A Model of Product Portfolio Design: Guiding Consumer Search Through Brand Positioning

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Abstract. We investigate a firm’s optimal product portfolio design on a Hotelling line that can affect consumers’ search decisions. Consumers form their perceptions of a brand from interactions with all products in the portfolio. We conceptualize the average location of the products as the brand position that represents the aggregate information about characteristics common to the product portfolio. Then, we propose a mechanism for why and how brand positioning induced by a firm’s product portfolio design can deliver credible information that guides consumer search. We show that niche positioning naturally conveys more information than mainstream positioning. A mainstream brand has incentives to opportunistically dilute its brand by offering a wide range of products. Even in a monopolistic market, a niche brand positioning may arise as an equilibrium because it serves as a commitment device that prevents brand dilution.

1. Introduction

A product portfolio is a compilation of all the products under the same brand name. How a firm chooses its product offerings can affect consumers’ perception of the brand’s overall image and, ultimately, its brand positioning. Although the advantages of honing a firm’s unique position in the market may seem obvious and well acknowledged, it is easy to take brand positioning for granted. It has rarely been the subject of a rigorous analysis of how a firm’s product portfolio design influences its brand positioning, which ultimately affects consumers’ choices. 

Marketing literature defines brand positioning as the way in which consumers perceive the brand (Kotler et al. 2014) and the overall view that consumers have of a brand—a view that is often formed by a unique bundle of associations in the minds of the target customers (Avery and Gupta 2014). In other words, brand positioning is the particular place in consumers’ minds that a firm seeks to own. Consumers form their perception of a brand from various interactions with the brand’s general line of products (that is, several different products under the same brand), which taken together, identify and refine a brand’s distinctiveness.

A consumer who owns Gucci products, for instance, may have in mind an overall image of Gucci as a brand that carries fashionable and sensual handbags and shoes. This perception is formed by the consumer’s experiences with Gucci’s products, and critically, this image of the brand will affect the consumer’s future apparel purchase decisions. When instead considering Hermes, which is another popular luxury brand, the consumer may have an entirely different brand image given the way the company has positioned itself: timeless elegance and classic luxury. Both Gucci and Hermes keep their brand positions and identities running consistently throughout their product portfolio. By doing so, Gucci and Hermes have effectively differentiated themselves from other luxury brands (Yoffie and Kwak 2001). Consumers who are interested in fashion luxury will seek out Gucci for their next spring collection without foreknowledge of the exact design or style of each individual product. Consumers who prefer classic styles may search for Hermes first.

Brand positioning provides critical information about the characteristics of a firm’s products, making it easier for consumers to locate their preferred products and choose where to purchase without blindly searching through several brands to find the right one for them. Suppose that a consumer wants to buy a new jacket for

She may not like what we’ve made that following season, but she’ll give us the first look because she is a customer of ours, and it’s our job to make sure we know what she’ll like, and give her some of that.

—Stuart Weitzman, the founder of Stuart Weitzman
the current fall season. Where does she start her shopping journey? She may go to her favorite brand first, even though she does not know the exact design or style of the jackets offered in this season. This suggests that brand images provide important information that guides the consumer’s search. She could potentially find something she likes in other brands’ stores that she decided not to visit. The main reason she did not visit those stores is that she has limited time and energy—in other words, search costs. This has also been recognized by marketers, as indicated in our opening quote by Stuart Weitzman. It is not easy to communicate to consumers what they can expect from a brand without a clear positioning. Brand positioning can thus serve as an efficient communication device that invites consumers to search and patronize.

To formalize this idea, we begin by studying a firm’s optimal product portfolio design and illuminate the relationship between the firm’s product portfolio design and brand positioning. Our view is that brand positioning is not merely about product positioning. Brand positioning is the overall image of a brand in a consumer’s mind. In other words, it is aggregate information about characteristics common to products under the same brand name. Therefore, a key feature of our model is that consumers form their perception of a brand’s position from interactions with different products under the same brand instead of any single product of the firm. To capture this intricate nature of brand positioning as aggregate information that is determined collectively by all of its products, we conceptualize the brand position as the average location of all products under the brand on a Hotelling line. Consumers are uncertain about the location of each product in the product portfolio, but they are aware of the average location of the brand’s products—that is, its brand position. Based on this information, consumers can decide whether to search for a specific brand by visiting the store. Therefore, a brand’s position can convey crucial information that guides the consumer’s search decisions.

We build a micromodel to examine how and why brand positioning can deliver credible information to consumers and study its implications on a firm’s optimal product portfolio design, which brings forth two specific brand positioning strategies: “niche” or “mainstream” positioning. We first consider a monopolist of two products that are substitutes. The firm chooses the location of its two products on the Hotelling line on which a continuum of consumers is distributed. A product located closer to a consumer is more likely to satisfy the consumer’s need. Consumers do not observe the individual product locations, but they can observe the brand position, which is the average location of two products. We assume there are two types of consumers in the market: regular consumers who incur a positive search cost and shoppers who can search for a brand without incurring any cost. Under this model setup, a brand is characterized by two parts: the brand position, captured by the average location of its two products, and the brand strength, captured by the spread of the two products. After observing the brand position, consumers form rational expectations about the spread of two products and decide whether to engage in a costly visit to the firm’s store. When a consumer visits, she discovers the locations of the two products and can determine how well they match her preferences. She then makes a purchase decision. If neither of the two products matches her needs, she inures only search costs—visiting a store but leaving empty handed. Brand positioning, therefore, serves as a crucial device of communication to convey important information to its customers and reduce uncertainty, thereby prompting consumers to initiate the search process.

We first establish that there are two types of brand positioning in equilibrium: mainstream positioning, where the brand is positioned close to the center of the Hotelling line to appeal to the majority of consumers, and niche positioning, where the brand is positioned near an end point to appeal to a small portion of consumers. Then, we show that the presence of heterogeneity in consumer search frictions (i.e., the coexistence of regular consumers who incur a positive search cost and shoppers who do not incur a search cost) creates the critical issue of the holdup problem. Specifically, the brand has an incentive to increase the spread between the two products in order to better serve shoppers who have heterogeneous tastes and visit the store at no cost, irrespective of brand positioning or spread. Regular consumers anticipate the brand’s opportunistic behavior of spreading out the products and update their beliefs about the spread; this, in turn, lowers their expected utility and discourages them from paying the search cost and visiting the brand.

We also find that a brand has little incentive to deviate from the expected level of spread when there are sufficiently many regular consumers who are willing to visit the brand, thus supporting a strong mainstream positioning. Otherwise, a mainstream-positioned brand has a greater incentive to spread its product locations, leading to brand dilution. Therefore, brand position information alone may be insufficient to convince a regular consumer to visit the brand. Given this, a brand sometimes needs to credibly communicate its product spread, specifically a smaller spread by committing to a niche positioning, to increase the regular consumers’ expected utility. As brand positioning becomes more niche, the spread of individual products becomes smaller as the range of possible spreads shrinks. This reduced spread of a specific brand’s position can convey more information about individual products. Hence, we provide a new explanation for consumers’ appeal of niche brands via the firm’s endogenous product portfolio design—niche brand positioning can serve as the commitment
mechanism that allows a brand to communicate credible information about its product spread to consumers.

Finally, we identify the conditions under which brands tend to adopt niche over mainstream positioning strategy after deriving the optimal level of nicheness (or spread). We find that search friction tends toward a proliferation of niche brands. In particular, unless there exist too many or too few shoppers, brands are better off with a niche brand position, one that appeals to a smaller range of consumers but has a higher chance of matching their preferences when search cost is high. In this way, the presence of shoppers can alter the equilibrium dynamics and dramatically change the firm’s profit. Our equilibrium analysis sheds light on the optimal conditions and economic trade-offs under which a firm should position its brand as either “mainstream” or “niche.” Therefore, we investigate a mechanism where brand position serves as a potential communication channel for consumers’ search decisions by providing a formal economic structure on the concept of brand positioning, as well as the firm’s optimal product portfolio design.

The article is organized as follows. Section 2 discusses the related literature. Section 3 develops the formal model and points out the main trade-off behind the model, and Section 4 analyzes it. In Section 5, we show the robustness of our main results to the alternative formulation of brand position formation, the three-product case, and provide an example that relaxes exogenous pricing. Finally, Section 6 concludes. All proofs are in the appendix.

2. Literature Review
Our study contributes to several streams of research about brand positioning in marketing and economics. First, there is a large body of literature studying the concept of brand positioning from the psychological perspective focusing on how consumers perceive, think, and feel about brands (Trout and Ries 1986, Davis 2000). Along this line of research, many papers point out that positioning is a process of emphasizing the brand’s distinctive and motivating attributes, establishing both the point of unique difference and the point of parity association with the category (Jan Alsem and Kosteljik 2008, Keller et al. 2011). We follow the definition of the traditional approach and provide a formal economic modeling structure to understand the concept of brand positioning from a product portfolio view.

Also, a vast literature investigates the issue of branding from an economic perspective. Especially in the context of umbrella branding, researchers have examined whether firms can credibly convey vertical information about quality (Sappington and Wernerfelt 1985; Wernerfelt 1988, 1991; Cabral 2000; Zhang 2015; Klein et al. 2019; Neeman et al. 2019; Yu 2021) or horizontal match (Sappington and Wernerfelt 1985, Kuksov 2007, Kuksov et al. 2013) through branding. The literature in umbrella branding studies whether a firm should use the existing or new brand when launching a new product. Both models in the umbrella branding and ours provide mechanisms about how to credibly convey information about the brand’s products. However, the umbrella branding literature studies whether the mere use of the existing brand can convey information about the new product. Thus, if the same brand is used for the products, the positioning of both products will coincide, which means that the brand position is identical to the position of its products. In contrast, in our model, the firm does not decide whether to use the existing or new brand for the new product. Instead, it decides how to design the characteristics of a line of products under the same brand and shapes the consumers’ perception of the brand. More importantly, we show that brand position is not necessarily the same as the positions of the brand’s individual products.

Several researchers study product positioning with strategic interactions with competition (Hauser and Shugan 1983, Moorthy 1988, Kuksov 2004, Lauga and Ofek 2011). Recent studies have analyzed the dynamic aspects of product positioning, namely repositioning of the product over time (Sweeting 2013, Jeziorski 2014). Villas-Boas (2018) analyzed a monopolist’s optimal repositioning strategy under changing consumer preferences. Cong and Zhou (2020) focused on the role of commitment to repositioning under competition. Our paper differs from the existing literature by highlighting the difference between brand positioning and individual product positioning.

Closer to our paper is the research on how product line design or assortment can affect consumer inference and search decisions. A stream of related studies (Wernerfelt 1995, Villas-Boas 2004, Kamenica 2008, Orhun 2009) highlights the importance of considering consumer information and communication when designing product lines and shows how the number of products affects consumer inference of the likelihood of fit in different contexts. Although the previous literature focuses on the number of products offered in the market, our focus is on the role of the aggregate information about characteristics common to all the products under the same brand and how this aggregate information—brand position—can affect consumer search decision.

Also, our paper is related to the literature on strategic disclosure of verifiable information (Crawford and Sobel 1982, Milgrom and Roberts 1986) and recent development in the information design (Kamenica and Gentzkow 2011; Bergemann and Morris 2016, 2019). In this literature, the focus is on how to optimally design information structure to foster efficient communication. Unlike papers in this area, we are not modeling the brand positioning as an optimal piece of information that the firm passes to consumers. Conceptually, we
study a different paradigm of the information environment, where the sender makes multiple decisions (i.e., style or design of individual products), but only the aggregate information (i.e., brand position) is observable to the receivers. We identify the product portfolio design problem as a natural application of this new information communication paradigm.

Finally, our paper builds on the stream of literature on consumer search, especially ordered search (Armstrong 2017). Armstrong et al. (2009) shows the value of being the first destination or being “prominent” when consumers engage in the costly search. Several papers have examined how firms can guide consumer search by using various instruments such as a lower price (Armstrong and Zhou 2011, Chen and He 2011, Zhou 2011), advertisement (Anderson and Renault 2006, Mayzlin and Shin 2011, Lu and Shin 2018), product design (Kuksov 2004, Bar-Isaac et al. 2012), service (Shin 2007, Janssen and Ke 2020), and targeting (Shin and Yu 2021). Our paper examines the role of brand positioning as another instrument through which the firm can influence consumer search decisions.8

3. Model
A multiproduct firm serves a unit mass of consumers who are represented by their horizontal preferences for product styles or designs. Consumers’ preferences are uniformly distributed on a Hotelling line in [−1, 1]. The firm chooses the design of its product portfolio by locating its products in [−1/2, 1/2] on this line.5 Each product either satisfies a consumer’s need or it does not. So, the realized utility of the consumer located at x from product i is \( u_i(x) = 1 \) or 0. If the consumer’s ideal design is closer to the product location, then the probability of the product satisfying the consumer’s need, \( \Pr(u_i(x) = 1) \), is higher. Specifically, the consumer receives utility \( u_i(x) \) from buying product \( i \in \{1, 2, \ldots, n\} \) located at \( x_i \), where

\[
\begin{align*}
  u_i(x) &= \left\{ \begin{array}{ll}
    1, & \text{with probability } \theta - t|x - x_i|, \\
    0, & \text{otherwise.}
  \end{array} \right.
\end{align*}
\]

Thus, even a consumer located exactly at the same location as product \( i \) may receive \( u_i(x) = 1 \) with probability \( \theta \leq 1 \).6 Conditional on the location of the consumer, \( x \in [-1/2, 1/2] \), \( u_i(x) \)'s are independent across different products. Hence, it is possible that a consumer realizes a match with a product located farther away from her location than another product located closer to her. Then, the consumer’s utility from the brand is \( u(x) = \max_{1 \leq i \leq n} u_i(x) \). The products are substitutes for one another, and if multiple products satisfy the consumer’s need, the consumer is indifferent and therefore, buys one at random. This stochastic nature of the matching utility allows for other factors that affect consumer utility beyond the preference matching on the focal dimension of product design. In reality, consumers may perceive brands in multiple dimensions (e.g., Keller 1993, 2003; John et al. 2006; Lehmann et al. 2008). Although we focus on brand positioning through product portfolio design in only one dimension, we recognize the multidimensional nature of consumer preferences through the parameter \( \theta \), which is allowed to be less than one to capture the possibility of a mismatch between a consumer’s ideal product on the focal dimension and the consumer’s need.

