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Searching for Rewards

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Abstract. Loyalty programs are widespread across various markets, offering members rewards based on their past purchases for future benefits. This study offers new insights into the strategic use of loyalty programs and their impact on market competition by exploring the dynamics of loyalty programs within a repeated ordered search framework, in which consumers sequentially search for the optimal product across multiple firms over two periods. Our findings highlight distinct roles for price discounts and rewards in influencing consumer decisions. Price discounts discourage further search in the current shopping period, whereas rewards encourage consumer loyalty by inducing prominence in subsequent visits. As search costs increase, firms tend to offer lower price discounts but higher rewards, and this results in a higher industry profit but lower consumer surplus. Compared with their absence, loyalty programs decrease both industry profit and consumer welfare, leading to a lose–lose outcome. Moreover, we demonstrate that, when the firms are heterogeneous in terms of their network sizes, those with larger networks tend to offer lower rewards yet achieve greater consumer loyalty in contrast to the firms with smaller networks, which compensate with higher rewards.

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1. Introduction

Loyalty programs are increasingly ubiquitous, demonstrating significant growth in memberships and their perceived value among consumers. A 2016 survey by Accenture (Wollan et al. 2017) revealed that more than 90% of companies implement some form of loyalty programs with membership in the United States growing at an impressive annual rate of 26.7%. Such rapid growth underscores the perceived value that these programs may offer to consumers, a sentiment strongly supported by a 2019 Wirecard study, which revealed that rewards play a crucial role in the purchasing decisions of the vast majority of consumers.[1]

The widespread appeal of loyalty programs spans various industries, underscoring their versatility and impact on business success. From the retail sector's points-for-purchases schemes, as exemplified by Sephora's Beauty Insider program,[2] to the travel incentives of airline frequent flyer programs such as Delta's SkyMiles and the fitness rewards offered by platforms such as ClassPass, loyalty programs are effectively tailored to meet the diverse needs of consumers. Fast food chains, including Starbucks, McDonald's, and Chipotle, have also successfully harnessed loyalty programs to foster repeat business, demonstrating the broad applicability and potential of these marketing tools.

Despite their evident popularity and immediate benefits to consumers, the long-term advantages of loyalty programs for firms remain ambiguous (Dowling and Uncles 1997, Bombaij and Dekimpe 2020). One classic argument about why loyalty programs could benefit firms in the long run is based on the switching costs (e.g., Caminal and Matutes 1990, Kim et al. 2001). Loyalty programs can effectively lock in customers by offering rewards for repeated purchases, potentially leading to increased profits over time. However, the effectiveness of such loyalty programs is often questioned in practice as critics believe that any advantage they provide can be easily copied by competitors. This can lead to a prisoner's dilemma scenario, in which the costs of maintaining loyalty programs outweigh the benefits, making it hard for firms to maintain long-term benefits from these programs (Deighton and Shoemaker 2000). Such skepticism is bolstered by observations from Accenture (Wollan et al. 2017) that highlight the hidden costs and managerial challenges associated with maintaining loyalty programs. This contrast in views presents the challenge companies face when implementing loyalty programs, highlighting an area that still demands more research and investigation.

In this paper, we revisit this discussion and examine the role of loyalty programs by explicitly considering consumers' costly search decisions within the framework of repeated ordered search. We show that rewards increase a firm's prominence in future search, making past customers more likely to inspect it first.[3] Therefore, price discounts and rewards have different roles in influencing consumer decisions. Price discounts discourage search in current shopping, whereas rewards promise prominence in future visits. As search costs rise, firms want to offer lower price discounts and higher rewards. In particular, this prominence effect gives rise to a prisoner's dilemma: firms adopt rewards because doing so is privately optimal, yet the resulting equilibrium reduces both industry profit and consumer welfare. By separating the prominence channel from standard switching-cost logic, our analysis provides a new rationale for the adoption of loyalty programs and clarifies how reward competition persists despite being socially inefficient.

Our analysis seeks to formalize and present a new rationale behind firms' adoption of loyalty programs: to gain prominence in future consumer searches. Imagine the scenario of "Clair," who visited New York City and stayed at a Hyatt Hotel. Before her visit, she searched several hotels and compared their prices and the rewards offered by their reward programs. Then, she chose to stay at the Hyatt Hotel and joined its loyalty program for a discount on future stays. Now, as she plans a trip to San Diego, Clair naturally starts her search with Hyatt, hoping to leverage her loyalty benefits. However, if Hyatt can't accommodate her specific needs, such as a room with two queen beds for a family trip, she's prompted to consider other brands. The narrative shifts as she plans another trip to Reykjavik, Iceland, where the absence of a Hyatt hotel directly leads her to explore alternative accommodations. These scenarios highlight several important points for the role of a reward program and the prominence that it can provide for the focal firm.

When consumers choose a product, they weigh both the current price and potential future rewards from loyalty programs. Such programs not only retain current customers by introducing switching costs but also attract new ones. Fast food chains such as McDonald's and Chipotle, for instance, use their loyalty apps as a strategy to draw new patrons.[4] Furthermore, loyalty programs elevate a brand's prominence in a consumer's future searches, exemplified by Hyatt becoming the first place to search for someone such as Clair planning her next trip. Consumers begin their search with the prominent brands, moving on to other options only if the initial choice fails to meet their expectations or value criteria.

The effect of the loyalty program is amplified by the brand's network size; the larger it is, the more enticing the loyalty program, encouraging customers to start their search with the brand. Whereas Hyatt's loyalty program initially steers Clair to consider Hyatt for her accommodations, its absence in Reykjavik for her Iceland trip starkly illustrates the constraints of a limited network. This scenario underscores the critical importance of network size; Hyatt's inability to serve Clair in Iceland because of a lack of local presence directly impacts her loyalty and search behavior. It is, therefore, in brands' interests to expand their network sizes to maximize loyalty program benefits. Indeed, many brands operate all over the world, and a single loyalty program applies to all of them. The more extensive the brand chain is, the more appealing its loyalty program becomes to consumers. Clair finds Hyatt's loyalty program more attractive when she can redeem rewards at more travel destinations.

Such scale economy in loyalty programs is vividly illustrated by Marriott's acquisition of Starwood Preferred Guest (SPG) and McDonald's ambitious goal to expand to 50,000 locations worldwide by 2027.[5] These examples reflect the broader principle that network size significantly impacts the appeal and effectiveness of loyalty programs: a concept we explore through our model to understand the nuances of loyalty programs in practice.

By analyzing the interactions between loyalty programs, consumer search behavior, and the size of brand networks, our paper seeks to illuminate the strategic considerations firms must weigh in designing these programs. Specifically, we investigate how firms use loyalty programs to compete for future prominence, identifying the critical differences between pricing and rewards. We further examine how these programs influence competitive dynamics, pricing strategies, future rewards, and the overall welfare of the market.

To answer these questions, we adopt a sequential searching framework developed by Wolinsky (1986) and Armstrong et al. (2009), by which consumers discover a specific match value for each firm at a search cost sequentially. We extend this framework to a two-period repeated ordered search model in which consumer preferences are independent across time. In this model, cities are filled with firms, each potentially having a branch, defined by an activation rate that captures the firm's network size or branch presence across cities. Consumers, traveling randomly to one of these cities in each period, engage with firms to learn about prices, rewards, and the scope of brand networks, necessitating discovery in both periods because of potential variations in product offerings.

We first characterize the pure-strategy symmetric equilibrium when all firms have the same network size, exploring how competition is affected by the search cost and the activation rate. Our analysis reveals that higher search costs and larger networks both encourage repeat purchases, yet their impacts on competition differ. As search costs increase, consumers' incentive to search diminishes, leading them to repeatedly purchase from the same firms, thereby placing higher value on loyalty rewards. This prompts firms to intensify competition by enhancing reward offerings. Interestingly, the relationship between search costs and the first period pricing is complex; prices may rise with moderate search costs but could drop when costs are high so as to lock in consumers. In the second period, the combination of greater obstacles to switching and enhanced rewards means firms can charge higher prices. Correspondingly, a higher search cost results in higher industry profits but a decrease in consumer surplus because of a shift toward purchases motivated more by rewards than by preference.

Then, we analyze how the industry-level activation rate affects market competition. A surprising finding is that reward levels remain constant regardless of these rates. The efficacy of loyalty programs is consistent across all cities in which a firm operates, and the size of a firm's network doesn't alter the likelihood of a member switching to a competitor within a city. This stability means that there's no incentive for firms to adjust their reward design. However, a higher activation rate leads to more members being locked in overall, so firms compete more intensively in the first period price and exploit them with higher prices in the second period. Thus, the industry profit drops because the market becomes more competitive. The total surplus also decreases because of lower match efficiency associated with the locked-in effect of rewards. The consumer surplus only increases with the activation rate when firms compete very intensively, that is, when the search cost and the activation rate are high; otherwise, it decreases as driven by match efficiency loss.

To further distinguish the roles of the price and the committed reward, we numerically analyze the equilibrium when the market is heterogeneous; that is, firms have different activation rates (network sizes). Whereas both price and reward can influence the number of consumers that a firm can attract, rewards specifically play a crucial role in customer retention, affecting the likelihood of repeat purchases. The first period price, on the other hand, seeks to optimize a firm's immediate and future profits through demand. In contrast to the scenario of a uniform industry-level activation rate, firms with extensive networks (high-type firms) tend to offer smaller rewards as their wide reach across numerous cities naturally attracts new members. These firms rely less on rewards for customer retention given their advantage in network size. Conversely, firms with smaller networks (low-type firms) use larger rewards, addressing their competitive shortfall. It is a stark contrast to the homogeneous activation rate case, suggesting that the variation in rewards between different firm types stems more from strategic decisions influenced by the firms' network sizes rather than from direct economic incentives dictated by market conditions. The strategy around the first period pricing varies significantly with market conditions. In scenarios in which attracting members yields long-term profitability—especially at high search costs—firms with larger networks may set lower prices to draw in customers early. However, these high-type firms typically charge more in the second period, leveraging their larger base of loyal members to secure higher revenues.

The rest of the paper is organized as follows: We begin with a literature review in the next section, followed by a detailed description of the model in Section 3. The main results, including equilibrium analysis for both homogeneous and heterogeneous network sizes among firms, are discussed in Sections 4 and 5. We conclude with a summary of our findings and discussion in Section 6. All the technical proofs are provided in the appendix.

2. Literature Review

Our research contributes to several strands of the literature, starting with the area of customer relationship management (CRM) and loyalty program research. Loyalty programs are widely used and one of the most important tools in CRM to enhance customer retention and profitability (Deighton and Shoemaker 2000, Belli et al. 2022). Several papers investigate the design of optimal reward programs with a particular emphasis on the referral reward (Biyalogorsky et al. 2001, Kamada and Öry 2020, Wolters et al. 2020, Fourie et al. 2023) or focusing on the role of switching costs created from a reward program as suggested by Caminal and Matutes (1990) and Kim et al. (2001).[6] Our model also builds on the switching costs (see Farrell and Klemperer 2007 for a comprehensive review). Von Weizsäcker (1984) and Klemperer (1987) investigate competition with exogenous switching costs. In contrast, rewards from loyalty programs can be regarded as a way to endogenize such switching costs (Caminal and Matutes 1990), and we also consider how loyalty programs introduce these costs endogenously.

Caminal and Matutes (1990) analyze a duopoly setting in which consumers' preferences may change randomly across two periods. In their framework, each firm can precommit to a price reduction for repeat purchases in the second period, effectively offering a reward to first period buyers that is redeemable only upon returning. Then, the loyalty program endogenously creates a switching cost in equilibrium: first period buyers face a higher effective price if they switch, and this, in turn, raises profits. Consistent with Caminal and Matutes (1990), in our model, such rewards also function as endogenous switching costs. However, we introduce a novel role for rewards by linking them to the concept of "prominence" within the context of a search model. Unlike the Caminal and Matutes (1990) framework, which does not incorporate search dynamics, our model distinctly separates the decision to visit from the decision to purchase. Here, a firm's prominence implies that consumers are more likely to visit it first although they may not ultimately make a purchase there. This separation is crucial as it allows us to explore how prominence, driven by reward, influences not just the choice of whether to switch but also the initial visit of consumers in a differentiated market. This is the critical conceptual difference between our paper and Caminal and Matutes (1990), which is not a search model.

Moreover, our findings suggest that the strategic deployment of rewards in search models can influence market equilibrium in ways not anticipated by traditional duopoly models such as that of Caminal and Matutes (1990). Specifically, whereas Caminal and Matutes (1990) suggest that rewards increase firm profits and, thus, potentially enhance social welfare, our search model framework predicts that reward programs may actually lead to a prisoner's dilemma for all firms and a loss–loss outcome for both firms and consumers. This divergence highlights the critical role of consumer search and decision processes in shaping market equilibrium, offering new insights into how firms' strategic lever of rewards impacts consumer behaviors and competitive outcomes.

Beyond switching costs, loyalty programs are analyzed as commitment devices (Kim et al. 2001, 2004). Kim et al. (2001) argue that firms choose reward types as a means of committing future prices. When there are large segments of price-insensitive consumers, firms are profitable to provide inefficient rewards to price high in the future. Furthermore, Kim et al. (2004) show that, in scenarios of limited capacity and low demand, firms can use rewards to moderate competition, reducing the incentive to undercut prices. These programs, by offering rewards for repeated purchases, not only aim to lock in customers but also present a strategic tool for firms to soften competition by committing to rewards.

