Targeted Advertising as Implicit Recommendation: Strategic Mistargeting and Personal Data Opt-Out

Z. Eddie Ning, Jiwoong Shin, Jungju Yu

June 21, 2024

Abstract

We study an advertiser’s targeting strategy and its effects on consumer data privacy choices, both of which determine the advertiser’s targeting accuracy. Targeted ads, serving as implicit recommendations when consumer preferences are uncertain, not only influence the consumer’s beliefs and purchasing decisions but also amplify the advertiser’s temptation towards strategic mistargeting—sending ads to poorly matched consumers. Our analysis reveals that advertisers may, paradoxically, choose less precise targeting as accuracy improves. Even if prediction is perfect, the advertiser still targets the wrong consumers, leading to strategic mistargeting. Nevertheless, consumer surplus can remain positive due to improved identification of well-matched consumers, thereby reducing the incentive for consumers to withhold information. However, the scenario shifts with endogenous pricing; better prediction leads to more precise targeting, although mistargeting persists. To exploit the recommendation effect of advertising, the advertiser raises prices instead of diluting recommendation power, lowering consumer welfare and prompting consumers to opt out of data collection. Furthermore, we investigate the impact of consumer data opt-out decisions under varying privacy policy defaults (opt-in vs. opt-out). These decisions significantly affect equilibrium outcomes, influencing both the advertiser’s targeting strategies and consumer welfare. Our findings highlight the complex relationship between targeting accuracy, privacy choices, and advertisers’ incentives.

Keywords: targeted advertising, consumer privacy, mistargeting, implicit recommendation, personal data opt-out, endogenous pricing

*Z. Eddie Ning is an Assistant Professor of Marketing at the University of British Columbia. Jiwoong Shin is a Professor of Marketing at the Yale University. Jungju Yu is an Assistant Professor of Marketing at the Korea Advanced Institute of Science and Technology. Emails: eddie.ning@sauder.ubc.ca, jiwoong.shin@yale.edu, jungju.yu@kaist.ac.kr. We thank Heski Bar-Issac, Dave Godes, Anthony Dukes, Fei Long, Sridhar Mowrthy, Dilip Soman, K. Sudhir, Yuting Zhu, Bobby Zhou, seminar participants at CEIBS, HKU, Maryland, NYU, Toronto, UBC, USC, Yale, Amazon Advertisement Science Seminar, Marketing Science Conference, and UTD FORMS conference for their useful comments.
1 Introduction

With the vast amount of online data being collected, predictive technologies like artificial intelligence (AI) empower advertisers to target consumers with increasing precision. Advertisers today have access to detailed information on individual consumers’ online behaviors, such as browsing patterns and social media interactions, as well as offline behaviors, like store visits and purchases tracked through smart devices. This granular, individual-level data allows advertisers to predict which products are suitable for which consumers and target them accordingly. Agrawal et al. (2018) frame the role of AI in business as a prediction machine that aids decision-making under uncertainty. As AI advances, it enables more accurate predictions at lower costs. Thus, as technology improves and data collection continues to expand, algorithms used by advertisers and platforms are expected to predict consumer preferences with greater accuracy. This represents the core promise of a bright future for the digital advertising industry. However, the reality is not playing out so well.

If consumers are uncertain about their product preferences, being targeted by a firm may convey valuable information. For example, a consumer may receive targeted ads about a new paid app on her phone. Without knowing all the features of the new app, she is uncertain about its utility. However, she may be aware that the firm has information about the fit between her and the app, such as whether she downloaded other apps designed for a similar consumer segment or whether other users like her enjoyed the new app. In such instances, consumers who are targeted may infer that algorithms predict the new app to be a suitable match for them, thereby enhancing their perceptions of the product’s value (Shin and Yu, 2021). This observed consumer behavior is supported by laboratory studies (Summers et al., 2016). In these studies, consumers exposed to identical ads responded differently depending on their awareness of being targeted. Those informed that ad targeting was based on their browsing history expressed higher purchase intentions compared to those informed otherwise. Hence, the mere act of targeting possesses persuasive power that extends beyond the content of the advertisement itself.

However, it is not immediately clear how a rational consumer should draw inferences in equilibrium, especially when firms might behave opportunistically. Consumers cannot observe the advertiser’s predictions about them or know the extent to which they have been targeted by ads. If consumers develop favorable inferences from being targeted and become more inclined to pur-
chase, advertisers might be tempted to target even those unlikely to benefit from the app while charging higher prices. In such scenarios, consumers should be concerned about the credibility of targeted ads, as firms might exploit positive inferences by mistargeting and charging higher prices as their algorithms become more accurate. Consequently, consumers may no longer make positive inferences, and the “persuasion by targeting” effect may not arise from rational beliefs.

Moreover, this line of logic may prompt consumers to seek ways to protect their privacy and disable ad targeting, thereby eliminating the need to make inferences from being targeted. In practice, recent regulations such as GDPR and CCPA allow consumers to choose whether to opt in or out of personal data collection (which advertisers need for targeting), thus empowering consumers to control their own data. As these privacy laws take effect, consumers’ ability to opt out may influence advertisers’ incentives, ultimately shaping both targeting strategies and the overall welfare of consumers. The design of privacy policies, such as the default privacy option, can also significantly impact the behaviors of both consumers and advertisers.

In this paper, we study the optimal targeting strategy of an advertiser with an imperfect prediction algorithm when consumers make inferences from being targeted and consumers’ incentives to opt out of data collection under two different privacy policies. Specifically, we focus on the implications of such targeting strategies on consumer welfare and privacy choices, which, in turn, affect the firm’s targeting strategy. How do advertisers use this algorithmic prediction in their targeting and pricing strategy at equilibrium? As advertisers can make individual-level predictions more accurately, how does it affect their incentives to exploit this advantage? Are consumers more likely to receive ads for products that better match their underlying preferences? What happens in the hypothetical limit as AI becomes omniscient? Can powerful AI predictions eliminate imprecise targeting and post-purchase regret? When do consumers have incentives to opt out of data collection and limit firms’ ability to predict their preferences?

To address these questions, we study a model between an advertiser and a continuum of consumers with two-sided private information. The advertiser first receives a noisy signal about each consumer’s match with the product and decides whether to target that consumer. Upon being targeted and seeing the ad, each consumer observes the price and privately receives a noisy signal about her match value. The consumer then infers her match value with the product and decides whether or not to buy.
We first show that when a consumer is uncertain about her preferences for an unknown product, a targeted ad carries an implicit message: the advertiser’s algorithm predicts that the product matches her preferences. Thus, a targeted ad can act as an implicit recommendation that influences a consumer’s purchasing decision. However, it also creates misaligned incentives; the advertiser may cheat by sending ads to a consumer even if the advertiser does not believe the product is a good fit for that consumer. Higher prediction accuracy strengthens the recommendation role of a targeted ad but also worsens this incentive problem. In equilibrium, despite the advertiser occasionally engaging in opportunistic behavior, a targeted advertisement can still be perceived as a credible implicit recommendation. Consumers may still draw positive inferences from such advertisements, although these inferences may not be as optimistic as those drawn in the absence of misaligned incentives. Essentially, while the advertiser might selectively and misleadingly target certain “incorrect” consumers, the advertisers predominantly direct its targeted advertisements towards consumers deemed a good fit for the advertised product.

We analyze two different pricing regimes: one where the product price is set exogenously and the other where it is chosen endogenously by the advertiser. First, under an exogenous price, misaligned incentives often lead the advertiser to send ads to consumers who are a poor fit, resulting in a less targeted advertising strategy. This effect becomes more prominent as the algorithm’s prediction accuracy increases. Consequently, consumers receive ads for unsuitable products and make incorrect purchase decisions, even if the advertiser’s AI can make perfectly accurate predictions. However, consumer surplus can still be positive despite the misaligned incentives because the firm can better identify consumers who are a good fit for the product.

Interestingly, under an endogenous price, the results are reversed. The advertiser may exploit higher prediction accuracy by raising prices instead of diluting the recommendation effect by sending ads to the “wrong” consumers. Moreover, as prediction accuracy improves, the parameter region for a fully-targeted advertising strategy increases, implying that higher accuracy leads to a more targeted advertising strategy and a higher price. When prediction accuracy becomes perfect, the firm extracts all consumer surplus through high prices, reducing consumer welfare to zero. Mistargeting, however, still exists, even with perfect prediction accuracy.

We further analyze these consumer privacy choices, considering the significant role of default settings, which have been empirically proven to profoundly influence decision-making (Johnson
and Goldstein, 2003; Madrian and Shea, 2001). An opt-in setting requires explicit permission from consumers to collect data, whereas an opt-out setting assumes permission by default unless consumers withdraw their consent. Under exogenous pricing, consumers find it optimal to permit data collection when faced with an opt-out setting. Given an opt-in setting, they permit data collection only under specific conditions. Because pricing is unaffected by their privacy choices, consumers are more inclined to permit data collection, anticipating more relevant advertising. In this scenario, an opt-out setting nudges consumers towards allowing data collection, inadvertently benefiting firms by providing more data for targeted advertising, thereby improving both firm profits and overall social surplus.

Under endogenous pricing, consumer dynamics are different. Consumers are more hesitant to allow data collection, regardless of the default privacy settings. This concern is driven by their anticipation of higher prices associated with data collection consent. With an opt-out setting, consumers tend to opt out within certain parameters, while they never permit data collection under an opt-in setting. Specifically, consumers opt out to access lower prices, even at the cost of receiving less relevant advertising. Therefore, firms also benefit more from an opt-out setting, which encourages consumers to allow data collection, fostering more profitable targeted advertising strategies. This study provides a nuanced understanding of the complex dynamics between consumer privacy decisions, advertising strategies, and pricing mechanisms across different default privacy settings, offering valuable insights into the dynamics at play.

Next, we discuss the related literature. Section 3 presents the model. Section 4 begins with an analysis of a simple benchmark case without the consumer’s private signal. Section 5 analyzes the main model with the consumer’s private signal and studies the equilibrium targeting strategy and the consumers’ data opt-out decision under exogenous pricing. Section 6 extends the main model to the advertiser’s endogenous pricing decision. Section 7 concludes.

2 Literature Review

Our paper relates to several streams of research in online targeted advertising, recommendation systems, and consumer privacy choices. First, the literature on online advertising has emphasized the importance of targetability, which refers to firms’ ability to identify individual consumer pref-
ferences using various customer data, such as demographics, browsing behaviors, or past purchases (Chen et al., 2009; Shen and Villas-Boas, 2018). Targeted advertising messages based on customer characteristics improve the performance of communications and consumer response (Goldfarb and Tucker, 2011; Rafieian and Yoganarasimhan, 2021). While one stream of research focuses on the role of advertising content in persuasion (Anderson and Renault, 2006; Chakraborty and Harbaugh, 2010; Mayzlin and Shin, 2011; Shin, 2005; Shin and Wang, 2024), several papers show an additional effect of targeting beyond the information contained in the advertising content (Anand and Shachar, 2009; Iyer et al., 2005).

The recent paper by Shin and Yu (2021) shares similarities with ours, analyzing how consumers infer their match value from firms’ equilibrium targeting strategies. They provide a detailed micro-mechanism of how consumers draw inferences from receiving targeted ads, focusing on competitive dynamics and advertising spillovers. In contrast, our paper highlights the strategic interactions between advertisers and consumers in scenarios with misaligned incentives, identifying the conditions under which firms intentionally target consumers who may not be well-suited for the products. This finding stands in stark contrast to the equilibria discussed in Shin and Yu (2021), where firms exclusively target well-matched consumers in their advertisements. Our paper expands on this by exploring the credibility of targeted advertising and the potential for advertisers to exploit persuasion effects, a topic not addressed in their work. We then explore the implications of these factors on consumers’ data privacy choices, which, in turn, impact targeting accuracy. Thus, our model treats targeting accuracy as an endogenous result of consumers’ strategic decisions, incorporating two-sided private information. This approach enables us to offer a unique perspective on consumers’ privacy choices, particularly their decisions to opt in or out of data sharing.

The examination of mistargeting and the credibility of targeting has garnered significant attention in recent academic research. Xu and Dukes (2022) addresses the credibility of personalized offerings by examining how firms can mitigate consumer skepticism by presenting list prices alongside personalized prices to enhance credibility.\(^1\) Li and Xu (2022) study how firms use their superior knowledge for personalized pricing, potentially leading consumers to infer product value. This scenario presents the possibility of firms exploiting their information advantage to charge higher prices.

\(^1\)Similarly, various studies explore how firms may exploit their superior knowledge of consumer preferences, a factor that poses credibility challenges in product line design (Xu and Dukes, 2019) and supply chain relationships (Mittendorf et al., 2022).
to consumers with lower preferences. Despotakis and Yu (2023) finds that expanding information dimensions in targeting strategies might reduce consumer engagement due to uncertainties about specific product benefits. Furthermore, Shin and Shin (2023) provides an alternative explanation for strategic mistargeting by highlighting how ad agencies may misallocate ads to mismatched advertisers for strategic reasons, anticipating better matches in future campaigns. Our study also explores the implications of consumers’ inferences from being targeted, along with the consequential mistargeting behavior by firms. However, our study examines consumers’ decisions to opt out of data collection, a critical issue in regulatory and academic discussions.

Additionally, our research touches on the literature concerning platform design, particularly optimal recommendation systems and content personalization. The marketing literature often focuses on AI-based features to improve recommendation systems (Choi et al., 2024; Dzyabura and Hauser, 2019), the effectiveness of data-driven content personalization (Yoganarasimhan, 2020), and platforms’ search design issues (Dukes and Liu, 2015; Zhong, 2022). However, a critical distinction in our study is the examination of firm commitment and its implications for advertising strategies.

In a related vein, Berman et al. (2022) explore a similar theme, where firms may have incentives to recommend products to consumers for whom the product is not an ideal match, a phenomenon termed “overselling” from a Bayesian persuasion perspective (Kamenica and Gentzkow, 2011; Bergemann and Morris, 2019). This overselling concept in their study parallels the idea of mistargeting in our research. The fundamental distinction between our research and that of Berman et al. (2022) lies in the treatment of firm commitment and the ensuing implications for advertising strategies. Berman et al. (2022) operate within an information design framework that presumes the firm or platform can commit to any recommendation policy. However, this assumption can be questionable and may not hold in practice, as firms are unable to publicly commit to exactly how they will use their customer data for targeted advertising. This critical perspective prompts us to study a model without the firm’s commitment power and investigate the credibility of the firm’s targeted practice, particularly its use of customer data in settings where commitment is not guaranteed.

Finally, several papers have studied the consumer’s privacy decision to opt out of data collection (Johnson and Goldstein, 2003; Madrian and Shea, 2001; Montes et al., 2019). The literature largely focuses on the economic trade-off between the benefit and cost of disclosing customer information. The benefit is that consumers can receive a more relevant product recommendation or advertise-
ment. However, at the same time, the firm can use this information for price discrimination. In these papers, consumers opt out of data collection to avoid such price discrimination. Ichihashi (2020) shows that a firm’s commitment to not using consumer information in its pricing decisions alleviates price-discrimination concerns. This encourages consumers to disclose information, which, in turn, improves the firm’s product recommendation quality. We contribute to this literature by showing that even under uniform pricing or no price discrimination, consumers may have incentives to opt out of data collection when firms can predict individual preferences with sufficient accuracy.

3 Model

We consider an online interaction between a firm that advertises a product and a unit mass of consumers. Consumers receive a utility of either 1 or 0, depending on whether the product matches a consumer’s need. Let $p$ denote the price of the product. We first assume the price to be exogenous but later endogenize the firm’s pricing decision. The firm chooses whether to send an ad to a consumer. So, each consumer may or may not receive a targeted ad from the advertiser, the realization of which we denote by $A \in \{a, \emptyset\}$. Upon receiving the ad, a consumer becomes aware of the product and decides whether or not to purchase it. A consumer who purchases the product obtains a utility of $1 - p$ if the product is a good match and $-p$ if it is a bad match. If the consumer decides not to buy, her utility is 0.

There are two types of consumers ($t \in \{H, L\}$): a fraction $\mu$ represents high-valuation consumers ($t = H$) who find a good fit with the product, while the remaining fraction $1 - \mu$ represents low-valuation consumers ($t = L$) who derive zero value from it. The probability that a consumer is a high-valuation type, denoted as $\mu = \Pr(t = H)$, is common knowledge for both the firm and the consumer. A high $\mu$ can be interpreted as the product having mass market appeal, while a small $\mu$ represents a more niche product. The advertiser has data on consumer $i$’s online behaviors to predict $t_i$, the true match between the product and consumer $i$. The advertiser’s algorithm produces a signal about $t_i$, denoted by $s \in \{s_H, s_L\}$. This firm’s signal is accurate with probability

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2 Hereafter, we interchangeably use the terms “firm” and “advertiser.”