Moreover, the stochastic matching utility, together with this conditional independence assumption, ensures that the consumer’s expected utility depends on the locations of the entire product portfolio as opposed to one particular product closest to her location. In particular, a consumer prefers several distinct products to be located closer to her because the total probability of finding a good match with the firm will be higher. Thus, the brand’s decision for its product portfolio influences consumers’ expected utility and their decisions.8

Consumers have a deterministic outside option that gives zero utility. Consumers prefer a matched product to the outside option, which is preferred to an unmatched product.7 In our main analysis, we abstract away the firm’s pricing decision. Although price image could be an important vertical dimension of brand positioning in reality, we focus on positioning decisions on one horizontal dimension while holding other possible dimensions constant. The current model, therefore, applies to brand positioning situations in which prices are held constant. We further endogenize the firm’s price through one example in Section 5.3 and show that our main model remains unchanged in this example. The consumers’ utility from a matched product is normalized to one, and that of an unmatched product is zero.

3.1. Brand Position Formation
Facing a brand with a portfolio of products, consumers find it prohibitively costly to learn about the exact design or style of all the products, which therefore, remains largely unobservable. However, consumers have a certain perception of a brand based on various interactions with the brand’s several different products. Such a perception is often shaped not by any one particular product but rather, by the designs of the brand’s entire product portfolio. In other words, brand position is a summary statistic that provides a piece of aggregate information about the brand’s product line. We operationalize this idea by defining the brand position \( B \) as the average location of all products under the brand. More specifically, a brand that carries \( n \) products chooses the location of each of its products \( x_k \) for \( 1 \leq k \leq n \) on the Hotelling line. In our main analysis, we assume the simplest case of two products \( (n = 2) \), and we extend the analysis to include three products in Section 5.2. Then, the brand position \( B \) is as
follows:

\[ B = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2}{2}. \]  

(2)

Based on the observed brand position \( B \), consumers form expectations about the exact designs of the brand’s products \( x_i \) and accordingly, decide whether to visit the store by paying a search cost. Then, we can rewrite \( x_1 = B - \Delta \) and \( x_2 = B + \Delta \), where \( \Delta = (x_2 - x_1)/2 \) represents the spread of the brand, defined by the distance between its two products. Without loss of generality, let \( \Delta \geq 0 \), or equivalently, \( x_2 \leq x_1 \). Therefore, there are two pieces of information relevant to consumer’s decisions: brand position \( B \) as the average location of products and brand spread \( \Delta \) as a deviation from \( B \). Consumers consider both the brand position \( B \) (which is observable) and their expectations about the spread \( \Delta \) (which is unobservable) to make their search decision.

A typical brand carries multiple product lines, so it is not feasible to convey all of its product information because of the limited bandwidth in communication. Instead, the consumers usually have an overall image of the brand, which is the brand positioning. For example, consumers have a brand image that Gucci is a fashion luxury brand through prior experience or advertisements without knowing the exact design or style of individual products for the current season. Also, readers know (through their past experiences) that CNN and Fox generally report news from a more liberal and conservative perspective, respectively. Based on such awareness, they form expectations about the range of news reported by each brand (CNN and Fox), which is the brand spread. Consumers who prefer more liberal articles will go to CNN without exactly knowing the tone of each individual news article, believing that they would find news articles tailored to a more liberal audience. The same thing happens for a more conservative audience that frequents the Fox News channel and web pages.

Based on these stylized patterns, we take it as given that consumers observe the brand position \( B \) without micro-modeling how \( B \) becomes known to consumers. By doing so, we are able to focus on analyzing the firm’s product portfolio design and how it gives rise to the brand positioning that affects consumer decisions. In reality, the process of consumers becoming aware of the brand’s position can take different forms, such as observations, advertisements, word of mouth, and prior purchase or use of products. In Section 5.1, we consider an alternative formulation of \( B \) based on the sequential arrival of shoppers and regular consumers and the regular consumers’ observational learning of the shoppers’ choices.

3.2. Consumer Type: Regular Consumers Vs. Shoppers

We assume that there are heterogeneous consumer types to examine the relationship between the optimal product portfolio design and the extent of search friction in the market. An \( \alpha \) fraction of consumers includes regular consumers who have a positive search cost \( s > 0 \). As described, they can only observe \( B \) but not \( \Delta \), so they form beliefs \( \hat{\Delta} \) about \( \Delta \). Only after paying a search cost can they see the true \( \Delta \) and the realized utilities \( u_1(x) \) and \( u_2(x) \). The remaining \( 1 - \alpha \) fraction of consumers includes shoppers with zero search cost who, therefore, observe everything at no cost.

We impose some restrictions on the search cost to simplify the equilibrium analysis and focus on the most interesting case. First, if the search cost is too high, no regular consumers will visit the brand even if the brand provides the highest possible benefits by locating its two products at the same location. To avoid this trivial case, we make the following assumption that ensures that regular consumers are willing to visit the brand if they are at the same location as the two products.

**Assumption 1.** \( s < 1 - (1 - \theta)^2 \).

Moreover, we want to simplify the analysis by requiring “no truncation” of demand for regular consumers. If demand truncation happens, it leads to unnecessary complications of corner cases and nonsmooth demand, which make the analysis rather tedious without adding insight. Particularly, we require the consumer located at 1 (or at −1) to not visit the brand even if both products are colocated as closely as possible at 1/2 (or at −1/2). This translates into the following assumption.

**Assumption 2.** \( s \geq 1 - (1 - \theta + t/2)^2 \).

All the subsequent equilibrium analysis is performed under Assumptions 1 and 2.

The game sequence is as follows. First, the brand decides the locations of its two products, \( x_1 \) and \( x_2 \), to maximize the expected profit. Second, regular consumers observe the brand \( B = (x_1 + x_2)/2 \) and decide whether to pay the search cost and visit the firm. Upon the visit, they discover the relevant information and decide which product to buy or to take the outside option. Shoppers always visit at zero cost and make purchase decisions after observing the exact matching values with both products. Given our assumption of exogenous prices in the main model, the brand’s de facto objective is to maximize the expected demand.

3.3. Breadth of Appeal Vs. Clarity of Information Communicated

Our model characterizes the image of a brand in two parts: (1) the average location of its two products \( B \), which is observed, and (2) the spread of two products \( \Delta \), which is not observed.

If \( B \) is close to zero, then the average location is close to the center of the Hotelling line where a greater mass of consumers (both shoppers and regular consumers) is concentrated. Therefore, we define a brand with \( B \) close
to zero as a mainstream brand, which can potentially appeal to the majority of consumers. On the other hand, if $B$ is close to either end of $\pm 1/2$, the brand could only appeal to a smaller mass of consumers away from the center of the Hotelling line. Therefore, we describe a brand with $B$ close to $\pm 1/2$ as a niche brand.

The brand’s spread $\Delta$ (and consumers’ expectation $\tilde{\Delta}$) crucially affects the regular consumers’ search decision and hence, the firm’s expected demand. If the spread is small, then both products are located close to $B$, and consumers located close to the brand have a high chance of having a good match and thus, may be willing to incur the search cost. So, if the firm can sustain a small-enough $\Delta$ in equilibrium, it can encourage more visits by neighboring regular consumers. Thus, we interpret a less diversified product portfolio with a small brand spread in equilibrium as being a “strong” brand. On the other hand, if the equilibrium product spread is large, then regular consumers’ probability of a good match with the brand will be smaller. So, it is possible that no regular consumers are willing to search for the brand. We specify a brand with a spread too large to serve any regular consumer as being a “weak” brand. In other words, brand strength—a brand being strong or weak—is defined as the inverse of the equilibrium spread of the product portfolio, which dictates whether the brand provides sufficient benefits to induce visits by some regular consumers. Although we adopt these terms that are seemingly binary to describe brand positioning (i.e., mainstream versus niche) and spread (i.e., strong versus weak), respectively, it must be noted that we analyze the firm optimal decision on the entire interval (i.e., $B \in [-1/2, 1/2]$ and $\Delta \in [0, 1/2 - |B|]$). Moreover, formal definitions of mainstream versus niche and strong versus weak are provided in Section 4.

As consumers have heterogeneous preferences, the brand has an incentive to keep a somewhat large spread and capture more demand by locating its products distant. Moreover, this would lead to a brand position closer to the center, $B = 0$, a mainstream positioning. On the other hand, upon observing a mainstream brand, consumers may be more uncertain about the exact spread of the brand (i.e., the true locations of the brand’s products). This is because there can be many different combinations that can all generate $B = 0$ as long as $x_1 = -x_2 \in [0, 1/2]$. So, a mainstream brand may only have a reduced scope of information communicated about the brand’s products, which thus, may discourage consumers’ search and hurt the brand.

By comparison, a niche positioning can convey clearer product information to consumers than a mainstream positioning. Particularly, in an extreme case, by observing $B = 1/2$, consumers can perfectly infer the positions of the two products as $x_1 = x_2 = 1/2$; similarly, by observing $B = -1/2$, consumers can perfectly infer the positions of the two products as $x_1 = x_2 = -1/2$. More generally, any brand position $B$ can be supported by $x_1 = B - \Delta$ and $x_2 = B + \Delta$ for $\Delta \in [0, \min\{1/2 + B, 1/2 - B\}]$. As $B$ goes from $0$ to $\pm 1/2$, the range of possible $\Delta$ gets smaller. By claiming a niche positioning, the brand can communicate credibly that all of its products are indeed consistent with the overall brand image. Then, consumers who are close to the brand position will knowingly visit the brand. Thus, in the presence of consumer search costs, the brand can convince consumers to incur search costs because of the increased information value from positioning.

Choosing a mainstream average location (i.e., the center of the line) can appeal to a broader set of shoppers. On the other hand, a tighter expected spread between the two products can appeal to the local regular consumers. This trade-off indicates that the brand may wish to position itself at the center to cater to a larger set of consumers while maintaining a smaller spread of the products to induce consumer search from regular consumers. However, as we show in our analysis, if the brand’s position is close to the center, it can face temptations to opportunistically dilute its brand by locating two products in opposite directions farther than expected. The trade-off between breadth of appeal and clarity of the information communicated is central to brand positioning, which endogenously arises in equilibrium from the firm’s product portfolio design. We explore this trade-off in the subsequent equilibrium analysis. We identify conditions (essentially depending on the degree of search frictions in the market) for the brand to choose a niche versus mainstream positioning.\(^\text{11}\)

4. Equilibrium Analysis

We start our analysis by characterizing a representative consumer’s search decisions and demand and then, formulating the equilibrium condition. After that, we analyze two benchmark cases, one with only shoppers and the other with only regular consumers. These cases can be obtained by setting $\alpha = 0$ and $\alpha = 1$, respectively, in the general model. These benchmark models help us isolate the effect of search frictions on the optimal brand positioning, which we subsequently explore.

4.1. Consumer’s Search Decision, Demand, and Equilibrium Concept

Given the firm’s choice of brand position $B$, we calculate the mass of regular consumers who search (or visit the store). Shoppers have zero search cost, so they always search.

By searching or visiting the brand, a regular consumer at location $x$ pays the search cost $s$ and discovers the matching utility with both products, $u_1(x)$ and
The regular consumer will visit the brand if and only if the benefit of search is greater than the cost of search: $E[u(x)] \geq s$. So, we can now calculate the total number of visitors (or those who search the brand) among regular consumers. We denote the set of visitors on the Hotelling line as $\mathcal{V}(B, \tilde{\Lambda})$, where

$$
\mathcal{V}(B, \tilde{\Lambda}) = \begin{cases} 
[B - \Gamma_1(\tilde{\Lambda}), B + \Gamma_1(\tilde{\Lambda})] & \text{if } \tilde{\Lambda} \leq \Delta_1, \\
[B - \Gamma_1(\tilde{\Lambda}), B - \Gamma_2(\tilde{\Lambda})] \cup [B + \Gamma_2(\tilde{\Lambda}), B + \Gamma_1(\tilde{\Lambda})] & \text{if } \Delta_1 < \tilde{\Lambda} < \Delta_2, \\
\emptyset & \text{if } \tilde{\Lambda} \geq \Delta_2,
\end{cases}
$$

where

$$
\Gamma_1(\tilde{\Lambda}) \equiv \sqrt{\tilde{\Lambda}^2 + (1 - s)/t^2 - (1 - \theta)/t}, \quad \Gamma_2(\tilde{\Lambda}) \equiv \sqrt{\left(\tilde{\Lambda} + (1 - \theta)/t\right)^2 - (1 - s)/t^2}, \quad \Delta_1 \equiv \sqrt{1 - s/t - (1 - \theta)/t}, \quad \Delta_2 \equiv [(1 - s) - (1 - \theta)^2]/[2t(1 - \theta)].
$$

Equation (A.9) shows that the total number of regular consumers who visit the brand can be characterized by three different cases, depending on how large the expected spread $\hat{\Lambda}$ is. First, if $\hat{\Lambda} \leq \Delta_1$, every consumer in between the two products is willing to visit the brand by incurring the search cost of $s$. We say the brand is strong in this case because it features a less diversified product portfolio with a small spread, as shown by Figure 1(a), where the gray line represents the set of regular consumers who visit the brand, $\mathcal{V}(B, \tilde{\Lambda}) = [B - \Gamma_1(\tilde{\Lambda}), B + \Gamma_1(\tilde{\Lambda})]$. Second, $\Delta_1 < \tilde{\Lambda} < \Delta_2$; some consumers located in between the two products find it not worthwhile to visit the brand because neither product is expected to provide a good match. We say the brand is medium in this case because it features a moderately diversified product portfolio with a medium spread, as shown by Figure 1(b), where regular consumers who visit the brand come from two disjoint intervals that are, roughly speaking, located around $B - \tilde{\Lambda}$ (i.e., $x \in [B - \Gamma_1(\tilde{\Lambda}), B - \Gamma_2(\tilde{\Lambda})]$) and $B + \tilde{\Lambda}$ (i.e., $x \in [B + \Gamma_2(\tilde{\Lambda}), B + \Gamma_1(\tilde{\Lambda})]$), respectively. Finally, when $\tilde{\Lambda} \geq \Delta_2$, as illustrated by Figure 1(c), the two products are so distant from each other that no regular consumer will visit the brand with $\mathcal{V}(B, \tilde{\Lambda}) = \emptyset$. We say the brand is weak in this case because it features a very diversified product portfolio with a large spread that is unattractive to regular consumers. For convenience, we will use these definitions of a brand being strong, medium, and weak throughout the paper.

