stochasticity, to clarify how these two key features interact. The complete analysis of the general case is available in the electronic companion; we focus here on the case in which high- and low-volume customers are equal in number ($\alpha = \frac{1}{2}$) to ease the exposition and analysis.

4.3.1. Only Preference Stochasticity, No Customer Heterogeneity in Purchase Quantity. The case without customer heterogeneity can be captured by setting $q = 1$ in the model, such that all customers buy one unit of the product. As Proposition 1(a) states, when $q = 1$, it is always optimal for firms to reward the competitor's customers, irrespective of the level of preference stochasticity. Figure 6 shows the total profits across the first and second periods with BBP and without BBP, with a discount rate $\delta \approx 1$. The per-period profits without BBP are close to $\frac{1}{2}$ with $\delta \approx 1$, and the total profits over two periods without BBP are close to 1.

Consistent with Fudenberg and Tirole's (2000) model ($\beta = 0$ and $q = 1$ in our model), we find that the profit from BBP is lower than profits without BBP when $\beta = 0$. Total profits reveal an inverted U-shaped curve when we extend our analysis to the case of general preference stochasticity in $\beta \in [0, 1]$. To understand the intuition behind this result, we decompose the total profits into profits from the first and second periods, as shown in Figure 6. The second-period profit increases monotonically with β, because preference instability softens competition. More precisely, firms do not wish to compete aggressively for customers who could be on their own turf naturally through changing preference; they do not offer aggressive prices to customers who may prefer their own product anyway. This competition-softening effect, similar to the "mistargeting" effect identified by Chen et al. (2001), raises second-period prices and profit.

Figure 6 **Profitability of BBP When $\alpha = 0.5$ and $q = 1$**



The effect of preference stochasticity on first-period profit is more subtle, including an indirect effect as a result of the consumer's consideration of future price. Consumers recognize that future prices will increase (both second-period repeat purchase and poaching prices increase in β), so their choices become less sensitive to changes in the first-period price. The lower price sensitivity shifts the first-period price upward as β increases.[16]

However, a countervailing direct effect exerts downward pressure on first-period prices. As β increases, the link between customers' choices in the first and second periods weakens, because consumer preferences become less correlated, and price elasticity in the first period is less affected by what happens in the second period. The direct effect interacts with the indirect effect and effectively weakens the upward pressure of the indirect effect. At the extreme, when $\beta = 1$, demand in the two periods becomes independent, and price elasticity increases to short-run elasticity without BBP; in turn, profits with and without BBP become identical.

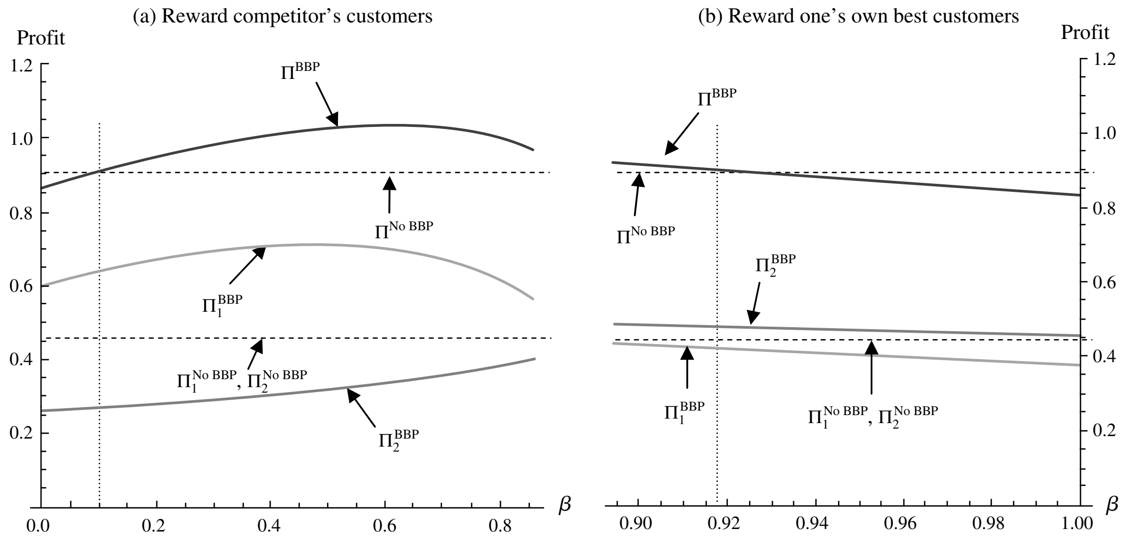
In summary, the net effect on first-period price (upward pressure from indirect effect and downward pressure from direct effect) leads to an inverted U-shaped curve for first-period profits (and total profits). Therefore, the total equilibrium profit is maximized at $\beta = 0.6435$, increasing for $\beta \in [0, 0.6435]$ and decreasing for $\beta \in [0.6435, 1]$.

4.3.2. No Stochastic Preference, Only Customer Heterogeneity in Purchase Quantity. When there is no changing preference ($\beta = 0$), we know from Proposition 1 that retailers offer the lowest price to their competitor's customers; therefore, $p_2^{iO} \le p_2^{iH} \le p_2^{iL}$. However, the more surprising result indicates that when heterogeneity in purchase quantities is sufficiently high, the total profit with BBP is greater than profits without BBP.

Proposition 2. *Suppose there is no preference stochasticity ($\beta = 0$). If heterogeneity in purchase quantities is sufficiently high ($q > 5$), both retailers increase their profits under BBP.*

[16] More precisely, the indirect effect of consumer's future consideration affects the first-period price in two ways: (1) second-period high repeat purchase prices decrease price elasticity in the first period, because customers know they will be ripped off by the same firm in the second period (ratchet effect); and (2) second-period low poaching price increases price elasticity in the first period, causing a downward pressure on prices. As β increases, both poaching and repeat purchase prices increase, and the correct expectation of high prices makes consumers less price sensitive in the first period (a higher repeat purchase price further decreases price elasticity; a higher poaching price weakens upward pressure on price elasticity). The indirect effect through a consumer's dynamic consideration makes first-period demand less elastic, causing firms to increase prices and profits in the first period as β increases.

