A Theory of Irrelevant Advertising: An Agency-Induced Targeting Inefficiency

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Abstract. Ad targeting technology has enabled a highly personalized delivery of online ads. Behind this development is the belief that better targeting will lead to more relevant ads. This paper challenges this lay belief by showing that irrelevant advertising can arise not necessarily from technological imperfection but also from the incentive problem embedded in the ad agency-advertisers relationship. We first demonstrate that the ad agency serving multiple advertisers may strategically allocate an ad impression to a lesser-matched, sometimes totally irrelevant, niche advertiser because future impressions can match better with the mainstream advertiser. We further find that, without a contractual obligation to serve both advertisers, the agency may not deliver completely irrelevant ads to consumers. However, another type of inefficiency can arise where the agency may not send any ad to potentially interested consumers who have a strictly positive match probability with advertisers. These inefficiencies arise due to contractual restrictions, either contractual obligations or budget constraints, when the agency serves multiple advertisers. As such, we endogenize the advertisers’ contractual requirement choices and show how the contractual obligation(s) can arise in equilibrium. Finally, we show that irrelevant ads will not disappear simply because more impressions are available in the market. Our analysis suggests that as the number of impressions increases, the irrelevant ads can persist, but the probability of receiving irrelevant ads decreases.

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1. Introduction

Digital advertising promises to leave behind the spray-and-pray tactics of the past and deliver ads closely aligned with true consumer interests. Advancements in targeting technology are helping firms place ads on various websites or mobile platforms based on customer demographics (e.g., age, gender, location), browsing habits (e.g., search terms, sites visited, past purchases), and social activity on platforms like Facebook or Twitter (e.g., postings, clicked ads, “likes,” “retweets,” “shares”). With increasingly individualized data, firms can target customers more efficiently, which suggests consumers must be exposed to ads that are more relevant to their actual needs and wants (Rafieian and Yoganarasimhan 2020, Shin and Yu 2021). This relevance is the core promise, the bright future, of the digital advertising industry.

The reality is not playing out so well.1 Targeting is far from perfect; the problem of irrelevant, if not downright annoying, advertising is still widespread, even growing.2 Ogury’s “The Reality Report 2019” surveyed 287,000 mobile users and found that 90% of users consider targeted mobile ads annoying. This figure was up from 79% of 2018.3 This is all puzzling, given that the access to consumer data and the quality of targeting technologies have only improved over time.

Consider the experience of one of the authors, who recently searched online for Subaru. He perused a few screens on the Subaru website and quickly found the car unsuitable. Nonetheless, a Subaru advertisement began to follow him everywhere: In his social media feeds, preceding YouTube videos, and alongside CNN news stories. Even after spending significantly more time searching for electric cars, such as a Toyota Prius or Tesla, the Subaru ads trailed him.

On the one hand, this is a typical “behavioral re-targeting” (or simply re-targeting) tactic. The past browsing history suggests some demonstrated interest in the Subaru brand. On the other hand, if advertisers can discern the author’s interest in Subaru, can they not discern his greater interest in Toyota Prius or Tesla? He had spent far more time searching Toyota’s websites. Clearly, technology alone cannot explain this inefficiency. What else
contributes? Here, we focus on an agency problem rooted in the online advertising ecosystem.

In the early days of online advertising, display ads were sold through a direct contract between the publisher and the advertiser (or agency). Advertising slots that remained unsold were taken to the ad exchange and auctioned off in real time as consumer impressions arrived. As technology has advanced, more publishers have chosen to sell their ad space through real-time bidding (RTB). The programmatic industry has grown at a phenomenal rate, with spending set to reach more than $150 billion worldwide in 2021. Almost all these online ads compete for one of a given number of slots available in various online venues, such as the homepage of newspaper sites or social networks feeds. The process, which usually occurs less than 100 milliseconds before the ad is placed, has fundamentally changed the landscape of the digital media market. Most advertisers, given the complexity of the process, contract with an agency to run their online advertising campaigns. As a result, with increasing frequency, the same agency bids in the auction on behalf of multiple advertisers. This may potentially create unintended consequences for consumers.