Given the total number of regular consumers who visit, $\mathcal{V}(B, \tilde{\Lambda})$, we can derive the total expected demand.
from both regular consumers and shoppers. The total expected demand \( D(B, \bar{\Delta}, \Delta) \) is a function of brand position \( B \), consumer’s expected spread \( \bar{\Delta} \) (which affects regular consumers’ visit decisions), and the actual spread \( \Delta \) (which affects all consumers’ purchase decisions):

\[
D(B, \bar{\Delta}, \Delta) = \frac{\alpha}{2} \int_{x \in \{B, \bar{\Delta}\}} \left[ 1 - (1 - \theta + t|B - \Delta - x|) \right] dx \\
\left(1 - \theta + t|B - \Delta - x|\right) dx + 1 - \frac{\alpha}{2} \int_{0}^{1} \left[ 1 - (1 - \theta + t|B + \Delta - x|) \right] dx \\
\left(1 - \theta + t|B - \Delta - x|\right) dx.
\]

(5)

Using \( D(B, \bar{\Delta}, \Delta) \), we formulate the firm’s problem next. The firm chooses the product portfolio of \( x_1 \) and \( x_2 \) or equivalently, \( B \) and \( \Delta \) based on the consumer’s expected spread \( \bar{\Delta} \) to maximize the expected demand, where the equilibrium brand position \( B^* \) and equilibrium brand spread \( \Delta^* \) are given by

\[
(B^*, \Delta^*) = \arg \max_{(B, \bar{\Delta}, \Delta) \in \{0 \leq \Delta \leq 1/2 - |B|\}} D(B, \Delta^*, \Delta).
\]

(6)

The right-hand side in Equation (6) involves a constrained optimization problem, where the constraint, \( 0 \leq \Delta \leq 1/2 - |B| \), is because of limited product space on the Hotelling line for product positioning. From the perspective of regular consumers who do not observe \( \Delta \), we have that their expected spread, \( \bar{\Delta} \), must satisfy \( 0 \leq \bar{\Delta} \leq 1/2 - |B| \). This signifies that the firm can use brand position \( B \) to directly influence consumers’ belief \( \bar{\Delta} \), which is a unique feature of our model. In equilibrium, we require that consumers’ expectation \( \bar{\Delta} \) coincides with the firm’s optimal choice, \( \Delta^* \).

We adopt the sequential equilibrium of the game (Kreps and Wilson 1982) as the solution concept of our game. Compared with perfect Bayesian equilibrium, sequential equilibrium uses trembling-hand perturbations to reach nodes that are off the equilibrium path and requires the players’ strategies and beliefs to be sequentially rational and consistent on these nodes. Hence, even if the game reaches off the equilibrium path, the players choose actions optimally from then on, consistent with the equilibrium strategy. More specifically, consumers are aware of the model primitives \( \alpha, \theta, \phi \), and \( s \) and have some expectations about the possible set of equilibria for \( B \), which they observe. If regular consumers observe the firm’s choice of \( B^* \) outside this set, then their beliefs about the unobserved spread \( \bar{\Delta} \) are consistent with the firm’s optimal choice, \( \Delta^*(B^*) \), given the deviated position choice of \( B^* \).

To solve for the sequential equilibria of the game, we proceed as the following two steps. First, we take the brand position \( B \) as given and consider the brand’s optimal decision on the spread \( \Delta \) that is consistent with consumers’ anticipation for the spread \( \Delta^* \). There are three cases to consider depending on whether the brand is strong, medium, or weak, according to Equation (A.9). The set of regular consumers who visit the brand, \( \mathcal{V}(B, \bar{\Delta}), \) is one interval when \( \Delta^* \leq \Delta_1 \), two disjoint intervals when \( \Delta_1 < \Delta^* < \Delta_2 \), or an empty set when \( \Delta^* \geq \Delta_2 \). For each of the three cases, we calculate the firm’s expected demand \( D(B, \Delta^*, \Delta) \) based on Equation (A.10) and determine the equilibrium \( \Delta^*(B) \) by taking derivatives of \( D(B, \Delta^*, \Delta) \) with respect to \( \Delta \) and solving the corresponding optimality conditions taking into account the constraint of \( 0 \leq \Delta \leq 1/2 - |B| \). Second, we maximize \( D(B, \Delta^*(B), \Delta^*(B)) \) with respect to \( B \) to solve for the equilibrium brand position, \( B^* \).

In the next section, we examine two benchmark cases where only one type of consumer exists in the market (shoppers only and regular consumers only). This helps us to identify the underlying forces for the firm’s optimal decision on product portfolio design.

4.2. Benchmarks: Shoppers Only (\( \alpha = 0 \)) and Regular Consumers Only (\( \alpha = 1 \))

As the first benchmark, we consider the case with only shoppers by setting \( \alpha = 0 \) in the main model. In this case, all consumers in the market do not incur search costs to visit the brand, and thus, both the brand position \( B \) and the spread of products \( \Delta \) are observable to consumers. Therefore, the problem reverts to a simple product positioning problem, and there is no special role of brand positioning beyond product differentiation. The later comparison between the main model and this benchmark can highlight the unique role of brand positioning. The equilibrium of the complete information game is presented in the following proposition.

Proposition 1 (Shoppers Only). If there are only shoppers in the market (\( \alpha = 0 \)), there exists a unique equilibrium with \( (B^*, \Delta^*) = (0, \max\{\Delta_{\tau=0}, 0\}) \), where \( \Delta_{\tau=0} \equiv [1 - (1 - \theta)/t] / 2 \). Moreover, \( \Delta_{\tau=0} \) increases in \( \theta \) and \( t \): \( \partial \Delta_{\tau=0}/\partial \theta \geq 0 \) and \( \partial \Delta_{\tau=0}/\partial t \geq 0 \).

If there are only shoppers in the market, the firm will design its product portfolio such that the brand positioning is in the center with \( B^* = 0 \), which is the standard result of a product positioning problem. As \( \theta \) increases, each product provides a good match for nearby consumers with a higher probability, and the firm does not see benefits of juxtaposing its products in proximity. Also, as \( t \) increases, consumers’ preferences become more heterogeneous. Both result in a larger optimal spread, which is quite intuitive.
Next, we study the alternative benchmark case with only regular consumers in the market (that is, $\alpha = 1$). This case is crucially different from the previous benchmark because the regular consumers only observe $B$ and not $\Delta$. Therefore, in equilibrium, the unobservable $\Delta$ must be credibly communicated through the firm’s choice of $B$.

**Proposition 2 (Regular Consumers Only).** If there are only regular consumers in the market ($\alpha = 1$), there exists a unique set of equilibria such that $\Delta^* = \max\{\Delta_{i=1}, 0\}$ and $B^*$ takes any value in $[-(1/2 - \Delta^*), 1/2 - \Delta^*]$, where $\Delta_{i=1}^* \equiv \sqrt{3(1-s) + 4(1-\theta)^2 - 4(1-\theta)} / (3t) \leq \Delta_1$. Therefore, some regular consumers will visit the brand in equilibrium, who come from one interval on the Hotelling line. Moreover, $\Delta_{i=1}^*$ decreases in search cost $s$ and increases in $\theta$: $\partial \Delta_{i=1}^* / \partial s \leq 0$ and $\partial \Delta_{i=1}^* / \partial \theta \geq 0$.

We find that, with only regular consumers, the unobserved product information $\Delta$ can be communicated credibly, which is a stark contrast to the main model later. Moreover, in equilibrium, the firm chooses a small spread of $\Delta^* \leq \Delta_1$. This is intuitive because the spread has to be sufficiently small for regular consumers to pay a visit to the store. Based on the observed $B$ and the expected small spread $\Delta^*$, the regular consumers close to $B$ will visit the store. Therefore, the firm will indeed choose a small spread to serve these regular consumers.

It is immediate to see that a higher search cost $s$ leads to a smaller $\Delta^*$ and thus, a stronger brand. The brand needs to compensate the consumer’s high search costs by placing two products closer, which increases their expected utility. Similar to the shoppers-only case, as $\theta$ increases the firm wants to keep its products more distant because each product alone is likely to match the needs of consumers close to the product. Also, notice that $B^* = 0 \in [-1/2 - \Delta^*, 1/2 - \Delta^*]$, and therefore, the main equilibrium positioning with $B^* = 0$ is always an equilibrium. Altogether, a strong ($\Delta^* \leq \Delta_1$) mainstream ($B^* = 0$) positioning can be an equilibrium in the case with only regular consumers.

We can now compare the equilibrium spread $\Delta^*$ across the two benchmark cases when there are only shoppers ($\alpha = 0$) and only regular consumers ($\alpha = 1$).

**Proposition 3.** The equilibrium spread, $\Delta^*$ in the case with only regular consumers ($\alpha = 1$), is less than or equal to that in the case with only shoppers ($\alpha = 0$): $\max\{\Delta_{i=1}^*, 0\} \leq \max\{\Delta_{i=0}^*, 0\}$.

For the regular consumers who must pay a search cost to visit the brand, the brand needs to provide enough benefits by locating two products sufficiently close. In contrast, for shoppers who can visit freely, the brand wants to spread out its products sufficiently to maximize market coverage. If both shoppers and regular consumers coexist in the market, the brand faces a trade-off in choosing the spread between serving the two types of consumers.

### 4.3. Equilibrium Product Portfolio Design

In this section, we analyze the main model with $\alpha \in (0, 1)$. In order to encourage some regular consumers to visit its store, the brand would want to locate its products close to each other by choosing a small spread. However, once regular consumers (who cannot observe the spread of the product directly but only anticipate the spread) visit the brand, the brand is tempted to deviate by increasing the spread to better serve the shoppers. This is the classic holdup problem. Regular consumers are rational and can anticipate the brand’s opportunistic behavior. This can be costly for the brand because regular consumers can be discouraged from visiting the brand in the first place. We will show that the brand may discipline itself by choosing its brand position $B$ close to either end of the Hotelling line. In other words, the brand may endogenously choose a niche positioning as a commitment device to preserve a small spread and thereby, serve both the shoppers and the regular consumers.

Importantly, this holdup problem does not arise under the two benchmark cases. When there are only shoppers, the brand cannot hold them up once they visit the store. When there are only regular consumers, there is no incentive for the brand to deviate by spreading the products farther apart. Regular consumers make their search decisions based on their own expectations. Therefore, such a deviation does not increase the number of regular consumers who visit the store. Moreover, such deviation (i.e., increasing the ex post spread) will reduce the match probability of those who visit the store, further decreasing the demand eventually. Thus, this holdup problem uniquely arises in the main model where there is heterogeneity in search frictions in the market.

We now solve the sequential equilibrium under the main model. Given any $B \in [-1/2, 1/2]$, there are three possible cases of $\Delta^* \leq \Delta_1$ (a strong brand), $\Delta_1 < \Delta^* < \Delta_2$ (a medium brand), and $\Delta^* \geq \Delta_2$ (a weak brand). For each case, we first relax the constraint $\Delta \leq 1/2 - |B|$ and solve the resulting unconstrained optimization problem in Equation (6), and then, we consider when the constraint $\Delta \leq 1/2 - |B|$ will be binding. In order to highlight the trade-off in the firm’s product portfolio design and brand position decision, our analysis focuses on the two more interesting cases of a strong brand and a weak brand.

We first analyze the first case of a strong brand. Given $\Delta^* \leq \Delta_1$, all visitors come from one interval on the Hotelling line. Based on Equations (A.9) and (A.10), we have that the first- and second-order optimality conditions
imply $\Delta^* = \max\{\Delta^*_a, 0\}$ where the unconstrained optimal spread $\Delta^*_a$ is

$$
\alpha\sqrt{[(1 + \alpha)(1 - \theta) - (1 - \alpha)t]^2 + (4 - \alpha^2)(1 - s)}
\Delta^*_a = -\frac{2[(1 + \alpha)(1 - \theta) - (1 - \alpha)t]}{(4 - \alpha^2)t},
$$

which does not depend on $B$. The following lemma generalizes the binary comparison in Proposition 3 to all $\alpha \in [0, 1]$.  

**Lemma 1.** The unconstrained optimal spread $\Delta^*_a$ decreases with $\alpha$: $\partial \Delta^*_a / \partial \alpha \leq 0$.

The constraint is not binding if $\max\{\Delta^*_a, 0\} \leq 1/2 - |B|$; otherwise, we have the constraint binding and $\Delta^* = 1/2 - |B|$. To summarize,

$$
\Delta^*(B) = \begin{cases} 
\max\{\Delta^*_a, 0\}, & \text{if } |B| \leq 1/2 - \max\{\Delta^*_a, 0\}, \\
1/2 - |B|, & \text{otherwise.}
\end{cases}
$$

This spread can be part of equilibrium only if it satisfies the assumed condition that $\Delta^*(B) \leq \Delta_1$ so that the firm’s optimal choice of the spread is consistent with the consumers’ expected spread.  

It is important to know whether a small spread can be maintained because a strong brand can serve both shoppers and regular consumers; on the contrary, the firm with a large spread loses regular consumers completely and thus, potentially suffers from a profit loss. The following proposition identifies the conditions under which a strong brand with a small spread can be sustained in equilibrium.  

**Proposition 4.**  
1. If $s \leq 1 - (1 - \theta + t)^2/4$ or $\alpha \geq \hat{\alpha}$, then $\Delta^*(B) \leq \Delta_1$ for all $B \in [-1/2, 1/2]$. Therefore, any positioning—regardless of whether it is mainstream or niche—can sustain a strong brand.  

2. Otherwise (i.e., if $s > 1 - (1 - \theta + t)^2/4$ and $\alpha < \hat{\alpha}$, then $\Delta^*(B) \leq \Delta_1$ if and only if $|B| \geq 1/2 - \Delta_1$. Therefore, only a niche positioning (i.e., $|B| \geq 1/2 - \Delta_1$) can sustain a strong brand.

This proposition suggests that a strong brand with a small spread $\Delta^* \leq \Delta_1$ can always be achieved by a niche positioning (i.e., $|B| \geq 1/2 - \Delta_1$). However, a mainstream positioning (i.e., around the center location of the Hotelling line such that $|B| < 1/2 - \Delta_1$) can only sustain a strong brand when $s$ is sufficiently small or $\alpha$ is sufficiently large.