3 Our analysis of exogenous price helps us understand the influence of the firm’s endogenous pricing motives beyond the firm’s incentives for targeted advertising on consumer welfare and equilibrium outcomes. Moreover, the analysis of exogenous price can represent many realistic situations. For instance, prices for mobile applications are largely fixed at 0.99 or 1.99.
\( \alpha \geq \frac{1}{2} \) such that \( \Pr(t_i = H|s = s_H) = \alpha \) and \( \Pr(t_i = L|s = s_L) = \alpha \). A higher \( \alpha \) implies that the advertiser has more data on a consumer or a more advanced algorithm for prediction. Given the prediction algorithm, the advertiser decides whether to send an ad to consumer \( i \). The cost of sending an ad is fixed and denoted by \( k \), which is assumed to be smaller than \( p \).

If a consumer is targeted (i.e., receives an ad), she can click on the ad and obtain noisy information about the product. By doing so, the consumer makes her own assessment, or impression, of product fit, which is a noisy signal observed privately by her, \( m \in \{m_g, m_b\} \). We denote the consumer’s private signal by impression \( m \) to differentiate it from the advertiser’s private signal \( s \). Hereafter, we interchangeably use both terms “private signal” and “impression.” The impression is accurate with probability \( \beta \geq \frac{1}{2} \), such that \( \Pr(t_i = H|m = m_g) = \beta \) and \( \Pr(t_i = L|m = m_b) = \beta \). A higher \( \beta \) implies that the consumer’s own impression of fit is more accurate. The advertiser’s prediction for a consumer’s type \( s \) and the consumer’s impression \( m \) are correlated through the consumer’s underlying type \( t_i \), but conditional on \( t_i \), they are independent. The consumer then decides whether to buy the product. We also assume that a consumer cannot buy the product if she does not receive an ad due to a lack of awareness.\(^4\)

The firm’s advertising strategy corresponds to its advertising decision based on the algorithm’s prediction of the consumer’s type \( s \in \{s_H, s_L\} \): \( \sigma(s_H|\mu, \alpha, \beta) \) and \( \sigma(s_L|\mu, \alpha, \beta) \in [0, 1] \), which represent the fraction of each segment that the firm targets.\(^5\) A targeted consumer’s strategy is the probability with which the consumer purchases the product, denoted by \( \delta_m(m, \bar{\sigma}|\mu, \alpha, \beta) \in [0, 1] \).

Importantly, consumers do not directly observe the firm’s advertising strategy \( \sigma(\cdot) \) and instead have rational expectations about it, denoted by \( \bar{\sigma} \). Therefore, the consumer’s strategy is a mapping from her private impression \( m \) and her belief of the advertiser’s targeting strategy \( \bar{\sigma} \). Because \( \mu, \alpha, \) and \( \beta \) are parameters of the model, the strategies of the firm and the consumers are abbreviated.

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\(^4\)Prior to receiving an ad, the consumer is unaware of a specific product, but otherwise, she is a rational agent who knows the primitives of the game, including the correct prior about the likelihood that any new product will be a good match with her preferences. This is an assumption commonly made in the literature on targeted advertising (e.g., Iyer et al. (2005)).

\(^5\)In practical scenarios, a firm, for instance, might launch an online advertising campaign deploying a thousand impressions without the ability to pinpoint specific users, exact timing, or particular webpages for ad delivery. Here, the decision on the number of ad impressions (or advertising coverage) allocated to each target segment, including both likely high and low types, effectively translates into selecting the probability that a randomly chosen user within each segment will encounter an advertisement. This approach reflects the often uncontrollable and probabilistic nature of ad delivery in real-world online advertising campaigns, where advertisers set broader parameters without micromanaging each impression’s delivery. In response, consumers also exhibit varied purchase behaviors following advertisement exposure. After receiving or being exposed to ads, consumers demonstrate a mix of responses: some decide to make a purchase, while others choose not to.
Figure 1: Timeline of the game sequence

Henceforth by $\sigma(s)$ and $\delta_c(m, \tilde{\sigma})$. Figure 1 illustrates the game sequence.

Only when $\sigma(s_H) \neq \sigma(s_L)$ does the advertiser’s targeting strategy vary by its prediction of the consumer’s preferences. We distinguish the two types of targeting strategies as follows:

**Definition 1.** The advertiser’s strategy, $\sigma(s)$, is individually targeted if $\sigma(s_H) \neq \sigma(s_L)$ is higher.

We solve for the Perfect Bayesian Equilibrium of this game.\(^6\)

**Consumer inference**

Upon receiving an ad ($A = a$), a consumer updates her posterior belief about product fit based on her own signal ($m$) and her belief about the advertiser’s targeting strategy ($\sigma$). We denote her posterior belief by $\tilde{\mu}(\tilde{\sigma}, m) = \Pr(t_i = H|\tilde{\sigma}, m)$. The firm’s advertising strategy depends on its

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\(^6\)We assume that the level of advertising is not observed by consumers. In any equilibrium where $0 < \sigma(s) < 1$ for at least one $s \in \{H, L\}$, both receiving and not receiving an advertisement are consistent with the firm’s equilibrium advertising strategy. For example, if a consumer believes that the firm sent out an advertisement 10% of the time for $s_H$, the fact that she did or did not receive an advertisement does not change her equilibrium belief. On the other hand, in an equilibrium in which $\sigma(s) = 0$ for both $s \in \{s_H, s_L\}$, consumers expect a zero probability of receiving an advertisement irrespective of the firm’s signal $s$. If a consumer unexpectedly receives an ad under these conditions — an event inconsistent with the presumed equilibrium — we must define the consumer’s off-equilibrium beliefs because Bayesian updating does not provide any guidance for pinning down consumers’ off-equilibrium beliefs when consumers actually observe an advertisement in this case. In this situation, we posit that the consumer believes the firm’s strategy did not depend on $s$. That is to say, receiving an ad does not alter the consumer’s prior beliefs as it’s assumed that the firm’s advertising strategy is not individually targeted.
private signal about the consumer’s type, $s$, which the consumer does not observe. Therefore, a consumer infers the firm’s signal $s$ using the anticipated advertising strategy $ar{\sigma}(s)$ and her private impression $m$ to update her belief about her true type $t_i$.

To derive a consumer’s posterior beliefs, we first analyze how a consumer updates her belief based on her impression $m$. With a slight abuse of notation, we denote $\mu_c(m)$ for $m \in \{m_g, m_b\}$ as such updated belief, which we call the consumer’s **private prior** based on impression $m$:

$$
\mu_c(m_g) = \frac{\mu \beta}{\mu \beta + (1 - \mu)(1 - \beta)}, \quad \mu_c(m_b) = \frac{\mu(1 - \beta)}{\mu(1 - \beta) + (1 - \mu)\beta}.
$$

Next, the mere fact that a consumer receives an ad ($A = a$) can influence her belief. When she receives an ad, she considers the following two possibilities: (1) the firm received a signal $s_H$ with accuracy $\alpha$ (the probability of this event is $\mu_c\alpha + (1 - \mu_c)(1 - \alpha)$) and sends an ad following its strategy $\bar{\sigma}(s_H)$, or (2) the firm received a signal $s_L$ with accuracy $\alpha$ (the probability of this event is $\mu_c(1 - \alpha) + (1 - \mu_c)\alpha$) but still sends an ad with probability $\bar{\sigma}(s_L)$. From the anticipated advertiser’s strategy $\bar{\sigma}(s) = (\bar{\sigma}(s_H), \bar{\sigma}(s_L))$, a consumer infers probabilities that her true type is $t_i = H$ or $L$, incorporating her private prior $\mu_c = \mu_c(m)$, which is a function of impression $m \in \{m_g, m_b\}$. Thus, the consumer’s posterior belief is:

$$
\tilde{\mu}(\sigma, m) = \Pr(t_i = H|\sigma, m) = \frac{\bar{\sigma}(s_H) \cdot \mu_c \cdot \alpha + \bar{\sigma}(s_L) \cdot \mu_c \cdot (1 - \alpha)}{\bar{\sigma}(s_H) \cdot (\mu_c \alpha + (1 - \mu_c)(1 - \alpha)) + \bar{\sigma}(s_L) \cdot (\mu_c(1 - \alpha) + (1 - \mu_c)\alpha),}
$$

where $\mu_c = \mu_c(m)$ is from Equation (1). The consumer’s posterior belief about her type $\tilde{\mu}(\sigma, m)$ depends on (i) her private signal $m_i$ through $\mu_c(m)$, (ii) anticipated firms’ advertising strategy $\bar{\sigma}(s)$, and (iii) information accuracy of the firm and consumer $\alpha$ and $\beta$.

**Lemma 1.** For any $m \in \{m_g, m_b\}$, $\tilde{\mu}(\sigma, m) \geq \mu_c(m)$ if and only if $\bar{\sigma}(s_H) \geq \bar{\sigma}(s_L)$. Moreover, the change in beliefs becomes larger as the firm’s prediction accuracy $\alpha$ increases: $\frac{\partial [\tilde{\mu} - \mu_c]}{\partial \alpha} \geq 0$.

A consumer updates her beliefs about her product match more positively when the advertising strategy is individually targeted (i.e., $\bar{\sigma}(s_H) \geq \bar{\sigma}(s_L)$). In contrast, suppose a consumer believes that the advertiser’s strategy is not individually targeted, i.e., $\frac{\bar{\sigma}(s_H)}{\bar{\sigma}(s_L)} = 1$. Then, the consumer’s posterior belief only depends on her own impression, $\tilde{\mu}(\sigma, m) = \mu_c(m)$. In that case, the consumer’s willingness to pay is determined by her own impression of product fit, which is either $\tilde{\mu}(\sigma, m_g) =$
$\mu_c(m_g)$ or $\bar{\mu}(\bar{\sigma}, m_b) = \mu_c(m_b)$.

**Persuadable consumer: recommendation effect and consumer decision**

Lemma 1 established that if the advertiser’s strategy is individually targeted, a targeted ad also performs the function of a recommendation to consumers: “You are receiving this ad because we believe this product is a good fit for you.” Such an implicit recommendation may or may not influence consumers’ buying decisions depending on the product price. For instance, suppose the price is prohibitively high such that $p > \bar{\mu}(\sigma, m_g)$. Then, even if $\bar{\sigma} = (\bar{\sigma}(s_H) = 1, \bar{\sigma}(s_L) = 0)$, which achieves the most favorable posterior belief of a targeted consumer, the consumer will never buy the product. Consequently, the advertiser never targets consumers. Similarly, if the price is sufficiently small, i.e., $p \leq \mu_c(m_b)$, a consumer will purchase the product as long as she receives an ad regardless of whether she believes the ad is individually targeted.

On the other hand, if the price falls in an intermediate range, the firm can influence the targeted consumers’ purchasing decisions — in other words, persuade — by modifying how individually targeted its advertising will be, if at all. In particular, consider a price in $\mu_c(m_b) < p \leq \bar{\mu}(\bar{\sigma}, m_b, |, \bar{\sigma}(s_H) = 1, \bar{\sigma}(s_L) = 0)$. When a consumer receives a bad impression of product fit $m_b$, her belief about the product fit $\mu_c(m_b)$ is lower than the price. If she believes that the ad is not individually targeted (i.e., $\bar{\sigma}(s_H) = \bar{\sigma}(s_L)$), her final posterior belief is still $\bar{\mu}(\bar{\sigma}, m_b) = \mu_c(m_b)$. So, she does not buy upon receiving the ad. However, if she believes that the ad is individually targeted (i.e., $\bar{\sigma}(s_H) > \bar{\sigma}(s_L)$), there always exists an advertising strategy $\bar{\sigma}(s_H), \bar{\sigma}(s_L)$ where $\frac{\bar{\sigma}(s_H)}{\bar{\sigma}(s_L)}$ is sufficiently large that the consumer’s posterior belief becomes $p \leq \bar{\mu}(\bar{\sigma}, m_b)$. Therefore, the consumer would consider buying upon receiving an individually targeted ad even if her impression is $m_b$.

Formally, consumers are **persuadable** if, under some $m \in \{m_g, m_b\}$, they would not buy the product if they believe the ad is non-targeted mass marketing but would buy the product if they believe the ad is sufficiently individually targeted.

It is crucial to note that our definition of **persuadable** is different from the one in the information design and Bayesian persuasion literature (Kamenica and Gentzkow, 2011; Bergemann and Morris, 2019), which explores a model wherein a sender can influence a receiver’s actions by selecting and committing to a specific information structure. Our model departs from their framework, given
that in our scenario, the firm has no practical ability to commit to any advertising strategy.\footnote{In contrast to the Bayesian persuasion literature, our model foregoes reliance on the sender’s commitment, thereby allowing the sender (i.e., the firm) to opportunistically dilute its advertising strategy. This unique dynamic, unexplored in traditional information design literature, is examined in our model. Herein, the advertisement functions as a signaling device rather than a Bayesian persuasion instrument. Consequently, our analysis focuses on the credibility of advertising strategies.}

**Lemma 2.** For any price \( p \in (\mu_c(m_b), \tilde{p}^U] \), there exists a threshold \( \xi^* \) such that for all advertising strategies which are more individually targeted \( \frac{\tilde{\sigma}(s_H)}{\tilde{\sigma}(s_L)} \geq \xi^* \), consumers are persuadable.

This lemma identifies the range of prices in which consumers are persuadable. By persuadable consumers, we refer to the price range given the model primitives (such as \( \mu, \alpha, \beta \)) in which consumers can be persuaded. In our analysis of the exogenous price, we focus on the case where the price is such that consumers are persuadable, and thus, targeted advertising can influence consumers’ behaviors. For the endogenous price case, we consider the range of all prices where consumers can be either persuadable or non-persuadable.

4 Benchmark: Consumers with an Uninformative Private Signal

We first analyze a simple benchmark case in which the consumers’ private signal of product fit is pure noise: \( \beta = 1/2 \). Upon receiving their uninformative private signal \( m \), consumers cannot make any informed assessment of product fit, so their posterior remains the same as the prior, i.e., \( \mu_c(m) = \mu \) for all \( m \in \{m_g, m_b\} \). However, a consumer will update her beliefs based on her expectation about the advertiser’s targeting strategy, which will be different from her prior \( \mu \) if and only if \( \tilde{\sigma}(s_H) \neq \tilde{\sigma}(s_L) \). Also, note that all targeted consumers with \( A = a \) have the same posterior beliefs about their true types \( t_i \in \{H, L\} \) since they do not directly observe the firm’s signal \( s \in \{s_H, s_L\} \). Hence, all targeted consumers’ strategy is the same, i.e., to make a purchase with probability \( \delta_c \).

Suppose consumers believe that the advertiser only targets high-type consumers, i.e., \( \tilde{\sigma}(s_H) = 1 \) and \( \tilde{\sigma}(s_L) = 0 \). Then, a consumer sees the ad as a clear recommendation, and her optimal response is to always buy because \( \tilde{\mu}(\tilde{\sigma}_A) = \tilde{\mu}^U \). However, given the consumer’s response to always purchase after receiving the ad, the advertiser sees the opportunity to take advantage of the implicit recommendation by targeting consumers even if the firm’s signal is \( s_L \). Then, the targeted consumers will unknowingly purchase the product. By doing so, the advertiser effectively cheats on its message.
by sending a perverse recommendation. Therefore, in equilibrium, we cannot have a pure-strategy separating equilibrium where $\sigma(s_H) = 1$ and $\sigma(s_L) = 0$.8

Also, there cannot be an equilibrium in which the advertising is not individually targeted, i.e., $\sigma(s_H) = \sigma(s_L) = 1$. Because a non-targeted ad cannot serve as a recommendation, consumers’ posterior belief is the same as the prior $\mu(\sigma) = \mu$, which is lower than the price $p$. Thus, there is no pure-strategy pooling equilibrium in which there is an ex-ante non-zero probability of transaction.9

We now consider a mixed and individually targeted strategy, where the advertiser targets the consumer with probability $\sigma(s_H), \sigma(s_L) \in [0, 1]$. Consumers purchase the product with the probability $\delta_c(\bar{\sigma}) \in [0, 1]$, which no longer depends on $m$ because the consumers’ signal is pure noise. For consumers to mix, they must be indifferent between purchasing and not purchasing after receiving an ad, which depends on their posterior beliefs. From Equation (2), the posterior belief is:

$$\bar{\mu}(\bar{\sigma}) = \frac{\bar{\sigma}(s_H) \cdot \mu \alpha + \bar{\sigma}(s_L) \cdot \mu (1 - \alpha)}{\bar{\sigma}(s_H) (\mu \alpha + (1 - \mu)(1 - \alpha)) + \bar{\sigma}(s_L) (\mu (1 - \alpha) + (1 - \mu) \alpha)}$$ \hspace{1cm} (3)

The consumer’s expected utility from purchasing is $E[U(\text{purchasing})] = \bar{\mu}(\bar{\sigma})(1-p) + (1-\bar{\mu}(\bar{\sigma}))(p)$, and from not purchasing is $E[U(\text{not purchasing})] = 0$. In equilibrium, these must be the same so that the consumer is indifferent, i.e., $\bar{\mu}(\bar{\sigma}) = p$, which pins down the firm’s mixed strategy.