Figure 7 **Profits When $\alpha = 0.5$ and $q = 2$**



Proof. See the appendix.

Thus, BBP can increase both competing firms' profits even when consumer preferences are stable if there is sufficient heterogeneity in quantities. This result does not occur simply because of heterogeneity in quantities and the ability to distinguish between high- and low-type customers. The asymmetric information about customer types is critical.

The intuition for this result is as follows: with asymmetric information, the poacher faces an adverse selection (lemon) problem when attracting the competitor's customers. Because the high types get lower prices from their current retailer, firms recognize that poaching disproportionately attracts L-type relative to H-type customers. Therefore, firms become less aggressive in attracting the competitor's customers as customer heterogeneity becomes significant (i.e., degree of adverse selection becomes greater). In other words, asymmetric customer information softens competition, so the benefit of price discrimination dominates the cost of increased competition typically associated with BBP when customer heterogeneity becomes significant.

4.3.3. Both Stochastic Preference and Customer Heterogeneity in Purchase Quantity. We next consider the full model by relaxing both the $q = 1$ and $\beta = 0$ assumptions. Specifically, we consider the $q = 2$ case to allow for customer heterogeneity, which helps build the intuition for the customer heterogeneity case with the least complexity.

In Figure 7, we show the total profits with and without BBP when $\alpha = 0.5$ and $q = 2$. As anticipated from Proposition 1, there are two regimes based on the pricing strategy in period 2: the "reward competitor's customers" regime and the "reward one's own best customers" regime. As a key takeaway, BBP can be profitable in both regimes. Specifically, profit with BBP is greater than profits without BBP in the range $\beta \in (0.115, 0.857)$ in the reward competitor's customers regime (Figure 7(a)). Similarly in the range, $\beta \in (0.894, 0.917)$, profit with BBP is greater than that without BBP in the reward one's own best customers regime (Figure 7(b)).

As a summary of this discussion, we state our third proposition.

Proposition 3 (Profitability of BBP). (a) Reward Competitor's Customers in Second Period: *When preference is stable ($\beta < \underline{\chi}(0.5, 2) = 0.857$), the total profit with BBP is greater than profits without BBP if preference stochasticity is in the range $\beta \in (0.115, 0.857)$.*

(b) Reward One's Own Best Customers in Second Period: *When customer preference is sufficiently stochastic ($\beta \geq \bar{\chi}(0.5, 2) = 0.894$), the total profit with BBP is greater than profits without BBP when preference stochasticity is in the range $\beta \in (0.894, 0.917)$.*

We see a stark reversal in profit patterns across the two periods between the reward one's own best and reward competitor's customers regimes. For the latter regime, we see a decreasing profit pattern over two periods: profits in the first period are higher than static levels, and profits in the second period are below the static levels. This profit-ordering reverses in the reward one's own best customers regime, so that we see an increasing profit pattern over time; that is, the first-period profits are below the static levels, whereas second-period profits are above.

The intuition for profits under the reward competitor's customer regime follows directly from the previous discussion regarding the direct and indirect effects of preference stochasticity alone and the discussion of information asymmetry with only heterogeneity. On

Table 2 Summary of Results

		Preference stochasticity			
		Low		Sufficiently high	
		Prices	Profits	Prices	Profits
Heterogeneity in quantity (information advantage about current customers)	Low	Reward competitor customers (Proposition 1(a))	Behavior-based pricing less profitable (Proposition 3(a))	Reward competitor customers (Proposition 1(b))	Behavior-based pricing more profitable (Proposition 3(a))
	Sufficiently high	Reward competitor customers (Proposition 1(a))	Behavior-based pricing more profitable (Proposition 2)	Reward current high-type customers (Proposition 1(b))	Behavior-based pricing more profitable unless preference is extremely unstable (Proposition 3(b))

the other hand, the intuition for the increasing price and profit pattern over time under the reward one's own best customers regime follows from the lock-in intuition, similar to switching-cost models (Klemperer 1995, Farrell and Klemperer 2007). Firms compete to attract more customers in the first period to gain an information advantage about customer types. By using this information advantage, firms effectively can lock in their most valuable (and profitable) H-type customers by rewarding them in the second period.[17] This lock-in intuition explains the profit pattern over time—that is, lower first-period profit and higher second-period profit. Moreover, we find that firms can be better off with BBP under this regime.

However, as preference stochasticity gets closer to one, the benefit of lock-in weakens because of the imperfect lock-in that arises from preference stochasticity, which in turn lowers the second-period profits. At high levels of preference stochasticity, overall profits can be lower with BBP when $\alpha = 0.5$ and $q = 2$. In general, the total profit can be higher with BBP even when preference stochasticity is very high because of the same adverse selection intuition in the pure heterogeneity case. For example, when $\alpha = 0.5$, BBP is always profitable if $q \geq 5$. The threshold level of preference stochasticity at which the total profit with BBP is greater than that without BBP is 0.917 when $\alpha = 0.5$ and $q = 2$.

This threshold is in fact a function of α and q. When the value of information asymmetry is greater (larger q), the threshold rises, and rewarding one's own best customers is a profitable strategy for a wider range of preference stochasticity. For example, when $q = 16$, which is the case suggested by the 80/20 rule when $\alpha = 0.5$,[18] the range in which BBP is profitable under the reward one's own best customers regime becomes much larger, $\beta \in (0.731, 0.961)$.