More precisely, when $s$ is relatively small or $\alpha$ is relatively large, there are enough regular consumers who are willing to visit the brand. Then, the firm can maintain a relatively small spread ($\Delta^* \leq \Delta_1$) regardless of its brand position $B \in [-1/2, 1/2]$. On the other hand, if $s$ is sufficiently large and $\alpha$ is sufficiently small, a brand positioned in the middle is tempted to increase the spread to better serve the large segment of shoppers (which is evident from Lemma 1 that $\partial \Delta^*_a / \partial \alpha \leq 0$). Moreover, a high search cost requires an extremely narrow spread to provide sufficient benefit from consumer search. Putting these two effects together, a mainstream brand’s unconstrained optimal choice of spread cannot coincide with the consumers’ expected spread, which can justify their search costs. This can cost the firm significantly because regular consumers will no longer trust it to serve products close to their tastes and decide not to visit. This is the holdup problem mentioned before.

Alltogether, the proposition implies that the firm may sometimes need to choose a niche positioning to overcome the holdup problem when it wants to serve regular consumers. The benefit of choosing a niche positioning is that the firm can attract both shoppers and regular consumers. However, by choosing a brand’s position far away from the center of the Hotelling line, the firm is unable to serve the shoppers optimally (as shown in Proposition 1). Given these trade-offs, the firm will choose the brand position $B^*$ optimally to maximize $D(B, \Delta^*(B), \Delta^*(B))$. Note that, in equilibrium, the firm chooses not only whether to be a mainstream or niche brand but also, how close to the central location a mainstream positioning will be and how close to an end a niche positioning will be.  

We show that two types of equilibria can arise under different parameter regions: mainstream and niche. First, as the next result shows, a mainstream brand equilibrium is always positioned at the center of the Hotelling line, $B^* = 0$, but it may sometimes feature a small or large spread.

**Proposition 5 (Mainstream Positioning).** Mainstream positioning can be an equilibrium either as a strong brand with a small spread or as a weak brand with a large spread.  

1. If $s \leq 1 - (1 - \theta + t)^2/4$ or $\alpha \geq \hat{\alpha}$, then a strong mainstream brand $(B^*, \Delta^*) = (0, \max\{\Delta^*_a, 0\})$ is an equilibrium.  

2. If $s > 1 - (1 - \theta)^2/4$ and $\alpha < \hat{\alpha}$, then a weak mainstream brand $(B^*, \Delta^*) = (0, \Delta^*_{\alpha=0})$ is an equilibrium, and the brand serves only the shoppers.

The composition of consumers (i.e., the size of $\alpha$), consumer’s search cost $s$, and match probability of each ideal product $\theta$ affect the brand’s optimal positioning decision. As implied by Proposition 4, when $s$ is relatively small or $\alpha$ is relatively high, there are enough regular consumers who are willing to visit the brand, supporting a strong brand for any positioning $B \in [-1/2, 1/2]$. Therefore, even a mainstream positioning can maintain a strong brand, and it is then an equilibrium to set $\Delta^* = \max\{\Delta^*_a, 0\}$ that attracts both regular consumers and shoppers to visit.
However, if consumer search cost is sufficiently high such that \( s > 1 - t(1 - \theta) \) and there exists a sufficiently large number of shoppers (\( \alpha < \overline{\alpha}_{\text{min}} \)), then only a weak brand with a large spread in product portfolio is feasible for a mainstream positioning, forgoing the regular consumers.\(^{17}\) When this happens, the firm may find it more profitable to choose a niche positioning and achieve a small spread. Although the brand would appeal to a smaller portion of the entire population, it can attract both regular consumers and shoppers. This benefit of monetizing on the regular consumers is not so significant if \( \alpha \) (the size of the regular consumers) is small. Therefore, appealing to a small group of regular consumers through niche positioning cannot justify forsaking the mainstream positioning when the firm can serve a large group of shoppers. This is the main trade-off between choosing the mainstream versus niche positioning. Also, we find that, given mainstream positioning, it is optimal to position at the center of the Hotelling line, \( B' = 0 \), in order to best serve the shoppers.

Next, we turn to the conditions under which a niche positioning can be the firm’s equilibrium choice in the following proposition. The firm chooses a niche brand position \( |B'| = 1/2 - \Delta_1 > 0 \) and one product exactly at one end point with \( |B'| + \Delta' = 1/2 \).

**Proposition 6** (Niche Positioning). If \( s > 1 - t(1 - \theta) \) and \( \overline{\alpha}_{\text{niche}} < \alpha < \overline{\alpha}_{\text{niche}}(< \overline{\alpha}') \), then the unique set of equilibria is strong niche brands: \( (B', \Delta') = (\pm(1/2 - \Delta_1), \Delta_1) \).\(^{18}\)

Following the discussions for Proposition 5, the firm finds niche positioning more profitable if the following two conditions are met: (1) a mainstream positioning suffers from a holdup problem, and (2) the benefit of attracting regular consumers as a niche positioning is large enough.

First, for condition (1), consumer’s search cost has to be sufficiently large or \( \theta \) not too large (\( s > 1 - t(1 - \theta) \)), as shown in Propositions 4 and 5. In this case, the regular consumers will not visit the brand if it is positioned in the center of the Hotelling line (\( B = 0 \)), which is illustrated in Figure 2(a). This is because of the holdup problem, where the existence of shoppers induces the firm to spread the two products apart. Still, regular consumers demand a smaller spread to justify their search costs. Then, as this proposition suggests, a niche positioning can serve as a commitment not to spread its products excessively. However, if there are many regular consumers (\( \alpha \geq \overline{\alpha}_{\text{niche}} \)), even the mainstream can sustain a strong brand with a small spread (Proposition 5) and serve regular consumers. Therefore, the holdup problem matters when the consumer’s search cost has to be sufficiently large (\( s > 1 - t(1 - \theta) \)), and there are not too many regular consumers (\( \alpha < \overline{\alpha}_{\text{niche}} \)).

Next, condition (2) is satisfied if \( \alpha > \overline{\alpha}_{\text{niche}} \) such that there are enough regular consumers in the market. Then, the brand finds it optimal to sacrifice the central location to position its brand toward an end point. Unlike mainstream positioning, which can only serve shoppers (depicted in Figure 2(a)), the brand can serve some regular consumers, which is illustrated in Figure 2(b).

Finally, we find that the firm chooses the product portfolio such that the niche brand is placed as close as possible to the center of the Hotelling line. This is because the firm can benefit from moving closer to the center position and thereby, serve more shoppers while keeping regular consumers’ expected benefit greater than search costs. Based on these two forces coming from incentives to serve regular consumers versus shoppers, each pulling the brand positioning in opposite directions, the firm chooses an optimal product portfolio design with \( (|B'|, \Delta') = (1/2 - \Delta_1, \Delta_1) \). This is precisely the equilibrium location \( B' \) where it must be close enough to the end points so that the regular consumers are willing to visit while at the same time, as close as possible to the center to maximize the demand from the shoppers.

**4.3.1. Mainstream Vs. Niche Positioning.** We have characterized two different types of equilibria—mainstream positioning and niche positioning. Propositions 5 and 6
together demonstrate the opposing forces induced by two segments of consumers and how their interplay fundamentally influences the firm’s product portfolio design and the resulting positioning choice between mainstream and niche. For regular consumers, the brand has an incentive to maintain a small spread within the product portfolio. However, the presence of shoppers tempts the brand to spread out its product locations to accommodate the shopper’s heterogeneous preferences more efficiently. The more shoppers there are, the stronger this temptation becomes. This will dilute the brand image and discourage regular consumers from visiting the brand. Therefore, the brand needs a mechanism to overcome the temptation if it aims to serve both shoppers and regular consumers. We showed that niche positioning can undertake this role by helping the firm credibly communicate a small spread to consumers and prevent brand dilution.

A niche brand can be more profitable than a mainstream brand under a specific condition (i.e., if \( s \) is in an intermediate interval \( (\theta_{\text{niche}}, \theta_{\text{niche}}) \), and the consumer search cost is sufficiently large; i.e., \( s > 1 - t(1 - \theta) \)). This condition is represented by the dotted area in Figure 3. This implies that even if the niche brand can provide more information about the brand’s products, under a wide range of parameter space, the brand prefers a mainstream positioning to a niche positioning. More specifically, whenever the strong mainstream brand (illustrated with horizontal lines in Figure 3) exists, it is an equilibrium. On the other hand, a weak mainstream brand (depicted by vertical lines in Figure 3) can only be an equilibrium if \( \alpha \) is sufficiently small. A weak mainstream brand is dominated by a niche brand in an intermediate region and by a strong mainstream brand when \( \alpha \) becomes sufficiently large. In summary, the figure shows that when the search cost is sufficiently high and there exist neither too many nor too few shoppers, a monopolistic firm can be better off with a niche brand position, which is a unique equilibrium.

Our findings provide an explanation for why some brands that target niche markets thrive and sometimes outperform other brands that aim to serve the mainstream market. For instance, Lululemon, which is regarded as a niche brand targeting young yoga lovers, has become the fifth largest market share in the entire athletic apparel industry (source: Statistica 2019/2020). The clarity in Lululemon’s niche brand positioning achieved through its product portfolio may have generated an intensive engagement from what may seem like a smaller niche market. Lululemon can assure consumers that they can find different yoga pants in various styles and colors. It is more likely for the shopper to find what she needs by visiting Lululemon than Nike, which also carries some yoga pants but is positioned as mainstream “sportswear” by offering a general line of the product portfolio.

Figure 3, as a visual summary of Propositions 5 and 6, focuses on the existence of different types of equilibria. It is a sequential game where a firm first chooses \( B \) based on the market situation such as \( s \) and \( \alpha \) to maximize its profit between mainstream and niche positioning. Therefore, the three distinct regions in Figure 3 for a strong mainstream, a weak mainstream, and a strong niche positioning do not overlap. However, we cannot rule out the coexistence of the medium brand. Nevertheless, for some cases, we still prove the uniqueness of the equilibrium (even considering the possibility of medium brand equilibrium), most prominently the uniqueness of the strong niche brand equilibrium in Proposition 6. For the mainstream brand, we can only show the uniqueness result for the strong mainstream brand equilibrium under \( s \leq 1 - (t + 1 - \theta)^2/4 \).

Our approach establishes the linkage between brand positioning and product portfolio design. The results from our analysis help explain some of the real-world
observations about how mainstream brands can be diluted. For example, fashion brands such as Ellen Tracy, Anne Klein, and Dana Buchman documented in a Harvard Business Review case on Eileen Fisher (Keinan et al. 2012) struggled because brands changed their product offerings to appeal to the younger generation in the early 2000s. Such attempts alienated the brands’ existing customers, and they turned into diluted and weak mainstream brands (Keinan et al. 2012). It is generally consistent with our model prediction that the brand can focus on the regular consumers who are close to the brand (just as existing core customers in Eileen Fisher’s case), but it sometimes cannot help giving in to temptations to appeal to the information- and fashion-savvy shoppers outside of its core segment. Moreover, our analysis predicts that a lower search cost for regular consumers will mitigate the temptations of dilution of mainstream brands. This prediction resonates with what we have observed in several industries, including fashion. In the past, fashion brands mostly depended on offline channels for sales as well as communications, which entailed a high search cost. However, as internet access and adoption have been rapidly increasing over the past decades, fashion-related information is no longer reserved only for fashion critics and experts, which means that the regular consumers’ search cost has dropped significantly. These are, by and large, consistent with our observations of several successful, strong mainstream brands, such as Uniqlo and GAP. It could maintain a strong mainstream positioning by offering a product portfolio evenly mixed between formal and casual designs in contrast to other brands that could not resist the temptation to dilute their brands.

5. Extensions

5.1. Product Portfolio Design with Demand-Weighted Brand Position Formation

The key conceptual innovation we formalize based on the existing studies on branding is the idea that brand positioning, as an outcome of consumers’ numerous interactions and experiences with various products under the brand, is a summary statistic of all the products in the firm’s product portfolio. One simple operationalization of this idea has been studied in the main model, where the brand positioning $B$ is the arithmetic mean of the product positions, $x_1$ and $x_2$. In reality, some interactions may have a more substantial effect on consumers’ perceptions of the brand than others. For example, consumers may observe certain products more often than others. In this extension, we consider an alternative formulation to reflect such situations with

$$B = \frac{S_1(x_1, x_2)}{S_1(x_1, x_2) + S_2(x_1, x_2)} x_1 + \frac{S_2(x_1, x_2)}{S_1(x_1, x_2) + S_2(x_1, x_2)} x_2,$$

(8)

where $S_i(x_1, x_2)$ is shoppers’ demand for product $i (i = 1, 2)$ given $x_1$ and $x_2$. Specifically, we have

$$S_i(x_1, x_2) = \frac{1 - \alpha}{2} \int_{-1}^{1} (\theta - t|x - x_i|) \left(1 - \frac{1}{2} \theta + \frac{1}{2} t|x - x_i|\right) dx.$$

(9)

This formulation has the following interpretation. Brand positioning $B$ only influences regular consumers’ search decisions, not shoppers. Shoppers have zero search costs, so they visit the store and make purchase decisions earlier than regular consumers. After that, regular consumers interact with shoppers randomly and observe what they have purchased. Particularly, a regular consumer may observe a sequence of shoppers’ choices before deciding whether to visit the store. As the interactions between agents are random, the probability that a regular consumer encounters a shopper of style $x_i$ is $S_i(x_1, x_2)/(S_1(x_1, x_2) + S_2(x_1, x_2))$ for $i = 1, 2$. Regular consumers have limited attention or memory, so they do not keep track of the styles of individual products; instead, they form an overall impression of the brand as the average style of all the products encountered as defined in Equation (8). This interpretation is more natural when the number of products under a brand is relatively large, which is oftentimes the case in reality. However, for analytical tractability and simple exposition and to be consistent with our main model, we analyze the case with two products.

Similar to what happened in the main model, regular consumers observe $B$, which is a deterministic function of $x_1$ and $x_2$ according to Equations (8) and (9), based on which they form an expectation of the two products’ locations, $x_1$ and $x_2$. The only difference is that in the main model, the dependence relationship between $B$, $x_1$, and $x_2$ takes a much simpler form of $B = (x_1 + x_2)/2$. Nevertheless, it is straightforward to show that the new formation in Equation (8) imposes a similar restriction on $x_1$ and $x_2$ given $B$ fixed. Particularly, as $|B|$ increases, the range of $x_1$ gets smaller. In extreme cases, if the brand position is at one end point ($B = \pm 1/2$), we must have both products also located at the end point ($x_1 = x_2 = \pm 1/2$); on the other hand, $B = 0$ can be supported by various combinations of $x_1$ and $x_2$ such that $x_1 = -x_2 \in [-1/2, 0]$.