Our study further connects with CRM research on how firms promote to loyal customers and the impact of loyalty on price competition.[7] Notably, Narasimhan (1988) is among the first to explore how price promotions differ between loyal and nonloyal customer segments. Also, Villas-Boas (2004) demonstrates that an increase in loyal customers could paradoxically lower future firm profitability because of customer uncertainty and value distribution. Kuksov and Zia (2021) recently examine how firms in a duopoly set search costs for nonloyal customers. We extend this literature by considering both exogenous search frictions and endogenous rewards under competition. We find that rewards from loyalty programs, serving as endogenous switching costs, intensify with exogenous search costs. This exploration of how external search frictions and internal reward strategies influence firm profits in a competitive context marks a novel addition to the literature.

Our model builds on the consumer sequential search theory in the product market. Wolinsky (1986) is the seminal paper that proposes a sequential search in a horizontally differentiated market in the context of random search order. Anderson and Renault (1999) analyze how search costs moderate the relationship between the degree of product differentiation and equilibrium prices. A more recent line of research examines more realistic market situations in which search is nonrandom and consumers search in a deliberate order (see Armstrong 2017 for an extensive literature review on this ordered search literature). Armstrong et al. (2009) demonstrate that, when consumers engage in costly search across firms, prominence, or being the first shopping destination, can be valuable as it can preempt demand. In this paper, we analyze a situation in which, for each consumer, only one firm (the one from which the consumer has cumulated rewards) is prominent and the others are symmetrically nonprominent. Thus, the consumer searches the prominent one first, and if it is not satisfactory, the consumer continues to search randomly among other nonprominent firms. Unlike classic one-period ordered search models, which require an asymmetric equilibrium to induce a nonrandom search order, our setting generates this asymmetry endogenously. Initial enrollment in loyalty programs differentiates firms in subsequent periods, thereby extending the logic of ordered search across multiple market interactions.

Subsequently, several papers investigate how firms can become prominent in consumer search. Firms use various instruments to direct consumer searches; for example, by charging lower price (Armstrong and Zhou 2011, Chen and He 2011), advertisement (Haan and Moraga-González 2011, Mayzlin and Shin 2011), brand positioning (Ke et al. 2023), targeting (Shin and

Yu 2021), offering service (Shin 2007, Janssen and Ke 2020), and providing price or product information (Choi et al. 2018, Au and Whitmeyer 2023, Lu 2023). The work most closely related to ours is by Armstrong and Zhou (2011), who consider several ways for a firm to gain prominence, notably through securing an initial sale to leverage the default bias, by which consumers tend to revert to previously chosen firms. This concept of default bias shares parallels with our model's consumer search behaviors. However, unlike Armstrong and Zhou (2011), who examine consumer search in a deterministic order and in which prominence is the same for all consumers, our model considers prominence as a reward-driven, endogenous outcome. Here, each firm becomes prominent only for its members in subsequent periods. We further show how firms seek to compete for prominence under different levels of search friction.

3. A Model of Repeated Ordered Search

We examine a market characterized by monopolistic competition with $N \rightarrow \infty$ firms (or brands). There are $B \in \mathrm{N}^+$ cities, in which each firm has an active branch in γB cities, in which the activation rate $\gamma \in (0,1]$ captures the breadth of each firm's network size. We assume a common γ for all firms in the basic model and later introduce heterogeneous γ across firms in Section 5. The presence of a firm's branch in specific cities is independent across firms. For each firm $j \in \{1,\ldots,N\}$, we denote $\mathbf{B}(j) \subset \{1,\ldots,B\}$ as the realized set of cities where it actively maintains branches. In these locations, the firm offers one product with the marginal production cost normalized to be zero.

There are $M \in \mathrm{N}^+$ consumers. We consider a two-period model, in which a representative consumer travels randomly to city $b(t) \in \{1,\ldots,B\}$ in period $t = 1,2$. The second period should be viewed as a future purchase occasion when the consumer might redeem loyalty rewards.[8] The travel destinations are independent across consumers. The consumer's match value with firm j's product at branch $b(t)$ is denoted by v_t^j, which follows a uniform distribution in [0, 1] and is independently distributed across both j and t.[9]

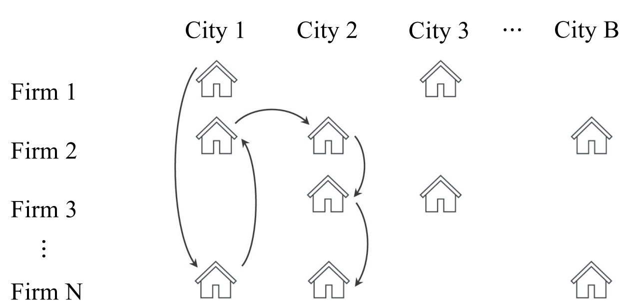
At first glance, it seems a strong assumption to have the two products offered by the same firm in two branches to be independent because, in reality, they may share some common attributes. It's important to note that such common attributes known before searching—such as a hotel chain's reputation—are accounted for in our model. For example, two hotels by Hyatt may share similar reputations in design features, but it is possible that consumers recognize this before searching and their search focuses on the idiosyncratic features of each hotel, such as locations or restaurant options.[10] Moreover, we assume that, even when consumers make repeated purchases from the same branch, the perceived value can vary across periods, necessitating further search. For instance, whereas the value offered by the Hyatt Hotel in New York City might be consistently high, the specific appeal of the hotel can change based on factors unique to each visit, such as the convenience of its location relative to event venues. A consumer might have chosen the Hyatt previously because it hosted a conference that the consumer attended, but its proximity to future events of interest remains uncertain until it is searched for again. Similarly, a customer's needs can change between visits; the customer might need a king-sized bed during a business trip but require two queen-sized beds to accommodate the customer's family on another trip. Even though Hyatt's general offerings remain the same, the customer's match value varies, prompting a new search to reassess the current value of the hotel's offerings according to the customer's changing needs.[11]

Without loss of generality, we can normalize the average number of consumers per city per firm, $M/(N \cdot B)$, to one. The model also incorporates a time discounting factor $\delta \in (0,1]$. Figure 1 illustrates the market structure with an example. Each house icon represents an active branch. In this example, firm 1 has a branch in cities 1 and 3; firm 2 has a branch in cities 1, 2, and B; and so on.

Each firm, from 1 to N, sets a consistent price p_t^j for its products across all its branches for each period $t = 1,2$. This approach mirrors practices of major brands such as McDonald's or fitness chains, which may adjust prices over time but maintain price consistency across different locations.[12] Moreover, firms offer a reward $r^j \geq 0$ to consumers who purchase in the first period, valid for use in any branch during the second period. This setup includes the possibility of a firm opting out of a loyalty program by setting $r^j = 0$, effectively making the reward offer inactive.[13]

In each period, consumers purchase at most one product from a firm. Consumers' utility from purchasing

Figure 1. Illustration of Market Structure and a Sample Consumer Search Trajectory



firm j's product in city $b(t)$ during period t is

$$u_t^j = \begin{cases} v_t^j - p_t^j + r^j, & \text{if } t = 2 \text{ and enrolled in firm } j\text{'s loyalty program}, \\ v_t^j - p_t^j, & \text{otherwise.} \end{cases}$$

Consumers have an outside option of not buying in each period, and this is normalized to zero. Before deciding on a purchase, consumers need to search among firms to discover prices and match values as this information is a priori unknown. Thus, in each period, they conduct a sequential search among firms with perfect recalls before making a purchase decision (Wolinsky 1986). The search cost per firm is assumed to be $s > 0$.

In the first period, when a consumer visits firm j in city $b(1)$, the consumer discovers the consumer's match value v_1^j, the price p_1^j, and reward r^j. Based on this information, the consumer decides whether to buy and enroll in the loyalty program. There is zero cost on the consumer side for enrolling, and this implies that consumers always enroll as long as $r^j > 0$.[14] In the second period, by visiting firm k in city $b(2)$, the consumer observes the consumer's match value v_2^k as well as the price p_2^k, and the consumer decides whether to make a purchase. If the consumer previously bought from the same firm ($k = j$), the consumer can redeem the reward of r^j if $k = j$. The connected lines with arrows in Figure 1 showcase one sample trajectory of consumer search. The consumer travels to city 1 in the first period by searching firm 1 first and then firm N before purchasing at firm 2; the consumer travels to city 2 in the second period by searching firm 2 first and then firm 3 and N.

3.1. Equilibrium Concept

In our analysis, we use the perfect Bayesian equilibrium (PBE) as our solution concept. Because all firms are ex ante the same, it is natural to focus on the symmetric equilibrium with $p_t^j = p_t^*$ and $r^j = r^*$ for $t = 1, 2$ and $\forall j \in \{1, \ldots, N\}$. In this equilibrium, consumers search randomly in the first period, but in the second period, prominence emerges endogenously because of reward programs, leading consumers to first search the firm from which they have a reward.

Consistent with existing literature (Wolinsky 1986, Anderson and Renault 1999), we assume passive beliefs for consumers' search within each period such that, if they observe a firm's deviation from the equilibrium behavior, they continue to believe that the other firms stick to their equilibrium prices. Moreover, unique to our repeated ordered search setting, we also need to specify consumers' off-equilibrium belief on a firm's second period price when they observe the firm's deviation on its first period price and/or reward. There are two cases to consider. The first case concerns a consumer who made a purchase from the deviating firm in the first period. The refinement by sequential equilibria (Kreps and Wilson 1982) requires that the consumer updates the consumer's belief of the second period price p_2^j based on the firm's deviation in the first period, p_1^j and/or r^j. The second case concerns a consumer who did not purchase from the deviating firm but instead decided to continue to search. We specify the off-equilibrium belief with the following assumption.[15]

Assumption 1. *If a consumer visited firm j in the first period but did not purchase from it after observing that it deviated on* p_1^j *and/or* r^j*, we assume that, prior to any visits in the second period, the consumer still believes that firm j will adhere to the second period equilibrium price* p_2^**.*

This assumption is based on a few key points. First, with an infinite number of firms in the market, it's realistic to expect that consumers might not remember every detail given limited attention or memory, especially about firms they only considered but didn't buy from. For instance, Clair, planning her next trip to San Diego, might not recall the specific offers from New York hotels she browsed but didn't book. It is worth noting that Assumption 1 does not necessarily introduce inconsistent off-equilibrium belief specification across consumers who purchased and who did not in the first period. In fact, we can rationalize the assumption by assuming that all consumers forget the first period price and reward by the second period; however, only those purchased can retrieve the relevant information through their transaction record, such as a receipt or confirmation. Second, note that our equilibrium concept is a PBE. Whereas sequential equilibrium is a refinement of PBE, applying the sequential equilibrium refinement uniformly across all consumers in our setting precludes the existence of a symmetric pure-strategy equilibrium. The reason is as follows: given that all other firms set the equilibrium price and reward (p_1^* and r^*), a single firm can profitably deviate by setting p_1^j infinitesimally below p_1^*. This deviation yields only a negligible change in current-period profit, but under the refinement, it causes all consumers who browsed but did not purchase from this firm to expect a slightly lower second period price $p_2^j < p_2^*$. As a result, these consumers would plan to visit the deviating firm immediately after their loyalty firm but before all others, generating a discrete increase in future demand.[16] Lastly, a more profound point is that Assumption 1 eliminates the possibility that firms could use their first period pricing to shape consumer expectations about second period prices, thereby guiding consumer search in the second period. In other words, Assumption 1 isolates rewards as the only instrument in equilibrium to direct consumer search for their next purchases, reflecting our thinking that loyalty programs are offered to direct consumer search for their future purchases.

4. Equilibrium Analysis

To begin our analysis, we define the reservation value w by equating the search cost s with the option value from searching (Weitzman 1979):

$$s = \int_w^1 (v - w)dv = \frac{1}{2}(1 - w)^2 \;\Rightarrow\; w = 1 - \sqrt{2s} \in (0, 1) \quad \text{for } s \in (0, 1/2).$$

By definition, the reservation value w represents a consumer's threshold for being indifferent between accepting a guaranteed offer immediately or continuing to search among firms for a better deal.

Next, we present the following lemma.

Lemma 1. *If a pure-strategy symmetric perfect Bayesian equilibrium exists, we must have* $r^* > 0$.

Lemma 1 implies that, in equilibrium, all firms offer loyalty programs, and thus, consumers search randomly in the first period, but in subsequent searches, they search the firm first with which they have memberships. We also make the assumption that $r^* < w$, which ensures that there exist some consumers who continue to search other firms after visiting the firm with which they have a membership. After we solve the equilibrium, we verify that, indeed, this assumption is also satisfied.

We solve the game by backward induction below. Particularly, we analyze the situation in which an individual firm j unilaterally deviates to p_1^j, p_2^j, and r^j.[17] As all of its branches behave in the same way, it suffices to consider one branch's optimization. We identify the condition that guarantees the firm has no incentive to deviate. This confirms the existence of the symmetric equilibrium and also pins down the equilibrium.

4.1. Second Period

In the second period, from firm j's perspective, consumers are categorized into two groups: (i) loyal customers, who made purchases and joined the loyalty program of firm j in the first period, and (ii) guest visitors, who neither bought from nor joined the loyalty program of firm j. We analyze the demand firm j receives from each type of consumer separately.

4.1.1. Demand from Loyal Customers. For firm j, the total demand in the first period across all its branches is denoted by $\gamma B D_1^j$, where D_1^j is the consumer demand for one branch of firm j in period 1. Consequently, in the second period, a portion γD_1^j of members travel to a specific city where one of the branches is located.