5. Conclusion

We address a basic but controversial question in customer management: should a firm use BBP, and if so, whom should it reward? We summarize our key results in Table 2. When heterogeneity in purchase quantity and preference stochasticity are *both* low, it is optimal to reward competitor's customers, and BBP is less profitable. This result is consistent with existing theoretical models. However, if either heterogeneity in customer quantities or preference stochasticity is sufficiently high, BBP can improve profits under competition, even when consumers and firms are strategic and forward-looking. Rewarding the firm's own best customers is optimal only when there is *both* heterogeneity in purchase quantities *and* preference stochasticity are sufficiently high.

This study thus reconciles the apparent conflict between practitioners' optimism and academic skepticism about using customer's past purchase information by nesting both existing analytical results and practitioner intuition. Both heterogeneity in customer value and the threat of customer switching as a result of stochastic preference are characteristics of a wide variety of markets; thus understanding the extent of such heterogeneity and preference stochasticity offers critical information about the value of using BBP and arriving at the right balance between customer acquisition and retention.

Certain limitations of our model suggest interesting directions for further research. First, we do not allow consumers to split their purchases across different

[17] In our model, the high types are indeed the most valuable customers for firms because the optimal prices are such that per-period profits will be higher for the high type (that is, $q \times p_2^{AH} > ((1+q)/2) \times p_2^{AO} > p_2^{AL}$ for all β and $q \geq 2$). In other words, the cost to retain high types does not exceed the benefit of retaining them. Furthermore, it is easy to see that second-period prices are all higher than the first-period price; $p_2^{AH} > p_1^{A}$.

[18] A q that solves for the 80/20 rule is obtained by solving $0.2q = 0.8(0.2q + 0.8)$.

firms or even across periods;[19] thus it is harder for firms to infer the customer's true potential and share of customer wallet. In our current model, the first-period purchases help firms unambiguously identify H- and L-type consumers. When consumers can split their purchases, the inference about consumer type would be only probabilistic, even if firms possess share-of-wallet information. Thus share-of-wallet information weakens the extent of information asymmetry in the model (see Du et al. 2008 as an example of how a firm may infer share of wallet).

Second, third parties also sell estimates of customer-spending potential, based on the observed characteristics of the household (e.g., zip code, demographics), but this information is far from perfect. Therefore, even with third-party information, the observed purchase history still provides the information asymmetry critical to the profitability of BBP. We therefore believe our findings are robust even with the introduction of noisy potential information, although it might weaken the information asymmetry. A systematic analysis of how share of wallet and potential estimates may affect information asymmetry, and their resulting effect on BBP, would offer an important next step from both the empirical and theoretical perspectives.[20]

Third, given our focus on BBP, we have abstracted away from modeling second-degree price discrimination. Our model is a reasonable abstraction of many real-world markets (e.g., airlines, department stores, grocery stores) in which firms only use BBP with past customer purchase history data but do not offer a menu of prices. This could be because consumers differ not in the quantity they use at any one time but in the frequency of their use. Firms need to aggregate the flow of multiple purchases across a certain time period into a "past quantity" stock. In this scenario, menu pricing is not feasible, and BBP is appropriate. Nevertheless, further research needs to consider how robust our results are if competitors could offer a menu of poaching prices. The second-degree price discrimination problem in a competitive setting with customer behavior information is a technical challenge that has not yet been solved generally and awaits further research (Stole 2007). We therefore leave the issue of combining BBP and second-degree price discrimination as an important, but challenging, area of further research.

[19] Consumers may strategically time their purchases. This could be related to sales force literature where in response to quotas and bonuses, salespeople sometimes shift their effort and book sales towards the end of the year to qualify for a bonus (Steenburgh 2008). We thank an anonymous reviewer for raising this issue.

[20] A potential formulation that accommodates the notion of share of wallet and customer potential might be $q = \text{Potential} \times S(\theta)$, where $S(\theta)$ the share of wallet of the customer, is a function of the relative preference between firms (i.e., based on location θ).

Fourth, our theoretical model provides empirically testable hypotheses for further research. We find that BBP is most likely effective when there is sufficient heterogeneity in customer quantities and preference stochasticity. Empirical research across categories which vary along these dimensions can help ascertain whether these predictions are valid. We therefore hope our model serves as an impetus for further theoretical and empirical research into BBP.

6. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mktsci.pubs.informs.org/.

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Appendix

Proof of Lemma 1 (Value of Information). We first address retailer A. Let z be the first-period market share. Then, $\Pr[\theta_2 \le z \mid \theta_1 \le z] \ge \Pr[\theta_2 > z \mid \theta_1 \le z] \Leftrightarrow (1-\beta) + \beta z \ge \beta(1-z)$. In turn, $\Pr[\theta_2 \le z \mid \theta_1 \le z] - \Pr[\theta_2 > z \mid \theta_1 \le z] \ge 0 \Leftrightarrow \beta \le 1/(2(1-z))$. Therefore, it always holds when $\beta \le 1/(2(1-z))$. In particular, when $z \ge \frac{1}{2}$, the inequality always holds for $\forall \beta \in (0,1]$. Next, let $\bar{z} = 1-z$ be retailer B's first-period market share and substitute it into Equation (2). We get exactly the same result. Q.E.D.

Proof of Proposition 1 (Reward One's Own Customers or Competitor's Customers).