Also, consumers mix such that the firm is indifferent between sending and not sending an ad. The expected payoff from sending an ad, $E\Pi(A = a|s) = \delta_c(p - k) + (1 - \delta_c)(-k)$, must be the same as not sending an ad $E\Pi(A = \emptyset|s) = 0$. Therefore, we have that $\delta_c = k/p$.

**Proposition 1.** Suppose consumers do not have an informative private signal about the product fit. Then, there is no equilibrium where the firm can obtain a positive expected profit. The set of equilibria characterized by $\sigma^* = (\sigma^*(s_H), \sigma^*(s_L))$ and $\delta^*_c$ are as follows: (i) the ads are individually targeted such that $\frac{\sigma^*(s_H)}{\sigma^*(s_L)} > 1$, and (ii) the advertising strategy $\sigma^*$ satisfies $\sigma^*(s_H) = \phi \cdot \sigma^*(s_L)$, where $\phi = \frac{(1 - \mu)p - (1 - \alpha)\mu (1 - p)}{\alpha \mu (1 - p) - (1 - \alpha)(1 - \mu)p} > 1$ for persuadable consumers, and (iii) $\delta^*_c = k/p$.

First, the firm’s equilibrium profit is zero,10 a result straightforward from the firm’s indifference condition. Second, even under a mixed equilibrium, the advertiser’s strategy is individually targeted,
i.e., $\phi = \frac{\sigma^*(s_H)}{\sigma^*(s_L)} > 1$. In particular, the firm sends an ad to a consumer it perceives as a high type ($s_H$) with a higher probability than to a low type ($s_L$).

Suppose the ads are not individually targeted in equilibrium, i.e., $\phi = 1$. Given that consumers are persuadable, consumers’ posterior will be the same as the prior, and $\bar{\mu}(\bar{\sigma}) = \mu < p$ and thus, no consumer will be persuaded to make a purchase. Therefore, even though the firm is facing consumers without heterogeneous private information and thus making the same purchasing decisions, it is in its best interest to individually target consumers, treating consumers differently.

In this mixed strategy equilibrium, the posterior belief must be equal to the price ($\bar{\mu}(\bar{\sigma}) = p$) from consumers’ indifference condition: $\mathbb{E}[U(\text{purchasing})] = \mathbb{E}[U(\text{not purchasing})] = 0$. Under a higher prediction accuracy, the same advertising strategy can be more persuasive, as it can improve consumers’ posterior belief $\bar{\mu}(\bar{\sigma})$, thus inducing all targeted consumers to always buy the product. However, this introduces an incentive for the advertiser to cheat by amplifying its strategic mistargeting (i.e., increasing $\sigma(s_L)$) until the posterior belief drops to $p$. The following proposition formalizes the relationship between prediction accuracy and the advertiser’s equilibrium strategy.

**Proposition 2.** For persuadable consumers, as prediction accuracy increases, the advertiser’s strategy becomes less individually targeted. That is, $|\phi| = \left| \frac{\sigma(s_H)}{\sigma(s_L)} \right| > 1$ strictly decreases in $\alpha$.

When the advertiser’s prediction of an individual’s preferences becomes more accurate, we should expect the advertiser’s targeting strategy to be more focused on the “right” consumers, all else being equal. However, all else is not equal. A targeted ad carries an implicit message of recommendation that affects the consumer’s evaluation of the product. Better prediction accuracy strengthens the power of such implicit recommendations, creating the incentive for the advertiser to intentionally dilute the message to capture a bigger market. Thus, ironically, the improvement in prediction can lead to less individually targeted advertising.

We also consider the limit case as the firm’s prediction accuracy $\alpha$ approaches 1, an extreme case where the AI can predict consumers’ preferences perfectly. The following corollary shows that the advertiser still targets the “wrong” consumers for whom the firm predicts to be a low-valuation type, and consumers sometimes make incorrect purchases, resulting in post-purchase regret.
Corollary 1. In the limit as $\alpha \to 1$, (i) the advertiser knowingly targets low-type consumers with a positive probability: $\sigma^*(s_L) > 0$. (ii) Given that high-type consumers are targeted, they do not buy the product with a probability $(p - k)/p$, whereas when low-type consumers are targeted, they buy with a probability $k/p$.

The corollary highlights the persistence of the firm’s strategic mistargeting even under perfect prediction technology. Thus, prediction technology alone cannot eliminate the misaligned incentives of the firm and the consumer.

While the model posits that consumers are aware of the firm’s prediction accuracy, $\alpha$, and the advertising cost, $k$, it is widely recognized that platforms like Instagram and Amazon provide advertisers with significantly greater precision than offline channels, such as cable TV or newspapers. This enhanced precision stems from detailed consumer activity online and the first-party data shared. Thus, it is a reasonable assumption that consumers have a general awareness of this precision level, even if the exact value of $\alpha$ remains unknown to them. We argue that this assumption offers a practical approximation of reality, capturing scenarios where consumers can broadly distinguish whether the precision is high or low without precise knowledge of its magnitude. Our extended analysis in Online Appendix-A further demonstrates that the strategies detailed in Proposition 1 continue to constitute an equilibrium, even if either $\alpha$ or $k$ is the firm’s private information.

5 Main Model: Consumer with an Informative Private Signal

One might posit that the firm’s strategic mistargeting behavior characterized in the previous section may not be robust if consumers have an informative private signal ($\beta > 1/2$). Nevertheless, we analyze a main model with consumers’ informative signals and show that the findings from the benchmark are robust. Given the existence of strategic mistargeting, consumers may want to withhold their data so that the firm cannot send them targeted advertising. We analyze consumers’ incentives to opt into or out of data collection under different default privacy policies.

In this model, consumers with a good impression ($m = m_g$) have a more positive posterior belief and thus a higher willingness to pay than consumers with a bad impression ($m = m_b$). Therefore, the firm has a weakly greater incentive to send an ad to a consumer of type $s_H$ than to another consumer of type $s_L$. This is because the firm’s signal is positively correlated with the consumer’s
impression through the consumer’s true type \( t_i \). So, in equilibrium, the firm will exhaust the entire segment of consumers with \( s_H \) before it advertises to anyone in the other segment \( s_L \). The following lemma formalizes these observations.

**Lemma 3.** Suppose consumers are persuadable. If \( \beta > 1/2 \), a totally mixed equilibrium with \( 0 < \sigma^*(s_L) \leq \sigma^*(s_H) < 1 \) does not exist. Moreover, if \( \sigma^*(s_L) > 0 \), then it must be \( \sigma^*(s_H) = 1 \).

Next, we analyze two different types of pure-strategy equilibria with \( \sigma^*(s_H) = 1 \): separating equilibria (where the firm only advertises to the high type \( s_H \)) and pooling equilibria (where the firm advertises to both types).

### 5.1 Pure strategy equilibrium: separating and pooling

#### Separating equilibrium

When consumers are persuadable (\( \mu_c(m_b) < p \leq \bar{\mu}^U \)), there can exist a pure-strategy separating equilibrium, where the firm only sends an ad if a consumer is of a high type, \( \sigma^s = (\sigma(s_H) = 1, \sigma(s_L) = 0) \).

The superscript \( s \) denotes a **separating** equilibrium. In equilibrium, consumers believe that the firm only targets them because \( s_i = s_H \). Thus, the ad serves as a clear recommendation. The separating equilibrium occurs when a targeted consumer’s posterior belief after having a bad impression \( m_b \) is less than the price, so she does not purchase; and her posterior belief after having a good impression \( m_g \) is above the price, so she purchases the product. The ad cost must be in an intermediate range so that the firm has incentives to target its ad to a high-type but not to a low-type consumer. The following proposition characterizes the separating equilibrium.

**Proposition 3.** Suppose consumers are persuadable: \( \mu_c(m_b) \leq p \leq \bar{\mu}(\sigma^s, m_g) = \bar{\mu}^U \).

If \( p \cdot \Pr(m_g|s_L) < k < p \cdot \Pr(m_g|s_H) \) and \( p > \bar{\mu}(\sigma^s, m_b) = \frac{\mu(1-\beta)\alpha}{\mu(1-\beta)\alpha + (1-\mu)(1-\alpha)\beta} \), there exists a separating equilibrium where \( \sigma^s = (\sigma^s(s_H) = 1, \sigma^s(s_L) = 0) \), and \( \delta^s_c(m_g) = 1, \delta^s_c(m_b) = 0 \).

In a separating equilibrium, advertising acts as a clear recommendation to consumers, and the firm attains a positive expected profit, both of which were unattainable without consumers’ private signals. Therefore, the presence of consumers’ private information about the product can discipline the firm’s opportunistic mistargeting and make the firm better off if both the cost of advertising and the price are in somewhat high ranges.
Pooling equilibrium

Next, there can exist a pure-strategy equilibrium, i.e., pooling equilibrium, where the firm’s advertising strategy is not targeted, i.e., $\sigma_{pool-m_g} = (\sigma(s_H) = 1, \sigma(s_L) = 1)$, and targeted consumers purchase only if their impression of the product fit is good, $\delta_c^{pool-m_g}(m_g) = 1$ and $\delta_c^{pool-m_g}(m_b) = 0$. Here, the superscript ‘pool-$m_g$’ denotes a pooling equilibrium where only a consumer with a good impression $m_g$ purchases, which we call pooling-$g$. The firm should find it profitable to send an ad even to a consumer with $s_L$. This condition is satisfied if the cost of advertising $k$ is sufficiently low or the advertiser’s prediction $\alpha$ is sufficiently low.

On the other hand, if the unit advertising cost is too high, another type of pooling equilibrium exists where the firm does not advertise at all, $\sigma(s_H) = \sigma(s_L) = 0$, which is called pooling-$\emptyset$. The following proposition characterizes the two pooling equilibria for persuadable consumers.

Proposition 4. Suppose consumers are persuadable: $\mu_c(m_b) < p \leq \tilde{\mu}(\sigma^*, m_g) = \tilde{\mu}^U$. There are two different types of pooling equilibria.

1. If $k < p \cdot \Pr(m_g|s_L)$ and $p < \mu_c(m_g)$, there exists a pooling equilibrium where $\sigma_{pool-m_g} = (\sigma(s_H) = 1, \sigma(s_L) = 1)$ and $\delta_c^{pool-m_g} = (\delta_c(m_g) = 1, \delta_c(m_b) = 0)$. The parameter space in which this equilibrium exists shrinks as the advertiser’s prediction accuracy $\alpha$ increases.

2. If $k \geq p$ or $p \cdot \Pr(m_g|s_H) \leq k < p$ and $p \geq \tilde{\mu}(\sigma^*, m_b)$, there exists another pooling equilibrium where $\sigma_{pool-\emptyset} = (\sigma(s_H) = 0, \sigma(s_L) = 0)$.

In the pooling-$g$ equilibrium, the firm engages in maximal mistargeting, and the firm’s advertising as a recommendation is completely diluted. These equilibria exist when the cost of advertising is sufficiently low and the price is not high enough. A cheap advertising cost amplifies the firm’s incentives to mistarget low-type consumers. If the price were high, the firm could be disciplined to engage in individually-targeted advertising because consumers would not be willing to pay a high price unless the ads are sufficiently precisely targeted. This hints that the firm’s endogenous pricing decision analyzed in Section 6 may have critical implications for the equilibrium outcome.

In the range of prices where consumers are not persuadable (either $p > \tilde{\mu}^U$ or $p < \mu_c(m_b)$), there also exist other pooling equilibria. If $p > \tilde{\mu}^U$, the price is so high that no consumer can be persuaded to make a purchase, irrespective of the firm’s targeting strategy. Therefore, we again
have the pooling-$\emptyset$ equilibrium. Also, if $p < \mu_c(m_b)$, a targeted consumer will make a purchase irrespective of her private impression $m$. Thus, it is profitable for the firm to target all consumers, i.e., $\sigma^{\text{pool-all}} = (\sigma(s_H) = 1, \sigma(s_L) = 1)$. We refer to this as the pooling-all equilibrium.

5.2 Semi-separating equilibrium

Given that consumers are persuadable, there can exist a “hybrid” type of Perfect Bayesian Equilibrium, where the advertiser mixes between sending and not sending an ad for consumers of type $s = s_L$ (i.e., $0 < \sigma^{\text{semi}}(s_L) < 1$) while always sending an ad for consumers of type $s = s_H$ (i.e., $\sigma^{\text{semi}}(s_H) = 1$). As a result, by being targeted, consumers imperfectly update their prior beliefs about the firm’s signal $s$ (and about their true match with the product $t_i$).

More specifically, when a consumer receives an ad, there are two possibilities: (1) the firm received a signal $s_H$ with accuracy $\alpha$ (the probability of this event is $\mu\alpha + (1 - \mu)(1 - \alpha)$) or (2) the firm received a signal $s_L$ with accuracy $\alpha$ (the probability of this event is $\mu(1 - \alpha) + (1 - \mu)\alpha$), but the firm “cheats” by sending an ad with probability $\sigma(s_L) > 0$. Therefore, the firm influences consumers’ posterior beliefs through its strategy $\sigma(s_L)$. In equilibrium, given their posterior beliefs $\tilde{\mu}(\sigma^{\text{semi}}, m)$, consumers choose an optimal purchase behavior about whether to purchase or not.

Figure 2 shows the game structure of this semi-separating equilibrium.

In this equilibrium, the firm mixes between sending an ad and not sending to a consumer of type $s = s_L$. The firm’s expected payoff from sending an ad is $E\Pi(A = a|s_L) = \Pr(m_g|s_L) \cdot \{p \cdot \delta_c(m_g) - k\} + \Pr(m_b|s_L) \cdot \{p \cdot \delta_c(m_b) - k\}$. The firm’s expected payoff from not sending an ad is $E\Pi(A = \emptyset|s_L) = 0$. For the existence of a semi-separating equilibrium, the firm should be indifferent, i.e., $E\Pi(A = a|s_L) = E\Pi(A = \emptyset|s_L) = 0.12$ We first characterize the mixing behaviors of the consumers in a semi-separating equilibrium.

**Lemma 4.** In a semi-separating equilibrium, the consumer with either $m = m_g$ or $m_b$ mixes between buying and not buying after receiving an ad. If $\delta_c(m_b) > 0$, it must be $\delta_c(m_g) = 1$. Also, if

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11 In this paper, we do not analyze another possible semi-separating equilibrium in which $\sigma^*(s_L) = 0$ and $\sigma^*(s_H) \in (0, 1)$. In this equilibrium, the firm makes zero expected profit because it is indifferent between sending and not sending ads to consumers with $s_H$. Moreover, this equilibrium can only exist for a specific $\mu$ given model primitives, $k$, $p$, and the firm’s strategy, i.e., $\tilde{\mu}(\sigma^*, m_g) = p$ (if $\delta_c(m_b) \in (0, 1)$ and $\delta_c(m_g) = 1$) or $\tilde{\mu}(\sigma^*, m_g) = p$ (if $\delta_c(m_b) = 0$ and $\delta_c(m_g) \in (0, 1)$). We focus on the other semi-separating equilibrium where the firm’s expected profit can be positive, i.e., $\sigma(s_H) = 1$ and $\sigma(s_L) \in (0, 1)$.

12 Also, the firm must find it optimal to send an ad to consumers with $s_H$. It is confirmed that $E\Pi(A = a|s_H) \geq E\Pi(A = a|s_L)$ because $\Pr(m_g|s_H) > \Pr(m_g|s_L)$.
\( \delta_c(m_g) < 1 \), it must be \( \delta_c(m_b) = 0 \).

The lemma suggests that there are two different types of semi-separating. (1) Only consumers with a bad impression mix: \( \delta_c(m_b) \in (0, 1) \) and \( \delta_c(m_g) = 1 \), which we call ‘semi-b’ case, and (2) only consumers with a good impression mix: \( \delta_c(m_g) \in (0, 1) \) and \( \delta_c(m_b) = 0 \), which we call ‘semi-g’ case. But not both types mix their behaviors. More specifically, if a consumer with a bad impression mixes such that \( \delta_c(m_b) > 0 \), it must be that a consumer with a good impression always buys, i.e., \( \delta_c(m_g) = 1 \). Also, if consumers with a good impression mix such that \( \delta_c(m_g) < 1 \), it must be that consumers with a bad impression never buy, i.e., \( \delta_c(m_b) = 0 \). We characterize those two semi-separating equilibria in the following proposition.