Consider a consumer who is a loyal customer of firm j. The consumer already observed p_1^j and r^j in the first period, based on which, the consumer forms an expectation of the firm's second period price, $\tilde{p}_2^j$. Recall that w denotes the consumer's reservation utility from continuing the search in period 2. The consumer chooses to search firm j first instead of other firms from which the consumer has no reward if and only if the following condition holds:

$$w - \tilde{p}_2^j + r^j \geq w - p_2^* \iff \tilde{p}_2^j \leq p_2^* + r^j.$$

We verify the above condition after determining $\tilde{p}_2^j$ below. Intuitively, the consumer understands that firm j offers the reward to attract loyal customers to visit it first in the second period, so it is unprofitable for the firm to set the second period price so high that no consumer visits. After visiting firm j, the consumer observes v_2^j and purchases if and only if $v_2^j - p_2^j + r^j \geq w - p_2^*$. Otherwise, the consumer continues to search and never comes back. Therefore, we have the loyal customers' demand for one branch of firm j in the second period as

$$D_{2L}^j = \gamma D_1^j[1 - (w - p_2^* + p_2^j - r^j)].$$

4.1.2. Demand from Guest Visitors. Consider a guest visitor of firm j. If the firm from which the visitor previously purchased is present in the market (with probability γ), the guest first visits that firm k, where the guest holds membership, and learns v_2^k. If the value $v_2^k - p_2^* + r^*$ is less than $w - p_2^*$ (with probability $\Pr(v_2^k - p_2^* + r^* < w - p_2^*) = w - r^*$), the visitor decides to continue searching. Given the visitor's continuation, with probability $1/(\gamma N - 1) \cdot w^n$, the guest visits n other firms before visiting firm j. The guest purchases from firm j if and only if $v_2^j - p_2^j \geq w - p_2^*$, which occurs with probability $1 - (w - p_2^* + p_2^j)$.

On the other hand, if the firm where the guest has membership isn't active (with probability $1 - \gamma$), the guest starts searching randomly among active firms. With probability $1/(\gamma N) \cdot w^n$, the guest visits n other firms before visiting firm j and purchases from firm j if and only if $v_2^j - p_2^j \geq w - p_2^*$, which also occurs with probability $1 - (w - p_2^* + p_2^j)$. In summary, the demand from guest visitors for firm j in the second period is

$$D_{2G}^j = \lim_{N\to\infty}\left(\frac{M}{B} - \gamma D_1^j\right)\left(\gamma(w - r^*)\sum_{n=0}^{\gamma N-2}\frac{1}{\gamma N - 1}w^n + (1-\gamma)\sum_{n=0}^{\gamma N-1}\frac{1}{\gamma N}w^n\right) \times [1 - (w - p_2^* + p_2^j)]$$
$$= \frac{(\gamma(w - r^*) + 1 - \gamma)(1 - (w - p_2^* + p_2^j))}{\gamma(1 - w)}.$$

4.1.3. Optimal Second Period Price. To solve the optimal price for the second period, we analyze the profit of firm j, which comprises demands from both loyal customers and guest visitors. Firm j's second period profit function is

$$\pi_2^j(p_2^j) = D_{2L}^j(p_2^j - r^j) + D_{2G}^j p_2^j$$
$$= \gamma D_1^j[1 - (w - p_2^* + p_2^j - r^j)](p_2^j - r^j)$$
$$+ \frac{(\gamma(w - r^*) + 1 - \gamma)(1 - (w - p_2^* + p_2^j))}{\gamma(1 - w)} p_2^j.$$

Maximizing this profit leads to the optimal second period price:

$$p_2^{j*}(D_1^j, r^j) = \frac{(1 - w + p_2^* + 2r^j)(1 - w)\gamma^2 D_1^j + (1 - \gamma + \gamma(w - r^*))(1 - w + p_2^*)}{2[(1 - w)\gamma^2 D_1^j + (1 - \gamma + \gamma(w - r^*))]}. \quad (1)$$

By substituting $p_2^{j*}(D_1^j, r^j)$ back to $\pi_2^j(p_2^j)$, we can obtain the firm's optimal second period profit, $\pi_2^{j*}(D_1^j, r^j) \equiv \pi_2^j(p_2^{j*}(D_1^j, r^j))$. By taking derivatives, one can easily prove the following lemma that illustrates the dependence relationship of $p_2^{j*}(D_1^j, r^j)$ and $\pi_2^{j*}(D_1^j, r^j)$ on D_1^j and r^j.

Lemma 2. *The optimal second period price $p_2^{j*}(D_1^j, r^j)$ increases with both demand D_1^j and reward r^j; the second period profit $\pi_2^{j*}(D_1^j, r^j)$ increases with demand D_1^j and decreases with reward r^j.*

The lemma suggests that, whereas a higher first period demand consistently benefits the firm by boosting both price and profit; an increase in the reward, though positively affecting the price, negatively impacts the profit in the second period. To get an intuition behind the optimal second period price, $p_2^{j*}(D_1^j, r^j)$, we can rewrite Equation (1) as the following:

$$p_2^{j*}(D_1^j, r^j) = \lambda(D_1^j)\left(\frac{1 - w + p_2^*}{2} + r^j\right) + (1 - \lambda(D_1^j))\frac{1 - w + p_2^*}{2},$$

where $\lambda(D_1^j) \equiv D_1^j/[D_1^j + (1 - \gamma + \gamma(w - r^*))/(\gamma^2(1 - w))]$. Note that the optimal prices for loyal customers and guest visitors alone are $(1 - w + p_2^*)/2 + r^j$ and $(1 - w + p_2^*)/2$, respectively. Therefore, $p_2^{j*}(D_1^j, r^j)$ can be seen as a linear combination of these two. The weight on price for loyal customers $\lambda(D_1^j)$ is the fraction of loyal customers among those who come to visit firm j. The part of the price for loyal customers, $(1 - w + p_2^*)/2 + r^j$, is higher because they receive a reward, r^j. As the first period demand, D_1^j, increases, indicating more loyal customers, the optimal price shifts closer to what is ideal for loyal customers, hence increasing. An increase in the reward, r^j, directly raises the price component for loyal customers, thereby lifting the overall optimal price, $p_2^{j*}(D_1^j, r^j)$.

Regarding profit, $\pi_2^{j*}(D_1^j, r^j)$, an increase in loyal customer number, D_1^j, allows the firm to raise prices, leading to a higher profit in the second period. This, in turn, incentivizes firms to compete by lowering their first period prices to acquire more consumers in the first period, which we formally analyze next. The reward r^j affects profit through two channels. First, with a higher reward, loyal customers are more likely to stop searching and make a purchase, leading to a higher demand. On the other hand, the firm needs to spend more on rewarding loyal customers. Overall, the latter effect dominates, and the optimal second period profit decreases as the reward increases.

4.2. First Period

In the first period, consumers search randomly. After visiting firm j and observing the value v_1^j, price p_1^j, and reward r^j, a consumer can infer its first period demand D_1^j. Based on this information, a consumer forms an expectation of the firm's second period price, $\tilde{p}_2^j = p_2^{j*}(D_1^j, r^j)$, given by Equation (1). Then, the consumer stops searching and makes a purchase from firm j if and only if

$$v_1^j - p_1^j + \delta \mathrm{E}[u_2|j] \geq w - p_1^* + \delta \mathrm{E}[u_2|k], \quad (2)$$

where k represents an alternative firm from which the consumer may buy if the consumer decides to continue to search; $\mathrm{E}[u_2|j]$ and $\mathrm{E}[u_2|k]$ are the consumer's expected utility in the second period if the consumer purchases from firm j and k in the first period, respectively. Equation (2) implies that the consumer's optimal search strategy in the first period still follows an index policy as the expected utilities $\mathrm{E}[u_2|j]$ and $\mathrm{E}[u_2|k]$ are independent of the consumer's first period match value realization.

By definition, we have

$$\mathrm{E}[u_2|j] = \gamma\left[-s + \int_{w - p_2^* + p_2^{j*}(D_1^j, r^j) - r^j}^{1} (v_2^j - p_2^{j*}(D_1^j, r^j) + r^j)dv_2^j + (w - p_2^* + p_2^{j*}(D_1^j, r^j) - r^j)(w - p_2^*)\right] + (1 - \gamma)(w - p_2^*),$$

$$\mathrm{E}[u_2|k] = \gamma\left[-s + \int_{w - r^*}^{1} (v_2^k - p_2^* + r^*)dv_2^k + (w - r^*)(w - p_2^*)\right] + (1 - \gamma)(w - p_2^*).$$

By substituting the above expressions of $\mathrm{E}[u_2|j]$ and $\mathrm{E}[u_2|k]$ back to Equation (2), we have that a consumer stops at firm j if and only if $v_1^j \geq \hat{v}(p_1^j, r^j)$, where

$$\hat{v}(p_1^j, r^j) \equiv w - p_1^* + p_1^j - \delta(\mathrm{E}[u_2|j] - \mathrm{E}[u_2|k])$$
$$= w - p_1^* + p_1^j - \frac{\gamma\delta}{2}(p_2^* - p_2^{j*}(D_1^j, r^j) + r^j - r^*) \times (p_2^* - p_2^{j*}(D_1^j, r^j) + r^j + r^* + 2 - 2w). \quad (3)$$

Similar to the case for guest visitors' demand in the second period, we can calculate the first period demand as

$$D_1^j = \frac{1-\hat{v}(p_1^j, r^j)}{\gamma(1-w)}. \quad (4)$$

Equation (4) indicates that D_1^j depends on $\hat{v}(p_1^j, r^j)$, which is affected by $p_2^{j*}(D_1^j, r^j)$. In other words, consumers' choices in the first period depend on their expectation of the next period price. Because $p_2^{j*}(D_1^j, r^j)$ itself also depends on D_1^j, by combining Equations (1), (3), and (4), one could obtain D_1^j as a function of p_1^j and r^j. However, it turns out that the resulting expression of D_1^j is rather complicated. On the other hand, by combining Equations (3) and (4), one can express p_1^j as a more manageable function of D_1^j and r^j explicitly:

$$p_1^j = p_1^* + (1-w)(1-\gamma D_1^j) + \frac{\gamma\delta}{2}(p_2^* - p_2^{j*}(D_1^j, r^j) + r^j - r^*) \times (p_2^* - p_2^{j*}(D_1^j, r^j) + r^j + r^* + 2 - 2w) \equiv p_1^j(D_1^j, r^j).$$

Thus, it is more straightforward to consider firm j's decisions in terms of D_1^j and r^j rather than p_1^j and r^j. In other words, one can equivalently view firm j making decisions on D_1^j and r^j in the first period.

Firm j's profit in the first period, $\pi_1^j(D_1^j, r^j) = D_1^j p_1^j (D_1^j, r^j)$. In the first period, the firm tries to maximize its total profit $\pi_T^j(D_1^j, r^j) = \pi_1^j(D_1^j, r^j) + \delta\pi_2^{j*}(D_1^j, r^j)$, where

$$(D_1^*, r^*) = \underset{(D_1^j, r^j)}{\arg\max}\, \pi_T^j(D_1^j, r^j).$$

We demonstrate that, when the search cost is not prohibitively high, there exists a unique symmetric equilibrium in which firms optimally offer positive rewards.

Proposition 1. *A unique symmetric equilibrium exists when* $0 < s < \bar{s} \equiv 1/[2(2+\gamma)^2]$. *In this equilibrium, the firms' equilibrium reward and prices are*

$$r^* = \sqrt{2s},$$
$$p_1^* = \frac{\sqrt{2s} - 4\gamma s + 2\sqrt{2}(1-2\delta)\gamma^2 s^{\frac{3}{2}}}{(1-\gamma\sqrt{2s})^2},$$
$$p_2^* = \frac{\sqrt{2s} + 2\gamma s}{1-\gamma\sqrt{2s}}.$$

Moreover, in this equilibrium, consumers randomly search in the first period, make purchases, and become members. In the second period, if consumers find their membership firms active in the cities they visit, they first visit them and continue searching randomly if necessary. If their membership firms are not active, they randomly search among all firms.

The proposition indicates that firms endogenously adopt loyalty programs to compete for future prominence. The upper bound of $\bar{s}$ on search cost s ensures that consumers prefer to search rather than take the outside option. Moreover, $r^* < w$ equates to $s < 1/8$, which is always satisfied given $s < \bar{s}$.

Next, we examine how these equilibrium outcomes change in response to the key market conditions by the following two propositions. The first proposition shows the impact of search costs on various market outcomes.

Proposition 2. *The impact of search cost, s, on first period price and profit,* p_1^* *and* π_1^**, varies depending on the discount factor* δ *and the activation rate* γ*. Specifically,*

i. *When* $0 < \delta \le 1/2$ *or* $1/2 < \delta \le 1$ *and* $0 < \gamma \le \hat{\gamma}$*, both* p_1^* *and* π_1^* *increase with s.*

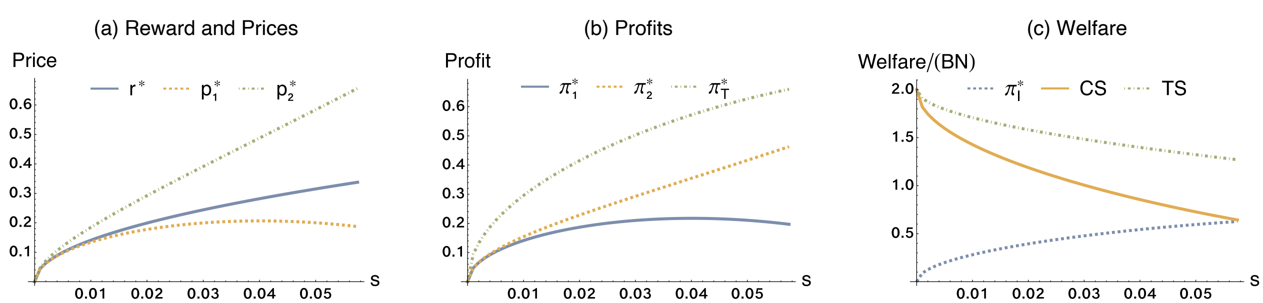
ii. *When* $1/2 < \delta \le 1$ *and* $\hat{\gamma} < \gamma \le 1$*,* p_1^* *and* π_1^* *first increase and then decrease with s.*

Here, $\hat{\gamma}$ *is uniquely determined by the equation of* $\delta\gamma^3 + 3\delta\gamma^2 - 2 = 0$.