(a) We start with $\beta < \underline{\chi}(\alpha, q)$,

$$\text{where } \underline{\chi}(\alpha, q) = \frac{2((3-q)(1-\alpha) + 2\alpha q^2)}{(3+q)(1-\alpha) + 4\alpha q^2}$$

and $\underline{\chi}(\alpha, q) < \bar{\chi}(\alpha, q)$ for $\forall \alpha \in (0,1)$, which ensures that $\tilde{\theta}_1^L > \tilde{\theta}_2^{AL}$ and $\tilde{\theta}_1^H > \tilde{\theta}_2^{AH}$, in equilibrium. Again, using Equations (1) and (2), we know that

$$\Pr{}^{AH} = \left(\frac{1-\beta}{\tilde{\theta}_1^H} + \beta\right) \frac{1 + q(p_2^{BO} - p_2^{AH})}{2},$$

$$\Pr{}^{AL} = \left(\frac{1-\beta}{\tilde{\theta}_1^L} + \beta\right) \frac{1 + p_2^{BO} - p_2^{AL}}{2},$$

$$\Pr{}^{BH} = \left(\frac{1-\beta}{1-\tilde{\theta}_1^L} + \beta\right) \frac{1 - q(p_2^{BH} - p_2^{AO})}{2}, \quad \text{and}$$

$$\Pr{}^{BL} = \left(\frac{1-\beta}{1-\tilde{\theta}_1^L} + \beta\right) \frac{1 - p_2^{BL} + p_2^{AO}}{2}.$$

Similar to the proof of Proposition 1, we solve the first-order conditions. When $\tilde{\theta}_1^j=\frac{1}{2}$, second-period prices are

$$p_2^{AH}=p_2^{BH}=\frac{1}{2q}+\frac{(2+\beta)(1+(q-1)\alpha)}{6(2-\beta)(1+(q^2-1)\alpha)},$$

$$p_2^{AL}=p_2^{BL}=\frac{1}{2}+\frac{(2+\beta)(1+(q-1)\alpha)}{6(2-\beta)(1+(q^2-1)\alpha)},\quad \text{and}$$

$$p_2^{AO}=p_2^{BO}=\frac{(2+\beta)(1+(q-1)\alpha)}{3(2-\beta)(1+(q^2-1)\alpha)},$$

which confirms that $\tilde{\theta}_1^L>\tilde{\theta}_2^{AL}$ and $\tilde{\theta}_1^H>\tilde{\theta}_2^{AH}$ when

$$\beta<\underline{\chi}(q)=\frac{2((3-q)(1-\alpha)+2\alpha q^2)}{(3+q)(1-\alpha)+4\alpha q^2}.$$

We only need to show that $p_2^{iO}\le p_2^{iH}\le p_2^{iL}$. First, we notice that

$$\underline{\chi}(q)=\frac{2((3-q)(1-\alpha)+2\alpha q^2)}{(3+q)(1-\alpha)+4\alpha q^2}$$

is an increasing function in $\alpha\in[0,1]$ for any $q\ge1$. Second, we have

$$p_2^{iO}\le p_2^{iH}\ \Leftrightarrow\ \frac{(2+\beta)(1+(q-1)\alpha)}{6(2-\beta)(1+(q^2-1)\alpha)}\le\frac{1}{2q}$$
$$\Leftrightarrow\ \beta\le\frac{2(5(1-\alpha)-q\alpha+6q^2\alpha)}{7(1-\alpha)+q\alpha+6q^2\alpha},$$

and

$$\frac{2(5(1-\alpha)-q\alpha+6q^2\alpha)}{7(1-\alpha)+q\alpha+6q^2\alpha}$$

for any $q\ge1$ and $\forall\alpha\in[0,1]$. Hence, the inequality always satisfies for all $q\ge1$. In addition, it is obvious that $p_2^{iH}\le p_2^{iL}\Leftrightarrow 1/(2q)\le\frac{1}{2}$ for all $q\ge1$. $\square$

(b) Next, we look at the case $\beta\ge\bar{\chi}(\alpha,q)$, which ensures that in equilibrium, the market is $\tilde{\theta}_1^L>\tilde{\theta}_2^{AL}$ and $\tilde{\theta}_1^H\le\tilde{\theta}_2^{AH}$, where

$$\bar{\chi}(\alpha,q)=(2\alpha q^2-(q+6)(1-\alpha)+\sqrt{(2\alpha q^2-(q+6)(1-\alpha))^2+12(1-\alpha)(4\alpha q^2+(q-3)(1-\alpha))})\cdot(4\alpha q^2+(q-3)(1-\alpha))^{-1}.$$

Again, from Equations (1) and (2), we know that

$$\Pr^{AH}=(1-\beta)+\beta\left(\frac{1+q(p_2^{BO}-p_2^{AH})}{2}\right),$$

$$\Pr^{AL}=\left(\frac{1-\beta}{\tilde{\theta}_1^L}+\beta\right)\frac{1+p_2^{BO}-p_2^{AL}}{2},$$

$$\Pr^{BH}=(1-\beta)+\beta\frac{1-q(p_2^{BH}-p_2^{Ao})}{2},\quad \text{and}$$

$$\Pr^{BL}=\left(\frac{1-\beta}{1-\tilde{\theta}_1^L}+\beta\right)\frac{1-p_2^{BL}+p_2^{AO}}{2}.$$

Plugging these into Equation (3), we obtain the second-period prices by solving the retailers' first-order conditions. Specifically, when $\tilde{\theta}_1^j=\frac{1}{2}$, the second-period prices are

$$p_2^{AH}=p_2^{BH}=\frac{2-\beta}{2q\beta}+\frac{(2+\beta)(1+(q-1)\alpha)}{6((2-\beta)(1-\alpha)+q^2\alpha\beta)},$$

$$p_2^{AL}=p_2^{BL}=\frac{1}{2}+\frac{(2+\beta)(1+(q-1)\alpha)}{6((2-\beta)(1-\alpha)+q^2\alpha\beta)},\quad \text{and}$$

$$p_2^{AO}=p_2^{BO}=\frac{(2+\beta)(1+(q-1)\alpha)}{3((2-\beta)(1-\alpha)+q^2\alpha\beta)}.$$