**Proposition 5.** Suppose consumers are persuadable: \( \mu_c(m_b) \leq p \leq \overline{\mu} = \overline{\mu}(\sigma^s, m_g) \). There exist two semi-separating equilibria where \( \sigma_{{\text{semi}}}(s_H) = 1 \) and \( 0 < \sigma_{{\text{semi}}}(s_L) < 1 \):

1. If \( p < \overline{\mu}(\sigma^s, m_b) \) and \( 0 < k < p \), then \( \sigma_{\text{semi-b}}(s_L) = \frac{p(1-\mu_c(m_b))(1-\alpha)(1-p)-\mu_c(m_b)\alpha}{p(1-\mu_c(m_b))(1-\alpha)(1-p)-\mu(1-\mu_c(m_b))\alpha} \), and only consumers with a bad impression \( (m_b) \) mix: \( \delta_{\text{semi-b}}(m_g) = 1 \), and \( \delta_{\text{semi-b}}(m_b) = k \cdot \frac{1}{p} - \frac{k(1-\alpha)(1-\beta)+(1-\mu)(1-\beta)}{\mu(1-\alpha)(1-\beta)+(1-\mu)(1-\beta)} \).

2. If \( p > \mu_c(m_g) \) and \( k < p \cdot \Pr(m_g|s_L) \), then \( \sigma_{\text{semi-g}}(s_L) = \frac{p(1-\mu_c(m_g))(1-\alpha)(1-p)-\mu_c(m_g)\alpha}{p(1-\mu_c(m_g))(1-\alpha)(1-p)-\mu(1-\mu_c(m_g))\alpha} \), and only consumers with a good impression \( (m_g) \) mix: \( \delta_{\text{semi-g}}(m_g) = k \cdot \frac{1}{p} - \frac{k(1-\alpha)(1-\beta)+(1-\mu)(1-\beta)}{\mu(1-\alpha)(1-\beta)+(1-\mu)(1-\beta)} \) and \( \delta_{\text{semi-g}}(m_b) = 0 \).
Figure 3 demonstrates all equilibria of the game and shows how they change as the firm’s targeting technology \( \alpha \) improves.\(^{13}\) First, a unique equilibrium exists in the entire parameter space except for a measure zero set.\(^{14}\) Figure 3(a) depicts how the region of each equilibrium changes in response to an increase in \( \alpha \), indicated by bold arrows. The posterior belief \( \tilde{\mu}(\sigma^s, m_g) \) increases and converges to \( \tilde{\mu}^U \), eventually forcing \( \tilde{\mu}^U \to 1 \). Moreover, the slope \( 1/\Pr(m_g|s_L) \) gets steeper. Therefore, as illustrated in Figure 3(b) where \( \alpha \to 1 \), the parameter region for the semi-separating equilibrium (semi-\( b \) type, in particular) increases, ultimately engulfing both separating and pooling equilibrium regions. In particular, the region for the separating equilibrium (of trapezoid shape) shrinks because its height diminishes as \( \tilde{\mu}(\sigma^s, m_g) \) increases and converges to \( \tilde{\mu}^U \to 1 \). In other words, as the firm’s targeting becomes more accurate, the firm’s incentive problem worsens, causing the separating equilibrium to disappear. On the other hand, the scope of semi-separating equilibria, in which the firm intentionally sends an ad to consumers with \( s_L \), increases, becoming more prevalent.

Note that in the limit where \( \alpha \to 1 \), the semi-\( b \) type semi-separating equilibrium still exists and is the unique equilibrium when \( \mu_c(m_b) < p < \tilde{\mu}^U \) and \( p \cdot (1 - \beta) < k < p \). Also, when the advertising cost \( k \) increases, the firm’s advertising strategy may shift towards more individualized targeting. For example, when \( k \) is sufficiently low, we observe either a pooling-\( g \) equilibrium or a semi-\( g \) equilibrium. However, when \( k \) increases, the firm moves either to a semi-\( b \) or a separating equilibrium, characterized by individually targeted advertising. Consequently, higher \( k \) values may prompt the firm to settle into an equilibrium characterized by more personalized advertising tactics.

We confirm that key findings derived from the benchmark without the customer’s informative signal remain robust in semi-separating cases, exhibiting the same qualitative properties: as \( \alpha \) increases, the firm engages in less individually-targeted advertising.

**Proposition 6.** In any semi-separating equilibrium, the firm engages in individually targeted advertising, i.e., \( \frac{\sigma(s_H)}{\sigma(s_L)} > 1 \). As the firm’s information becomes more precise, the firm engages in less targeted advertising, i.e., \( \frac{\sigma(s_H)}{\sigma(s_L)} \) decreases as \( \alpha \) increases.

\(^ {13}\)This figure depicts the case where \( \tilde{\mu}(\sigma^s, m_g) > \mu_c(m_g) \), i.e., the consumer’s posterior belief is more positive when being targeted carries more information about the consumer’s true match-type than the consumer’s private signal. This is the case if the firm’s signal is much more informative than the consumer’s own signal, i.e., \( \alpha \gg \beta \).

\(^ {14}\)There can exist multiple equilibria on the boundary between different equilibrium regions. We detail these boundaries in the proof of Lemma A-2 in the Online Appendix Section D.
The following limiting result, where $\alpha \to 1$, is qualitatively similar to Corollary 1, focusing on the semi-$b$ equilibrium. Under an exogenous price, even though there are misaligned incentives between the firm and the consumers, we show that consumer surplus can be positive.

**Corollary 2.** As $\alpha \to 1$, if $\mu_c(m_b) < p < \frac{k}{1-\beta}$ and $k < p$, we have the following results:

1. The firm knowingly targets consumers with $s_L$ with probability $\frac{1-p}{p} \frac{\mu(1-\beta)}{(1-\mu)\beta}$, which is decreasing in $\beta$. Targeted consumers mistakenly buy the product with probability $1-\beta + \beta \left( \frac{k}{p} - \frac{p-k}{p} \frac{\mu(1-\beta)}{(1-\mu)\beta} \right)$ and realize a negative utility of $-p$;

2. If $\mu$ is sufficiently large, the consumer surplus is positive for a sufficiently large $\beta$.

Some consumers will end up making wrong purchasing decisions even with private information. In particular, even when the advertiser’s prediction becomes perfectly accurate (i.e., $\alpha \to 1$), the effect still persists. In fact, the extent of mistargeting enlarges. Targeted consumers will sometimes buy the product and suffer a utility loss.\(^{15}\) Nevertheless, consumer surplus can be positive, especially when $\beta$ and $\mu$ are large enough. When consumers’ private information becomes more accurate, the firm will engage in less mistargeting (i.e., $\frac{\partial \sigma^{semi-b}(s_L)}{\partial \beta} < 0$). Moreover, consumers can avoid making wrong purchases by relying on private signals. So, the consumer surplus can be positive despite the incentives of the firm and the consumers being misaligned.

\(^{15}\)Similarly, if $\mu_c(m_g) \leq p < 1$ and $k < (1-\beta) \cdot p$, we have the semi-$g$ equilibrium. The firm sends an ad to consumers with $s_L$ with probability $\sigma^{semi-g}(s_L) \rightarrow \frac{1-p}{p} \frac{\mu\beta}{(1-\mu)(1-\beta)}$. Among these consumers, $\Pr(m_g | s_L) \delta_c(m_g) \rightarrow \frac{k}{p}$ fraction purchases and eventually realize a negative utility.
Next, we analyze the effect of the firm’s advertising cost $k$ and its prediction accuracy $\alpha$ on the firm’s profits. Previous discussion and Figure 3 demonstrate that a higher $k$ can shift the equilibrium so that the firm engages in more individually targeted advertising. It is important to note that within the same equilibrium regime, the relationship between the advertising cost and the firm’s profit within each equilibrium is more subtle. An interesting observation about the semi-$g$ equilibrium is that consumers’ probability of purchasing increases with the cost of advertising $k$, as shown in Proposition 5. As a result, the firm’s equilibrium profit increases with $k$ in the semi-$g$ equilibrium region. The higher cost of advertising reduces the firm’s ability to exploit the recommendation effect through mistargeting. This finding implies that a higher cost of advertising can discipline the firm’s mistargeting behaviors. In practice, firms can influence the advertising costs they pay, for example, by bidding for more expensive slots or retargeting consumers who have previously seen the ad. Thus, the semi-$g$ equilibrium suggests that firms may have incentives to use more expensive methods of targeting as a way to make their ads more credible.

The firm’s profits under pooling equilibria remain independent of $\alpha$ as the firm does not rely on individual consumer signals. In the separating equilibrium, the firm’s profit increases with $\alpha$ because the firm’s signal becomes more precise without engaging in mistargeting. Under the semi-$g$ equilibrium, the firm’s profit also increases with $\alpha$ since only consumers with $m_g$ purchase with less than probability one, and higher $\alpha$ amplifies this purchase probability in equilibrium. On the other hand, under the semi-$b$ equilibrium, the firm’s profit can decrease with $\alpha$ when the consumer’s information is sufficiently accurate (when $\mu > 1/2$ and $\beta > \mu/(2\mu - 1)$). In this equilibrium, part of the firm’s profit is derived from consumers with bad impressions $m_b$. When $\alpha$ is higher, consumers with $m_b$ form more negative beliefs about the firm’s signal, leading to a lower probability of purchasing.

\[16\] Details of the analysis of the effect of $k$ and $\alpha$ are provided in Online Appendix Sections B and C, respectively.

\[17\] First, in separating or pooling equilibria, neither the firm’s advertising nor the consumer’s purchasing strategies are sensitive to variations in $k$, resulting in a weak, inverse relationship between $k$ and firm profits within these regimes. Second, in semi-separating equilibria (including both semi-$b$ and semi-$g$), while $k$ does not change the firm’s advertising strategy, it does impact consumer purchasing behavior due to its effect on the firm’s indifference condition. A higher $k$ leads consumers to adjust their purchasing probability, offsetting $k$’s direct negative effect on profits variably depending on the specific equilibrium. We found that the indirect positive effect in the semi-$g$ equilibrium is strong enough that the firm’s equilibrium profit increases with $k$ but not in the semi-$b$ equilibrium.

\[18\] We thank an anonymous reviewer for making this observation.
5.3 Personal data opt-out

In order to send targeted advertising, advertisers require consumers’ personal data. Taking into account the firm’s advertising incentives, we examine consumers’ privacy choices, which endogenously determine the firm’s targeting accuracy and associated welfare implications in equilibrium. At the beginning of the game, consumers decide whether to give permission for data collection, and subsequently, the previously analyzed game is played as a subgame. If a consumer forbids data collection, the advertiser cannot access the consumer’s personal data, effectively leading to $\alpha = \frac{1}{2}$. Conversely, permitting data collection allows the advertiser to predict with $\alpha > \frac{1}{2}$. If a consumer is indifferent between the two, we assume that the consumer stays with the default privacy option.

Given the significant role of defaults on decision-making, as evidenced by empirical studies in fields like organ donation (Johnson and Goldstein, 2003), insurance choice (Johnson et al., 1993), and pension savings (Madrian and Shea, 2001), we examine two scenarios with different defaults: the opt-in setting and the opt-out setting. Depending on the default privacy option, consumers may be automatically enrolled in data sharing and have to explicitly withdraw consent if they do not want their data to be collected (opt-out), or data is not collected by default and consumers have to give explicit consent before the firm can collect data (opt-in). Deviating from the default requires effort, whereas adhering to it is effortless. Thus, under the opt-out setting, a consumer opts out of data collection if and only if the consumer strictly prefers to do so. Similarly, under the opt-in setting, a consumer opts into data collection if and only if the consumer strictly prefers to do so.

If consumers eventually opt out of data collection, then regardless of the default option, only pooling equilibria can exist because the advertiser’s private signal about individual consumers’ preferences is a complete noise. Borrowing from Proposition 4, the equilibrium of the subgame when the consumers’ data is not shared is characterized as follows:

**Lemma 5.** Suppose consumers are persuadable ($\mu_c(m_b) < p \leq \bar{\mu}^U$), and consumers opt out of the firm’s data collection. If $k < p \cdot \Pr(m_g|s_L)$ and $p \leq \mu_c(m_g)$, the advertiser mass advertises in equilibrium (i.e., the pooling-g). Otherwise, there is no advertising (i.e., the pooling-$\emptyset$) in equilibrium.

If the cost of advertising is cheap ($k \leq p \cdot \Pr(m_g|s_L)$) and the price is not too high ($p \leq \mu_c(m_g)$), the unique equilibrium is the pooling equilibrium with mass advertising, irrespective of consumers’ privacy choices (see Figure 3 for illustration). Outside this parameter region, the equilibrium can
differ depending on the consumer’s privacy choice. If consumers do not allow data collection, only the no-advertising pooling equilibrium exists, yielding a zero consumer surplus. Conversely, allowing data collection enables a positive expected surplus outside the pooling-*g region, either through semi-*b or separating equilibrium. In the semi-*b equilibrium, the overall consumer surplus is positive. Specifically, consumers with a bad impression (*m_b) are indifferent between buying and not buying and expect a zero surplus, while those with a good impression (*m_g) always buy the product and expect a positive surplus.

Therefore, under the opt-out setting, it is optimal for consumers to retain the default option, permitting the firm to track their information. In the opt-in setting, consumers allow data tracking only if it leads to either the separating or the semi-*b equilibrium, as detailed in Propositions 3 and 5. This occurs within an intermediate range of advertising costs, which broadens as prediction accuracy improves. The results are summarized in the following proposition.

**Proposition 7.** Suppose that the consumers are persuadable.

1. In opt-out setting, consumers adhere to the default and permit the firm’s access to their data.
2. In opt-in setting, consumers choose data collection within an intermediate range of advertising costs *k* (i.e., regions corresponding to semi-*b or separating equilibrium). This range widens as the prediction accuracy *α* improves, increasing the likelihood of opting in.
3. Under both settings, the consumer surplus is positive within an intermediate range of *k*, which broadens as *α* increases.

Proposition 7 shows that, with an exogenous price, consumers’ decision to withhold information hinges on the default privacy option. In the opt-out setting, where the default allows data collection, consumers lack incentive to opt out because opting out would lead the firm to send fewer ads across a broader range of parameters, limiting purchase opportunities. Moreover, with exogenous pricing, consumers face no risk of higher prices when they opt in and match well with the product. Conversely, in the opt-in setting, allowing data collection can be advantageous as it enables targeted advertising that offers purchasing opportunities for well-matched products, yielding a positive surplus. This benefit intensifies as the prediction accuracy (*α*) increases, expanding the parameter range where consumers gain a positive surplus as the firm’s predictions improve.
We now compare profit and welfare under the two default privacy policies. Note that the default option only affects consumer behavior if the consumer is indifferent between allowing and not allowing the firm’s data collection. Thus, consumer surplus does not depend on the default option, except through the cognitive cost of switching out of the default option, which we assume to be negligibly positive. However, the default privacy option can affect the firm’s profits. In scenarios such as the semi-$g$ equilibrium, where the conditions $(\mu_c(m_g) < p < \bar{\mu}^U$ and $k < p \cdot \Pr(m_g|s_L))$ hold, consumer surplus is zero regardless of the privacy setting. If consumers had opted out of data collection, a pooling-$\emptyset$ equilibrium would occur, also resulting in zero consumer surplus. Consequently, consumers generally adhere to the default option. However, from a firm’s perspective, the default option significantly influences profitability. Under the semi-$g$ equilibrium with permitted data collection, the firm enjoys positive profits. Conversely, if consumers opt out, leading to a pooling-$\emptyset$ equilibrium, the firm’s profits drop to zero. Therefore, the firm benefits from an opt-out setting that encourages data collection, enhancing both its profits and potentially the social surplus. In contrast, the opt-in setting expands the parameter range where the firm earns zero profit, making it less favorable for the firm. The opt-out setting proves more advantageous by nudging consumers towards allowing data collection, thereby boosting both firm profit and social welfare.

**Corollary 3.** The firm’s profit and social surplus are higher under the opt-out setting than under the opt-in setting.

It is important to note that our model only represents consumers’ privacy concerns via the utility derived from product purchases; it doesn’t incorporate any explicit disutility associated with giving up personal information, receiving ads, or deviating from default privacy settings. Additionally, it does not account for explicit utility gained from data sharing, often a prerequisite for full service access on various platforms. In our current model, incentives for data collection decisions are exclusively tied to the transaction utility of the product in question.\(^{19}\)

\(^{19}\)When consumers derive an additional net disutility $v$ from privacy protection (i.e., refusing data collection), our model’s predictions change. If $v$ is positive, reflecting privacy concerns, consumers are more inclined to avoid data collection. This shifts their behavior away from adhering to the default in the opt-out setting. Under both the opt-out and the opt-in settings, consumers will allow data collection only if the benefit of doing so outweighs the disutility $v$, leading to a narrowed parameter region where data collection is permitted, which contracts further as $v$ increases. Conversely, with a negative $v$ (i.e., full service is contingent upon data sharing), consumers are more likely to permit data collection, deviating from the default in the opt-in setting. This results in an acceptance of data collection across all settings, regardless of $v$’s magnitude.
6 Endogenous Pricing

We have assumed the firm’s price to be exogenously given and shown that strategic mistargeting by the firm increases as prediction accuracy improves. Under this assumption, consumers find it optimal to allow data collection under the more profitable opt-out privacy setting. However, this result critically relies on the exogenous price assumption, which limits the firm’s ability to adjust prices and influence consumers’ expected utility and their data collection opt-in/opt-out decisions. Now, we allow the firm to set prices optimally at the beginning of the game.\textsuperscript{20} Subsequently, the game we have previously analyzed is played as a subgame. A consumer observes the price in the ad if she receives one.