Figure 4 plots the possible range of $x_1$ given $B \in [-1/2, 1/2]$ implied by Equation (8) and compares it with that in the main model. The range of $x_2$ given $B$ can be obtained by symmetry. To understand the figure, let us take a look at an example with $B = 0.3$, as marked by blue dotted lines. Given $x_1 \leq x_2$ by stipulation, the upper bound on $x_1$ is $B$ for both the main model and the extension, which is marked by the 45° line in the figure. On the other hand, the lower bounds...
on \( x_1 \) are different between the main model and the extension, and we have that the lower bound in the extension (black solid line) is higher than that in the main model (gray dashed line) because in the extension, the brand position puts more weight on the product with higher demand (product 1 in this example) and consequently, \( x_1 \) is closer to \( B \). From Figure 4, we can see that although Equation (8) looks quite different from \( B = (x_1 + x_2)/2 \) in the main model, the restriction imposed on product locations \( x_1 \) and \( x_2 \) given \( B \) is very similar.

Moreover, notice that after regular consumers visit the store and observe \( x_1 \) and \( x_2 \), their purchase decisions no longer depend on \( B \) and thus, are the same as that in the main model. In other words, \( B \) only influences regular consumers’ search decisions. These observations lead to the following proposition, which shows that the alternative formulation of brand position formation in Equation (8) would not change the mainstream brand equilibria.

**Proposition 7.** There exist parameter regions for \( t, \theta, s \), and \( \alpha \) where the equilibria of weak and strong mainstream brands exist and are, respectively, the same as those in the main model in the sense that the firm’s choices of product locations, as well as consumers’ search and purchase decisions, are the same.

Although Proposition 7 shows that the equilibria involving a mainstream brand are the same as those in the main model, the equilibrium existence condition will be different because when a firm deviates on \( x_1 \) and \( x_2 \), what regular consumers observe is \( B \) given by Equation (8), which is different from the main model, and consequently, their expectations of \( \bar{x}_1 \) and \( \bar{x}_2 \) are also different.

Lastly, for the niche brand equilibrium, it is difficult to analytically characterize the equilibrium existence condition. Instead, we can showcase that the equilibrium exists by combining an analytical proof for identifying feasible equilibrium with numerical analysis (for identifying one numeric example to verify that the equilibrium exists).

**Proposition 8.** There exist parameter regions for \( t, \theta, s \) and \( \alpha \) where the where the unique equilibrium is niche positioning \( (B^*, \Delta^*_L, \Delta^*_R) \), which satisfies \( B^* + \Delta^*_L = 1/2 \) and \( \Delta^*_L + \Delta^*_R = 1/2 \).

The proposition shows that the firm’s optimal product portfolio design involves niche brand positioning such that \( x^*_1 = 1/2 - 2 \cdot \Delta^*_L \) and \( x^*_2 = 1/2 \) or equivalently, \( (B^*, \Delta^*_L, \Delta^*_R) \) where \( B + \Delta^*_L = 1/2 \) and \( \Delta^*_L + \Delta^*_R = 2 \cdot \Delta^*_L \). Note these product positions are exactly the same as the niche brand equilibrium in the main model characterized in Proposition 6. Therefore, this proposition confirms that our main result in Proposition 6 is robust to this alternative formulation of brand position.

To summarize, combining Propositions 7 and 8 together, we find that all equilibria identified in the main model (weak mainstream, strong mainstream, and niche) exist under the extension model, although the equilibrium exists under different conditions. Moreover, when the equilibria exist, they are the same as those in the main model.

### 5.2. Three-Product Case

The main model analyzes a simple case where the firm has two products. This section analyzes a case when the firm carries three products and shows that our main results are robust and thus, generalizable beyond the simple two-product cases. In this extension, the firm chooses the styles, or locations, of three products \( x_1, x_2, \) and \( x_3 \). The shoppers of size \( 1 - \alpha \) observe individual product locations without incurring any positive cost. Regular consumers of size \( \alpha \) only observe the brand position \( B = (x_1 + x_2 + x_3)/3 \) after they incur a search cost \( s > 0 \) to visit the firm’s store. After observing the brand position \( B \), regular consumers decide whether to visit the brand. Without loss of generality, it is assumed that \( x_1 \leq B \leq x_2 \leq x_3 \). We can define \( x_1 = B - \Delta_1, x_2 = B + \Delta_2, \) and \( x_3 = B + \Delta_3 \), which implies \( \Delta_1 \geq \Delta_3 \geq \Delta_2 \geq 0 \) and \(-\Delta_1 + \Delta_2 + \Delta_3 = 0\). If \( \Delta_2 = 0 \), then we have \( \Delta_1 = \Delta_3 \) such that the middle product is located precisely at \( B \) and two other products are equally distant (in the opposite directions) from \( B \). We refer to this special case with \( \Delta_2 = 0 \) as a symmetric positioning.

Repeating the steps of analysis of the two-product model, we show that the qualitative results are robust to the case with more products. We briefly state the
results and relegate a more detailed analysis to the appendix.

**Proposition 9 (Two Benchmarks).** Suppose that the firm carries three products. Then, the equilibrium of the two benchmark models is characterized as follows.

1. Suppose $\alpha = 0$ (shoppers only). There exists a unique equilibrium with $B^* = 0$, $A_2^* = 0$, and $A_1^* = \max\{A_{i=0}^*, 0\}$, where $\Delta_{i=0}^* = \frac{1}{\sqrt{2^2 + 16(1-\theta) + 9(1-\theta)^2 - 5(1-\theta)}}$.

2. Suppose $\alpha = 1$ (regular consumers only). If $s$ is not too small, there exists a set of symmetric equilibria such that $A_2^* = 0$, $A_1^* = \alpha$, $A_{i=1}^* = 1$, and $B^* \in [-(1/2 - \alpha_{i=1}^*), 1/2 - \alpha_{i=1}^*]$.

Similar to the main model with two products, we have $\Delta_{i=0}^* > \Delta_{i=1}^*$ such that the equilibrium spread is greater when the firm faces shoppers only than when it serves only regular consumers. In the appendix, we show that there exists $\bar{A} > 0$ such that the equilibrium spread $\Delta_{i=0}^* > \bar{A}$. Thus, had there been any regular consumers, she would not visit the firm. It implies that in a more general model with both shoppers and regular consumers $\alpha \in [0, 1]$, the firm can face a holdup problem if there are sufficiently many shoppers in the market. Therefore, the firm can lose the opportunity to serve regular consumers. This leads to the following result analogous to Proposition 6 for the two-product case.

**Proposition 10 (Niche Positioning).** If $\alpha$ is in an intermediate range, then the firm’s expected revenue can be greater as a niche positioning brand than as a mainstream positioning brand.

### 5.3. Relaxes Exogenous Pricing: An Example

In our main analysis, we have assumed that the prices of the two products are exogenously given, and the normalized consumer utility of a matched product is one. One might worry that our main analysis and results will not be robust if the firm chooses its product prices endogenously. In this section, we present one example with the firm’s choice of prices and a slight modification in our utility to demonstrate the robustness of our main analysis without endogenous pricing.

The firm sets price $p_i$ for each product $i$. Consumers’ match utility distribution is modified from Equation (1) as follows:

$$u_i(x) = \begin{cases} 
\max\{v_i - p_i, 0\}, & \text{with probability } \theta - t|x - x_i|, \\
\max\{-p_i, 0\}, & \text{otherwise}.
\end{cases}$$

Here, $v_i$ is consumer-specific consumption value, which follows a binary distribution with $v_i = v_0$ with probability $\varphi$ and $v_i = v_0 + 1$ with probability $1 - \varphi$. The term $v_i$ represents the ex ante heterogeneity among consumers, which is observable to consumers at the beginning of the game. The firm sets the prices and the positions of the two products simultaneously, and those prices are not observable to regular consumers before they pay the search cost and visit the store.23

We find that, in the modified game in which the firm chooses its product prices, there exists an equilibrium in which the firm sets $p_i = v_0$ for both products, and the firm’s decisions for brand position $B^*$ and the brand spread $\Delta$ are equivalent to the equilibrium branching decision under exogenous pricing in Propositions 5 and 6, with an appropriate scaling of parameters $\alpha$.24 In this equilibrium, no regular consumers with lower value $v_i = v_0$ will incur the positive search cost. Under the equilibrium price, the regular consumers with $v_i = v_0 + 1$ have the following utility distribution:

$$u_i(x) = \begin{cases} 
1, & \text{with probability } \theta - t|x - x_i|, \\
0, & \text{otherwise}.
\end{cases}$$

This is equivalent to the utility distribution in Equation (1) under the exogenous pricing assumption. So, the fraction of effective regular consumers who may visit the firm is $q_{\text{price}} = \frac{\alpha(1-\varphi)}{\alpha(1-\varphi) + \varphi(1-\alpha)}$. The shoppers’ decision is also the same as the model with exogenous pricing. These equivalence properties show that our main analysis of the firm’s branching decision under exogenous pricing is robust to this particular example with endogenous pricing.

### 6. Conclusion

We study a monopolist’s product portfolio design and link it to brand positioning in equilibrium. A brand’s positioning is one of a firm’s most valuable assets. It can have a significant impact on consumers’ purchase decisions, helps firms create market differentiation by providing or articulating critical information about product characteristics, and it facilitates consumer search and aids them in determining where to purchase products without having to search through multiple brands. However, the critical issue of brand positioning remains an underresearched topic in economics and marketing. There is a lack of a framework for thinking about the relationship between brand positioning and product positioning in a unified and consistent way.

In this paper, we first propose one mechanism for how a firm’s product portfolio design can determine brand positioning and how brand positioning can deliver credible information to consumers. We also provide a rationale for a firm’s specific positioning strategy: “niche” or “mainstream.” Consumers are uncertain about each product location of the brand but are aware of the average location of the brand’s products—in other words, its
brand position. Based on this information, consumers can make decisions about whether to visit a brand (or firm), whereas brands can simultaneously determine their individual product locations. We identify the conditions under which brands tend to adopt a niche positioning strategy over a mainstream strategy and find that niche brands prevail when the fraction of regular consumers in the population is within the intermediate range and search cost is sufficiently high. We also find that a mainstream-positioned brand has a greater incentive to spread its individual product locations, which leads to brand dilution, whereas the niche-positioned brand has less incentive to do so. The niche-positioned brand can thus serve as a commitment tool for a brand not to spread its individual product location and thereby, invite more consumers to visit the store. Therefore, our results shed light on the coexistence of different positioning strategies and provide firms with insights into forming an optimal product portfolio and positioning strategy.

The current research proposes one possible mechanism of how brand positioning can affect consumer information, search decisions, and market outcomes. In doing so, we adopted a simple framework of single-dimensional brand positioning through the product portfolio of a monopolist, which cannot fully capture the intricacy of multidimensional characteristics of brand positioning in consumers’ perceptual space. A natural extension would be to construct a more general framework of product portfolio design in multiple dimensions, especially including a price image. Also, we have examined a monopolistic firm’s problem. We believe our main finding (that even a monopolist sometimes finds it optimal to choose a niche positioning to commit not to dilute its own brand) can be a valuable addition to the branding literature. Nevertheless, it would be interesting to investigate, for example, how strategic considerations can change the optimal product portfolio design under competition. Extending the current framework to a competitive situation can be a fruitful and important venue for further investigation, which will broaden the implications of our study. Also, we investigated the credibility and effects of a particular piece of information (i.e., brand positioning in this paper). Another important issue is, in line with a growing body of research in information design, endogenizing the firm’s choice of the piece of information to be communicated. Finally, our main analysis assumes that consumers become aware of the firm’s brand position B as a summary statistic about the firm’s individual products. We have abstracted away from how consumers become aware of the brand position, which is another fruitful avenue for future research. We hope that our modeling framework of brand position will serve as a basis for advancing further discussions and stimulating more studies on this fundamental concept of marketing for further research.

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Appendix

Proof of Proposition 1. Under $\alpha = 0$, by Equation (A.10), $D(B, \Delta', \Delta)$ does not depend on $\Delta'$. For simplicity, we use the shorthand notation of $D$ for $D(B, \Delta', \Delta)$ when there is no confusion. We have

$$\frac{\partial D}{\partial B} = -2t(1 - \theta + t)B' = 0 \text{ and}$$

$$\frac{\partial^2 D}{\partial B^2} = -2t(1 - \theta + t) < 0 \Rightarrow B' = 0.$$ 

$$\frac{\partial D}{\partial \Delta} \bigg|_{\Delta = \Delta'} = -4t^2 \Delta \left[ \Delta' - \frac{1}{2} \left( 1 - \frac{1}{\theta} \right) \right] = 0 \text{ and}$$

$$\frac{\partial^2 D}{\partial \Delta^2} \bigg|_{\Delta = \Delta'} = -8t^3 \left[ \Delta' - \frac{1}{4} \left( 1 - \frac{1}{\theta} \right) \right] \leq 0 \Rightarrow \Delta' = \max \left\{ \frac{1}{2} \left( 1 - \frac{1}{\theta} \right), 1 \right\}.$$ 

Notice that we always have $\Delta' \leq 1/2$. The comparative statics of $\Delta'$ with respect to $t$ and $\theta$ are straightforward to obtain. □

Proof of Proposition 2. Given any $B \in [-1/2, 1/2]$, let us consider three cases depending on $\Delta'$ according to Equation (A.9). For each case, we will first relax the constraint $\Delta \leq 1/2 - |B|$ and solve the resulting unconstrained optimization problem in Equation (6), and then, we consider when will the constraint $\Delta \leq 1/2 - |B|$ be binding. For simplicity, we use the shorthand notation of $D$ for $D(B, \Delta', \Delta)$ when there is no confusion.