We confirm that $\tilde{\theta}_1^L>\tilde{\theta}_2^{AL}$ and $\tilde{\theta}_1^H\le\tilde{\theta}_2^{AH}$ when $q>1$ and $\beta\ge\bar{\chi}(q)$. First, we verify that

$$p^{iL}\ge p^{iO}\ \Leftrightarrow\ p^{iL}-p^{iO}=\frac{1}{2}-\frac{(2+\beta)(1+q)}{6(q^2+1)(2-\beta)}\ge0$$
$$\Leftrightarrow\ 2(3q^2-q+2)\ge(3q^2+q+4)\beta.$$

Note that $(3q^2+q+4)\beta\le(3q^2+q+4)$ for $\beta\in[0,1]$. Hence, the result that $p^{iL}\ge p^{iO}$ for all $\beta\in[0,1]$ and $q\ge1$ derives directly from $2(3q^2-q+2)>(3q^2+q+4)\Leftrightarrow 3q(q-1)\ge0$. Second, we show that $p^{iH}\le p^{iO}$. Notice that

$$p^{iH}\le p^{iO}\ \Leftrightarrow\ \frac{2-\beta}{q\beta}\le\frac{(2+\beta)(1+q)}{6(q^2+1)(2-\beta)}$$
$$\Leftrightarrow\ \beta\ge\bar{\chi}(q).$$

We only need to show that there exists $\beta\in[0,1]$, such that

$$\beta\ \ge\ \bar{\chi}(\alpha,q)=(2\alpha q^2-(q+6)(1-\alpha)+\sqrt{(2\alpha q^2-(q+6)(1-\alpha))^2+12(1-\alpha)(4\alpha q^2+(q-3)(1-\alpha))})\cdot(4\alpha q^2+(q-3)(1-\alpha))^{-1}.$$

Lemma 2. *For all $\alpha\in(0,1)$, there always exists*

$$q>\max\left\{\frac{-1+\alpha+\sqrt{1+46\alpha-47\alpha^2}}{8\alpha},\frac{9(1-\alpha)}{5+3\alpha}\right\},$$

such that $\bar{\chi}(\alpha,q)<1$.

Proof. Note that

$$\begin{aligned}\bar{\chi}(\alpha,q)&\le\frac{2(2\alpha q^2-(q+6)(1-\alpha))+\sqrt{12(1-\alpha)(4\alpha q^2+(q-3)(1-\alpha))}}{4\alpha q^2+(q-3)(1-\alpha)}\\&<\frac{2(2\alpha q^2-(q+6)(1-\alpha))+4q+\sqrt{12(q-3)(1-\alpha)^2}}{4\alpha q^2+(q-3)(1-\alpha)}\\&<\frac{2(2\alpha q^2-(q+6)(1-\alpha))+4q+4(q-3)(1-\alpha)}{4\alpha q^2+(q-3)(1-\alpha)}\\&<\frac{2(2\alpha q^2-(q+6)(1-\alpha))+8q}{4\alpha q^2+(q-3)(1-\alpha)}.\end{aligned}$$

Let us define $G(\alpha,q)\equiv(2(2\alpha q^2-(q+6)(1-\alpha))+8q)/(4\alpha q^2+(q-3)(1-\alpha))>\bar{\chi}(\alpha,q)$ for all $\alpha\in(0,1)$ and q. We can easily see that $G(\alpha,q)$ is monotonically decreasing in q because

$$\frac{\partial G(\alpha,q)}{\partial q}=\frac{-6-2\alpha(6-9\alpha+2q(5q+3(6+q)\alpha-18))}{(q-3+(3+q(4q-1))\alpha)^2}<0$$

for all $\alpha\in(0,1)$ when $q>(-1+\alpha+\sqrt{1+46\alpha-47\alpha^2})/(8\alpha)$. Also, $q=(9(1-\alpha))/(5+3\alpha)$ is the unique cut-off value of q, such that $G(\alpha,q)=1$. By the monotonicity of $G(\alpha,q)$ and $\bar{\chi}(\alpha,q)<G(\alpha,q)\le1$, we recognize that $\bar{\chi}(\alpha,q)<1$ when $q>\max\{(-1+\alpha+\sqrt{1+46\alpha-47\alpha^2})/(8\alpha),(9(1-\alpha))/(5+3\alpha)\}$. There always exists

$$q>q^*=\max\left\{\frac{-1+\alpha+\sqrt{1+46\alpha-47\alpha^2}}{8\alpha},\frac{9(1-\alpha)}{5+3\alpha}\right\},$$

such that $\bar{\chi}(\alpha,q)<1$. $\square$

Lemma 1 shows that when q is sufficiently large, there exists $\beta \in [0,1]$, such that

$$\begin{aligned}\beta &\geq \bar{\chi}(\alpha, q)\\ &= \Big(2\alpha q^2-(q+6)(1-\alpha)\\ &\quad+\sqrt{(2\alpha q^2-(q+6)(1-\alpha))^2+12(1-\alpha)(4\alpha q^2+(q-3)(1-\alpha))}\Big)\\ &\quad\cdot(4\alpha q^2+(q-3)(1-\alpha))^{-1}. \quad \text{Q.E.D.}\end{aligned}$$

Derivation of First-Period Price

We follow Fudenberg and Tirole's (2000) proof strategy, based on the envelope theorem. Because we are focusing on the symmetric game, we expect that $p_1^A = p_1^B$, and $\tilde{\theta}_1^j = \frac{1}{2}$. In other words, marginal customers of type j in the first period reside at the center, and therefore, the two retailers split demand equally in the first period. We subsequently confirm that this symmetric outcome is an equilibrium, but the level of the first-period equilibrium price depends on the elasticity of demand to a change in price. Because consumers are forward-looking, elasticity is affected by consumer expectations about prices in the second period. In addition, the H- and L-type consumers face different prices in the second period, so the price elasticities of the two types in the first period differ and the optimal first-period prices require retailers to balance the effects of a change in price on the demand of the two types of customers.