Moreover, other equilibrium outcomes, including r^*, p_2^**; second period profit* π_2^**; total profit* π_T^**; and industry profit* $\pi_I^* \equiv \gamma NB\pi_T^*$ *all increase with s, whereas consumer surplus CS and total surplus TS both decrease with s.*

Figure 2 illustrates case (ii) in Proposition 2. As the search cost increases, consumers are less inclined to explore multiple options. Given that loyal customers first search firms with which they have memberships (when active), the likelihood of repeat purchases rises. Therefore, firms are more incentivized to attract consumers with higher rewards. In this context, the search cost acts as an exogenous switching cost for loyal customers, whereas rewards represent an endogenous switching cost determined by firms. Our findings reveal that endogenous switching costs effectively complement exogenous ones. Rewards, precommitted at the outset, serve as another channel for firms to compete

Figure 2. (Color online) Comparative Statics Examples on s When $\delta = 1$ and $\gamma = 0.95$



with each other beyond pricing. With elevated search costs, firms compete more fiercely through loyalty programs. This predicts that, as online travel agencies or price comparison sites lower search costs, the strategic value of reward-induced prominence declines, and firms may benefit more from alternative tools for consumer retention.

The impact of search friction on the first period price exhibits a nonlinear pattern for large values of δ and γ. One can interpret the first period price as any effort to acquire new customers, such as sign-up bonuses, introductory pricing, or welcome offers. Specifically, Proposition 2 shows that, as search friction becomes higher, the first period price, p_1^*, initially rises before declining as illustrated by Figure 2(a). In fact, when search costs are relatively low, first period price competition is fierce. An increase in search costs reduces price competition, allowing firms to charge a higher price p_1^*. On the other hand, when search costs are relatively high, it becomes a more important consideration to lock in consumers for second period exploitation through lower first period prices. In this case, an increase in search costs strengthens firms' incentives to lock in consumers, leading to a lower equilibrium price p_1^*. Lastly, when either δ or γ is relatively low, firms prioritize immediate profits over the second period profits through lock-in, leading to a consistent rise in the first period price as search costs increase.

For the second period pricing, as search costs increase, consumers' propensity to explore decreases, allowing firms to charge higher prices. This observation aligns with the finding from Wolinsky (1986), in which higher search friction reduces market competition and leads to higher prices. Moreover, as rewards rise in response to greater search friction, loyal customers are more inclined to repurchase from the same firms, enabling these firms to further increase their prices. Consequently, the second period price rises with search costs as illustrated by Figure 2(a).

The first period profit mirrors the response of p_1^* to changes in search cost. The second period profit, the total profit, and industry profit increase as the search cost is higher. This uptrend is attributed to consumers' diminished willingness to engage in extensive search efforts, effectively augmenting each firm's market power. As a result, the total surplus diminishes alongside increasing search frictions. Initially, this reduction in surplus is due to the increased costs faced by consumers. Subsequently, the increase in repeat purchases is driven more by higher rewards than by consumer preference, leading to a greater incidence of mismatches and lower total surplus. A similar rationale applies to the decline in consumer surplus as illustrated in Figure 2, (b) and (c), respectively.

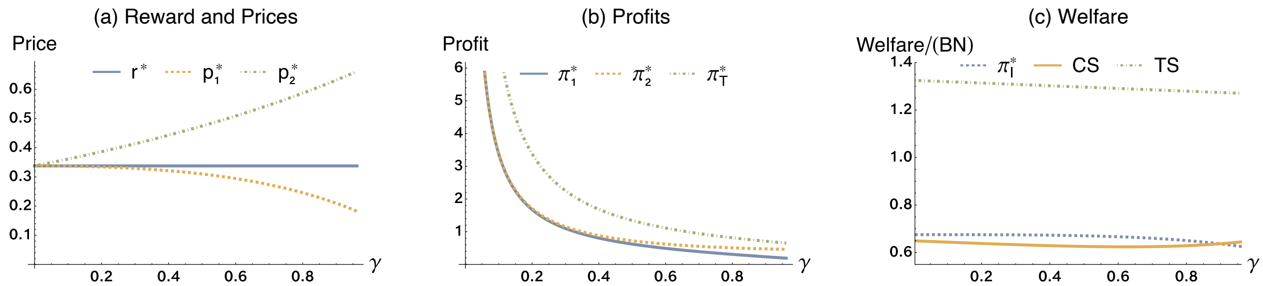
Next, we investigate the impact of the activation rate (which captures the industry-level network size) on the equilibrium outcomes.

Proposition 3. *An increase in the activation rate, γ, leads to a decrease in p_1^*, whereas r^* remains unchanged and p_2^* increases. Profits for the first period and second period π_1^*, π_2^*; total profit π_T^*; industry profit π_I^*; and total surplus TS all decrease with γ. Consumer surplus CS reacts differently depending on s; it decreases when $0 < s \le s_1$ or $s_2 \le s < 1/8$ but shows a decrease followed by an increase when $s_1 < s < s_2$, where the threshold s_1 and s_2 are uniquely determined by the equations $1 - 3\sqrt{2s} - 18s + 6\sqrt{2}s^{\frac{3}{2}} = 0$ and $\sqrt{2} - 6\sqrt{s} + 12s^{\frac{3}{2}} = 0$, respectively, with $s_1 \approx 0.022$, $s_2 \approx 0.078$.*

The proposition shows that, counterintuitively, the reward remains constant regardless of the activation rate. Essentially, rewards serve as a price discount to prevent loyal customers from seeking alternatives in each city, functioning effectively at the local level regardless of a firm's network size. The prevalence or scarcity of a firm's branches across cities does not influence the effectiveness of rewards within each city; they operate independently at each branch location. With an increase in the activation rate, consumers in the second period are more likely to remain with the firms at which they hold memberships, resulting in a decline in the number of guest visitors. This allows firms to charge a higher price to exploit loyal customers in the second period. However, the incentive to acquire more customers in the first period heightens competition, leading to a reduction in first period prices as the activation rate increases. See Figure 3(a) for the impact of activation rate on equilibrium rewards and prices.

Furthermore, the activation rate directly affects the number of active firms within each city. A higher activation rate translates to more firms per city. With a constant flow of consumers to each city, each firm's market share shrinks. Therefore, the first period profit, the second period profit, and the total profit all decrease. The industry profit also decreases because firms compete too fiercely for customer acquisition (see Figure 3(b) for profit results). The total surplus diminishes with the activation rate for reasons akin to its decrease with search costs: more repeat purchases are motivated by rewards rather than genuine product preference, leading to welfare losses because of higher incidents of mismatches. The impact on consumer surplus is subtle and contingent on the search cost magnitude. At moderate search costs, two opposing effects emerge. Initially, increased mismatches from a higher activation rate suppress consumer surplus. However, as the rate climbs further, reduced prices in the first period benefit consumers, thereby enhancing consumer surplus. On the other hand, at

Figure 3. (Color online) Comparative Statics Examples on γ When $\delta = 1$ and $s = 0.057$



very low search costs, extensive consumer search in the second period and the likelihood of brand switching prompt firms to be very conservative in pricing low in the first period, reducing consumer surplus as the activation rate increases. Conversely, at very high search costs, increasing mismatch decreases the consumer surplus (as in Proposition 2) monotonically with the activation rate. Figure 3(c) illustrates the impact of activation rate on the firm profits, total surplus, and consumer surplus.

4.2.1. Discussion: The Effects of Search Cost and Activation Rate. It might appear that higher search costs and increased activation rates impose similar influence on market competition by promoting repeated purchases and reducing the propensity for consumers to switch brands in the second period. Thus, fewer switching behaviors in the second period increase each firm's market power but intensify competition in the first period as member acquisition becomes more profitable in the long run. However, there are notable distinctions in their impact on competition.

First, whereas rewards increase with search costs, they remain unaffected by changes in the activation rate. Although both factors lead to more repeat purchases, heightened search costs discourage consumers from exploring alternatives within a city, whereas increased activation rates primarily make directed searches occur more frequently across all different cities. Rewards, thus, play a pivotal role in each city where a firm operates with firms opting to increase rewards only when there's a high likelihood of members ceasing their search to make repeat purchases (for example, when search costs become higher). This does not occur with variations in the activation rate, resulting in rewards remaining constant. Second, higher activation rates make the market more competitive by reducing the firm's market shares, whereas increased search costs, by increasing each firm's market power even in the first period, mitigate the competitive incentive for aggressively acquiring new customers.

These analyses underscore the distinct impacts of search cost and activation rate on market equilibrium as depicted in the corresponding Figures 2 and 3, highlighting how these factors shape market competition and equilibrium outcomes differently.

4.3. Impact of Loyalty Programs

To investigate the impact of loyalty programs, we consider a benchmark scenario in the absence of loyalty programs, in which firms only decide their prices in each period. This setting renders prior consumer behaviors irrelevant to future decisions because the match values of products are independent across time in the absence of a loyalty program, allowing the game to be simplified into two separate stage games.

Lemma 3. *In the absence of loyalty programs, there is a unique symmetric equilibrium in which the prices are* $p_1^B = p_2^B = \sqrt{2s}$.

Then, we further compare the outcomes with and without loyalty programs as summarized in the following proposition, in which the superscript "B" denotes the benchmark case.

Proposition 4. *Comparison of equilibria with and without loyalty programs:*

- $p_1^* < p_1^B$, $p_2^* > p_2^B$, *and* $p_2^* - r^* < p_2^B$.
- $\pi_1^* < \pi_1^B$, $\pi_2^* > \pi_2^B$, $\pi_T^* < \pi_T^B$, $\pi_I^* < \pi_I^B$, $CS < CS^B$, *and* $TS < TS^B$.

This analysis reveals that loyalty programs introduce an additional dimension of competition for firms. By offering rewards, firms effectively reduce consumer propensity to switch in the second period, encouraging more aggressive pricing strategies in the first period to secure consumer loyalty.

In the absence of loyalty programs, a consumer randomly searches in the second period. However, with loyalty programs in place, consumers prioritize firms at which they hold memberships. Firms, accordingly, differentiate on their pricing: offering a reduced rate, $p_2^* - r^*$, for members and a standard rate, p_2^*, for nonmembers. Here, p_2^* is always higher than the price without a loyalty program ($p_2^* > p_2^B$) because firms want to exploit loyal customers who visit them first. Nonetheless, the effective price for members, after accounting for rewards, is lower than p_2^B (i.e., ($p_2^* - r^* < p_2^B$), indicating that firms engage in competition by offering rewards to members. This allows firms to leverage their prominence among members to impose higher prices and secure greater profits than those without loyalty programs.

In the first period, with loyalty programs, firms have incentives to reduce their prices to attract members, anticipating future exploitation. This contrasts with scenarios without loyalty programs, in which consumers search randomly in the second period. Consequently, both the first period's price and profit are lower than those from the benchmark case of without loyalty programs ($p_1^* < p_1^B$ and $\pi_1^* < \pi_1^B$). Loyalty programs, thus, spur competition over prominence, not solely through rewards but also via the first period price. This intensified price competition results in diminished total and industry profits ($\pi_T^* < \pi_T^B$, $\pi_I^* < \pi_I^B$), generating a prisoner's dilemma: in equilibrium, each firm has an incentive to offer rewards, yet all would be better off without them. Whereas this stands in contrast to classic switching-cost models such as Caminal and Matutes (1990), in which loyalty programs typically raise total profits, our search-based framework produces the opposite outcome because rewards here also serve to induce prominence, shifting competitive pressure to the first period.

Whereas loyalty programs are pervasive in practice, their profitability is far from universal. In fact, the effectiveness and profitability of reward programs have been challenged both theoretically and empirically in numerous studies (Dowling and Uncles 1997, Sharp and Sharp 1997, Mägi 2003, Shugan 2005, Leenheer et al. 2007, Meyer-Waarden 2007, Villanueva et al. 2007, Liu and Yang 2009, Iyengar et al. 2022). As noted by Chen et al. (2021), such dynamics can make loyalty programs resemble a prisoner's dilemma, a view echoed by Hilton Honors' Jeff Diskin: "Loyalty programs have been at the core of how we attract and retain our best customers for over a decade. But they are only as cost-effective as our competitors let them be" (Deighton and Shoemaker 2000). Our model provides a theoretical foundation for these observations by showing how reward-induced prominence, unlike pure switching costs, can intensify early period competition enough to lower profits industry-wide.

To understand the comparison of consumer and social welfare results, notice that, without loyalty programs, consumer purchases are based purely on high product valuation following random searches. In contrast, loyalty programs encourage members to prioritize affiliated firms, sometimes leading to purchases even when product fit is suboptimal. This discrepancy suggests that rewards may deter consumers from discovering products that better match their preferences, resulting in welfare losses. Consumers are also worse off even though they enjoy lower prices in the first period as well as additional rewards. To summarize, loyalty programs lead to a lose–lose situation for both consumers and firms.

5. Heterogeneous Network Sizes

In this section, we extend the basic model to allow for two distinct activation rates of firms in the market, denoted by $\theta \in \{H, L\}$. To focus on the main economic insights, we present only the key equilibrium expressions in this section and relegate some of the intermediate algebraic derivations to Online Appendix B. Specifically, $\alpha \in (0,1)$ fraction of firms are of high type with $\theta = H$ that operate a branch in $\gamma_H B$ cities, where a high activation rate $\gamma_H \in (0,1]$ symbolizes an extensive network coverage. The remaining $1-\alpha$ fraction of firms are of low type with $\theta = L$ that operate a branch in $\gamma_L B$ cities, where a low activation rate $\gamma_L \in (0, \gamma_H]$ represents more limited network sizes. Again, whether a firm operates a branch in specific cities is independent across firms. Consumers do not observe the firms' types a priori. Upon visiting firm j in city $b(1)$ in the first period, consumers observe the firm's type θ_j along with the consumer's match value v_1^j, the price $p_{1\theta_j}^j$, and reward $r_{\theta_j}^j$.