First, $\Delta' \leq \Delta_1$, in which case all visitors come from one interval on the Hotelling line. Based on Equations (A.9) and (A.10), we have the first- and second-order optimality conditions:

$$\frac{\partial D}{\partial \Delta} \bigg|_{\Delta = \Delta'} = 2t^2 \Delta \left[ \sqrt{\frac{1-s}{t^2}} + (\Delta')^2 - 2 \Delta' - \frac{2(1 - \theta)}{t} \right] = 0,$$

$$\frac{\partial^2 D}{\partial \Delta^2} \bigg|_{\Delta = \Delta'} = 2t^2 \left[ \sqrt{\frac{1-s}{t^2}} + (\Delta')^2 - 4 \Delta' - \frac{2(1 - \theta)}{t} \right] \leq 0.$$ 

This implies $\Delta' = \max \left\{ \sqrt{3(1-s) + 4(1-\theta)^2} - 4(1-\theta) \right\} / (3t, 0) = \max \{\Delta'_1, 0\}$, which does not depend on $B$. This
Consider $\Delta'(B)$ given by Equation (A.1). By symmetry, it is without loss of generality to consider $B \geq 0$. We have the following four observations. (1) $\partial_B D(B, \Delta', \Delta) = 0$. (2) From Equation (A.1), it is obvious that $\Delta'(B) \leq 0$ for $B \geq 0$. (3) By the first-order optimality condition, we have that $\partial_B D(\Delta'(B), \Delta'(B)) \geq 0$, where the equality holds when $|B| + \Delta'(B)/2 < 1/2$ and the inequality holds when $|B| + \Delta'(B) = 1/2$. (4) $\partial_B D(\Delta'(B), \Delta'(B)) = s \Delta'(B)/\sqrt{\Delta'(B)^2 + (1-s)/t^2} > 0$. These four observations together imply

$$
\frac{dD(B, \Delta'(B), \Delta'(B))}{dB} = \frac{\partial_B D(B, \Delta'(B), \Delta'(B))}{dB} + \frac{\partial_B D(\Delta'(B), \Delta'(B))}{dB} \Delta'(B) \leq 0.
$$

(A.5)

This implies that the equilibrium brand position $B^*$ takes any value such that $|B^*| \leq 1/2 - \max\{\Delta_{\alpha=0}^*, 0\}$ and correspondingly, the equilibrium spread $\Delta = \max\{\Delta_{\alpha=0}^*, 0\}$. Moreover, later in Proposition 3, we show that $\max\{\Delta_{\alpha=0}^*, 0\} \leq \max\{\Delta_{\alpha=1}^*, 0\} \leq 1/2$. Notice that this equilibrium exists for the entire parameter space under Assumptions 1 and 2.

**Proof of Proposition 3.** We prove that $\max\{\Delta_{\alpha=0}^*, 0\} \geq \max\{\Delta_{\alpha=1}^*, 0\}$. Notice that $\Delta_{\alpha=1}^*$ decreases with $s$ and $s \geq 1 - (1 - \theta + t/2)^2$ under Assumption 2. This implies that $\Delta_{\alpha=1}^*$ takes the maximum value of $\left[\sqrt{3(1 - \theta + t/2)^2 + 4(1 - \theta^2)} - 4(1 - \theta)\right]/(3t)$ at $s = 1 - (1 - \theta + t/2)^2$. Therefore, we only need to prove that

$$
\max\{\Delta_{\alpha=0}^*, 0\} \geq \max\left\{\frac{\sqrt{3(1 - \theta + t/2)^2 + 4(1 - \theta^2)} - 4(1 - \theta)}{3t}, 0\right\}.
$$

To prove this, we only need to prove that $\Delta_{\alpha=0}^* \geq \max\left\{\frac{\sqrt{3(1 - \theta + t/2)^2 + 4(1 - \theta^2)} - 4(1 - \theta)}{3t}, 0\right\}$. Further, notice that $\Delta_{\alpha=0}^* \geq \frac{\sqrt{3(1 - \theta + t/2)^2 + 4(1 - \theta^2)} - 4(1 - \theta)}{3t}$, and notice that $\Delta_{\alpha=0}^* \geq \frac{\sqrt{3(1 - \theta + t/2)^2 + 4(1 - \theta^2)} - 4(1 - \theta)}{3t}$, and notice that $\Delta_{\alpha=0}^* \geq \frac{\sqrt{3(1 - \theta + t/2)^2 + 4(1 - \theta^2)} - 4(1 - \theta)}{3t}$, implies that $\Delta_{\alpha=0}^* \geq \frac{\sqrt{3(1 - \theta + t/2)^2 + 4(1 - \theta^2)} - 4(1 - \theta)}{3t}$. This completes the proof.

**Proof of Lemma 1.** First differentiating $D$ with respect to $\Delta$ results in $\frac{dD}{d\Delta} = 2t \cdot \Delta_{\alpha=0}^* \cdot \left[1 - (1 + aT)\theta + t \left(1 - 2\Delta_{\alpha=0}^* - \Delta_{\alpha=0}^* \right)\right] = 2t \cdot \Delta_{\alpha=0}^* \cdot \left[1 - (1 + aT)\theta + t \left(1 - 2\Delta_{\alpha=0}^* - \Delta_{\alpha=0}^* \right)\right]$. Differentiating both sides with respect to $\alpha$, we have $\frac{d}{d\alpha} \cdot \left(1 - (1 + aT)\theta + t \left(1 - 2\Delta_{\alpha=0}^* - \Delta_{\alpha=0}^* \right)\right) = \frac{d\Delta_{\alpha=0}^*}{d\alpha} \cdot \left[1 - (1 + aT)\theta + t \left(1 - 2\Delta_{\alpha=0}^* - \Delta_{\alpha=0}^* \right)\right]$, where $\frac{d\Delta_{\alpha=0}^*}{d\alpha} = \frac{\Delta_{\alpha=0}^*}{\sqrt{(\Delta_{\alpha=0}^*)^2 + 1/4}}$. The left-hand side is negative if and only if $t^2(\Delta_{\alpha=0}^*)^2 + 1 - s < (1 - \theta + t)^2$. Given $\Delta_{\alpha=0}^* \leq \frac{1}{2}$, we only need to show $\frac{d}{d\alpha} \cdot \left[1 - (1 + aT)\theta + t \left(1 - 2\Delta_{\alpha=0}^* - \Delta_{\alpha=0}^* \right)\right] > 0$.
\[ (\Leftrightarrow s > 1 - (1 - \theta + \frac{1}{2}(1 - \theta + \frac{1}{2})). This holds by Assumption 2. \]

**Proof of Proposition 4.** Using \( \Lambda'_{\alpha} \leq \Lambda_1 \) and \( d\Lambda'/da < 0 \), it is straightforward to show that if \( \Lambda'_{\alpha \leq 0} > \Lambda_1 \), then there exists \( \delta \in (0, 1) \) such that \( \Lambda'_{\alpha} \leq \Lambda_1 \) if and only if \( \alpha \geq \delta \). Note that \( \Lambda'_{\alpha \leq 0} > \Lambda_1 \) if and only if \( s > 1 - (1 + \theta - \frac{3}{2}) \). Also, if \( s \leq 1 - (1 + \theta - \frac{3}{2}) \), then \( \Lambda'_{\alpha \leq 0} \geq \Lambda_1 \). Thus, the results in parts (1) and (2) follow immediately from Equation (7). \( \square \)

**Proof of Proposition 5.** Two propositions will be proven in the following steps. (1) We rule out equilibria involving a medium brand in which the equilibrium spread \( \Delta ' \) satisfies \( \Delta_1 < \Delta < \Delta_2 \). (2) Equilibria for a mainstream positioning are analyzed for a strong brand with a small spread (\( \Delta \leq \Delta_1 \)) and a weak brand with a large spread (\( \Delta \geq \Delta_2 \)). (3) An equilibrium with a niche positioning is identified. (4) We compare the profits and characterize the equilibrium product portfolio decision.

**Lemma A.1.** A medium brand cannot be an equilibrium given mainstream positioning if either \( s \leq 1 - (1 + \theta - \frac{3}{2}) \) or \( s > 1 - t(1 - \theta) \) and \( \alpha \leq \alpha_{niche} \). A medium brand cannot be an equilibrium given niche positioning if \( \alpha \leq \alpha_n \) or \( \alpha > \alpha_n \).

**Proof.** Suppose an equilibrium existed such that an equilibrium spread \( \Delta ' \) satisfies \( \Delta_1 < \Delta < \Delta_2 \). Then, the regular consumers who visit the brand come from two disjoint intervals. Similar to the analysis of a strong brand in the main text, we first consider an unconstrained optimization problem. A medium brand implies that \( |B| \) cannot be too close to an end (i.e., \( |B| < 1/2 - \Delta_1 \)).

In the proof of Proposition 2, we have proved that \( d\Lambda'/\Lambda_{\alpha \leq 0} > 0 \). The proof of Proposition 1 shows that if \( \Lambda'_{\alpha \leq 0} > \Lambda_1 \) or \( s \leq 1 - (1 + \theta - \frac{3}{2}) \) and \( d\Lambda'/\Lambda_{\alpha \leq 0} > 0 \). Also, if \( \Lambda'_{\alpha \leq 0} \geq \Delta_2 \) or \( s > 1 - t(1 - \theta) \), then \( d\Lambda'/\Lambda_{\alpha \leq 0} > 0 \). Note that \( \Lambda'_{\alpha \leq 0} = \alpha_0 d\Lambda'/\Lambda_{\alpha \leq 0} = (1 - \alpha) d\Lambda'/\Lambda_{\alpha \leq 0} \). So, if \( s \leq 1 - (1 + \theta - \frac{3}{2}) \) and \( d\Lambda'/\Lambda_{\alpha \leq 0} > 0 \), then \( d\Lambda'/\Lambda_{\alpha \leq 0} < 0 \), which means that the medium brand cannot be an equilibrium.

A medium brand may be feasible with a niche positioning where \( 1/2 - \Delta_2 \leq |B| < 1/2 - \Delta_1 \) and \( \Delta_2 > 1/2 - B \). It remains to characterize the optimal choice of \( B \). For this, we compute \( dD(B, \Delta_2, \Delta_2)/dB \). In particular, we rule out any equilibrium with a medium brand by finding a condition that \( dD(B, \Delta_2, \Delta_2)/dB > 0 \), which would imply that the firm wants to position its brand close enough to an end until it reaches \( |B| = 1/2 - \Delta_1 \). When there is no confusion, \( D(B, \Delta_2, \Delta_2) \) abbreviated by \( D'(B) \). Equation (4) applies here, with \( D'(B) = \alpha \cdot D'(B)_{\alpha = 1} + (1 - \alpha) \cdot D'(B)_{\alpha = 0} \).

First, for \( \alpha = 1 \), we have \( dD'(B)_{\alpha = 1} = \frac{1}{\sqrt{\Lambda'_{\alpha} + \lambda^2}} \frac{\lambda'(\Lambda'_{\alpha} + \lambda^2) - \lambda_0(1 - \alpha)(\Lambda'_{\alpha} + \lambda^2)}{\sqrt{\Lambda'_{\alpha} + \lambda^2}} < 0 \). It is equivalent to \( \alpha \leq \theta \frac{(\Lambda'_{\alpha} + \lambda_0^2)^2 - \lambda_0^2 - (\Lambda'_{\alpha} + \lambda_0^2)}{\sqrt{\Lambda'_{\alpha} + \lambda^2}} \). For any \( \alpha = 0 \), we have \( dD'(B)_{\alpha = 0} = \frac{1}{\lambda} \frac{\lambda_0(1 - \alpha)(\Lambda'_{\alpha} + \lambda^2)}{\sqrt{\Lambda'_{\alpha} + \lambda^2}} > 0 \). This proves that \( d\Lambda/\Lambda_{\alpha \leq 0} > 0 \). Moreover, it is easy to show that \( d\Lambda/B_{\alpha \leq 0} = 0 \) and \( \Delta_2(B) = \Lambda_2(B) = 1 \). This implies that \( dD/\Lambda_{\alpha \leq 0} > 0 \).

Second, for \( \alpha = 0 \), we have \( dD(B)_{\alpha = 0} = \frac{4\Lambda_2(B) + 4(1 - \theta)\Delta_2'(B) - 1(1 - \theta)}{2} \). Because \( \Delta_2'(B) < \Delta_2 \), we look at \( dD/\Lambda_{\alpha = 0} \), where \( \Delta_2'(B) < \Delta_2 \), so to show \( dD'(B)/d\alpha_{\alpha = 0} \), we only need to show that \( \Delta_2 \leq \sqrt{2 + \delta}(1 - (1 + \theta - \frac{3}{2}) \right) \), which holds if \( s > 1 - t(1 - \theta) \). This shows that \( dD'(B)/d\alpha_{\alpha = 0} < 0 \).

Putting together the two benchmark cases of \( \alpha = 1 \) and \( \alpha = 0 \), there exist \( \alpha \leq \alpha_n \) such that \( dD(B, \Delta_2(B), \Delta_2'(B)) \leq 0 \) for \( \alpha < \alpha_n \) and \( dD(B, \Delta_2(B), \Delta_2'(B))/d\alpha > 0 \) for \( \alpha > \alpha_n \). In summary, the medium brand cannot be an equilibrium for both mainstream and niche positioning under the following conditions: \( \alpha \leq \alpha_n \) or \( \alpha > \alpha_n \) and (2) \( s \leq 1 - (1 + \theta - \frac{3}{2}) \) or \( s > 1 - t(1 - \theta) \) and \( \alpha \leq \alpha_{niche} \). \( \square \)

**Lemma A.2.** Given a mainstream positioning (i.e., \( |B| < 1/2 - \Delta_1 \)), a weak brand with a large spread (\( \Delta \geq \Delta_2 \)) can be sustained in an equilibrium only if \( s > 1/2 - \Delta_1 \).