6.1 Pricing equilibrium analysis

In any pure-strategy equilibrium, the strategies of the players—specifically the firm’s advertising strategy and the consumers’ purchasing strategy—are independent of price by definition. Consequently, the firm tends to benefit from a marginal price increase. However, in a semi-separating equilibrium, as outlined in Proposition 5, strategies may vary with price changes, meaning profits do not necessarily rise with price increases. Nonetheless, we demonstrate that in both types of semi-separating equilibria, the firm’s expected profit is continuously (weakly) increasing with price $p$ (see Lemma A-1 in the Appendix). Thus, the advertiser’s profits are continuous across all price points within each equilibrium type. However, profits may change discontinuously at certain boundaries between different equilibrium types, as indicated by Lemma A-2 in the Appendix.

We determine the firm’s optimal pricing strategy by evaluating the firm’s profits across various prices, characterized by three critical thresholds on $k$.

Proposition 8. Let $\beta = \sqrt{\mu^2 - 6\mu + 5 - \mu^{-1}} / 2(1 - 2\mu)$. There exist thresholds $k_1 \leq k_2 \leq k_3$ such that:

1. For $k < k_1$, if $\beta < \beta_1$, the optimal price is $p^* = \mu_c(m_b)$ and it results in the pooling-all equilibrium. Otherwise (i.e., $\beta \geq \beta_1$), the optimal price is $p^* = \mu_c(m_g)$ and it results in the pooling-g equilibrium.

\textsuperscript{20}The firm sets its price prior to observing the signal about individual consumer types $t \in \{H, L\}$. Although price discrimination based on predicted types could be considered, it would inadvertently introduce price as an additional signaling device, complicating the analysis, which is beyond this paper’s scope. Moreover, in real-world scenarios such as sales on platforms like Instagram and Facebook, firms typically set prices before acquiring signals about individual consumer types, aligning our assumption with common practice.
2. If $k_1 \leq k < k_2$, any price $p \in \left[ \max \{ \mu_c(m_b), \frac{k}{\Pr(m_b|s_L)} \}, \mu_\sigma(m_g) \right]$ is optimal, and it results in the semi-$g$ equilibrium.

3. If $k_2 \leq k < k_3$, the optimal price is $p^* = \mu_\sigma(m_g)$ and it results in the separating equilibrium.

4. If $k \geq k_3$, the optimal price is $p^* = \mu_c(m_b)$ and it results in the semi-$b$ equilibrium.\(^{21}\)

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\(^{21}\) If $k$ is very large, i.e., $k \geq k_4 = \max(k_3, \mu_\sigma(m_b))$, the unique equilibrium is pooling-$\emptyset$ equilibrium in which the firm does not send any ads. So, $p^* = \mu_\sigma(m_b)$ is weakly optimal because no transaction occurs irrespective of the price.

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Figure 4: Equilibrium prices (colored in Red)
price and the corresponding equilibrium are provided in the Appendix.

Proposition 8 provides two critical insights. First, a semi-separating equilibrium persists across an intermediate range of cost $k$ ($k_{1} \leq k < k_{2}$), where multiple prices can be optimal for the firm. Under these conditions, any semi-$g$ equilibrium identified with exogenous pricing remains as an equilibrium with endogenous pricing. In these equilibria, the firm engages in strategic mistargeting of consumers it classifies as low-valuation.

Second, for high costs ($k \geq k_{2}$), the firm opts for a high price and fully individualized targeting. When the price is exogenously fixed, the firm responds to higher prediction accuracy by engaging in more mistargeting (Proposition 6). In contrast, when the firm can choose its own price, the advertiser can discipline itself to execute a more precisely targeted advertising campaign by announcing a high price. The firm deploys a more individually targeted advertising strategy to improve the persuasive power of the targeted advertising, which allows it to charge a higher price as prediction accuracy increases. Therefore, advertising serves as a clearer recommendation for consumers who may, unfortunately, face a higher price as $\alpha$ increases under endogenous pricing, as detailed in the following proposition.

**Proposition 9.** If the firm chooses its price optimally,

1. For lower $k$ ($k < k_{1}$), the firm engages in mass advertising. Neither price nor advertising strategy depends on $\alpha$ in equilibrium.

2. For intermediate $k$ ($k_{1} \leq k < k_{2}$), there exist equilibria with strategic mistargeting (i.e., semi-$g$ equilibria).

3. For high $k$ ($k \geq k_{2}$), the firm engages in a maximally individually-targeted advertising, and price increases with $\alpha$.

As $\alpha$ increases, the area for semi-$g$ equilibrium shrinks, while the regions for separating or semi-$b$ equilibria, where the firm engages in maximally individually-targeted advertising, expand. In this sense, a higher $\alpha$ leads to more individually-targeted advertising under endogenous pricing. Moreover, the impact of $\alpha$ on consumer welfare is ambiguous. While consumers may benefit from

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22When $k_{1} \leq k < k_{2}$, we have a semi-$g$ equilibrium where consumers with a good impression $m_{g}$ are indifferent between purchasing and not purchasing, implying consumer surplus is zero. In this case, the firm cannot extract any surplus from changing the price. Thus, any price that makes the consumers indifferent can be an equilibrium price.
the firm’s increasingly individualized advertising, they may concurrently encounter higher prices. The subsequent corollary outlines the dynamics as the firm’s information precision approaches 1. This corollary shows that strategic mistargeting persists in the limit, leading to a consumer surplus that diminishes to zero under a wide range of parameters.

**Corollary 4.** In the limit as \( \alpha \to 1 \), there exists \( k \) such that:

1. If \( k < k \), the equilibrium features mass advertising. The optimal price is \( p^* = \mu_c(m_b) \) for \( \beta < \overline{\beta} \) with the pooling-all equilibrium, and \( p^* = \mu_c(m_g) \) for \( \beta \geq \overline{\beta} \) with the pooling-g equilibrium.

2. If \( \overline{k} \leq k < 1 - \beta \), any \( p^* \in \left[ \frac{k}{1-\beta}, 1 \right] \) is optimal with the semi-g equilibrium if \( k < 1 - \beta \). Otherwise (i.e., \( k \geq \max\{\overline{k}, 1 - \beta\} \)), \( p^* = 1 \).

3. Moreover, consumer surplus approaches 0 in all cases except when \( p^* = \mu_c(m_b) \) under the pooling-all equilibrium (i.e., \( k < \overline{k} \) and \( \beta < \overline{\beta} \)).

Figure 4(b) depicts the limit case as \( \alpha \to 1 \) for \( \beta < \overline{\beta} \). First, the optimal price must approach 1 for \( k \geq \max\{\overline{k}, 1 - \beta\} \) under the semi-b equilibrium, so the consumer welfare approaches 0. Consumer welfare is also 0 under the semi-g equilibrium. Consumer surplus is positive only under the pooling-all equilibrium, where the consumer purchases irrespective of her signal \( m \), and the price is \( p = \mu_c(m_b) \). This equilibrium can arise only when the advertising cost is small (\( k < \overline{k} \)) and the consumer signal is not sufficiently informative (\( \beta < \overline{\beta} \)). If instead \( \beta \geq \overline{\beta} \), the advertiser charges a higher price \( p = \mu_c(m_g) \) in the pooling-g equilibrium, where consumer welfare is again 0.

The welfare implications in Corollary 4 are in stark contrast to those under the exogenous price case in Corollary 2. Under exogenous pricing, despite the firm’s widespread strategic mistargeting, consumer welfare is strictly positive under a wide range of parameters (where semi-separating equilibria exist). In contrast, when the firm optimally sets prices, it generally refrains from “cheating” (except within the semi-g equilibrium region) but instead raises the price to extract all consumer surplus. This provides an important implication for the firm if consumers make their own privacy choice, which is the focus of the next section.
6.2 Personal data opt-out

As in the exogenous pricing case, consumers decide whether to permit the firm’s data collection. For analytical tractability, we focus on the limiting case as $\alpha \to 1$. If a consumer restricts data collection, the firm cannot access her data, resulting in a prediction accuracy of $\alpha = 1/2$. Conversely, if data collection is permitted, the firm’s predictive accuracy is $\alpha \geq 1/2$. We explore two default privacy settings: the “opt-out” setting, which allows data collection by default, and the “opt-in” setting, which restricts it unless explicitly permitted by the consumer.

First, consumers make their privacy decision, with an infinitesimal cost associated with changing the default setting. Then, as outlined in Section 6, the firm sets its pricing strategy, differentiating prices based on consumers’ data sharing choices.\(^{23}\) When consumers forbid data collection, only pooling equilibria (either “pooling-g” or “pooling-all”) occur, as the firm lacks access to signals about consumer types. The optimal price for these consumers is set at either $\mu_c(m_g)$ or $\mu_c(m_b)$.

From Corollary 4, as $\alpha \to 1$, consumer surplus from data collection becomes zero, except in the pooling-all equilibrium. In the opt-out setting, consumers only opt out when the expected surplus from doing so is greater than from allowing data collection. Therefore, consumers opt out if: (1) the surplus under the default option of permitting data collection is zero (seen in pooling-g, semi-g, or semi-b equilibria), and (2) opting out leads to the pooling-all equilibrium, where they gain a positive expected surplus.

Under the opt-in setting, consumers opt into data collection only if the expected surplus from doing so exceeds that under the default setting, which prohibits data collection. A strictly positive surplus from data collection is anticipated only in the pooling-all equilibrium. However, if it is optimal for the firm to mass advertise at a price of $p^* = \mu_c(m_b)$ with access to personal data, the firm would likely adopt the same strategy without access to data. Consequently, scenarios where consumer surplus is higher with data collection than without do not arise.

Proposition 10. In the limit as $\alpha \to 1$,

1. Under the opt-out setting, consumers opt out of data collection for some intermediate range of $k$: $k \in [\bar{k}, \mu_c(m_b)]$.

\(^{23}\)A common reason consumers opt out of data collection is the belief that firms charge lower prices to anonymous consumers (Ichihashi, 2020).
2. Under the opt-in setting, consumers adhere to the default and never opt into data collection.

With exogenous pricing, consumers never opt out of data collection in the opt-out setting, but opt in for an intermediate range of $k$ under the opt-in setting (Proposition 7). In contrast, with endogenous pricing, the dynamics reverse: consumers opt out for an intermediate range of $k$ in the opt-out setting and never opt in under the opt-in setting.

When consumers permit data collection, the range of pooling equilibria contracts as $\alpha$ approaches 1, as shown in Figure 4(b). On the other hand, when consumers opt out, the pooling-all equilibrium dominates a broader parameter range. Specifically, for $k \in [\overline{k}, \mu_c(m_b)]$, consumers who opt out enter the pooling-all equilibrium, securing a price of $p = \mu_c(m_b)$, the unique situation resulting in positive consumer welfare under both default options.

Under endogenous pricing, consumers are more hesitant to consent to data collection compared to exogenous pricing, regardless of the default privacy setting. This difference arises because, with exogenous pricing, prices remain unchanged by consumer privacy decisions, encouraging consumers to permit data collection for more relevant advertising. However, with endogenous pricing, consumers expect higher prices if they allow data collection, prompting them to opt out to secure lower prices at the expense of receiving relevant advertising.

We compare profit and welfare under the two default privacy settings, noting that the default option only impacts consumer surplus if consumers are initially indifferent to data collection choices. As observed in Proposition 10, the default setting influences consumer privacy decisions when $k < \overline{k}$ or $k > \mu_c(m_b)$. For $k < \overline{k}$, a pooling equilibrium emerges regardless of privacy choices, and the firm’s profit remains unaffected by the default setting. In contrast, for $k > \mu_c(m_b)$, allowing data collection leads to a semi-$b$ equilibrium with $p^* \to 1$, resulting in positive firm profits. Without data collection, however, the firm abstains from advertising in equilibrium, resulting in zero profit. Thus, as with exogenous pricing, the firm benefits more from an opt-in default that promotes consumer consent to data collection.

**Corollary 5.** Under endogenous pricing, the firm’s profit and social surplus are higher under the opt-out setting than under the opt-in setting.

Interestingly, there is significant overlap between the parameter region where consumers opt out of data collection under the opt-out setting ($k \in [\overline{k}, \mu_c(m_b)]$) and where mistargeting occurs
\( (k \in [k, 1 - \beta]) \). Depending on the prior \( \mu \), the presence of mistargeting can be either a necessary or sufficient condition for consumers to opt out of data collection.

**Corollary 6.** Suppose the default privacy option is allowing data collection (the opt-out setting).

1. For \( \mu \geq \frac{1}{2} \), if there exists equilibrium with mistargeting (i.e., semi-g with \( 0 < \sigma^*(s_L) < 1 \)), consumers always opt out of data collection.

2. For \( \mu \leq \frac{1}{2} \), if consumers opt out of data collection in equilibrium, there must exist equilibrium with mistargeting (i.e., semi-g with \( 0 < \sigma^*(s_L) < 1 \)).

3. As \( \beta \to 1 \), both the region for opting out and the region for mistargeting disappear.

As the consumer’s signal precision improves (\( \beta \to 1 \)), both \( \mu_c(m_b) = \frac{(1-\beta)\mu}{(1-\beta)\mu + \beta(1-\mu)} \) and \( 1 - \beta \) approach zero. Consequently, the regions where consumers opt out and where mistargeting occurs in equilibrium both vanish. This indicates that both the occurrence of mistargeting and the incentives for consumers to opt out of data collection are dependent on the quality of their signals. This relationship offers valuable managerial insights into consumer privacy choices and potential firm responses, highlighting the complementarity between the advertiser’s predictive and informational provision technologies. As the advertiser’s algorithm improves in accuracy, it should also facilitate better consumer self-assessment of product fit to discourage data opt-outs, which would otherwise undermine the algorithm’s effectiveness. For example, the advertiser could improve the accuracy of the consumers’ own assessment of product fit (\( \beta \)) by providing more informative ads, offering more information upon clicking the ads, or making the consumer’s information search easier.\(^{24}\)

### 7 Conclusion

This study explores a firm’s optimal targeting strategy, considering its ability to predict consumer preferences through various data sources. Being targeted can act as an implicit product recommen-\(^{24}\) As noted in Section 5, our model exclusively captures consumer utility derived from purchasing the product. Introducing a net (dis)utility \( v \) from withholding their personal data modifies model predictions. With a positive \( v \), consumers are more motivated to avoid data collection, deviating from the default in the opt-out setting. According to Proposition 10-2, consumers consistently refuse data collection under any default option, regardless of \( v \)’s magnitude. Conversely, with a negative \( v \), consumers are inclined to permit data collection, moving away from the default in the opt-in setting. Following Proposition 10-1, regardless of the default option, consumers withholding data for an intermediate range of \( k \), which shrinks as \( v \) increases. Comparing these predictions to those under exogenous pricing, we can see that the firm’s ability to set prices based on prediction accuracy makes consumers more likely to withhold their personal data, irrespective of \( v \).
dation by the firm, influencing consumer purchase decisions. However, with increased prediction accuracy, while the recommendation effect of targeted ads is enhanced, it also increases the potential for a misalignment between the firm’s interests and consumer welfare. This paper examines the credibility of targeted advertising by analyzing when it effectively serves as an implicit recommendation, how prediction accuracy and the firm’s pricing decisions influence consumer privacy choices, and how these choices, in turn, affect targeting accuracy.

Under exogenous pricing, being targeted can serve as either a perfect signal (in a separating equilibrium) or a noisy signal (in semi-separating equilibria). Interestingly, as the firm’s prediction accuracy increases, it actually engages in less targeted advertising. Even with perfect prediction capabilities, the firm may intentionally target some consumers incorrectly, leading them to receive ads for products that are not well-suited. Despite the negative implications of the firm’s strategic mistargeting, consumer surplus can still be positive, particularly because higher prediction accuracy enables the firm to more precisely identify consumers with high valuations.

We then demonstrate how the firm’s ability to optimally select its pricing can significantly alter the effects of prediction accuracy on its strategy. Under endogenous pricing, while mistargeting persists, the firm may raise prices when advertising costs are high. This strategy enables the firm to commit to serving the appropriate consumers, rather than diluting the effectiveness of its targeted advertising. However, when the firm can perfectly predict each consumer’s product valuation, the higher prices charged by the firm reduce consumer surplus to zero.