This extended model allows us to examine how the disparity in network size, such as between large hotel chains such as Marriott (with 9,300 properties across 110 countries) and smaller networks such as the Loews Hotels (part of the Leading Hotels of the World collection, which includes around 400 hotels in more than 80 countries) impacts the optimization of loyalty programs. We analyze situations in which this size disparity becomes either larger or smaller, exploring how such heterogeneity affects loyalty program rewards, pricing strategies, and ultimately firm profits. In a similar spirit as Assumption 1, we introduce an additional assumption to address consumers' potential memory or attention limitations regarding firms they've visited without making a purchase in this setting.

Assumption 2. *If a consumer visited a firm in the first period without making a purchase, in the second period, the consumer no longer remembers the firm's type* θ_j.

This assumption effectively homogenizes all firms from which the consumer did not purchase, significantly streamlining the consumer decision-making process for subsequent searches. Specifically, it suggests that, if consumers opt to search beyond the firm with which the consumer has a reward in the second period, the consumer will search randomly among all other firms. Absent this assumption, the decision of which firms to visit next—be it previously visited high-type firms, low-type firms, or those of unknown type—becomes notably more complex.

5.1. Equilibrium Analysis

Similar to the main model, we solve the game by backward induction. Our analysis focuses on an equilibrium in which all firms of the same type θ set the same rewards r^*_θ and price $p^*_{t\theta}$ for both periods. Similar to the main analysis, we characterize the equilibrium by considering potential deviations by any firm of type θ to alternative reward and pricing strategies $(r^j_\theta, p^j_{1\theta}, p^j_{2\theta})$.

5.1.1. Second Period Analysis. Again, there are two kinds of consumers in the second period: (i) loyal customers and (ii) guest visitors. We first calculate firm j's demand from these two types of consumers separately and then derive the optimal second period prices for them.

5.1.1.1. Demand from Loyal Customers. Consider the total number of active firms in the second period as $N_A \equiv (\alpha\gamma_H + (1-\alpha)\gamma_L)N$. The proportions of high- and low-type firms active in a city are defined respectively as

$$\beta_H = \frac{\alpha\gamma_H}{\alpha\gamma_H + (1-\alpha)\gamma_L} = \frac{\alpha\gamma_H}{A_2},$$
$$\beta_L = \frac{(1-\alpha)\gamma_L}{\alpha\gamma_H + (1-\alpha)\gamma_L} = \frac{(1-\alpha)\gamma_L}{A_2},$$

where $A_2 \equiv \alpha\gamma_H + (1-\alpha)\gamma_L$ is the composite activation rate in the second period, which measures the overall market presence of both high- and low-type firms in the second period, taking into account their respective shares in the market and their individual probabilities of operating an active branch in any given city.

Define w_2 as the second period reservation value, the utility level at which a consumer is indifferent between receiving a sure payoff w_2 and continuing searching randomly from the firms with which the consumer does not have membership. We obtain this reservation value by equating the marginal cost of searching, s, with the marginal benefit of searching one additional firm:

$$s = \beta_H \int_{p^*_{2H}+w_2}^{1} (v^j_2 - p^*_{2H} - w_2)dv^j_2 + \beta_L \int_{p^*_{2L}+w_2}^{1} (v^j_2 - p^*_{2L} - w_2)dv^j_2,$$

which yields

$$w_2 = 1 - \frac{\alpha\gamma_H}{A_2}p^*_{2H} - \frac{(1-\alpha)\gamma_L}{A_2}p^*_{2L} - \sqrt{2s - \frac{\alpha(1-\alpha)\gamma_H\gamma_L(p^*_{2H}-p^*_{2L})^2}{A_2{}^2}}.$$

Similar to the main model, for firm j, the total demand in the first period across all its branches is captured by $\gamma_\theta D^j_{1\theta}B$. This translates to $\gamma_\theta D^j_{1\theta}$ members destined for a particular city where the firm operates a branch. Consider a loyal customer who holds a membership with firm j. The customer observes $p^j_{1\theta}$ and r^j_θ in the first period, based on which the customer forms an expectation of the firm's second period price, $\tilde{p}^j_{2\theta}$. The consumer searches firm j first if and only if $w - \tilde{p}^j_{2\theta} + r^j_\theta \ge w_2$. Conditional on searching there, a purchase occurs strictly when the utility from buying, $v^j_2 - p^j_{2\theta} + r^j_\theta$, exceeds the reservation utility, w_2, leading to the following expression for the second period demand of type θ from loyal customers:

$$D^j_{2\theta L}(p^j_{2\theta}) = \gamma_\theta D^j_{1\theta}[1 - (w_2 + p^j_{2\theta} - r^j_\theta)]. \tag{5}$$

5.1.1.2. Demand from Guest Visitors. In contrast to the main model, guest visitors now fall into four distinct segments. These segments are determined based on their memberships with either high- (H) or low-type (L) firms and the operational status (active or not) of the firms with which they are affiliated.

For a guest visitor who is a member of an active high-type firm, the guest first visits the affiliated firm. If the guest's net utility $(v^k_2 - p^*_{2H} + r^*_H)$ is less than the reservation value w_2, the guest proceeds to search other firms. Given the guest's continuation, with probability $1/(N_A - 1)\cdot(\beta_H(w_2 + p^*_{2H}) + \beta_L(w_2 + p^*_{2L}))^n$, the guest visits n other firms before visiting firm j and will purchase from firm j if and only if $v^j_2 - p^j_{2\theta} \ge w_2$. A similar decision-making pathway applies to guest visitors of active low-type (L) firms. Guest visitors whose affiliated firm is not active in the current city, whether high- or low-type, follow analogous search paths, generating two additional segment-specific demand components.

To avoid burdening the exposition with intermediate algebra, the explicit expressions for these four segment-level demands ($D^j_{2\theta G_{HA}}$, $D^j_{2\theta G_{LA}}$, $D^j_{2\theta G_{HN}}$, and $D^j_{2\theta G_{LN}}$) are derived in Online Appendix B. Summing these four terms yields the total demand from guest visitors faced by a type-θ firm:

$$D^j_{2\theta G} = D^j_{2\theta G_{HA}} + D^j_{2\theta G_{LA}} + D^j_{2\theta G_{HN}} + D^j_{2\theta G_{LN}}$$
$$= \frac{(1 - w_2 - p^j_{2\theta})\begin{pmatrix} D^*_{1H}\alpha\gamma_H{}^2(w_2 + p^*_{2H} - r^*_H) \\ + D^*_{1L}(1-\alpha)\gamma_L{}^2(w_2 + p^*_{2L} - r^*_L) \\ + D^*_{1H}\alpha\gamma_H(1-\gamma_H) + D^*_{1L}(1-\alpha)\gamma_L(1-\gamma_L) \end{pmatrix}}{A_2(1 - (\beta_H(w_2 + p^*_{2H}) + \beta_L(w_2 + p^*_{2L})))}. \tag{6}$$

5.1.1.3. Optimal Second Period Price. To solve for the optimal price for the second period, we analyze the profit of firm j of type θ, which includes demands from both loyal customers and guest visitors from Equations (5) and (6). Firm j's second period profit function is

$$\pi^j_{2\theta}(p^j_{2\theta}) = (p^j_{2\theta} - r^j_\theta)D^j_{2\theta L} + p^j_{2\theta}D^j_{2\theta G}.$$

The firm's optimal second period price solves

$$p_{2\theta}^{j*}(D_{1\theta}^j, r_\theta^j) = \arg\max_{p_{2\theta}^j} \pi_{2\theta}^j(p_{2\theta}^j).$$

This maximization yields a unique closed-form expression for $p_{2\theta}^{j*}(D_{1\theta}^j, r_\theta^j)$. Because the resulting formula is lengthy and adds little intuition, we report the full expression in Online Appendix B.

Substituting $p_{2\theta}^{j*}(D_{1\theta}^j, r_\theta^j)$ back into $\pi_{2\theta}^j(p_{2\theta}^j)$ yields the firm's optimal second period profit, $\pi_{2\theta}^{j*}(D_{1\theta}^j, r_\theta^j) \equiv \pi_{2\theta}^j(p_{2\theta}^{j*}(D_{1\theta}^j, r_\theta^j))$, which we use in the first period analysis below.

5.1.2. First Period Analysis. In the first period, a consumer randomly searches among all firms. Upon visiting a firm j and learning about the match value v_1^j, first period price $p_{1\theta}^j$, and reward offered r_θ^j, consumers form expectations regarding the firm's pricing and reward strategy for the second period, denoted by $\tilde{p}_{2\theta}^j = p_{2\theta}^{j*}(D_{1\theta}^j, r_\theta^j)$. We define the first period reservation value as w_1. Formally,

$$s = \beta_H \int_{p_{1H}^* - \delta E[u_{2H}|H] + w_1}^{1} (v_1^j - p_{1H}^* + \delta E[u_{2H}|H] - w_1) dv_1^j + \beta_L \int_{p_{1L}^* - \delta E[u_{2L}|L] + w_1}^{1} (v_1^j - p_{1L}^* + \delta E[u_{2L}|L] - w_1) dv_1^j,$$

where $E[u_{2H}|H]$ and $E[u_{2L}|L]$ are the consumer's expected utility in the second period depending on whether the consumer purchased from a high- or a low-type firm in the first period, respectively. where $E[u_{2\theta}|j]$ is the consumer's expected utility in the second period conditioning on that the consumer has purchased from firm j of type θ in the first period. We have

$$E[u_{2\theta}|j] = \gamma_\theta \left[-s + \int_{w_2 + p_{2\theta}^{j*}(D_{1\theta}^j, r_\theta^j) - r_\theta^j}^{1} (v_2^j - p_{2\theta}^{j*}(D_{1\theta}^j, r_\theta^j) + r_\theta^j) dv_2^j + (w_2 + p_{2\theta}^{j*}(D_{1\theta}^j, r_\theta^j) - r_\theta^j) w_2 \right] + (1 - \gamma_\theta) w_2.$$

Therefore,

$$w_1 = 1 - \frac{\alpha \gamma_H \Psi_{1H} + (1-\alpha)\gamma_L \Psi_{1L}}{A_2} - \sqrt{2s - \frac{\alpha(1-\alpha)\gamma_H \gamma_L (\Psi_{1H} - \Psi_{1L})^2}{A_2{}^2}},$$

where $\Psi_{1H} \equiv (p_{1H}^* - \delta E[u_{2H}|H])$ and $\Psi_{1L} \equiv (p_{1L}^* - \delta E[u_{2L}|L])$.

The consumer stops at firm j if and only if $v_1^j \geq \hat{v}_\theta(p_{1\theta}^j, r_\theta^j) \equiv w_1 + p_{1\theta}^j - \delta E[u_{2\theta}|j]$. Therefore, the first period demand is

$$D_{1\theta}^j = \frac{1 - \hat{v}_\theta(p_{1\theta}^j, r_\theta^j)}{(1 - \beta_H(w_1 + \Psi_{1H}) - \beta_L(w_1 + \Psi_{1L}))A_2}.$$

The price $p_{1\theta}^j$ can then be written as a function of the first period demand $D_{1\theta}^j$ and the reward r_θ^j, providing a direct link between pricing, demand, and the reward structure:

$$p_{1\theta}^j(D_{1\theta}^j, r_\theta^j) \equiv 1 - w_1 + \delta E[u_{2\theta}|j] - A_2(1 - \beta_H(w_1 + \Psi_{1H}) - \beta_L(w_1 + \Psi_{1L}))D_{1\theta}^j.$$

Firm j's profit in the first period is then represented as $\pi_{1\theta}^j(D_{1\theta}^j, r_\theta^j) = D_{1\theta}^j p_{1\theta}^j(D_{1\theta}^j, r_\theta^j)$. The optimal strategy for firm j in the first period involves maximizing the total profit $\pi_{T\theta}^j(D_{1\theta}^j, r_\theta^j)$, which includes both the first and second period profits. Thus, the firm's decision problem in the first period is

$$(D_{1\theta}^*, r_\theta^*) = \arg\max_{(D_{1\theta}^j, r_\theta^j)} \pi_{T\theta}^j(D_{1\theta}^j, r_\theta^j) = \pi_{1\theta}^j(D_{1\theta}^j, r_\theta^j) + \delta \pi_{2\theta}^{j*}(D_{1\theta}^j, r_\theta^j).$$

The $D_{1\theta}^*$and r_θ^* should satisfy two optimality conditions for each type, respectively. Unfortunately, there is no closed-form solution; instead, we solve the equilibrium numerically, which we present next.