**Proof.** No regular consumer will visit the firm, and accordingly, the firm’s optimal branding decision is equivalent to the firm’s decision in the first benchmark with only the shoppers. Therefore, the firm sets \( (B', \Lambda') = (0, \Lambda_{\alpha \leq 0}) \). Then, regular consumers’ decision not to visit the firm is optimal if only \( \Lambda_{\alpha \leq 0} > \Delta_2 \), which holds if \( s > 1 - t(1 - \theta) \). This is a necessary condition for a weak mainstream brand to be an equilibrium as the firm’s potential deviation to a niche positioning needs to be checked (which is done at the end of the proof of Proposition 5). \( \square \)

**Lemma A.3.** The optimal niche positioning is \( (B', \Lambda') = (1/2 - \Delta_1, \Delta_1) \).

**Proof.** In Lemma A.1, we ruled out a medium brand case for a niche positioning. The only remaining possibility for an equilibrium with a niche positioning is a strong brand with a small spread with \( \Delta \leq \Delta_1 \). For \( |B| \geq 1/2 - \Delta_1 \), we have \( \Delta_2(B) = 1/2 - |B| \) given by Equation (7). In this case, by the proofs of Propositions 1 and 2, we know that both the shoppers’ demand and the regular consumers’ demand decrease with \( |B| \), and therefore, the total demand, \( D(B, \Delta_2(B), \Delta_2'(B)) \), achieves the maximum at \( |B| = 1/2 - \Delta_1 \). \( \square \)

We have thus far identified optimal positioning for each of the three types of positioning: a strong mainstream, a weak mainstream, and strong niche positioning. It remains to check for profitable deviations from each of these cases. In particular, from a mainstream positioning, the firm can deviate to theorlal niche positioning and vice versa. The demand under each of the three identified candidates for equilibrium is, respectively, denoted by \( D_{\text{max}}(\alpha_{\text{niche}}) = (1 - \alpha) \cdot D'(0, \Lambda_{\alpha \leq 0})_{\alpha = 0}, D_{\text{niche}}(\alpha_{\text{niche}}) = \alpha \cdot D'(1/2 - \Delta_1, \Delta_1)_{\alpha = 0}, D_{\text{niche}}(\alpha_{\text{niche}}) = (1 - \alpha) \cdot D'(1/2 - \Delta_1, \Delta_1)_{\alpha = 0}. \) There are the following three cases:

1. A strong mainstream brand. When it exists as the optimal and consistent positioning decision, there is no profitable
deviation to a niche positioning. Therefore, if either $s < 1 - (t + 1 - \theta)^2/4$ or $\alpha \geq \tilde{\alpha}$, then a strong brand with $(B, \Delta') = (0, \Delta_{\text{main}}')$ is an equilibrium.

2. A weak mainstream brand. The firm only serves the shoppers. If the firm deviates to a niche positioning $(B', \Delta') = (\pm (1/2 - \Delta_{\text{main}}'), \Delta_{\text{main}}')$, regular consumers update their beliefs accordingly, and some of them visit. Therefore, the firm serves both shoppers and regular consumers. This is not a profitable deviation (i.e., $D_{\text{main}}' > D_{\text{main}}')$ if and only if $\alpha$ is sufficiently small, $\alpha < \pi_{\text{main}}$. So, if $s > 1 - t(1 - \theta)$ and $\alpha < \pi_{\text{main}} := \min\{\tilde{\alpha}, \pi_{\text{main}}\}$, then a weak mainstream brand $(B, \Delta') = (0, \Delta_{\text{main}}')$ is an equilibrium.

3. A strong niche brand. Whenever a strong mainstream positioning exists, it is profitable to deviate to a mainstream positioning. If a weak mainstream brand exists, niche positioning is more profitable if $s > 1 - t(1 - \theta)$ and $\Delta_{\text{niche}} := \max\{\tilde{\alpha}, \pi_{\text{niche}}\} < \alpha < \pi_{\text{niche}}$. This completes the proof. □

**Proof of Proposition 6.** The proof follows immediately from the proof of Proposition 5, case (3) when $s > 1 - t(1 - \theta)$ and $\Delta_{\text{niche}} < \alpha < \pi_{\text{niche}}$. Finally, for this result, the interval $\alpha \in (\Delta_{\text{niche}}, \pi_{\text{niche}})$ must be nonempty. We are unable to show these properties for the general parameter space. Instead, we present a numeric analysis to show that these intervals are nonempty under some parameter setting. Take $s = 0.9, \theta = 0.8, t = 0.7$. Then, $\alpha \equiv \pi_{\text{niche}} = 0.475, \Delta_{\text{niche}} = 0.271$, and $\pi_{\text{main}} = 0.047$. So, our main result holds for all parameter regions $\alpha \in (0.271, 0.475)$. □

**Proof of Proposition 7.** First, notice that when the search cost $s$ is sufficiently high so that regular consumers are not willing to visit the store, we have that the weak mainstream brand equilibrium exists. On the other hand, when the search cost $s$ is sufficiently low so that regular consumers are willing to visit the store even if the firm places the two products at the locations that maximize shoppers’ demand, we have that a strong mainstream brand equilibrium must exist.

Given a mainstream equilibrium exists, we next show that the equilibrium must be the same as that in the main model. The statement regarding the weak mainstream brand equilibrium is straightforward because only shoppers whose behaviors are not influenced by the definition of $B$ make purchases. Next, consider the strong mainstream brand equilibrium. By definition, we have $B = x_1 + x_2 = 0$, given which the firm is solving the same problem as that in the main model when deciding $x_1 = \Delta_{\text{main}}$; moreover, regular consumers’ expectations, $x_1$ and $\tilde{x}_2$, satisfy that $x_1 + \tilde{x}_2 = B = 0$, the same as that in the main model. □

**Proof of Proposition 8.** The firm’s choice of the two product locations $x_1$ and $x_2$ can be converted into the brand position $B$ and a vector of spreads $\Delta = (\Delta_{\text{L}}, \Delta_{\text{R}})$, where $x_1 = B - \Delta_{\text{L}}$ and $x_2 = B + \Delta_{\text{R}}$. Then, Equation (8) becomes

$$S_1(B, \Delta) \cdot \Delta_{\text{L}} - S_2(B, \Delta) \cdot \Delta_{\text{R}} = 0. \quad (A.6)$$

This equation characterizes the relationship between $B$ and $\Delta$. Especially for a given $B$ observed by regular consumers, the possible vectors $\Delta$ that can support $B$ must satisfy the equation.

It is useful to differentiate Equation (A.6) with respect to $\Delta_{\text{R}}$ and $B$ as follows:

$$\frac{\partial S_1}{\partial \Delta_{\text{L}}} \Delta_{\text{R}} - \frac{\partial S_1}{\partial \Delta_{\text{R}}} \cdot \Delta_{\text{L}} \bigg|_{\Delta = \Delta'} = 0 \quad (A.7)$$

and

$$\frac{\partial S_1}{\partial B} \Delta_{\text{R}} - \frac{\partial S_1}{\partial \Delta_{\text{R}}} \cdot \Delta_{\text{L}} \bigg|_{\Delta = \Delta'} = \frac{\partial S_1}{\partial \Delta_{\text{L}}} \Delta_{\text{R}} \bigg|_{\Delta = \Delta'}. \quad (A.8)$$

Given $B$ and consumers’ expectation for the spread $\Delta$, the set of regular consumers who will incur the search cost $s > 0$ and visit the firm is characterized the same as in Equation (A.9) in the main analysis. We denote the set of visitors on the Hotelling line as $\mathcal{V}(B, \Delta)$, where

$$\mathcal{V}(B, \Delta) = \begin{cases} \{B' - \Gamma_1(\Delta), B' - \Gamma_1(\Delta), B' + \Gamma_2(\Delta), B' + \Gamma_2(\Delta)\} & \text{if } \Delta_{\text{L}} + \Delta_{\text{R}} \leq \Delta_{\text{L}}, \\ \{B' - \Gamma_1(\Delta), B' - \Gamma_2(\Delta)\} & \text{if } \Delta_{\text{L}} + \Delta_{\text{R}} > \Delta_{\text{L}} \text{ or } \Delta_{\text{L}}^{\text{opt}} \leq \Delta_{\text{L}} \end{cases} \quad (A.9)$$

where $B' = B - (\Delta_{\text{L}} + \Delta_{\text{R}})/2, \Gamma_1(\Delta) = \sqrt{(\Delta_{\text{L}} + \Delta_{\text{R}})^2/4 + (1 - s)/t^2}$,

$$\quad - (1 - \theta)/t, \Gamma_2(\Delta) = \sqrt{(\Delta_{\text{L}} + \Delta_{\text{R}})^2/4 + (1 - s)/t^2} - (1 - s)/t^2.$$ The cutoff spreads $\Delta_{\text{L}}$ and $\Delta_{\text{R}}$ are the same cutoffs that determine the range of strong, weak, and medium brands.

Let $D(B, \Delta', \Delta)$ be the firm’s expected demand when the regular consumers observe $B$ and expect $\Delta = (\Delta_{\text{L}}', \Delta_{\text{R}}')$, and the firm chooses the spread $\Delta = (\Delta_{\text{L}}', \Delta_{\text{R}})$:

$$D(B, \Delta', \Delta) = \frac{1 - \alpha}{2} \int_1^t [1 - (1 - \theta + t)B + \Delta_{\text{R}} - x](1 - \theta + t)B + \Delta_{\text{R}} - x]dx + \frac{\alpha}{2} \int_{x \in \mathcal{V}(B, \Delta')} [1 - (1 - \theta + t)B + \Delta_{\text{R}} - x](1 - \theta + t)B + \Delta_{\text{R}} - x]dx. \quad (A.10)$$

We want to find conditions under which a niche positioning equilibrium exists in which the firm chooses the location of the two products $x_1 = 1/2 - \Delta_{\text{L}}$ and $x_2 = 1/2$. Equivalently, $B + \Delta_{\text{L}} = 1/2$ and $B + \Delta_{\text{R}} = 2 - \Delta_{\text{L}}$. These two conditions, together with Equation (A.6), pin down the observed $B$ and the deterministic $\Delta_{\text{L}}$ and $\Delta_{\text{R}}$. We first solve for the unconstrained optimal $\Delta'$ consistent with the consumers’ expectation, provided that they observe $B$. That is,}

$$\frac{\partial D(B, \Delta')}{\partial \Delta'} \bigg|_{\Delta = \Delta'} = 0. \quad (A.11)$$
Then, we also consider constraints $B + \Delta_t < 1/2$ and Equations (A.6), (A.7), and (A.8). Given the characterized $\Delta(B)$, the firm chooses $B = B^*$ in equilibrium that maximizes the total expected demand $D(B^*, \Delta(B^*), \Delta'(B^*))$.

We perform numerical analysis to verify that the main result of Proposition 6 is robust to the current formulation of the demand-weighted brand position using the same numeric examples offered in the proof of Proposition 6 (i.e., $s = 0.9$, $\theta = 0.8$, $t = 0.7$, and for $\alpha = 0.271$ and $\alpha = 0.475$). The steps are as follows.

1. The nonexistence of a strong mainstream brand is verified. That is, given $B - \Delta^*_t < 1/2 - 2\Delta_1$ (or equivalently, $B < 10\frac{(0.8)^3}{72} - 10\frac{(0.8)^2}{72} < \frac{10}{72}$, by Equation (A.6)), there exists no $\Delta^*$ that satisfies Equations (A.6) and (A.7).

2. Similarly, the nonexistence of a strong medium brand is verified. That is, given $B - \Delta^*_t < 1/2 - 2\Delta_1$ (or equivalently, $B < 10\frac{(0.8)^3}{72} - 10\frac{(0.8)^2}{72} < \frac{10}{72}$), there is no $\Delta^*$ that satisfies the same conditions as the previous case.

3. It is verified that given a strong niche brand (i.e., $B + \Delta^*_t = 1/2$ and $B - \Delta^*_t > 1/2 - 2\Delta_1$ (or equivalently, $B < 10\frac{(0.8)^3}{72} - 10\frac{(0.8)^2}{72} < \frac{10}{72}$), the optimal position is attained when $B - \Delta^*_t = 1/2 - 2\Delta_1$. These production locations are exactly the same as the niche branding equilibrium identified in Proposition 6. For this, we check that for any $B$ in this range, $D(B, \Delta^*_t, \Delta_1)\Delta \Delta_{\Delta^*_t} > 0$. Moreover, $D(B, \Delta^*_t, \Delta_1)/\Delta \Delta_{\Delta^*_t} > 0$, where $\Delta^*_t$ is such that $\Delta^*_t = 1/2 - B$ and $\Delta^*_t$ satisfies Equation (A.7). Therefore, given a strong niche brand, the brand wants to move $B$ (and their products) closer to the center of the Hotelling line.

4. Lastly, for a medium niche brand (i.e., $B < 10\frac{(0.8)^3}{72} - 10\frac{(0.8)^2}{72} < \frac{10}{72}$ and $\Delta^*_t = 1/2 - B$), first it is verified that $D(B, \Delta^*_t, \Delta_1)/\Delta \Delta_{\Delta^*_t} > 0$. Moreover, it is checked that Equation (A.6) does not hold for a medium brand. Thus, a medium niche brand does not exist.

Therefore, given a niche brand, the optimal positioning is $(B^*, \Delta^*_t, \Delta^*_t) = (0.312226, 0.248724, 0.187774)$, where $-34.01317 + 17.642 V_{05} = 0.312226$ and $1/2 - (-34.01317 + 17.642 V_{05}) = 0.187774$. By the linearity of all the conditions considered in the analysis in $\alpha$, this shows that the same niche positioning is optimal for the interval $\alpha 

Given a specific brand position, the expected demand in this alternative model with demand-weighted brand position is identical to that under the main model with symmetric weights of $1/2$. Therefore, the result in Proposition 6 holds here as well.

Proof of Proposition 9.