Building on the analysis of consumer surplus, we also examine how different default privacy settings influence consumer decisions on data collection. Under exogenous pricing, consumers always consent to data collection in an opt-out setting and only permit data collection under specific conditions in an opt-in setting. This occurs because pricing remains unaffected by their privacy choices, leading consumers to favor data collection in anticipation of more relevant advertising. An opt-out setting subtly encourages consumers to permit data collection, which inadvertently benefits the firm by improving targeting accuracy, thereby increasing both firm profits and social surplus.

Under endogenous pricing, consumer behavior notably changes. Consumers become more reluctant to permit data collection across all default privacy settings, driven by the anticipation of higher prices post-consent. In an opt-out setting, consumers frequently withdraw their consent within specific parameters, and in an opt-in setting, they consistently decide against permitting
data collection. This strategy is aimed at securing lower prices, though it results in less relevant advertising. In this model, firms find the opt-out default setting more advantageous because it encourages consumers to consent to data collection, facilitating more effective and profitable targeted advertising strategies.

Our analysis aligns with various privacy-related regulations, demonstrating that default settings significantly influence consumer behavior and outcomes. It shows that under both exogenous and endogenous pricing structures, an ‘opt-out’ setting nudges consumers towards permitting data collection. This tendency inadvertently benefits firms by providing more data for targeted advertising, enhancing both firm profits and overall social surplus. By examining and emphasizing the influence of default choices on consumer privacy decisions, our work offers crucial insights for developing policy and regulatory frameworks in data privacy.

In summary, our study sheds light on the relationship between predictive technologies and firm targeting strategies, revealing that mistargeting is driven not only by technological limitations but also by advertisers’ economic incentives. Even with enhanced predictive capabilities, accurate targeting may not be achieved due to these misaligned incentives, which could deter consumer consent for data collection. These findings hold significance for regulators aiming to safeguard consumer welfare amidst increasing data collection and targeted advertising trends. Our research prompts important questions about optimal data collection policies, the availability of consumer data privacy choices, and the appropriateness of differential pricing based on these choices, which are critical considerations from both managerial and regulatory viewpoints. A notable limitation of our model is its assumption of firms’ use of prediction algorithms without exploring their decision-making on investing in or adopting these technologies. Future research can investigate the optimal levels of prediction accuracy and advertisement informativeness, and how these factors interact with current privacy policies.

Funding and Competing Interests

All authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.
Appendix

Proof of Lemma 1

The first part that $\mu(\sigma, m) \geq \mu_c(m)$ if and only if $\sigma(s_H) \geq \sigma(s_L)$ follows directly from Equation (2).

Also, we have $\frac{\partial (\mu - \mu_c)}{\partial \alpha} \geq 0$ if and only if $\frac{\partial \mu}{\partial \alpha} = \frac{\mu_c(1-\mu_c)(\sigma(s_H)^2-\sigma(s_L)^2)}{(\mu_c(\alpha \cdot \sigma(s_H)+(1-\alpha) \cdot \sigma(s_L))+(1-\mu_c)(\alpha \cdot \sigma(s_L)+(1-\alpha) \cdot \sigma(s_H)))^2} \geq 0$

if and only if $\sigma(s_H) \geq \sigma(s_L)$.

Proof of Proposition 1

The steps of the analysis provided in the main text show most of the proof of Proposition 1. It only remains to show $\sigma^*(s_H) = \phi \cdot \sigma^*(s_L)$. It follows directly from Equation (3) such that $p \cdot (\sigma(s_H)(\mu \alpha + (1-\mu)(1-\alpha)) + \sigma(s_L)(\mu(1-\alpha) + (1-\mu)\alpha)) = (\sigma(s_H) \cdot \mu \alpha + \sigma(s_L) \cdot \mu(1-\alpha))$. This simplifies to $\sigma(s_H) = \phi \cdot \sigma(s_L)$, where $\phi = \frac{(1-\mu)(\mu-\mu_c)(1-p)}{\mu_c(\alpha(1-p)-(1-\mu)(1-\alpha)p)}$.

Proof of Proposition 2

Differentiating $\phi$ with respect to $\alpha$: $\frac{\partial \phi}{\partial \alpha} = -\frac{(p-\mu)(\mu(1-p)+(1-\mu)p)}{\mu_c(\alpha(1-p)-(1-\mu)(1-\alpha)p)^2} \leq 0$ because $p \in (\mu, \bar{\mu}(\sigma^*))$. ■

Proof of Corollary 1

The first part follows directly from taking the limit $\lim_{\alpha \to 1} \phi = \frac{(1-\mu)p}{\mu_c(1-p)}$, which is greater than 1 because $p \in (\mu, \bar{\mu}(\sigma^*))$. In the benchmark, $k/p$ fraction of consumers buy the product. ■

Proofs of Lemma 3 and Lemma Lemma 4

Suppose there exists an equilibrium where the firm totally mixes, i.e., $0 < \sigma^*(s_L) \leq \sigma^*(s_H) < 1$. Given consumers' purchase strategy $\delta^*_c(m_g)$ and $\delta^*_c(m_b)$, the firm must be indifferent between sending an ad or not sending an ad for both consumers with $s_H$ and $s_L$. The firm’s indifference conditions for both cases are $p \cdot (\Pr(m_g|s_H) \cdot \delta^*_c(m_g) + \Pr(m_b|s_H) \cdot \delta^*_c(m_b)) - k = 0$ and $p \cdot (\Pr(m_g|s_L) \cdot \delta^*_c(m_g) + \Pr(m_b|s_L) \cdot \delta^*_c(m_b)) - k = 0$, respectively. It is straightforward to see that, however, the left-hand side of the former condition is greater than or equal to the left-hand side of the latter condition. This is because $\Pr(m_g|s_H) > \Pr(m_g|s_L)$ given that $\alpha, \beta > 1/2$. Moreover, as we will show, $\delta^*_c(m_g) \geq \delta^*_c(m_b)$. Therefore, the term in the former condition places a strictly greater weight
on $\delta^*_{\sigma}(m_g)$ (which is weakly greater than $\delta^*_{\sigma}(m_b)$). This shows that the indifference condition cannot hold simultaneously unless $\delta^*_{\sigma}(m_g) = \delta^*_{\sigma}(m_b)$.

If $\delta^*_{\sigma}(m_g) = \delta^*_{\sigma}(m_b)$, the firm’s expected profit is zero because the firm is indifferent between sending an ad or not. Also, for the firm to adopt a totally mixed strategy, consumers must also mix: $\delta^*_{\sigma}(m_g) \in (0, 1)$. A consumer’s indifference condition is $\tilde{\mu}(\sigma, m) - p = 0$ for both $m = m_g$ and $m_b$ as defined in Equation (1) and (2). It is easy to check that $\tilde{\mu}(\sigma, m_g) > \tilde{\mu}(\sigma, m_b)$ because given $\beta > 1/2$, a consumer’s posterior is more positive if her own impression is better. So, if $\delta^*_{\sigma}(m_b) \geq 0$, $\delta^*_{\sigma}(m_{g}) = 1$. Also, if $\delta^*_{\sigma}(m_g) \in (0, 1)$, $\delta^*_{\sigma}(m_b) = 0$ must hold (which proves Lemma 4). So, $\delta^*_{\sigma}(m_g) = \delta^*_{\sigma}(m_b)$ cannot hold unless $\delta^*_{\sigma}(m_g) = \delta^*_{\sigma}(m_b) = 0$, or $= 1$. Moreover, if $\sigma^*(s_L) > 0$, $\sigma^*(s_H) = 1$.

Proof of Proposition 5

(1) For semi-$b$ case, where consumers with $m_b$ is indifferent, the indifference condition for consumers with $m_b$, $\tilde{\mu}(\sigma^*_{semi}, m_b) = \mu_{c} + (1-\alpha)\mu_{c}^* = p$ must hold. This helps us to pin down the firm’s mixing probability for a consumer with $s_L$ is $\sigma^*_{semi-b}(s_L) = \frac{p(1-\mu_c)(1-\alpha)-(1-p)\mu_{c}^*}{(1-p)\mu_{c}+(1-p)\mu_{c}^*}$, where $\mu_{c} = \mu_{c}(m_b)$. Also, $\delta^*_{\sigma}(m_{g}) = 1$ must hold. To ensure the existence of $\sigma^*_{semi-b}(s_L) \in (0, 1)$, it is necessary and sufficient to have $\tilde{\mu}(\sigma^*, m_b) = \mu_{c}(m_b) < p < \tilde{\mu}(\sigma^*, m_{g})$. Also, because the firm adopts $\sigma^*(s_L) < 0$, $Pr(m_g | s_L) = \frac{1}{\tilde{\mu}(m_g | s_L)}$ and $\delta^*_{\sigma^*, m_g}(m_g) \in (0, 1)$. Solving the indifference condition gives us $\delta^*_{\sigma^*, m_g}(m_g) = \frac{k}{p} \cdot \frac{1}{Pr(m_g | s_L)}$, which is less than 1 if and only if $\frac{k}{p} < Pr(m_g | s_L)$. The indifference condition of consumers with $m_g$ is the same as that of consumers with $m_b$ in the previous case, except that their private prior is $\mu_{c} = \mu_{c}(m_g)$. To ensure that $\sigma^*_{semi-g}(s_L) \in (0, 1)$ exists such that $\tilde{\mu}(\sigma^*_{semi-g}(s_L), m_g) = p$, we need $\tilde{\mu}(\sigma^*, m_g) = \mu_{c}(m_g) < p < \tilde{\mu}(\sigma^*, m_{g})$.

Proof of Proposition 6

In the semi-$g$ equilibrium, $\sigma(s_L) = \frac{p(1-\mu_{c}(m_g))(1-\alpha)-(1-p)\mu_{c}(m_g)\alpha}{(1-p)\mu_{c}(m_g)(1-\alpha)-(1-p)\mu_{c}(m_g)\alpha}$. Differentiating it with respect to $\alpha$, we have $-\frac{p^2(1-\mu_{c}(m_g))^2-(1-p)^2\mu_{c}^2(m_g)}{(1-p)\mu_{c}(m_g)(1-\alpha)-(1-p)\mu_{c}(m_g)\alpha}$, which is $\geq 0$ if and only if $(1-p)\mu_{c}(m_g) \leq p(1-\mu_{c}(m_g))$. This condition holds from the assumption that $\mu_{c}(m_g) \leq p$. In the semi-$b$
equilibrium, \( \sigma(s_L) = \frac{p(1-\mu_c(m_b))(1-\alpha) - (1-p)\mu_c(m_b)\alpha}{(1-p)\mu_c(m_b)(1-\alpha) - p(1-\mu_c(m_b))\alpha} \). Differentiating it with respect to \( \alpha \), we have

\[
\frac{p^2(1-\mu_c(m_b))^2 - (1-p)^2\mu_c(m_b)^2}{(1-p)\mu_c(m_b)(1-\alpha) - p(1-\mu_c(m_b))\alpha} \leq 0,
\]

which is \( \geq 0 \) if and only if \( (1-p)\mu_c(m_b) \leq p(1-\mu_c(m_b)) \). This condition holds from the assumption that \( p \geq \mu_c(m_b) \). Since \( \sigma(s_H) = 1 \) in both semi-separating equilibria, this proves that \( |\sigma(s_H) - \sigma(s_L)| \) decreases in \( \alpha \).

**Proof of Corollary 2**

In this region, we have the semi-\( b \) equilibrium in which the firm’s strategy converges to \( \sigma^*(m_g) = 1 \) and \( \sigma^*(s_L) = \frac{1-p}{p} \cdot \frac{\mu(1-\beta)}{(1-\mu)^\beta} \). Also, among those targeted consumers, only consumers with \( m_b \) mix with \( \delta^*_c(m_b) = 1 - \beta + \beta \left( \frac{k}{p} - \frac{p-k}{p} \cdot \frac{\mu(1-\beta)}{(1-\mu)^\beta} \right) \). Thus, a fraction of consumers of type \( t_i = L \) who purchases the product and suffers a utility loss is \( 1 - \beta + \beta \cdot \delta^*_c(m_b) = 1 - \beta + \beta \left( \frac{k}{p} - \frac{p-k}{p} \cdot \frac{\mu(1-\beta)}{(1-\mu)^\beta} \right) \).

The consumer surplus is \( CS = \mu \cdot (\beta + (1-\beta) \cdot \delta^*_c(m_g)) \cdot (1-p) + (1-\mu) \cdot (1-\beta + \beta \cdot \delta^*_c(m_b) \cdot (0-p)) \).

At \( \beta = 1 \), \( CS|_{\beta=1} = \mu(1-p) - (1-\mu)k \), which is \( \geq 0 \) if and only if \( \mu \geq \frac{k}{1+k-p} \). So, if \( \mu \) is above a certain threshold, by continuity, the consumer surplus is positive for \( \beta \) close to 1.

**Proof of Lemma 5**

It follows directly by plugging \( \alpha = 1/2 \) into Proposition 4.

**Proof of Proposition 7**

The first part follows directly from Lemma 5, which shows that the consumer surplus from opting out of data is zero. So, consumers weakly prefer opting in. Second part is also straightforward because if \( k \leq p \cdot \Pr(m_g|s_L) \), either pooling-\( g \) or semi-\( g \) equilibrium exists, each of which has zero consumer surplus. Also, if \( k > p \cdot \Pr(m_g|s_H) \) and \( p > \overline{\mu}(\sigma^*, m_b) \), pooling-\( \emptyset \) is the unique equilibrium, which also gives zero consumer surplus. On the other hand, if \( p \cdot \Pr(m_g|s_L) < k \leq p \cdot \Pr(m_g|s_H) \), either semi-\( b \) or separating equilibrium exists where consumer surplus is positive. Lastly, as \( \alpha \) increases, the region for parameter \( k \) identified above expands, i.e., \( \Pr(m_g|s_L) \) and \( \Pr(m_g|s_H) \) decreases and increases in \( \alpha \), respectively. Also, the boundary for \( p \), i.e., \( \overline{\mu}(\sigma^*, m_b) \), increases in \( \alpha \).

**Proof of Proposition 8**

We prove the proposition in a few following steps. First, we show that the firm’s expected profit under semi-\( b \) equilibrium is weakly increasing in \( p \) (Lemma A-1) and that the firm’s equilibrium
profit is continuous in \( p \) except for some boundaries between different types of equilibrium (Lemma A-2) below. The proofs for these Lemmas are in the Online Appendix Section D.

**Lemma A-1.** Within the semi-b equilibrium, the firm’s profit is increasing in \( p \). However, within the semi-g equilibrium, it is invariant in \( p \).

**Lemma A-2.** Equilibrium profit is continuous in \( p \) except when \( p = \mu_c(m_b) \) and \( k \in (p \cdot \Pr(m_g|s_L), p) \), or \( p = \tilde{\mu}(\sigma^*, m_b) \) and \( k/p \in (\Pr(m_g|s_L), \Pr(m_g|s_H)) \), or \( p = \mu_c(m_g) \) and \( k < p \cdot \Pr(m_g|s_L) \).

Using these two lemmas, we prove Proposition 8. First consider the choices between \( p = \mu_c(m_b) \) and \( p = \mu_c(m_g) \) in the range of \( k \leq \min\{\mu_c(m_g) \cdot \Pr(m_g|s_L), \mu_c(m_b)\} \). Setting \( p = \mu_c(m_b) \) is optimal if and only if: \( \mu_c(m_b) - k \geq \Pr(m_g) \cdot \mu_c(m_g) - k \), or equivalently, \( \frac{\mu_c(1-\beta)}{\mu(1-\beta) + (1-\mu)\beta} \geq \mu \beta \). Note that the right-hand side increases in \( \beta \) and the left-hand side decreases in \( \beta \). Also note that the right-hand side is larger at \( \beta = 1 \) and the left-hand side is larger at \( \beta = 1/2 \). Thus, there exists \( \beta \in (1/2, 1) \) such that \( p = \mu_c(m_b) \) is optimal if and only if \( \beta \leq \beta \). The closed-form solution for \( \beta \) is \( \beta = \frac{\sqrt{\mu^2 - 6\mu + 5} - \mu - 1}{2(1-2\mu)} \). For a given \( \beta \), there are four potential candidates for the optimal price:

1. **Candidate 1:** First, when \( \beta \leq \beta \), \( p = \mu_c(m_b) \) under the pooling-all equilibrium. It can only be optimal for \( k < \mu_c(m_b) \). The firm’s profit in this case is: \( \Pi_1(k) = \Pi_{\text{pool-all}} = \mu_c(m_b) - k \). On the other hand, when \( \beta > \beta \), \( p = \mu_c(m_g) \) under the pooling-g equilibrium. It can only be optimal for \( k < \mu_c(m_g) \cdot \Pr(m_g|s_L) \). The firm’s profit in this case is: \( \Pi_1(k) = \Pi_1(k) = \Pi_{\text{pool-m}_g}(p) = \Pr(m_g) \cdot \mu_c(m_g) - k \).