5.1.3. Numerical Analysis: Optimal Rewards and First Period Price. This section outlines our approach to solving the model under a variety of market conditions, providing insights into the practical implications of our theoretical framework. To explore the model's predictions across different scenarios, we conduct a numerical analysis. We set $\alpha = 0.5$ and $\delta = 1$ and examine the model outcomes under three specific conditions by varying γ_H, γ_L, and s:

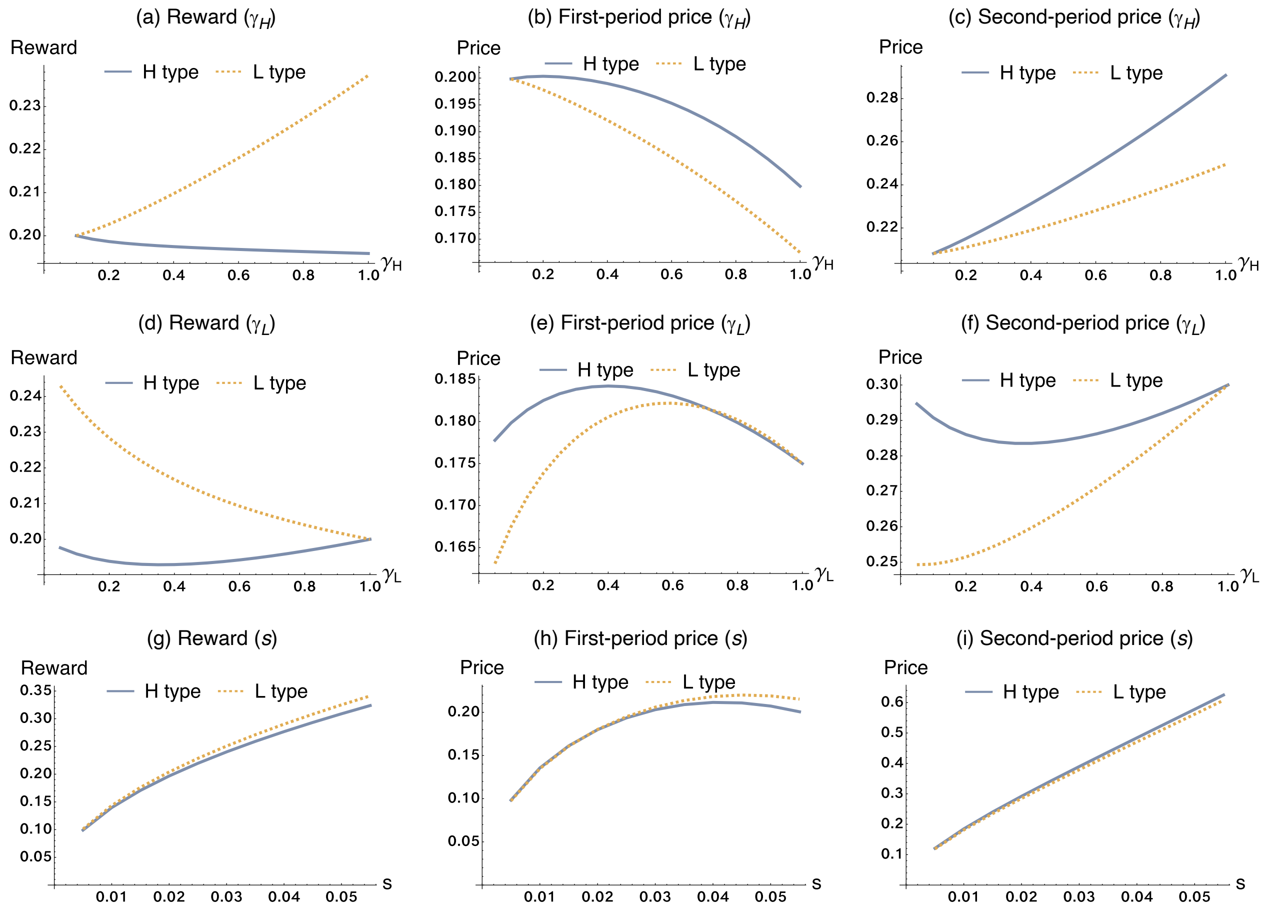
- With $s = 0.02$, $\gamma_L = 0.1$, we vary γ_H from 0.1 to 1 in increments of 0.05.
- With $s = 0.02$, $\gamma_H = 1$, we adjust γ_L from 0.05 to 1 in increments of 0.05.
- Setting $\gamma_H = 1$, $\gamma_L = 0.8$, we change s from 0.005 to 0.055 in increments of 0.005.

The results from these analyses, presented in the subsequent figures (see Figure 4), illustrate the robustness of our model across a spectrum of market conditions.

Result 1 (Comparison of High- and Low-Type Firms). *$r_H^* < r_L^*$ and $p_{2H}^* > p_{2L}^*$. Also, $p_{1H}^* < p_{1L}^*$ when s is relatively large and $\gamma_H - \gamma_L$ is small, whereas $p_{1H}^* > p_{1L}^*$ otherwise. Moreover, $D_{1H}^* > D_{1L}^*$. $\pi_{TH}^* > \pi_{TL}^*$.*

The comparison of the high- and low-type firms shows that high-type firms (with larger networks) set lower rewards r_H^* than low-type firms r_L^*, charge higher second period prices p_{2H}^* than p_{2L}^*, and attract more first period demand D_{1H}^*, leading to higher total profits π_{TH}^* compared with low-type firms. Also, the first period price p_{1H}^* is lower for high-type firms when search costs are significant and the difference in

Figure 4. (Color online) Equilibrium Changing with γ_H, γ_L, and s



network sizes is minimal, indicating a strategic trade-off between initial price reduction for member acquisition and subsequent reward setting for customer retention.

The numerical analysis reveals intriguing contrasts in optimal reward levels between high-type (larger network) and low-type (smaller network) firms. Specifically, low-type firms set higher rewards ($r_L^* > r_H^*$) despite having smaller networks. This is in stark contrast to the previous result, which suggests that optimal rewards would not vary with network size under homogeneous market competition (Proposition 3). Specifically, in a homogeneous setting, all firms share identical network sizes, so they have the same competitive advantage from network coverage, and thus, the equilibrium reward level is the same for all firms and independent of the network size. However, in a heterogeneous scenario, the disparity in network sizes becomes a critical factor that can affect their customer acquisition, leading to different reward levels and prices depending on their network size. High-type firms with extensive networks can afford to offer smaller rewards as their wide reach across numerous cities naturally attracts new members. Thus, they leverage their extensive network size to acquire new customers efficiently in the first period with lower rewards, allowing them to exploit their loyal customer base with higher prices in the second period ($p_{2H}^* > p_{2L}^*$). On the other hand, low-type firms compensate for their smaller networks with higher rewards to acquire customers in the first period.

The finding that firms with larger network sizes offer a lower reward is consistent with what happened after Marriott's acquisition of SPG in 2016. Before the acquisition, Marriott managed approximately 4,500 properties. After acquiring Starwood, Marriott's portfolio expanded to more than 9,300 properties across 110 countries, thereby becoming the world's largest hotel chain at that time. This significant event showcased how such a drastic change in the market conditions can affect the optimal reward programs provided by firms within the industry. As our model predicted, Marriott streamlined the benefits of its rewards program following the merger, resulting in heightened customer dissatisfaction. This included alleviated status qualification thresholds, scaled-back perks across various membership levels, and difficulties in consolidating rewards points and statuses, all of which contributed to increased customer frustration.[18]

Whereas both price reductions and reward increases can attract more customers, they serve distinct roles in a firm's strategy, and they are not necessarily completely interchangeable. In particular, rewards not only attract first period customers but also enhance second period retention and profitability, emphasizing the unique value of rewards in sustaining customer loyalty and encouraging repeat business.

Interestingly, first period price dynamics (p^*_{1H} versus p^*_{1L}) reflect the balance between search costs and the differences in network sizes. When securing early membership is crucial for long-term gains, larger network firms may opt for lower initial prices to attract customers quickly. This strategy is particularly effective when search costs are high, locking in loyalty, and when the network size disparity is minimal, prompting more aggressive competition. In such cases, high-type firms, leveraging their extensive networks, price more competitively than low-type firms. Conversely, when high-type firms hold a substantial network advantage or when loyalty becomes less valuable because of lower search costs, they might prioritize immediate gains over member acquisition, leading to higher first period prices. These patterns highlight the distinct roles of pricing and rewards in shaping competitive strategies and market outcomes in a heterogeneously networked market.

The comparative statics based on the numerical analysis further illustrate how market conditions influence competition between firms with varying network sizes. These findings are summarized below and illustrated by Figure 4.

Result 2 (Comparative Statics on γ_H, γ_L, and s).

i. *As γ_H increases, p^*_{1H} increases then decreases, and p^*_{1L} decreases; r^*_H decreases, and r^*_L increases; p^*_{2H} and p^*_{2L} increase; both π^*_{TH} and π^*_{TL} decrease.*

ii. *As γ_L increases, both p^*_{1H} and p^*_{1L} increase then decrease; r^*_H decreases then increases, and r^*_L decreases; p^*_{2H} decreases then increases, and p^*_{2L} increases; both π^*_{TH} and π^*_{TL} decrease.*

iii. *As s increases, first period prices increase then decrease, and rewards, second period prices, and total profits increase.*

The numeric analysis highlights how changes in network sizes (γ_H and γ_L) and search costs (s) impact firm strategies and market outcomes. For high-type firms, as their network size (γ_H) grows, they initially increase first period prices because of their enhanced market presence but eventually decrease prices to intensify member acquisition as competition with other high-type firms becomes their focus. Low-type firms consistently lower their prices in response to high-type firms' dominance. The rewards strategy adapts accordingly with high-type firms decreasing rewards as low-type firms increase theirs to maintain competitiveness. Second period prices for both firm types rise with γ_H, reflecting a market shift toward homogeneity with predominant high-type firms, and this escalates competition and affects overall industry profit and consumer welfare negatively.

As the network size of low-type firms (γ_L) increases, there are more active low-type firms in the second period, affecting both high- and low-type firms' strategies. Initially, with fewer low-type firms, high-type firms dominate, leading to intense competition among themselves. However, as γ_L rises, the market becomes more diversified, softening competition and prompting all the firms (both high and low types) to adjust their first period prices and rewards strategically. This differentiation peaks at intermediate γ_L levels and is more homogeneous when either γ_L is very low or very high.[19] Consequently, industry profits and consumer welfare fluctuate, highlighting a nuanced interplay between market structure and firm strategies. High-type firms adapt by adjusting rewards and prices to maintain their competitive edge and customer base, whereas low-type firms strive to become more competitive, affecting overall market outcomes.

The comparative statics related to search costs align with findings from homogeneous firms scenarios (Proposition 2): first period prices initially increase and then decrease as search costs rise, whereas both rewards and second period prices increase monotonically. Consequently, total profit increases as consumer and total surplus decline, mirroring earlier results. A notable additional insight in the heterogeneous setting is the asymmetric impacts across firm types. Higher search costs amplify the advantage of high-type firms as their greater likelihood of future prominence is more easily converted into purchases when frictions are larger. This widens the profitability gap between high- and low-type firms, underscoring the joint role of network size and search costs in shaping competitive outcomes.

6. Conclusion

This study presents an economic analysis of loyalty programs, examining their effects on consumer search behavior, market competition, and firm profitability across different network sizes. Theoretically, we model consumers' costly search within a repeated ordered-search framework, offering a new rationale for the adoption of loyalty programs: to gain prominence in future searches. Managerially, our analysis underscores that the effectiveness of loyalty programs hinges on striking the right balance between up-front pricing incentives and long-term rewards, which together determine both consumer loyalty and firm profitability.

Our results show that rewards function as endogenous switching costs that amplify the role of search frictions, generating distinctive pricing dynamics. Firms compete aggressively in the first period to build membership and then leverage loyalty to soften competition in the second, resulting in rising price paths. We also show that higher search costs intensify the use of rewards, that loyalty and search frictions are strategic complements, and that greater activation rates heighten initial competition but ultimately erode industry profits. Interestingly, under homogeneous competition, optimal reward levels remain consistent across different network sizes, highlighting the uniform effectiveness of loyalty programs regardless of scale. By contrast, in heterogeneous markets, large firms rely more on their network reach, whereas smaller firms must compete through stronger rewards.

Our model abstracts from several features of real-world loyalty programs, which also points to directions for future work. First, we adopt a two-period structure in which rewards are redeemed immediately, simplifying redemption timing to isolate the mechanism by which loyalty influences search and price competition. Second, we collapse overlapping memberships into a representative single program, interpreted as the collection of firms with which a consumer holds loyalty status, thereby abstracting from within-group choice. Third, we abstract from important program design features such as redemption thresholds, expiration dates, or category-specific restrictions, which, in practice, may allow firms to avoid head-to-head competition for prominence. Whereas our framework adopts a simplified memory assumption and focuses on prominence rather than switching-based lock-in, this parsimonious structure allows us to isolate how search-order effects alone shape firms' incentives to offer loyalty rewards. Future research could relax these abstractions to better capture the design trade-offs firms face and the competitive strategies they may adopt.

These abstractions, although limiting realism in some respects, enable analytical clarity and allow us to derive general insights into loyalty programs as endogenous switching costs. Importantly, our results contrast with Caminal and Matutes (1990): instead of declining prices, ordered search yields rising price trajectories and a prisoner's dilemma in which all firms adopt loyalty programs even though collective profits fall. Taken together, these findings highlight the strategic value of loyalty as a lever for gaining prominence in markets in which being searched first matters, also laying a foundation for future extensions to richer redemption structures, overlapping memberships, and multiperiod settings.

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Appendix

Proof of Lemma 1. We prove $r^* > 0$ by contradiction. Suppose $r^* = 0$, under which, consumers search randomly in both periods. It is profitable for an individual firm j to deviate by setting $r^j > 0$ but very small. This leads to a demand jump in the second period because consumers who purchased from firm j in the first period start their search at firm j in the second period. □

Proof of Proposition 1. Following the arguments in the main text, we characterize optimal strategies with necessary conditions. Then, we prove that r^*, p_1^*, and p_2^* indeed construct the unique equilibrium.