Applying the implicit function theorem, we arrive at the following lemma:

Lemma 3. *The first-period price sensitivities for L- and H-type consumers are*

$$\frac{d\tilde{\theta}_1^L}{dp_1^A} = -\frac{F_{\theta H}^H - q\cdot F_{\theta H}^L}{F_{\theta L}^L F_{\theta H}^H - F_{\theta H}^L \cdot F_{\theta L}^H} \quad \textit{and} \quad \frac{d\tilde{\theta}_1^H}{dp_1^A} = -\frac{q\cdot F_{\theta L}^L - F_{\theta L}^H}{F_{\theta L}^L F_{\theta H}^H - F_{\theta H}^L \cdot F_{\theta L}^H},$$

where

$$F_{\theta L}^L = 2-\delta\left(\frac{\partial E^{AL}}{\partial \tilde{\theta}_1^L} - \frac{\partial E^{BL}}{\partial \tilde{\theta}_1^L}\right), \quad F_{\theta L}^H = -\delta\left(\frac{\partial E^{AH}}{\partial \tilde{\theta}_1^L} - \frac{\partial E^{BH}}{\partial \tilde{\theta}_1^L}\right),$$

$$F_{\theta H}^L = -\delta\left(\frac{\partial E^{AL}}{\partial \tilde{\theta}_1^H} - \frac{\partial E^{BL}}{\partial \tilde{\theta}_1^H}\right), \quad F_{\theta H}^H = 2-\delta\left(\frac{\partial E^{AH}}{\partial \tilde{\theta}_1^H} - \frac{\partial E^{BH}}{\partial \tilde{\theta}_1^H}\right).$$

Proof. We define the implicit functions from Equation (7) as follows:

$$\begin{aligned}F^L(p_1^A, \tilde{\theta}_1^L, \tilde{\theta}_1^H) &= 2\tilde{\theta}_1^L - \{1+p_1^B - p_1^A\\ &\quad+\delta(E^{AL}(\tilde{\theta}_1^L, \tilde{\theta}_1^H) - E^{BL}(\tilde{\theta}_1^L, \tilde{\theta}_1^H))\} = 0,\end{aligned}$$

and

$$\begin{aligned}F^H(p_1^A, \tilde{\theta}_1^L, \tilde{\theta}_1^H) &= 2\tilde{\theta}_1^H - \{1+qp_1^B - qp_1^A\\ &\quad+\delta(E^{AH}(\tilde{\theta}_1^L, \tilde{\theta}_1^H) - E^{BH}(\tilde{\theta}_1^L, \tilde{\theta}_1^H))\} = 0.\end{aligned}$$

We differentiate both equations and rearrange them as follows:

$$\frac{d\tilde{\theta}_1^L}{dp_1^A} = -\frac{(\partial F^L/\partial p_1^A + (\partial F^L/\partial \tilde{\theta}_1^H)\cdot(d\tilde{\theta}_1^H/dp_1^A))}{\partial F^L/\partial \tilde{\theta}_1^L}, \quad \text{and}$$

$$\frac{d\tilde{\theta}_1^H}{dp_1^A} = -\frac{(\partial F^H/\partial p_1^A + (\partial F^H/\partial \tilde{\theta}_1^L)\cdot(d\tilde{\theta}^L/dp_1^A))}{\partial F^H/\partial \tilde{\theta}_1^H}.$$

Solving these two equations simultaneously, we obtain the following results:

$$\frac{d\tilde{\theta}_1^L}{dp_1^A} = -\frac{(F_p^L - (F_{\theta H}^L/F_{\theta H}^H)\cdot F_p^H)}{(F_{\theta L}^L - (F_{\theta H}^L\cdot F_{\theta L}^H)/F_{\theta H}^H)} = -\frac{F_{\theta H}^H F_p^L - F_{\theta H}^L\cdot F_p^H}{F_{\theta H}^H F_{\theta L}^L - F_{\theta H}^L \cdot F_{\theta L}^H} \quad \text{and}$$

$$\frac{d\tilde{\theta}_1^H}{dp_1^A} = -\frac{(F_p^H - (F_{\theta L}^H/F_{\theta L}^L)\cdot F_p^L)}{(F_{\theta H}^H - (F_{\theta H}^L\cdot F_{\theta L}^H)/F_{\theta L}^L)} = -\frac{F_{\theta L}^L F_p^H - F_{\theta L}^H\cdot F_p^L}{F_{\theta L}^L F_{\theta H}^H - F_{\theta H}^L \cdot F_{\theta L}^H}.$$

Using

$$F_p^L = \frac{\partial F^L}{\partial p_1^A} = 1, \quad F_P^H = \frac{\partial F^H}{\partial p_1^A} = q,$$

we recognize that

$$F_{\theta L}^L = \frac{\partial F^L}{\partial \tilde{\theta}_1^L} = 2-\delta\left(\frac{\partial E^{AL}}{\partial \tilde{\theta}_1^L} - \frac{\partial E^{BL}}{\partial \tilde{\theta}_1^L}\right),$$

$$F_{\theta L}^H = \frac{\partial F^H}{\partial \tilde{\theta}_1^L} = -\delta\left(\frac{\partial E^{AH}}{\partial \tilde{\theta}_1^L} - \frac{\partial E^{BH}}{\partial \tilde{\theta}_1^L}\right), \quad \text{and}$$

$$F_{\theta H}^H = \frac{\partial F^H}{\partial \tilde{\theta}_1^H} = 2-\delta\left(\frac{\partial E^{AH}}{\partial \tilde{\theta}_1^H} - \frac{\partial E^{BH}}{\partial \tilde{\theta}_1^H}\right). \quad \text{Q.E.D.}$$