2. **Candidate 2:** any \( p \in \left[ \frac{k}{\Pr(m_g|s_L)}, \tilde{\mu}(\sigma^*, m_g) \right] \) under the semi-g equilibrium. It can only be optimal for \( \mu_c(m_g) \cdot \Pr(m_g|s_L) \leq k \leq \tilde{\mu}(\sigma^*, m_g) \cdot \Pr(m_g|s_L) \). The firm’s profit in this case is: \( \Pi_2(k) = \Pi_{\text{semi-g}}(p) = \Pr(s_H) \cdot k \cdot \left( \frac{\Pr(m_g|s_H)}{\Pr(m_g|s_L)} - 1 \right) \).

3. **Candidate 3:** \( p = \tilde{\mu}(\sigma^*, m_g) \) under the separating equilibrium. It can only be optimal for \( \tilde{\mu}(\sigma^*, m_g) \cdot \Pr(m_g|s_L) \leq k \leq \tilde{\mu}(\sigma^*, m_g) \cdot \Pr(m_g|s_H) \). The firm’s profit in this case is: \( \Pi_3(k) = \Pi^*(p) = \Pr(m_g|s_H) \cdot \Pr(s_H) \cdot \tilde{\mu}(\sigma^*, m_g) - \Pr(s_H) \cdot k \).

4. **Candidate 4:** \( p = \tilde{\mu}(\sigma^*, m_b) \) under the semi-b equilibrium. It can only be optimal for \( \tilde{\mu}(\sigma^*, m_b) \cdot \Pr(m_g|s_L) \leq k \leq \tilde{\mu}(\sigma^*, m_b) \cdot \Pr(m_g|s_H) \). The firm’s profit in this case is: \( \Pi_4(k) = \Pi_{\text{semi-b}}(p) = \Pr(s_H) \left[ \Pr(m_g|s_H) - \Pr(m_g|s_L) \cdot \frac{\Pr(m_g|s_L)}{\Pr(m_g|s_L)} \right] (\tilde{\mu}(\sigma^*, m_b) - k) \).

Now, we show that given the model primitives \( \mu, \alpha \) and \( \beta \), only one of \( p = \mu_c(m_g) \) (under the pooling-g equilibrium) and \( p = \mu_c(m_b) \) (under the pooling-all) can be the optimal price in the entire
interval for $k \in [0, 1]$. First, if $\mu_c(m_b) < \mu_c(m_g) \cdot \Pr(m_g|s_L)$, then the region where the pooling-all equilibrium can exist is a subset of the region for the pooling-g equilibrium. Moreover, it implies that the profit is greater under the pooling-g equilibrium between the two pooling equilibria because $\mu_c(m_g) \cdot \Pr(m_g|s_L) \leq \mu_c(m_b) \cdot \Pr(m_g)$. Therefore, pooling-all equilibrium is dominated by pooling-g equilibrium in its entire region of existence.

Second, if $\mu_c(m_b) \geq \mu_c(m_g) \cdot \Pr(m_g|s_L)$, there are two sub-cases. Suppose $\mu_c(m_b) \geq \mu_c(m_g) \cdot \Pr(m_g)$. Then, pooling-g equilibrium is dominated by pooling-all equilibrium in its entire region. The only remaining case is $\mu_c(m_g) \cdot \Pr(m_g|s_L) \leq \mu_c(m_b) \leq \mu_c(m_g) \cdot \Pr(m_gB)$. In this case, for $k \leq \mu_c(m_g) \cdot \Pr(m_g|s_L)$, where both pooling equilibria can exist, pooling-g equilibrium dominates. Between the two equilibria, for $k > \mu_c(m_g) \cdot \Pr(m_g|s_L)$, only the pooling-all equilibrium can exist. However, in this region, semi-g equilibrium (Candidate 2 defined above) dominates pooling-all equilibrium. This is because the profit under the pooling-g and semi-g equilibrium coincide precisely at $k = \mu_c(m_g) \cdot \Pr(m_g|s_L)$ and $p = \mu_c(m_g)$. Moreover, the profit of pooling-g equilibrium decreases in $k$ at a higher rate. Therefore, for $k \geq \mu_c(m_g) \cdot \Pr(m_g|s_L)$, the profit of semi-g equilibrium is greater than the profit under the pooling-all equilibrium. This proves that only one of $p = \mu_c(m_g)$ (under the pooling-g equilibrium) and $p = \mu_c(m_b)$ (under the pooling-all) can be the optimal price.

Next, note that regardless of $\beta$, we have $\frac{d\Pi_1}{dk} < \max\{\frac{d\Pi_2}{dk}, \frac{d\Pi_3}{dk}, \frac{d\Pi_4}{dk}\}$. This proves Proposition 8-1. Also, we can have $k_1 < 0$, so that the statement in Proposition 8-1 becomes trivial.

Consider $k \in [\bar{\mu}(\sigma^*, m_b) \cdot \Pr(m_g|s_L), \bar{\mu}(\sigma^*, m_g) \cdot \Pr(m_g|s_L)]$. This is the overlapping region between candidate 2 and candidate 4. Note that $\Pi_2(k) = \bar{\mu}(\sigma^*, m_b) \cdot \Pr(m_g|s_L)) = \Pi_4(k = \bar{\mu}(\sigma^*, m_b) \cdot \Pr(m_g|s_L))$ if $\bar{\mu}(\sigma^*, m_b) \geq \mu_c(m_g)$, and $\Pi_2(k = \bar{\mu}(\sigma^*, m_b) \cdot \Pr(m_g|s_L)) > \Pi_4(k = \bar{\mu}(\sigma^*, m_b) \cdot \Pr(m_g|s_L))$ if $\bar{\mu}(\sigma^*, m_b) < \mu_c(m_g)$. Thus, $\Pi_2(k = \bar{\mu}(\sigma^*, m_b) \cdot \Pr(m_g|s_L)) \geq \Pi_4(k = \bar{\mu}(\sigma^*, m_b) \cdot \Pr(m_g|s_L))$ in this range. In addition, we have $\frac{d\Pi_4}{dk} > 0 > \frac{d\Pi_2}{dk}$. Thus, either candidate 1 or candidate 2 is optimal in this range. Let $\bar{k}_2 = \bar{\mu}(\sigma^*, m_g) \cdot \Pr(m_g|s_L)$ if $\bar{k}_1 < \bar{\mu}(\sigma^*, m_g) \cdot \Pr(m_g|s_L)$, and let $\bar{k}_2 = \bar{k}_1$ if $\bar{k}_1 \geq \bar{\mu}(\sigma^*, m_g) \cdot \Pr(m_g|s_L)$. This proves Proposition 8-2. Also, if $\bar{k}_1 = \bar{k}_2$, the statement in Proposition 8-2 becomes trivial.

Finally, consider the choice between candidate 3 and candidate 4. Note that $\frac{d\Pi_3}{dk} < \frac{d\Pi_4}{dk}$ because $\left[\Pr(m_g|s_H) \cdot \Pr(m_b|s_H) \cdot \Pr(m_g|s_L)\right] < 1$. Thus, $\Pi_3$ and $\Pi_4$ have a single crossing. Note that from the above we have $\Pi_3(k = \bar{\mu}(\sigma^*, m_g) \cdot \Pr(m_g|s_L)) = \Pi_2(k = \bar{\mu}(\sigma^*, m_g) \cdot \Pr(m_g|s_L)) > \bar{\mu}(\sigma^*, m_g) \cdot \Pr(m_g|s_L)$, and thus the crossing happens at some $k$ above $\bar{\mu}(\sigma^*, m_g) \cdot \Pr(m_g|s_L))$. Letting $\bar{k}_3$
denote this point of crossing proves Proposition 8-3 and 4. Also, if $k_3 > \bar{\mu}(\sigma^*, m_b)$, the statement in 8-4 is trivial. There is no advertising under any price, so all prices are weakly optimal.

\section*{Proof of Corollary 4}

Consider the limiting case $\alpha \to 1$ of the four optimal price candidates in the proof of Proposition 8.

1. \textit{Limit of Candidate 1 (when }$\beta \leq \beta$\textit{): } $p = \mu_c(m_b)$. It can only be optimal for $k < \mu_c(m_b)$. Let $\hat{\Pi}_1(k)$ denote the firm’s profit in this case, we have: $\hat{\Pi}_1(k) = \mu_c(m_b) - k$. On the other hand, when $\beta > \beta$, $p = \mu_c(m_g)$. It can only be optimal for $k < (1 - \beta)\mu_c(m_g)$. Let $\Pi_1(k)$ denote the firm’s profit in this case, we have: $\hat{\Pi}_1(k) = \Pr(m_g) \cdot \mu_c(m_g) - k$.

2. \textit{Limit of Candidate 2: } any $p \in \left[\max \{\mu_c(m_g), \frac{k}{1-\beta}\}, 1\right]$. It can only be optimal for $k \leq 1 - \beta$. Let $\hat{\Pi}_2(k)$ denote the firm’s profit in this case, we have: $\hat{\Pi}_2(k) = \mu \cdot k \cdot \frac{2\beta-1}{1-\beta}$.

3. \textit{Limit of Candidate 3: } $p = 1$. It can only be optimal for $k \geq 1 - \beta$. Let $\hat{\Pi}_3(k)$ denote the firm’s profit in this case, we have: $\hat{\Pi}_3(k) = \mu \cdot (\beta - k)$.

4. \textit{Limit of Candidate 4: } $p = 1$. It can only be optimal for for $k \geq 1 - \beta$. Let $\hat{\Pi}_4(k)$ denote the firm’s profit in this case, we have: $\hat{\Pi}_4(k) = \mu \cdot \frac{2\beta-1}{\beta} \cdot (1 - k)$.

Note that the limit of Candidates 3 and 4 have the same range ($k \geq 1 - \beta$), and one can confirm that $\hat{\Pi}_4(k) \geq \hat{\Pi}_3(k)$ for $k \geq 1 - \beta$, so we can ignore candidate 3 in the limit. Because $\frac{d\Pi_3}{dk} < \frac{d\Pi_4}{dk}$ and $\frac{d\Pi_2}{dk} < \frac{d\Pi_4}{dk}$, if candidate 1 is optimal for $k$, then candidate 1 must be optimal for all $k' < k$. This proves the existence of $\bar{k}$. If $\beta > \beta$, then $\bar{k} = (1 - \beta) \cdot \mu_c(m_g)$. For $\beta \leq \beta$, to get the value of $\bar{k}$, we find the value of $k$ where $\hat{\Pi}_1(k)$ crosses $\hat{\Pi}_2(k)$ for $k \leq 1 - \beta$ or $\hat{\Pi}_3(k)$ for $k \geq 1 - \beta$, which is $\bar{k} = \max \left\{ \frac{(1-\beta)\mu_c(m_b)}{(1-\beta)+\mu_c(m_b)}, \frac{\beta \mu_c(m_b) - \mu_c(m_g)}{\beta - \mu_c(m_g)} \right\}$. We can see that as $\beta \to \frac{1}{2}$, $\mu_c(m_b) \to \mu$, and so $\bar{k} \to \mu$. Thus, we must have $\bar{k} < 1 - \beta$ for $\mu < \frac{1}{2}$ and $\beta$ sufficiently close to $\frac{1}{2}$. This concludes the proof.

\section*{Proof of Proposition 10}

Consumers opt out if and only if consumer surplus becomes strictly higher by doing so. From Lemma 5, the optimal price under opt-out can only be $p = \mu_c(m_g)$ or $p = \mu_c(m_b)$ and consumer surplus is 0 under $p = \mu_c(m_g)$. From Corollary 4, consumer surplus is positive only when price is $p = \mu_c(m_b)$. Thus, combining Corollary 4 and Lemma 5 produces the range of $k$ in the first part of the proposition. The second part is also trivial. When $\beta \to 1$, $\mu_c(m_b) = \frac{(1-\beta)\mu}{(1-\beta)\mu + \beta(1-\mu)} \to 0$. Thus, the consumer opt-out region disappears.
References


A.1 When α is private

Suppose α is drawn from a distribution, $F(\alpha)$, with a mean of $\bar{\alpha}$. The firm observes the realization of $\alpha$, but consumers only know the distribution $F(\alpha)$. Let $\sigma_\alpha$ denote the firm’s advertising strategy when its prediction accuracy is $\alpha$. Let $\bar{\sigma}_\alpha$ denote consumers’ belief of the firm’s advertising strategy when its prediction accuracy is $\alpha$.

Following the same rationale as presented in the benchmark, there cannot be a pure strategy equilibrium with a positive probability of transaction. If the posterior belief is strictly above the price, then consumers should never buy, and the firm should never advertise, which means that the posterior belief should be lower than the price. If the posterior belief is strictly below the price, then consumers should never buy, and the firm should never advertise.

In a mixed equilibrium, the posterior belief should be exactly at the price, and consumers mix such that the firm is indifferent between sending an ad and no ad. The expected payoff from sending an ad $\mathbb{E}(A = a|s) = \delta_c(p - k) + (1 - \delta_c)(-k)$ must be the same as not sending an ad $\mathbb{E}(A = \emptyset|s) = 0$. Therefore, we have that $\delta_c = k/p$, same as in the benchmark.

Consumers’ posterior belief can be written as the following by adopting equation (3):

$$
\bar{\mu}(\bar{\sigma}_\alpha) = \frac{\int_0^1 \bar{\sigma}(s_H) \cdot \mu\bar{\alpha} + \bar{\sigma}(s_L) \cdot \mu(1 - \alpha) \, dF(\alpha)}{\int_0^1 \bar{\sigma}(s_H) (\mu\bar{\alpha} + (1 - \mu)(1 - \alpha)) + \bar{\sigma}(s_L) (\mu(1 - \alpha) + (1 - \mu)\alpha) \, dF(\alpha)}
$$

(1)

Focusing the subset of strategies where $\sigma_\alpha$ and $\bar{\sigma}_\alpha$ are invariant to $\alpha$, we can write the above equation as

$$
\bar{\mu}(\bar{\sigma}) = \frac{\bar{\sigma}(s_H) \cdot \mu\bar{\alpha} + \bar{\sigma}(s_L) \cdot \mu(1 - \bar{\alpha})}{\bar{\sigma}(s_H) (\mu\bar{\alpha} + (1 - \mu)(1 - \bar{\alpha})) + \bar{\sigma}(s_L) (\mu(1 - \bar{\alpha}) + (1 - \mu)\bar{\alpha})}
$$

(2)

Note that this is the same as equation (3) with $\bar{\sigma}$ replacing $\alpha$.

For consumers to mix, we must have $\bar{\mu}(\bar{\sigma}) = p$. This implies the equilibrium strategy satisfies

$$
\sigma^*_H = \tilde{\sigma} \cdot \sigma^*_L, \text{ where } \tilde{\sigma} = \frac{\mu(1 - \mu)p - (1 - \bar{\alpha}) \mu(1 - p)}{\mu(1 - \mu) - (1 - \bar{\alpha})(1 - \mu)p} > 1.
$$

Thus, any equilibrium of the benchmark is also an equilibrium when $\alpha$ is the firm’s private information, as long as the expected prediction accuracy remains the same. There can also exist other equilibria, analysis of which is beyond the scope of this paper.

A.2 When $k$ is private:

Now we consider the case where consumers are uncertain about the firm’s advertising cost, $k$. Let $k$ be a random variable drawn from the set $\{k_1, k_2, ..., k_N\}$, where $k_1 > k_2 > ... > k_N$. Let $\gamma_n$ denote the probability that the cost is $k_n$, for $1 \leq n \leq N$. Only the firm observes the realized $k$.

Let $\sigma_k$ denote the firm’s advertising strategy when its cost is $k_n$. Let $\bar{\sigma}_n$ denote consumers’ belief of the firm’s advertising strategy when its cost is $k_n$. 
Following the previous rationale, a pure strategy equilibrium with a positive probability of transaction cannot exist.

For the firm to mix, the expected payoff from sending an ad $\mathbb{E}\Pi(A = a|s) = \delta_c(p - k) + (1 - \delta_c)(-k)$ must be the same as not sending an ad $\mathbb{E}\Pi(A = \emptyset|s) = 0$. If consumers buy with probability $\delta_c = k_n/p$, then the firm mixes under cost $k_n$, while the firm never sends an ad regardless of its predictions for $k > k_n$ and always sends ads regardless of its predictions for $k < k_n$.

For consumers to mix, the posterior belief should be exactly at the price. Consumers’ posterior belief can be written as the following by adopting equation (3):

$$
\bar{\mu}(\sigma_n) = \frac{\sum_n[\bar{\sigma}_n(s_H) \cdot \mu \alpha + \bar{\sigma}_n(s_L) \cdot \mu (1 - \alpha)] \gamma_n}{\sum_n[\bar{\sigma}_n(s_H)(\mu \alpha + (1 - \mu)(1 - \alpha)) + \bar{\sigma}_n(s_L)(\mu (1 - \alpha) + (1 - \mu)\alpha)] \gamma_n}
$$

(3)

It is then straightforward to check that $\delta_c = k_N/p$ and $\sigma^*_N(s_H) = \phi \cdot \sigma^*_N(s_L)$ constitute an equilibrium, where $\phi$ is the same as in Proposition 1. This equilibrium is analogous to the benchmark equilibrium, with $k_N$ replacing $k$.