Case A.1. $\alpha = 0$. A shopper located at $x \in [-1, 1]$ expects to receive utility $\mathbb{E}[u(x)|x] = 1 - (1 - \theta + t|x - B + \Delta_1|)(1 - \theta + t|x - B - \Delta_1|)$ with the firm's expected revenue is $D_{u=0}(B, \Delta_1, \Delta_3)$, which the firm maximizes by setting $B, \Delta_1, \Delta_2$, and $\Delta_3$. The first-order condition should satisfy $\partial D(B, \Delta_1, \Delta_3)/\partial B = 0, \partial D(B, \Delta_1, \Delta_3)/\partial \Delta_1 = 0$, and $\partial D(B, \Delta_1, \Delta_3)/\partial \Delta_3 = 0$:

$$\frac{\partial D(B, \Delta_1, \Delta_3)}{\partial B} = -tB^3 + tB^3(3 - \Delta_1^2 - \Delta_1^2 - \Delta_3^2) + 6t(1 - \theta)$$

$$\frac{\partial D(B, \Delta_1, \Delta_3)}{\partial \Delta_1} = \frac{t(\Delta_3 - 2\Delta_1)}{2}\left(\Delta_1^2 + (6\Delta_3 + 4t(1 - \theta))\right)$$

$$\frac{\partial D(B, \Delta_1, \Delta_3)}{\partial \Delta_3} = -\frac{t(\Delta_1 - 2\Delta_3)}{2}\left(\Delta_1^2 + (6\Delta_3 + 4t(1 - \theta))\right)$$

First, note that $\frac{\partial D(B, \Delta_1, \Delta_3)}{\partial \Delta_1} = 0$ if and only if $B = 0$ and $\Delta_1 \Delta_3 (\Delta_1 - \Delta_3) = 0$. This is because the expression inside the parentheses is non-negative, and it can be zero only if the terms multiplied by $B$ vanish and so does the term $t^2\Delta_1 \Delta_3 (\Delta_1 - \Delta_3)$. Given that $B = 0$, we have $D_{u=0}(B, \Delta_1, \Delta_3)$ (i.e., there is symmetry between $\Delta_1$ and $\Delta_3$). This implies that there the optimal levels of $\Delta_1$ and $\Delta_3$ will coincide (i.e., $\Delta_1^* = \Delta_3^*$). Plugging this property back into the second or third equation results in $\frac{\partial D(B, \Delta_1^*, \Delta_3^*)}{\partial \Delta_1} = 1 \Delta^* \cdot g_{\alpha=0}(\Delta^*) = 0$, where $g_{\alpha=0}(\Delta) = t^2 - 2\Delta^2 - t^2 - 5\Delta \cdot t(1 - \theta) + 2t(1 - \theta) - 2(1 - \theta)^2$, for which $\Delta^* = \min(0, \Delta_{\alpha=0})$, where $\Delta_{\alpha=0} = \frac{\sqrt{6\Delta^2 + 6\Delta\Delta + 2\Delta^3 - 5\Delta^2 - 5\Delta\Delta^2 - 10\Delta^3 - 10\Delta^2}}{4}$.

Case A.2. $\alpha = 1$. Assume that $\alpha = 1$ and $\Delta_1 = \Delta_3 = \Delta^*$. A regular consumer located at $x$ expects to receive utility $\mathbb{E}[u(x)|x] = 1 - (1 - \theta + t|x - B + \Delta_1|)(1 - \theta + t|x - B - \Delta_1|)$ with the firm's expected demand $D_{\alpha=1}(B, \Delta_1, \Delta_3)$, which the two endpoints of the interval are solutions to the equation $\mathbb{E}[u(x)|\Delta] = s$. If the anticipated spread is too large (i.e., $\Delta > \Delta^* := \frac{1 - \theta}{t}$), then no regular consumer will engage in search. Assuming symmetry (i.e., $\Delta_2 = 0$; hence, $\Delta_1 = \Delta_3 = \Delta^*$), we can obtain $\Delta = B - f(\Delta)$ and $\Delta = B + f(\Delta)$, where $0 < f(\Delta) < 1$ and $25$.

$$f(\Delta) := \frac{t}{1 - \theta} + \frac{\sqrt{27(1 - s) - \sqrt{27^2(1 - s)^2 - 108\cdot t^6 \cdot \Delta^6}}}{3\sqrt{2}\cdot t \cdot \Delta^2}$$

For simplicity, we focus on a symmetric case from here on. The total consumer demand $D_{\alpha=1}(B, \Delta_1, \Delta_3; \Delta_1, \Delta_3) :=$
In a market consisting of a fraction of regular consumers and 1 − α of shoppers, the expected revenue of the firm is $D_\alpha = 0$ for $D_\alpha = 1$. The expected demand is $D_\alpha(\beta, \Delta_1, \Delta_2; \bar{A}_1, \bar{A}_2) = \frac{\alpha}{1-\alpha} \int_x f(x) dx$.

The firm will choose $\bar{B}, \Delta_1$, and $\Delta_2$ to maximize this expected demand.

If $\alpha$ is sufficiently small so that the firm cannot serve all of the regular consumers, then as a mainstream positioning brand, the maximum expected demand is $D_\alpha(0, \Delta_\alpha) = (1 - \alpha) \cdot D_0(a_0(0, \Delta_\alpha))$. On the other hand, if the firm chooses a niche positioning $\bar{B} = 1 - \bar{N}, \bar{\Delta} = 1 - \bar{N}$, the expected demand is $D_\alpha(0, \Delta_\alpha) = 0$.

The equilibrium spread $\Delta_\alpha$ under $B = 0$ should satisfy $\frac{\alpha}{1-\alpha} \cdot g_{a_0}(\Delta) = 0$, where the left-hand side of the equation is decreasing in $\Delta$. We will find a lower bound of the left-hand side to obtain a lower bound of $\Delta_\alpha$ and compute conditions under which this lower bound is greater than $\bar{\Delta}$.

Note that $f(0) > 0$, and therefore $g_{a_0}(\Delta) = \frac{\alpha}{1-\alpha} \cdot \frac{\sqrt{2(1+\beta)\Delta + \beta(1-\beta)(1+\beta)} - 2\sqrt{2(1+\beta)}}{\sqrt{2(1+\beta)}} < 0$. The solution to the equation $t = \frac{\alpha}{1-\alpha} \cdot g_{a_0}(\Delta) = 0$ can be computed and denoted by $\Delta_\alpha^* = \frac{16(1-\beta)}{\sqrt{2(1+\beta)}} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{\sqrt{2(1+\beta)\Delta + \beta(1-\beta)(1+\beta)} - 2\sqrt{2(1+\beta)}}{\sqrt{2(1+\beta)}}$.

Additionally, $\Delta_\alpha^* > \bar{\Delta}$ if and only if $\alpha$ is sufficiently small and $s$ large enough.

By finding ati a number that satisfies sufficient conditions for the result stated in this proposition to hold: $\theta = 0.8, t = 0.5, s = 0.97$, and $0.375 \leq \alpha \leq 0.388$. □

Endnotes

1. Gucci and Hermes have been quoted in a Harvard Business case Gucci Group N.V. (Yoffie and Kwak 2001) as examples of luxury fashion brands with opposite positioning. According to the case, Gucci even tried to take Hermes’ positioning (“classic, timeless luxury”), but it failed and found a new position of “fashion luxury with a sensual appeal.”
Mean location is just one operationalization to capture the fact that brand positioning is determined collectively by all of its products under the same brand in a tractable way. Such “aggregate information” represents the average style of each individual product or characteristic common to products under the same brand name. For example, Gucci products tend to be more fashionable, whereas Hermes is more classic. In Section 5.1, we show the robustness of our results by examining another specification of the brand positioning, which is the weighted average of the product locations where the weights are shoppers’ demand for the two products.

Also, see Bronnenberg et al. (2019) for an excellent review of the economics of brand and branding. Recent studies have focused on measuring the brand value in a static setting (Goldfarb et al. 2009) and a dynamic environment (Borkovsky et al. 2017).

In particular, Rhodes (2014) is closely related to our study. Rhodes (2014) studies a retail setting where consumers decide whether to search for firms carrying multiple products that are not substitutes. It finds that carrying multiple products can provide the firm with the commitment power to maintain low prices, thus encouraging consumer visits. Both his and our papers can be considered as providing different mechanisms on how firms can guide consumer search through the assortment of products. Although Rhodes (2014) is more about retailing, our paper is about branding.

We impose a restriction that the support for the firm’s product positions is a subset of the consumer market. This assumption eliminates the cases of truncated demand at end points and thereby, simplifies our equilibrium analysis.

To ensure that \( \theta - 1(x - x_i) \in [0, 1] \) for any \( x \in [-1, 1] \) and \( x_i \in [-1/2, 1/2] \), we impose the restriction that \( 0 < 3t/2 \leq \theta \leq 1 \). In fact, the farthest possible distance between a consumer and a product is \( 3/2 \), so we need \( \theta - 3t/2 \geq 0 \). Also, the consumer can be located precisely at either product, so \( \theta = 1 \).

In a standard deterministic Hotelling model where \( u_i(x) = \theta - 1(x - x_i) \), the consumer’s utility is determined by a product closer to the consumer’s location, irrespective of the design of remaining products in the product portfolio.

Our stochastic utility setup where consumers realize a binary utility after incurring a positive search cost is similar to Athey and Ellison (2011). In their model of ordered sponsored search ads, each link \( i \) will generate a binary utility for users, where the probability of the good outcome \( q_i \) is drawn from a common distribution \( F \). The exact outcome of each link is realized only after the user incurs a positive click cost. So, although in expectation a link with a higher \( q_i \) is more likely to generate a good outcome, the realized outcome need not always be. Similarly in our model, a product located at \( x_i \) provides a good match to the consumer at \( x \) with probability \( \theta - 1(x - x_i) \). A product located closer to the consumer is more likely to satisfy the consumer’s need, but it sometimes realizes to be a less preferred option.

In a more general model setup, we can have consumers’ utility of the outside option as \( u_0 \in [0, 1] \). One can show that the general model is equivalent to our current model if we scale the search cost in the general model by \( 1/(1 - u_0) \).

Additional examples also demonstrate that a Hotelling line (as opposed to other demand models without opposite ends, such as a circular city or logit model) provides a natural and appropriate environment to capture the heterogeneous horizontal preferences for consumers such as designs or styles in a spectrum. A circular city or a logit model with symmetric demand distribution is not conducive to an analysis of an important managerial decision on whether to become a mainstream or niche brand.

Our model setup—specifically, our formulation of brand position—constrains a niche brand to have a small-enough spread. This restriction may reflect reality. Most brands perceived by consumers as niche indeed carry products of similar styles that are consistently positioned in a niche market. For instance, Lefty’s specializes in a niche market of left-handed products. If Lefty’s offered some products for right-handed users, Lefty’s would no longer be the same niche brand for left-handed consumers that it used to be. Other examples include Studio Tomboy and Urban Outfitters, which are perceived as “bold and casual” and “hip and contemporary,” respectively. Both brands offer niche designs mostly consistent with their images. Had Studio Tomboy or Urban Outfitters carried some formal dresses, consumers’ perception of the brands would be altered and no longer be so niche anymore.

For any given brand position \( B \in [-1/2, 1/2] \), we have \( v(B, \Lambda) \in (-1, 1) \) under Assumption 2, and therefore, the demand truncation never happens.

However, the threshold \( \Delta_s \) decreases in \( s \) at a faster rate. Therefore, as we state in the results in Section 4.3, as \( s \) increases, the brand is harder to maintain a small spread in equilibrium.

In the appendix, we analyze all three cases including a medium brand and identify conditions under which a medium brand cannot be an equilibrium. See Lemma A.1 in the proof of Proposition 5.

In the appendix (proof to Proposition 5), we follow a similar procedure to analyze the other two cases with \( \Delta_1 < \Delta' < \Delta_2 \) and \( \Delta' \geq \Delta_2 \).

We can further prove the uniqueness of this equilibrium under \( s \leq 1 - (t + 1 - \theta)^2/\Lambda \).

The condition \( s > 1 - (1 - \theta) \) implies \( \Delta_{\text{small}} > \Delta_1 \) such that the spread \( \Delta_{\text{small}} \) is part of an equilibrium for a mainstream brand where no regular consumers visit the brand.

We have \( \alpha_{\text{small}} \geq \alpha_{\text{main}} \). Therefore, the interval of \( \alpha \) for a niche positioning is optimal in Proposition 6 is to the right of the interval \([0, \alpha_{\text{main}}]\) for which a weak mainstream brand is optimal in part (2) of Proposition 5.

Note that Propositions 5 and 6 together do not cover the entire parameter space in Figure 3; notably, equilibrium in one intermediate region for \( s \) and two intermediate intervals of \( \alpha \) is not identified. This is because we have identified a sufficient condition to rule out the existence of a medium brand equilibrium where regular consumers visit the brand from two disjoint intervals. Therefore, in these missing regions, a medium brand equilibrium may exist. However, an analysis of a medium brand would add significant technical complexities, and thus, we focus on the other two cases. This decision helps us clearly understand the important trade-offs in the firm’s product portfolio design decision and how it affects the role of brand positioning as a communication mechanism.

It is assumed that if both products provide a good match, consumers choose one of them with equal probabilities.

Notice that we do not model regular consumers’ interactions with other regular consumers because this will lead to a loop in the definition of \( B \), where regular consumers’ search and purchase decisions depend on their observation of \( B \), which in turn, depends on their search and purchase decisions. This kind of self-fulfilled expectation could lead to multiple equilibria and greatly complicates the problem.

In fact, there is a closed-form expression for the range of \( x \) given \( B \) implied by Equation (8), but it is very complex as it is a root of a third-order polynomial equation.

The assumption that consumers do not observe the prices until they visit the store is conventional in the consumer search literature reviewed in Section 2. This assumption allows us to check for the robustness of our results without underlining brand positioning as the main channel of information communication. Alternatively, if prices were observable to consumers prior to visiting a store, then it may be possible that the prices convey some information about
individual products. Analyzing such a model in addition to our main model with implicit communication through brand positioning is beyond the scope of this paper, which we leave for future research.

24 It is straightforward to argue that the firm will set the equilibrium price for each product as either \( p_1 = 1 \) or \( p_3 \). If the equilibrium price is set at \( p_3 \), only the regular consumers with \( \nu = 0 \) will make a purchase if they find a match, in which case they derive zero utility. Consequently, it is not worthwhile for them to pay the search cost and visit the store in the first place. It becomes a trivial case, so we focus on a more interesting case where \( p_1 = p_3 \) when \( \nu_0 \) is sufficiently high and/or \( \nu \) is sufficiently high.

25 This requires \( s < 1 - \frac{1}{3\sqrt{3}} \cdot 1 - \frac{1}{3\sqrt{3}} \cdot 1 = \frac{(1 - \frac{1}{3\sqrt{3}})^2}{3\sqrt{3}} \) to ensure that the function is real valued. Further, it requires an assumption such that \( V(B, \lambda, \lambda) \) is one interval. Also, it is that if \( i \) is an increasing function whenever it is real valued.

26 The last inequality is because of \( \sqrt{-27(1-s)} \geq \sqrt{-27(1-s) + 2\sqrt{27(1-s)^2} - 27s/16} \leq 0 \), and therefore, squaring the terms results in \( \sqrt{-27(1-s)^2} \geq \sqrt{-27(1-s) + 2\sqrt{27(1-s)^2} - 27s/16} \geq 0 \).

27 Note that here we do not show that \((B, \Lambda, \Lambda) = (1/2 - \bar{X}, \bar{X})\) is the optimal niche positioning decision, which is analytically cumbersome for the three-product case. However, as we compare profits under the optimal mainstream positioning with those under a possibly suboptimal niche positioning, we make a conservative statement in the proposition.

References