It is convenient to rewrite Equation (10) for firm A's overall profits using the functions Π_2^{AA}, Π_2^{AB} to represent its second-period profit from its own previous customers and from retailer B's previous customers, respectively,

$$\begin{aligned}\Pi^A &= p_1^A(\alpha q \tilde{\theta}_1^H + (1-\alpha)\tilde{\theta}_1^L)\\ &\quad+\delta\{\Pi_2^{AA}(p_2^{Ai}(p_1^A, p_1^B), p_2^{Bi}(p_1^A, p_1^B), \tilde{\theta}_1^L(p_1^A, p_1^B), \tilde{\theta}_1^H(p_1^A, p_1^B))\\ &\quad+\Pi_2^{AB}(p_2^{Ai}(p_1^A, p_1^B), p_2^{Bi}(p_1^A, p_1^B), \tilde{\theta}_1^L(p_1^A, p_1^B), \tilde{\theta}_1^H(p_1^A, p_1^B))\},\end{aligned} \tag{13}$$

where

$$\begin{aligned}\Pi_2^{AA}(p_2^{Ai}, p_2^{Bi}, \tilde{\theta}_1^L, \tilde{\theta}_1^H) &= (p_2^{AH})\alpha q \tilde{\theta}_1^H \Pr^{AH}(p_2^{AH}, p_2^{BO}, \tilde{\theta}_1^H)\\ &\quad+(p_2^{AL})(1-\alpha)\tilde{\theta}_1^L \Pr^{AL}(p_2^{AL}, p_2^{BO}, \tilde{\theta}_1^L),\\ \Pi_2^{AB}(p_2^{Ai}, p_2^{Bi}, \tilde{\theta}_1^L, \tilde{\theta}_1^H)&\\ = p_2^{AO}\{&\alpha q(1-\tilde{\theta}_1^H)(1-\Pr^{BH}(p_2^{AH}, p_2^{BO}, \tilde{\theta}_1^H))\\ &+(1-\alpha)(1-\tilde{\theta}_1^L)(1-\Pr^{BL}(p_2^{AL}, p_2^{BO}, \tilde{\theta}_1^L))\}.\end{aligned}$$

Note that

$$\Pr^{AH} = \Pr\left[\theta_2 \leq \frac{1+q(p_2^{BO}, p_2^{AH})}{2} \,\middle|\, \theta_1 \leq \tilde{\theta}_1^H\right] \quad \text{and}$$

$$\Pr^{BH} = \Pr\left[\theta_2 > \frac{1+q(p_2^{BH}, p_2^{AO})}{2} \,\middle|\, \theta_1 > \tilde{\theta}_1^H\right]$$

are functions of $p_2^{AH} - p_2^{BO}, \tilde{\theta}_1^H$; in addition,

$$\Pr^{AL} = \Pr\left[\theta_2 \leq \frac{1+q(p_2^{BO} - p_2^{AL}}{2} \,\middle|\, \theta_1 \leq \tilde{\theta}_1^L\right] \quad \text{and}$$

$$\Pr\left[\theta_2 > \frac{1+(p_2^{BL} - p_2^{AO})}{2} \,\middle|\, \theta_1 > \tilde{\theta}_1^L\right]$$

are functions of $p_2^{AL}, p_2^{BO}, \tilde{\theta}_1^L$.

Because retailer A's own second-period prices are set to maximize A's second-period profit, we can use the envelope theorem ($\partial\Pi_2^{AA}/\partial p_2^{Ai}=0$, $\partial\Pi_2^{AB}/\partial p_2^{AO}=0$) to write the first-order conditions for this maximization as

$$\begin{aligned}
&(\alpha q\tilde{\theta}_1^H+(1-\alpha)\tilde{\theta}_1^L)+p_1^A\left(\alpha q\frac{\partial\tilde{\theta}_1^H}{\partial p_1^A}+(1-\alpha)\frac{\partial\tilde{\theta}_1^L}{\partial p_1^A}\right)\\
&\quad+\delta\Bigg[\Bigg\{\frac{\partial\Pi_2^{AA}}{\partial p_2^{BO}}\frac{dp_2^{BO}}{d\tilde{\theta}_1^H}+\frac{\partial\Pi_2^{AB}}{\partial p_2^{BL}}\frac{dp_2^{BL}}{d\tilde{\theta}_1^H}+\frac{\partial\Pi_2^{AB}}{\partial p_2^{BH}}\frac{dp_2^{BH}}{d\tilde{\theta}_1^H}\\
&\qquad\qquad+\frac{\partial\Pi_2^{AA}}{\partial\tilde{\theta}_1^H}+\frac{\partial\Pi_2^{AB}}{\partial\tilde{\theta}_1^H}\Bigg\}\frac{\partial\tilde{\theta}_1^H}{\partial p_1^A}\\
&\quad+\Bigg\{\frac{\partial\Pi_2^{AA}}{\partial p_2^{BO}}\frac{dp_2^{BO}}{d\tilde{\theta}_1^L}+\frac{\partial\Pi_2^{AB}}{\partial p_2^{BL}}\frac{dp_2^{BL}}{d\tilde{\theta}_1^L}+\frac{\partial\Pi_2^{AB}}{\partial p_2^{BH}}\frac{dp_2^{BH}}{d\tilde{\theta}_1^L}\\
&\qquad\qquad+\frac{\partial\Pi_2^{AA}}{\partial\tilde{\theta}_1^L}+\frac{\partial\Pi_2^{AB}}{\partial\tilde{\theta}_1^L}\Bigg\}\frac{\partial\tilde{\theta}_1^L}{\partial p_1^A}\Bigg]=0.
\end{aligned}$$