In this equilibrium, the firm only advertises at the lowest cost, i.e., $\sigma^*_n(s) = 0$ for all $n < N$. There can also exist an equilibrium where the firm mixes under cost $k_n$ for any $n$ if $\gamma_n$ is sufficiently large. A full analysis is beyond the scope of this paper.

B Effects of advertising cost $k$ on the firm’s equilibrium profit

It is straightforward that the firm’s profit decreases in $k$ in any pooling and separating equilibrium. This is because in each of these equilibria, the firm’s advertising amount and demand are independent of $k$. Thus, a higher $k$ simply means a greater amount of advertising expenditure.

As for semi-separating equilibria, recall that the firm’s profit under semi-$b$ and semi-$g$ equilibrium are $\Pi^\text{semi-}b = \Pr(s_H)\left(\Pr(m_g|s_H) - \Pr(m_b|s_H) \cdot \frac{\Pr(m_g|s_H)}{\Pr(m_b|s_H)}\right)(p - k)$ and $\Pi^\text{semi-}g = \Pr(s_H) \cdot k \cdot (\frac{\Pr(m_g|s_H)}{\Pr(m_b|s_H)} - 1)$. Thus, the firm’s profit decreases in $k$ under the semi-$b$ equilibrium but increases under the semi-$g$ equilibrium. This variation in outcomes stems from the differences in consumers’ likelihood to purchase, which is influenced by their initial perceptions of the advertisement (either $m_g$ or $m_b$). In the semi-$g$ where consumers with $m_g$ mix, and those are more likely to be targeted and hence to purchase, amplifying the indirect positive effect on profits as $k$ increases (since they purchase with a higher probability). On the other hand, in the semi-$b$, consumers with $m_b$ mix, and are less likely to be targeted, resulting in an indirect positive effect on profits that is not sufficiently large enough to counterbalance the direct costs associated with $k$. In that sense, the indirect positive effect in semi-$g$ equilibrium is strong enough that the firm’s equilibrium profit increases in $k$ but not in semi-$b$ equilibrium.

C Effects of targeting accuracy $\alpha$ on the firm’s equilibrium profit

The firm’s profit under the pooling equilibria (pooling-all, pooling-$g$ or pooling-$\emptyset$) does not depend on $\alpha$ because the firm does not use the firm’s signal about individual consumers. In the separating equilibrium, the firm’s profit increases in $\alpha$ because the firm’s signal becomes more precise without engaging in mistargeting.

For the remaining semi-separating equilibria, we differentiate the profit functions from above with respect to $\alpha$ one by one.
First, under the semi-\(g\) equilibrium,

\[
\frac{\partial \Pi^{\text{semi-}g}}{\partial \alpha} = k \left( \frac{\partial \Pr(s_H)}{\partial \alpha} \cdot \left( \frac{\Pr(m_g | s_H)}{\Pr(m_g | s_L)} - 1 \right) + \Pr(s_H) \cdot \frac{\partial}{\partial \alpha} \left( \frac{\Pr(m_g | s_H)}{\Pr(m_g | s_L)} \right) \right) \\
= k \cdot \mu \cdot \frac{(1 - \mu)(2\beta - 1)(1 - \beta + \mu(2\beta - 1))}{(\mu(\alpha - \beta) + \alpha(1 - \beta))^2} > 0.
\]

So, the firm’s profit is monotonically increasing in \(\alpha\) under the semi-\(g\) equilibrium.

Second, under the semi-\(b\) equilibrium,

\[
\frac{\partial \Pi^{\text{semi-}b}}{\partial \alpha} = (p - k) \left( \frac{\partial \Pr(s_H)}{\partial \alpha} \cdot \left( \Pr(m_g | s_H) - \Pr(m_b | s_H) \cdot \frac{\Pr(m_g | s_L)}{\Pr(m_b | s_L)} \right) \\
+ \Pr(s_H) \cdot \frac{\partial}{\partial \alpha} \left( \Pr(m_g | s_H) - \Pr(m_b | s_H) \cdot \frac{\Pr(m_g | s_L)}{\Pr(m_b | s_L)} \right) \right) \\
= (p - k) \mu(1 - \mu)(2\beta - 1)(\mu + \beta - 2\mu \beta) \left( \frac{\alpha(\beta - \mu) + \mu(1 - \beta)^2}{\alpha(\beta - \mu) + \mu(1 - \beta)^2} \right),
\]

which is greater than zero if and only if \(\mu \leq 1/2\) or \(\mu > 1/2\) and \(\beta > \frac{\mu}{2\mu - 1}\). Otherwise, if \(\mu > 1/2\) and \(\beta > \frac{\mu}{2\mu - 1}\), we have \(\frac{\partial \Pi^{\text{semi-}b}}{\partial \alpha} < 0\). Under the semi-\(b\) equilibrium, the firm’s profit comes from two segments of the targeted consumers: consumers whose private impression of the ad is positive \((m_g)\) who buy the product with probability one and those whose impression is negative \((m_b)\) who purchase the product with a probability less than one. The profit from the former increases in \(\alpha\) because the firm can better identify consumers with a good fit for the product. However, the profit from the latter can decrease in \(\alpha\) because the probability with which the latter consumers buy decreases in \(\alpha\). After receiving a bad signal from the ad, especially when \(\beta\) is large, the consumer further infers that the firm’s private signal about her could have been bad.

D Lemma A-1 and Lemma A-2 in the Proof of Proposition 8

**Lemma A-1.** Within the semi-\(b\) equilibrium, the firm’s profit is increasing in \(p\). However, within the semi-\(g\) equilibrium, it is invariant in \(p\).

**Proof.** In both types of semi-separating equilibria, the firm is indifferent between sending and not sending an ad to a consumer with \(s_L\). This implies that the firm’s expected profit from this consumer is zero. Thus, it suffices to compute the expected profit from consumers with \(s_H\).

In the “semi-\(b\)” equilibrium, a consumer’s purchase decision is described by \(\delta^{\text{semi-}b}_c(m_g) = 1\) and \(\delta^{\text{semi-}b}_c(m_b) = \frac{k}{p} - (1 - \frac{k}{p}) \cdot \frac{\Pr(m_g | s_L)}{\Pr(m_b | s_L)}\). Therefore, we can write the advertiser’s profit: \(\Pi^{\text{semi-}b}(p) = \Pr(s_H) \left[ \Pr(m_g | s_H) + \Pr(m_b | s_H) \cdot \delta^{\text{semi-}b}_c(m_b) \right] \cdot p - \Pr(s_H) \cdot k = \Pr(s_H) \left[ \Pr(m_g | s_H) - \Pr(m_b | s_H) \cdot \frac{\Pr(m_g | s_L)}{\Pr(m_b | s_L)} \right] \cdot (p - k)\) which strictly increases in \(p\) due to a positive correlation between \(s\) and \(m\).

In the “semi-\(g\)” equilibrium, a consumers’ purchasing strategies are \(\delta^{\text{semi-}g}_c(m_g) = \frac{k}{p} \cdot \frac{1}{\Pr(m_g | s_L)}\) and \(\delta^{\text{semi-}g}_c(m_b) = 0\). So, we can write the advertiser’s profit: \(\Pi^{\text{semi-}g}(p) = \Pr(s_H) \cdot \Pr(m_g | s_H) \cdot \delta^{\text{semi-}g}_c(m_g) \cdot p - \Pr(s_H) \cdot k = \Pr(s_H) \cdot k \cdot \left( \frac{\Pr(m_g | s_H)}{\Pr(m_g | s_L)} - 1 \right)\) which is invariant in \(p\). Thus, the firm’s profit for semi-separating equilibrium is continuously (weakly) increasing in \(p\). \(\square\)
Lemma A-2. Equilibrium profit is continuous in $p$ except when $p = \mu_c(m_b)$ and $k \in (p \cdot \Pr(m_g|s_L), p)$, or $p = \bar{\mu}(\sigma^*, m_b)$ and $k/p \in (\Pr(m_g|s_L), \Pr(m_g|s_H))$, or $p = \mu_c(m_g)$ and $k < p \cdot \Pr(m_g|s_L)$.

**Proof.** Under exogenous pricing, multiple equilibria can exist on the boundary between different types of equilibrium. We describe here the set of equilibria on each boundary. We focus only for the case of $\alpha$ sufficiently high such that $\bar{\mu}(\sigma^*, m_b) > \mu_c(m_g)$.

**Case 1. Boundary between Pooling-g and semi-g.** Consider $p = \mu_c(m_g)$ and $k < p \cdot \Pr(m_g|s_L)$. The limit of the pooling-g equilibrium as $p$ approaches $\mu_c(m_g)$ from below is $\sigma^{\text{pool-g}} = (\sigma(s_H) = 1, \sigma(s_L) = 1)$ and $\delta_c^{\text{pool-g}} = (\delta_c(m_g) = 1, \delta_c(m_b) = 0)$ from Proposition 4. The limit of the semi-b equilibrium as $p$ approaches $\mu_c(m_g)$ from above is $\sigma^{\text{semi-b}} = (\sigma(s_H) = 1, \sigma(s_L) = 1)$ and $\delta_c^{\text{semi-b}} = (\delta_c(m_g) = 1, \delta_c(m_b) = 0)$ from Proposition 5. Note that $\delta_c^{\text{semi-b}}(m_g) < 1$ for $k < p \cdot \Pr(m_g|s_L)$.

Thus, we have $\sigma = (\sigma(s_H) = 1, \sigma(s_L) = 1)$ and $\delta_c = (\delta_c(m_g), \delta_c(m_b) = 0)$ is an equilibrium for any $\delta_c(m_g) \in \left[\frac{k}{p} \cdot \frac{1}{\Pr(m_g|s_L)} \right]$, 1. Because the firm’s profit increases in $\delta_c(m_g)$, it is maximized by the pooling-g equilibrium at this boundary. There is a discontinuous drop in the firm’s profit as we move from the pooling-g to the semi-g equilibrium, as $\delta_c(m_g)$ drops discontinuously from 1 to $\frac{k}{p} \cdot \frac{1}{\Pr(m_g|s_L)}$.

**Case 2. Boundary between Pooling-g and semi-b.** Consider $p \in [\mu_c(m_b), \mu_c(m_g)]$ and $p = \frac{k}{\Pr(m_g|s_L)}$. The limit of the pooling equilibrium is $\sigma^{\text{pool-g}} = (\sigma(s_H) = 1, \sigma(s_L) = 1)$ and $\delta_c^{\text{pool-g}} = (\delta_c(m_g) = 1, \delta_c(m_b) = 0)$ from Proposition 4. The limit of the semi-separating equilibrium is $\sigma^{\text{semi-b}} = (\sigma(s_H) = 1, \sigma(s_L) = 1)$ and $\delta_c^{\text{semi-b}} = (\delta_c(m_g) = 1, \delta_c(m_b) = 0)$ from Proposition 5. Note that $\sigma^{\text{semi-b}}(s_L)$ must be lower than 1 for consumers with a bad impression to be indifferent. Thus, we have that $\sigma = (\sigma(s_H) = 1, \sigma(s_L))$ and $\delta_c = (\delta_c(m_g) = 1, \delta_c(m_b) = 0)$ is an equilibrium for any $\sigma(s_L) \in \left[\frac{p(1-\mu_c(m_b)(1-\alpha)-(1-p)\mu_c(m_g)\alpha)}{(1-p)\mu_c(m_b)(1-\alpha)-(1-p)\mu_c(m_g)\alpha}, 1\right]$. The firm’s profit is the same for all equilibria because the advertiser is indifferent to target consumers with $s_L$.

**Case 3. Boundary between Pooling-g and Separating.** Consider $p \in [\bar{\mu}(\sigma^*, m_b), \mu_c(m_g)]$ and $p = \frac{k}{\Pr(m_g|s_L)}$. The limit of the pooling equilibrium is $\sigma^{\text{pool-g}} = (\sigma(s_H) = 1, \sigma(s_L) = 1)$ and $\delta_c^{\text{pool-g}} = (\delta_c(m_g) = 1, \delta_c(m_b) = 0)$ from Proposition 4. The limit of the separating equilibrium is $\sigma^{\text{sep}} = (\sigma(s_H) = 1, \sigma(s_L) = 1)$ and $\delta_c^{\text{sep}} = (\delta_c(m_g) = 1, \delta_c(m_b) = 0)$ from Proposition 5. The limit of the semi-separating equilibrium is $\sigma^{\text{semi-b}} = (\sigma(s_H) = 1, \sigma(s_L) = 1)$ and $\delta_c^{\text{semi-b}} = (\delta_c(m_g), \delta_c(m_b) = 0)$.

5. **Boundary between semi-g and Separating.** Consider $p \geq \bar{\mu}(\sigma^*, m_b)$ and $p = \frac{k}{\Pr(m_g|s_L)}$. The limit of the separating equilibrium is $\sigma^* = (\sigma(s_H) = 1, \sigma(s_L) = 0)$, and $\delta_c^{\text{sep}} = (\delta_c(m_g) = 1, \delta_c(m_b) = 0)$ from Proposition 3. The limit of the semi-separating equilibrium is $\sigma(s_H) = 1$, and $\sigma(s_L) =$
Consider \( p = \tilde{\mu}(\sigma^*, m_b) \) and \( k/p \in [\Pr(m_g | s_L), \Pr(m_g | s_H)] \).

The limit of the pooling-all equilibrium is \( \sigma_{\text{pool-all}} = (\sigma(s_H) = 1, \sigma(s_L) = 1) \), and \( \delta_c^{\text{pool-all}} = (\delta_c(m_g) = 1, \delta_c(m_b) = 1) \) from Proposition 4. The limit of the pooling-g equilibrium is \( \sigma_{\text{pool-g}} = (\sigma(s_H) = 1, \sigma(s_L) = 1) \), and \( \delta_c^{\text{pool-g}} = (\delta_c(m_g) = 1, \delta_c(m_b) = 0) \) from Proposition 4. Note that when \( p = \mu_c(m_b) \) and \( \delta_c = (\delta_c(m_g) = 1, \delta_c(m_b) = 0) \), consumers with a bad impression are indifferent between buying and not buying. Thus, \( \sigma = (\sigma(s_H) = 1, \sigma(s_L) = 1) \), and \( \delta_c = (\delta_c(m_g) = 1, \delta_c(m_b) = 0) \) is an equilibrium for any \( \delta_c(m_b) \in [0, \frac{k/p - \Pr(m_g | s_L)}{Pr(m_b | s_L)}] \). The firm’s profit strictly increases in \( \delta_c(m_b) \) and is maximized at the pooling-all equilibrium. There is a discontinuous drop in profit as \( p \) increases above \( \mu_c(m_b) \).

8. **Boundary between pooling-all and semi-b.** Consider \( p = \mu_c(m_b) \) and \( k \in [p \cdot \Pr(m_g | s_L), p] \).

The limit of the pooling-all equilibrium is \( \sigma_{\text{pool-all}} = (\sigma(s_H) = 1, \sigma(s_L) = 1) \), and \( \delta_c^{\text{pool-all}} = (\delta_c(m_g) = 1, \delta_c(m_b) = 1) \) from Proposition 4. The limit of the semi-separating equilibrium is \( \sigma_{\text{semi-b}} = (\sigma(s_H) = 1, \sigma(s_L) = 1) \) and \( \delta_c^{\text{semi-b}} = (\delta_c(m_g) = 1, \delta_c(m_b) = \frac{k/p - \Pr(m_g | s_L)}{Pr(m_b | s_L)} \) from Proposition 5. Note that \( \delta_c(m_b) = \frac{k/p - \Pr(m_g | s_L)}{Pr(m_b | s_L)} < 1 \) for \( k < p \). Thus, \( \sigma = (\sigma(s_H) = 1, \sigma(s_L) = 1) \), and \( \delta_c = (\delta_c(m_g) = 1, \delta_c(m_b)) \) is an equilibrium for any \( \delta_c(m_b) \in [\frac{k/p - \Pr(m_g | s_L)}{Pr(m_b | s_L)}, 1] \). Profit increases in \( \delta_c(m_b) \) and is maximized at the pooling-all equilibrium. There is a discontinuous drop in profit as \( p \) increases above \( \mu_c(m_b) \).

9. **Upper Boundary of semi-g** Consider \( p = \tilde{\mu}(\sigma^*, m_g) \) and \( k \leq p \cdot Pr(m_g | s_L) \). The limit of the separating equilibrium is \( \sigma^s = (\sigma(s_H) = 1, \sigma(s_L) = 0) \), and \( \delta_c^s = (\delta_c(m_g) = \frac{k}{p \cdot Pr(m_g | s_L)}, \delta_c(m_b) = 0) \) from Proposition 5.

Combining the above analyses (cases 1-9) proves the lemma.