- First, we characterize the symmetric equilibrium. That is, $p_1^j = p_1^*$, $p_2^j = p_2^*$, and $r^j = r^*$ in equilibrium. According to Equation (4), we can derive that $D_1^* = \frac{1}{\gamma}$. D_1^* and r^* should satisfy two first order conditions:

$$\left.\frac{\partial \pi_T}{\partial r^j}\right|_{D_1^j = D_1^*, r^j = r^*} = \frac{\delta}{2(1-\gamma r^*)^2}\big(1-\gamma(1+r^*-w)\big)\big(1+p_2^* - \gamma p_2^* r^* + 2\gamma r^{*2} + r^*(-2+3\gamma(-1+w)) - w\big) = 0,$$

and

$$\left.\frac{\partial \pi_T}{\partial D_1^j}\right|_{D_1^j = D_1^*, r^j = r^*}$$
$$= -1 + p_1^* + w - \frac{\delta\gamma}{8(1-\gamma r^*)^3}[-8\gamma^3 r^{*5} + 3p_2^{*2}(-1+\gamma r^*)^3 + 4\gamma^2 r^{*4}(6+\gamma(-1+w)) + \gamma r^{*3}(-24-12\gamma(-1+w) + 19\gamma^2(-1+w)^2) + r^{*2}(8+12\gamma(-1+w) - 25\gamma^2(-1+w)^2 - 12\gamma^3(-1+w)^3) + 2p_2^*(-1+\gamma r^*)(3+2\gamma^2 r^{*3} + \gamma r^{*2}(-4-7\gamma(-1+w)) + 2r^*(1+5\gamma(-1+w)+\gamma^2(-1+w)^2) - 3w) + r^*(-4+5\gamma(-1+w)+4\gamma^2(-1+w)^2)(-1+w) + (-1+w)^2] = 0.$$

Also, according to Equation (1), the equilibrium second period price should be

$$p_2^{j*}(D_1^*, r^*) = \frac{p_2^*(1-\gamma r^*) + (1+\gamma r^*)(1-w)}{2(1-\gamma r^*)} = p_2^*.$$

Optimal strategies r^*, p_1^*, and p_2^* are determined as

$$r^* = \sqrt{2s},$$
$$p_1^* = \frac{\sqrt{2s} - 4\gamma s + 2\sqrt{2}(1-2\delta)\gamma^2 s^{\frac{3}{2}}}{(1-\gamma\sqrt{2s})^2},$$
$$p_2^* = \frac{\sqrt{2s} + 2\gamma s}{1-\gamma\sqrt{2s}}.$$

When $\gamma = 1$, there is another potential solution that $r^* = w$, $p_1^* = 1 - \delta - w$, and $p_2^* = 1 + w$. It is easy to show that the second period price is so high that consumers will not engage in random search. This does not construct an equilibrium.

• Second, we ensure that consumers are willing to search in the first period and in the second period no matter whether the firms with which they have memberships are active or not:

$$E[u_1] = w - p_1^* + \delta[\gamma[-s + \int_{w-r^*}^{1} (v_2^k - p_2^* + r^*)dv_2^k + (w-r^*)(w-p_2^*)] + (1-\gamma)(w-p_2^*)] > 0,$$
$$E[u_{2a}] = -s + \int_{w-r^*}^{1} (v_2^k - p_2^* + r^*)dv_2^k + (w-r^*)(w-p_2^*) > 0,$$
$$E[u_{2na}] = w - p_2^* > 0.$$

The above conditions hold when $0 < s < \bar{s} \equiv \frac{1}{2(2+\gamma)^2}$. Equivalently, $\frac{1+\gamma}{2+\gamma} < w < 1$.

• Then, we discuss second order conditions:

$$\frac{\partial^2 \pi_T}{\partial r^{j2}}\bigg|_{D_1^j = D_1^*, r^j = r^*} = -\frac{\delta(1-2\gamma+2\gamma w)}{(1-\gamma(1-w))^2},$$

where $1 - 2\gamma + 2\gamma w > 1 - 2\gamma + 2\gamma \cdot \frac{1+\gamma}{2+\gamma} = \frac{2-\gamma}{2+\gamma} > 0$. Therefore, it is lower than zero.

$$\frac{\partial^2 \pi_T}{\partial r^{j2}}\frac{\partial^2 \pi_T}{\partial D_1^{j\,2}} - \left(\frac{\partial^2 \pi_T}{\partial r^j \partial D_1^j}\right)^2\bigg|_{D_1^j = D_1^*, r^j = r^*} = \frac{2\delta\gamma(1-2\gamma+2\gamma w))(1-w)}{(1-\gamma(1-w))^4} \times ((1-\gamma(1-w))^2 + \delta\gamma^2(1-w)^2(1-2\gamma+2\gamma w)^2) > 0.$$

The second order conditions hold.

• Finally, we check that there is only one critical point given r^*, p_1^*, and p_2^*. □

Proof of Proposition 2.

$$\frac{\partial r^*}{\partial s} = \frac{1}{\sqrt{2s}} > 0.$$
$$\frac{\partial p_2^*}{\partial s} = \frac{2 - (1-\sqrt{2s}\gamma)^2}{(1-\sqrt{2s}\gamma)^2\sqrt{2s}}.$$

As $0 < s < \bar{s} \equiv \frac{1}{2(2+\gamma)^2}$, $0 < \frac{2}{2+\gamma} < 1 - \sqrt{2s}\gamma < 1$. Then, $\frac{\partial p_2^*}{\partial s} > 0$.

$$\frac{\partial p_1^*}{\partial s} = \frac{-\sqrt{2} + 6\gamma\sqrt{s} + 2(-1+2\delta)\gamma^2(3\sqrt{2} - 2\gamma\sqrt{s})s}{2(-1+\sqrt{2s}\gamma)^3\sqrt{s}}.$$

As $\frac{\partial p_1^*}{\partial s}$ decreases with s and $\frac{\partial p_1^*}{\partial s}\big|_{s=0^+} > 0$, the relationship of p_1^* and s depends on $\frac{\partial p_1^*}{\partial s}\big|_{s=\bar{s}}$. When $0 < \delta \le \frac{1}{2}$ or $\frac{1}{2} < \delta \le 1$ and $0 < \gamma \le \hat{\gamma}$, the p_1^* increases with s, whereas when $\frac{1}{2} < \delta \le 1$ and $\hat{\gamma} < \gamma \le 1$, the p_1^* increases then decreases with s, where $\hat{\gamma}$ is the unique root of equation $\delta\gamma^3 + 3\delta\gamma^2 - 2 = 0$.

Substitute equilibrium strategies, equilibrium profits, and welfare are

$$\pi_1^* = D_1^* p_1^* = \frac{\sqrt{2s} - 4\gamma s + 2\sqrt{2}(1-2\delta)\gamma^2 s^{\frac{3}{2}}}{\gamma(1-\gamma\sqrt{2s})^2},$$
$$\pi_2^* = (1-w+r^*)(p_2^* - r^*) + \frac{\left(\gamma(w-r^*) + 1 - \gamma\right)}{\gamma}p_2^* = \frac{\sqrt{2s} - 2\gamma s + 4\sqrt{2}\gamma^2 s^{\frac{3}{2}}}{\gamma\left(1-\gamma\sqrt{2s}\right)},$$
$$\pi_T^* = \pi_1^* + \delta\pi_2^* = \frac{(1+\delta)\sqrt{2s} - 4(1+\delta)\gamma s + 2\sqrt{2}(1+\delta)\gamma^2 s^{3/2} - 8\delta\gamma^3 s^2}{\gamma(1-\gamma\sqrt{2s})^2},$$
$$\pi_I^* = \gamma NB\pi_T^* = \frac{\gamma NB((1+\delta)\sqrt{2s} - 4(1+\delta)\gamma s + 2\sqrt{2}(1+\delta)\gamma^2 s^{3/2} - 8\delta\gamma^3 s^2)}{\gamma(1-\gamma\sqrt{2s})^2},$$
$$CS = M\left[w - p_1^* + \delta\left[\gamma\left(-s + \int_{w-r^*}^{1}(v_2^k - p_2^* + r^*)dv_2^k + (w-r^*)(w-p_2^*)\right) + (1-\gamma)(w-p_2^*)\right]\right]$$
$$= \frac{M}{(1-\gamma\sqrt{2s})^2}\big[1 + \delta - 2\sqrt{2}(1+\delta+\gamma+\delta\gamma)\sqrt{s} + \gamma(8 + 7\delta + 2(1+\delta)\gamma)s - 2\sqrt{2}(2+\delta)\gamma^2 s^{3/2} + 6\delta\gamma^3 s^2\big],$$
$$TS = \pi_I^* + CS = NB((1+\delta)(1-\sqrt{2s}) - \delta\gamma s).$$

Then,

$$\frac{\partial \pi_1^*}{\partial s} = \frac{1}{\gamma}\frac{\partial p_1^*}{\partial s} = \frac{-\sqrt{2} + 6\gamma\sqrt{s} + 2(-1+2\delta)\gamma^2(3\sqrt{2} - 2\gamma\sqrt{s})s}{2(-1+\sqrt{2s}\gamma)^3\gamma\sqrt{s}}.$$

Refer to the comparative statics of p_1^* on s:

$$\frac{\partial \pi_2^*}{\partial s} = \frac{\sqrt{2} - 4\gamma\sqrt{s} + 14\sqrt{2}\gamma^2 s - 16\gamma^3 s^{\frac{3}{2}}}{2\gamma(-1+\sqrt{2s}\gamma)^2\sqrt{s}}.$$

When $0 < \gamma \le \frac{2}{5}$, the numerator decreases with s. It is higher than $\sqrt{2} - 4\gamma\sqrt{s} + 14\sqrt{2}\gamma^2 s - 16\gamma^3 s^{\frac{3}{2}}\big|_{s=\bar{s}} = \frac{2\sqrt{2}(4+2\gamma+6\gamma^2+\gamma^3)}{(2+\gamma)^3} > 0$. When $\frac{2}{5} < \gamma \le 1$, the numerator decreases then increases with s. It is no lower than $\sqrt{2} - 4\gamma\sqrt{s} + 14\sqrt{2}\gamma^2 s - 16\gamma^3 s^{\frac{3}{2}}\big|_{s=\frac{1}{72\gamma^2}} = \frac{91}{54\sqrt{2}} > 0$. Thus, $\frac{\partial \pi_2^*}{\partial s} > 0$.

$$\frac{\partial \pi_T^*}{\partial s} = \frac{1}{2\gamma(1-\sqrt{2s}\gamma)^3\sqrt{s}}\big[\sqrt{2} - 6\gamma\sqrt{s} + 6\sqrt{2}\gamma^2 s - 4\gamma^3 s^{\frac{3}{2}} + \delta(\sqrt{2} - 6\gamma\sqrt{s} + 6\sqrt{2}\gamma^2 s - 36\gamma^3 s^{\frac{3}{2}} + 16\sqrt{2}\gamma^4 s^2)\big].$$

The expression in the square brackets decreases with s, so it is higher than

$$\begin{aligned}&\sqrt{2}-6\gamma\sqrt{s}+6\sqrt{2}\gamma^2 s-4\gamma^3 s^{\frac{3}{2}}+\delta(\sqrt{2}-6\gamma\sqrt{s}+6\sqrt{2}\gamma^2 s\\&-36\gamma^3 s^{\frac{3}{2}}+16\sqrt{2}\gamma^4 s^2)\big|_{s=\bar{s}}\\&=\frac{4\sqrt{2}\left(2(2+\gamma)+\delta\left(4(1-\gamma^3)+\gamma(2-\gamma^3)\right)\right)}{(2+\gamma)^4}>\frac{8\sqrt{2}(2+\gamma)}{(2+\gamma)^4}>0.\end{aligned}$$

The total profit increases with the search cost:

$$\begin{aligned}\frac{\partial\pi_I^*}{\partial s}=&\frac{\gamma NB}{2\gamma(1-\sqrt{2s}\gamma)^3\sqrt{s}}\left[\sqrt{2}-6\gamma\sqrt{s}+6\sqrt{2}\gamma^2 s-4\gamma^3 s^{\frac{3}{2}}\right.\\&\left.+\delta(\sqrt{2}-6\gamma\sqrt{s}+6\sqrt{2}\gamma^2 s-36\gamma^3 s^{\frac{3}{2}}+16\sqrt{2}\gamma^4 s^2)\right]>0.\end{aligned}$$

Refer to the comparative statics of π_T^* on s.

$$\begin{aligned}\frac{\partial CS}{\partial s}=&\frac{M}{(1-\sqrt{2}\gamma\sqrt{s})^3\sqrt{s}}\left[-\sqrt{2}+6\gamma\sqrt{s}-6\sqrt{2}\gamma^2 s+4\gamma^3 s^{\frac{3}{2}}\right.\\&\left.-\delta(\sqrt{2}-5\gamma\sqrt{s}+3\sqrt{2}\gamma^2 s-14\gamma^3 s^{\frac{3}{2}}+6\sqrt{2}\gamma^4 s^2)\right].\end{aligned}$$

The expression in the square brackets increases with s, so it is lower than

$$\begin{aligned}&-\sqrt{2}+6\gamma\sqrt{s}-6\sqrt{2}\gamma^2 s+4\gamma^3 s^{\frac{3}{2}}-\delta(\sqrt{2}-5\gamma\sqrt{s}+3\sqrt{2}\gamma^2 s\\&-14\gamma^3 s^{\frac{3}{2}}+6\sqrt{2}\gamma^4 s^2)\big|_{s=\bar{s}}\\&=\frac{2\sqrt{2}\left(-4(2+\gamma)-\delta\left(\gamma(1-\gamma^3)+4\gamma(1-\gamma^2)+\gamma+8\right)\right)}{(2+\gamma)^4}\\&<-\frac{8\sqrt{2}}{(2+\gamma)^3}<0.\end{aligned}$$

The consumer surplus decreases with the search cost.

$$\frac{\partial TS}{\partial s}=-NB\left(\delta\gamma+\frac{1+\delta}{\sqrt{2s}}\right)<0.\quad\square$$

Proof of Proposition 3.