Then, the first-order condition of Equation (11) at $\tilde{\theta}_1^H=\tilde{\theta}_1^L=\frac{1}{2}$ simplifies to

$$p_1^A=p_1^B=\frac{((1-\alpha)+\alpha q)-2\delta(\Omega^H(-\partial\tilde{\theta}_1^H/\partial p_1^A)+\Omega^H(-\partial\tilde{\theta}_1^L/\partial p_1^A))}{2(\alpha q(-\partial\tilde{\theta}_1^H/\partial p_1^A)+(1-\alpha)(-\partial\tilde{\theta}_1^L/\partial p_1^A))},$$

where

$$\begin{aligned}
\Omega_L&=\frac{\partial\Pi_2^{AA}}{\partial p_2^{BO}}\frac{dp_2^{BO}}{d\tilde{\theta}_1^L}+\frac{\partial\Pi_2^{AB}}{\partial p_2^{BL}}\frac{dp_2^{BL}}{d\tilde{\theta}_1^L}+\frac{\partial\Pi_2^{AB}}{\partial p_2^{BH}}\frac{dp_2^{BH}}{d\tilde{\theta}_1^L}+\frac{\partial\Pi_2^{AA}}{\partial\tilde{\theta}_1^L}+\frac{\partial\Pi_2^{AB}}{\partial\tilde{\theta}_1^L},\\
\Omega_H&=\frac{\partial\Pi_2^{AA}}{\partial p_2^{BO}}\frac{dp_2^{BO}}{d\tilde{\theta}_1^H}+\frac{\partial\Pi_2^{AB}}{\partial p_2^{BL}}\frac{dp_2^{BL}}{d\tilde{\theta}_1^H}+\frac{\partial\Pi_2^{AB}}{\partial p_2^{BH}}\frac{dp_2^{BH}}{d\tilde{\theta}_1^H}\\
&\quad+\frac{\partial\Pi_2^{AA}}{\partial\tilde{\theta}_1^H}+\frac{\partial\Pi_2^{AB}}{\partial\tilde{\theta}_1^H}.\quad\text{Q.E.D.}
\end{aligned}$$

Proof of Proposition 2. We know that if $\beta=0$, it is optimal to reward competitor customers for all $q>1$. Hence, when $\alpha=\frac{1}{2}$, Equation (3) can be rewritten as

$$\begin{aligned}
\Pi_2^A&=\frac{p_2^{AH}q}{2}\left(\frac{1+q(p_2^{BO}-p_2^{AH})}{2}\right)+\frac{p_2^{AL}}{2}\left(\frac{1+p_2^{BO}-p_2^{AL}}{2}\right)+\frac{p_2^{AO}}{2}\\
&\quad\cdot\left\{q\left(\frac{1+q(p_2^{BH}-p_2^{AO})}{2}-\tilde{\theta}_1^H\right)+\left(\frac{1+p_2^{BL}-p_2^{AO}}{2}-\tilde{\theta}_1^L\right)\right\},\\
\Pi_2^B&=\frac{p_2^{BH}q}{2}\left(\frac{1-q(p_2^{AO}-p_2^{BH})}{2}\right)+\frac{p_2^{BL}}{2}\left(\frac{1-p_2^{AO}-p_2^{BL}}{2}\right)+\frac{p_2^{BO}}{2}\\
&\quad\cdot\left\{q\left(\tilde{\theta}_1^H-\frac{1+q(p_2^{BO}-p_2^{AH})}{2}\right)+\left(\tilde{\theta}_1^L-\frac{1+p_2^{BO}-p_2^{AL}}{2}\right)\right\}.
\end{aligned}$$

The first-order conditions for the retailers' maximization yield

$$p_2^{AH}=\frac{3+q(2q-1)+4q\bar{\theta}_1}{6q(1+q^2)},\quad p_2^{AL}=\frac{2+q(3q-1)+4\bar{\theta}_1}{6(1+q^2)},$$

$$p_2^{AO}=\frac{3(1+q)-4\bar{\theta}_1}{3(1+q^2)},\quad p_2^{BH}=\frac{3(q-1)+6(1+q^2)-4q\bar{\theta}_1}{6q(1+q^2)},$$

$$p_2^{BL}=\frac{3(1+q)+3(1+q^2)-4\bar{\theta}_1}{6(1+q^2)},\quad\text{and}\quad p_2^{BO}=\frac{4\bar{\theta}_1-(1+q)}{3(1+q^2)},$$

where $\bar{\theta}_1=(q\tilde{\theta}_1^H+\tilde{\theta}_1^L)/2$. In turn, firms A's and B's overall profit functions can be rewritten as

$$\Pi^A=\frac{p_1^A}{2}\{q\tilde{\theta}_1^H+\tilde{\theta}_1^L\}+\delta\left[\frac{49}{144}+\frac{62q+80\bar{\theta}_1(\bar{\theta}_1-(1+q))}{144(1+q^2)}\right],$$

$$\begin{aligned}
\Pi_2^B&=\frac{p_1^B-c}{2}\{q(1-\tilde{\theta}_1^H)+(1-\tilde{\theta}_1^L)\}\\
&\quad+\delta\left[\frac{49}{144}+\frac{62q+80\bar{\theta}_1(\bar{\theta}_1-(1+q))}{144(1+q^2)}\right].
\end{aligned}$$

By using Lemma 1 and Equation (11), we obtain $p_1^A=p_1^B=((3+\delta)(1+q))/(3(1+q^2))$. We get the profit result directly from plugging prices into the equation

$$\begin{aligned}
\Pi^A-\Pi^s&=\frac{41\delta}{144}+\frac{36(1+q^2)+46q\delta}{144(1+q^2)}-\frac{(1+q)^2(1+\delta)}{4(1+q^2)}\\
&=\frac{(q-5)(5q-1)\delta}{144(1+q^2)}>0\quad\text{if } q>5.\quad\text{Q.E.D.}
\end{aligned}$$

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