$$\frac{\partial r^*}{\partial\gamma}=0.$$

$$\frac{\partial p_2^*}{\partial\gamma}=\frac{4s}{(1-\sqrt{2s}\gamma)^2}>0.$$

$$\frac{\partial p_1^*}{d\gamma}=-\frac{8\sqrt{2}\delta\gamma s^{\frac{3}{2}}}{(1-\sqrt{2s}\gamma)^3}<0.$$

$$\frac{\partial\pi_1^*}{\partial\gamma}=\frac{-\sqrt{2s}+6\gamma s-6\sqrt{2s}\gamma^2 s+4\gamma^3 s^2-\delta(4\sqrt{2s}\gamma^2 s+8\gamma^3 s^2)}{\gamma^2(1-\sqrt{2s}\gamma)^3}.$$

The numerator is lower than $-\sqrt{2s}+6\gamma s-6\sqrt{2s}\gamma^2 s+4\gamma^3 s^2$, which decreases with s when $0<\gamma\le\frac{2}{3}$ or decreases then increases with s when $\frac{2}{3}<\gamma\le 1$. Thus, $-\sqrt{2s}+6\gamma s-6\sqrt{2s}\gamma^2 s+4\gamma^3 s^2$ is lower than $-\sqrt{2s}+6\gamma s-6\sqrt{2s}\gamma^2 s+4\gamma^3 s^2\big|_{s=0}=0$ and $-\sqrt{2s}+6\gamma s-6\sqrt{2s}\gamma^2 s+4\gamma^3 s^2\big|_{s=\bar{s}}=-\frac{8}{(2+\gamma)^4}$. Therefore, $\frac{\partial\pi_1^*}{\partial\gamma}<0$.

$$\frac{\partial\pi_2^*}{\partial\gamma}=\frac{-\sqrt{2s}+4\gamma s+2\sqrt{2s}\gamma^2 s}{\gamma^2(1-\sqrt{2s}\gamma)^2}<0.$$

The numerator decreases with s when $0<\gamma\le\frac{\sqrt{7}-1}{3}$ or decreases then increases with s when $\frac{\sqrt{7}-1}{3}<\gamma\le 1$. So $-\sqrt{2s}+4\gamma s+2\sqrt{2s}\gamma^2 s$ is lower than $-\sqrt{2s}+4\gamma s+2\sqrt{2s}\gamma^2 s\big|_{s=0}=0$ and $-\sqrt{2s}+4\gamma s+2\sqrt{2s}\gamma^2 s\big|_{s=\bar{s}}=-\frac{2(2-\gamma^2)}{(2+\gamma)^3}<0$. Thus, $\frac{\partial\pi_2^*}{\partial\gamma}<0$. and $\frac{\partial\pi_T^*}{\partial\gamma}=\frac{d\pi_1^*}{d\gamma}+\delta\frac{d\pi_2^*}{d\gamma}<0$.

$$\frac{\partial\pi_T^*}{\partial\gamma}=\frac{\partial\pi_1^*}{\partial\gamma}+\delta\frac{\partial\pi_2^*}{\partial\gamma}<0$$

$$\frac{\partial\pi_I^*}{\partial\gamma}=\gamma BN\frac{\partial\pi_T^*}{\partial\gamma}<0.$$

$$\frac{\partial TS}{\partial\gamma}=-NB\delta s<0.$$

And

$$\frac{\partial CS}{\partial\gamma}=\frac{M\delta s(1-3\sqrt{2s}\gamma-18\gamma^2 s+6\sqrt{2s}\gamma^3 s)}{(-1+\sqrt{2s}\gamma)^3},$$

which increases with γ, and $\frac{\partial CS}{\partial\gamma}\big|_{\gamma=0}<0$. When $0<s\le\frac{1}{18}$, $0<\gamma\le 1$, whereas when $\frac{1}{18}<s<\frac{1}{8}$, $0<\gamma\le\frac{\sqrt{2}-4\sqrt{s}}{2\sqrt{s}}$. The relationship of CS on γ depends on $\frac{\partial CS}{\partial\gamma}\big|_{\gamma=min\left\{1,\frac{\sqrt{2}-4\sqrt{s}}{2\sqrt{s}}\right\}}$. Consumer surplus decreases when $0<s\le s_1$ or $s_2\le s<\frac{1}{8}$ but shows a decrease followed by an increase when $s_1<s<s_2$, where the threshold s_1 and s_2 are uniquely determined by the equation $1-3\sqrt{2s}-18s+6\sqrt{2}s^{\frac{3}{2}}=0$ and $\sqrt{2}-6\sqrt{s}+12s^{\frac{3}{2}}=0$, respectively, both of which fall within $[0,\frac{1}{8}]$. $\square$

Proof of Proposition 4. In the absence of loyalty programs, the equilibrium profits and welfare are

$$\begin{aligned}\pi_1^B&=\frac{\sqrt{2s}}{\gamma},\\ \pi_2^B&=\frac{\sqrt{2s}}{\gamma},\\ \pi_T^B&=\frac{(1+\delta)\sqrt{2s}}{\gamma},\\ \pi_I^B&=\gamma NB\pi_T^B=NB(1+\delta)\sqrt{2s},\\ CS^B&=M\left(w-p_1^B+\delta(w-p_2^B)\right)=M(1+\delta)(1-2\sqrt{2s}),\\ TS^B&=\pi_I^B+CS^B=NB(1+\delta)(1-\sqrt{2s}).\end{aligned}$$

Then, we have

$$p_1^*-p_1^B=-\frac{4\sqrt{2}\delta\gamma^2 s^{\frac{3}{2}}}{(1-\sqrt{2s}\gamma)^2}<0.$$

$$p_2^*-p_2^B=\frac{4\gamma s}{1-\sqrt{2s}\gamma}>0.$$

$$p_2^*-r^*-p_2^B=\frac{\sqrt{2s}(3\sqrt{2s}\gamma-1)}{1-\sqrt{2s}\gamma}.$$

Because $3\sqrt{2}\bar{s}\gamma - 1 < -\frac{2(1-\gamma)}{2+\gamma} \leq 0$, $p_2^* - r^* - p_2^B < 0$.

$$\pi_1^* - \pi_1^B = -\frac{4\sqrt{2}\delta\gamma s^{\frac{3}{2}}}{(1-\sqrt{2s}\gamma)^2} < 0.$$

$$\pi_2^* - \pi_2^B = \frac{4\sqrt{2s}\gamma s}{1-\sqrt{2s}\gamma} > 0.$$

$$\pi_T^* - \pi_T^B = -\frac{8\delta\gamma^2 s^2}{(1-\sqrt{2s}\gamma)^2} < 0.$$

$$\pi_I^* - \pi_I^B = -\frac{8\delta\gamma^3 s^2 NB}{(1-\sqrt{2s}\gamma)^2} < 0.$$

$$TS - TS^B = -\delta\gamma sNB < 0.$$

$$CS - CS^B = \frac{\delta\gamma s(6\gamma^2 s + 2\sqrt{2s}\gamma - 1)M}{(1-\sqrt{2s}\gamma)^2}.$$

As $6\gamma^2 s + 2\sqrt{2s}\gamma - 1$ increases with s, it is lower than $6\gamma^2 s + 2\sqrt{2s}\gamma - 1\big|_{s=\bar{s}} = -\frac{4(1-\gamma^2)}{2+\gamma^2} \leq 0$. Therefore, $CS - CS^B < 0$. □

Endnotes

[1] See https://thelbma.com/research/wirecard-consumer-incentives-2019.

[2] The Sephora Beauty Insider program offers points for every dollar spent, and these can be exchanged for exclusive products and beauty experiences; see https://thelbma.com/research/wirecard-consumer-incentives-2019.

[3] Although prominence and switching costs both generate repeat business advantages, the economic mechanism in our model is fundamentally different. Switching costs work through consumers' intertemporal consumption utility: a consumer stays because leaving is painful. In contrast, prominence in an ordered-search environment operates through information and search frictions: a consumer revisits a firm first because it becomes the most salient or convenient starting point, not because switching is costly. Hence, prominence is not a second order switching cost but a distinct mechanism that shifts expected demand through the search process itself. We highlight this distinction because the prominence channel alone, without any switching-utility frictions, is sufficient to generate the prisoner's dilemma characterized in our equilibrium.

[4] See *Ad Age*, "How McDonald's and other restaurant brands are driving loyalty apps in the face of inflation," May 24, 2023, https://adage.com/article/marketing-news-strategy/how-mcdonalds-chipotle-and-others-are-promoting-loyalty-apps/2496401.

[5] See "McDonald's eyes speedy ramp-up to 50,000 restaurants by 2027," https://hospitality.economictimes.indiatimes.com/news/restaurants/mcdonalds-eyes-speedy-ramp-up-to-50000-restaurants-by-2027/105803480.

[6] Luo (2024) proposes a novel view of loyalty programs as financing instruments.

[7] There is also a stream of literature on behavior-based price (BBP) discrimination (Villas-Boas 1999, Fudenberg and Tirole 2000, Fudenberg and Villas-Boas 2006), and it investigates a related topic of whether the firm should reward its own customers or new customers through pricing (Shin and Sudhir 2010, De Nijs and Rhodes 2013, Caillaud and De Nijs 2014). However, our model deviates from the typical BBP framework by showing that, with rewards, firms offer lower prices to existing consumers (who purchased in the first period), charging higher prices to new consumers. Furthermore, whereas intensified competition under BBP typically emerges in the second period, in our model, this competitive tension occurs in the first period.

[8] We acknowledge that most real-world loyalty programs require multiple purchases before redemption and consumers often hold memberships in several programs simultaneously. The two-period structure serves as a metaphorical simplification with the second period representing the eventual redemption point. Also, recent industry practices lend support to this abstraction: several major hotel chains (e.g., IHG, Hilton, Marriott, Hyatt) now allow "points + cash" redemptions, enabling members to apply loyalty points toward their very next stay. Such immediate or near-immediate redemption options are consistent with the prominence-based role of rewards highlighted in our model. Likewise, whereas consumers often hold multiple memberships, we abstract away from this possibility of overlapping memberships by treating the consumer as affiliated with a representative single program. Importantly, this representative single firm should not be interpreted literally as just one program. Instead, it is a conceptual proxy for the collection of firms with which a consumer holds loyalty status. For example, if a consumer belongs to several loyalty programs, our model treats this group collectively as the single firm with which the consumer is affiliated, whereas firms outside this group are modeled as nonmembership firms. These abstractions, although stylized, allow us to isolate the strategic role of loyalty programs in shaping search behavior and price competition, maintaining tractability.

[9] Notice that, intuitively, v_t^j depends on $b(t)$; however, to simplify notation, this dependence relationship is not signified explicitly but rather through the superscript of t.

[10] In Online Appendix A, we further extend our basic model to allow for correlations between consumers' match values, v_1^j and v_2^j, of the same firm j across two periods. We find that the equilibrium reward remains unchanged, whereas the second period price increases with correlation. The relationship between the equilibrium first period price and correlation could be nonmonotonic. Moreover, higher correlation enhances firm profitability and boosts social welfare by facilitating better matching in the second period; however, it leads to a reduction in consumer surplus.

[11] A similar dynamic can be observed in less conventional settings. For instance, a customer might visit Starbucks for coffee on one occasion and return for a muffin on another. Given that Starbucks' offerings can be perceived as varying between visits, the customer must undertake a new search to determine the current value of the products according to the customer's changing needs.

[12] We require each firm to set the same price for all its branches in each period. This assumption is less restrictive than it appears for three reasons. First, given that all branches of the firm are symmetric, the prices for all branches could be the same in equilibrium even if we allow the firm to set different prices for different branches. Second, because each consumer visits only one branch per period, differing prices across branches wouldn't be noticeable to them. Essentially, the single-price per period assumption stipulates consumers' off-equilibrium belief about the prices of all other branches of the same firm to be the same as the branch they visited. Last, the real-world pricing strategies of widespread brands, which aim for uniformity across regions to maintain brand consistency and customer fairness, provide a practical foundation for our model's assumption.

[13] Notice that we can also allow $r^j < 0$, under which, there is no consumer signing up for firm j's loyalty program, and consequently, the program becomes inactive. Therefore, it is without loss of generality to restrict $r^j \geq 0$.

[14] In fact, a positive but small hassle cost for consumers to enroll in reward programs would have no impact on the equilibrium outcome because consumers expect positive payoff from the reward program and always choose to enroll as long as the hassle cost does not exceed the benefit from the enrollment.

[15] Assumption 1 imposes a simple bounded rationality view of consumer memory: consumers recall the transaction price of the firm from which they purchased but do not retain or retrieve the prices of firms they merely browsed. This asymmetric recall is consistent with evidence that individuals encode information related to completed transactions more reliably than incidental information (e.g., Mullainathan 2002). Although restrictive, this assumption allows us to maintain tractability without altering the qualitative forces driving the equilibrium.

[16] For completeness, in Online Appendix C, we also analyze the passive belief specification in which deviations in the first period do not alter consumers' expectations of second period pricing. This specification again leads to no symmetric pure-strategy equilibrium, underscoring why Assumption 1 is a reasonable choice.

[17] Upon firm j's deviation, other firms may respond optimally afterward, potentially deviating from equilibrium price p_2^*. However, as there are infinite firms in each city, firm j's deviation does not affect demand for other firms. Therefore, they adhere to the equilibrium strategies, and consumers hold rational expectations on that.

[18] See The Points Guy (2018), "One month in: Combined Marriott and SPG program still facing issues" (https://thepointsguy.com/news/one-month-in-combined-marriott-still-facing-issues/); Points with a Crew (2018), "The 3 worst devaluations/changes from the new Marriott/SPG loyalty program" (https://www.pointswithacrew.com/3-worst-devaluations-changes-new-marriott-spg-loyalty-program/); and Travel Sort (2018), "New Marriott and SPG rewards program: 5 reasons to hate it" (https://travelsort.com/new-marriott-and-spg-rewards-program-pros-cons/).

[19] When there are fewer low-type firms, high-type firms compete with other high types intensively. So the market becomes effectively homogeneous, dominated by the high types. Conversely, when γ_L approaches γ_H, then both types become homogeneous.

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