A typical transaction begins with a user visiting a website. Embedded in the visit are data like user demographics, browsing history, location, and the page being loaded. This information goes from the publisher to an ad exchange, which presents it with accompanying metadata, to multiple advertisers who automatically submit real-time bids to place their ads. The criteria for bidding on particular types of consumers can be very complex, considering everything from detailed behavioral profiles to conversion prospects. Given their expertise and huge stores of consumer data, advertising agencies are especially well equipped to figure out the relevance and potential value of each impression for their clients. This capability is used to determine the bid for the respective advertising slot on behalf of each client. At the same time, companies that hire advertising agencies expect better marketing campaign outcomes through improved efficiency. In theory, modern technologies in the advertising industry should eliminate all irrelevant advertising. We challenge this lay belief, demonstrating that irrelevant advertising is an inescapable reality arising not from technological imperfections, but from an incentive problem inherent in the advertising industry’s institutional arrangement. Advertising agencies, after all, serve multiple advertisers, and they strategically allocate impressions to maximize profits subject to their contractual obligations. Even with perfect targeting technology, this still creates irrelevant advertising.

We analyze a game where one strategic advertising agency serves two competing advertisers for potentially relevant impressions that arrive at a platform sequentially. We are interested in an asymmetric competition, where one advertiser is more mainstream than the other. Both advertisers have a budget for their ad spending, and any single advertiser cannot fully cover all the impressions available in the market. One critical feature is that the agency is contractually obligated to allocate one impression to each advertiser. It faces a tradeoff: It can allocate the first impression to the mainstream advertiser who, generally, has a higher match probability. Or it can save this advertiser for a future impression, which may have an even higher match probability. This tradeoff arises because the ad agency needs to serve both advertisers who have a budget constraint. That is, each impression’s assigned advertiser exits the marketplace, giving up the potential surplus from a future impression: one that may be a better match and lead to higher sales probability. This forgone future surplus is an opportunity cost from allocating an impression myopically; a strategic, forward-looking agency may modify its allocation of impressions accordingly.

We find that when the asymmetry between two advertisers becomes either extremely large or small, the ad agency is more likely to allocate an impression to the niche advertiser, whose match probability with an impression is generally low. It is intuitive that, as the asymmetry becomes small (or two advertisers become similar), the need to consider the future opportunity decreases. The agency simply allocates the impression to an advertiser that has a higher match probability, thus allocating more to the niche advertiser as it becomes comparable to the mainstream advertiser. The other case, allocating an impression to an extremely niche advertiser, is not intuitive. To save a future and potentially more-relevant impression for the mainstream advertiser, the ad agency “dumps” the current impression to a less relevant (or even completely irrelevant) advertiser. The agency’s strategic consideration creates this inefficiency, manifested most obviously in completely irrelevant advertising for consumers even in the presence of perfect targeting accuracy.

After demonstrating the existence of agency-induced inefficiency under contractual obligations, we examine how the contractual obligations in the agency-advertiser relationship affect the agency’s allocation decision. We allow each advertiser to decide whether to impose such a contractual requirement at the beginning of the game, considering its own profitability. We first show that even without being contractually bound, the agency may still strategically allocate an impression to a less relevant advertiser in the first period to save a more relevant but budget-constrained advertiser for the future impression. This implies that the future opportunity cost arises from not only the contractual obligation but also the budget constraint of advertisers. Therefore, those opportunity costs from either contractual obligations or budget constraints can lead to less relevant advertising in the context of the agency-advertisers relationship.

However, when there is no contractual necessity to deliver an impression, completely irrelevant ads from an
advertiser with a zero-match probability will disappear. The agency will rather spare both advertisers for a future impression when the first-period impression is not sufficiently attractive to any advertiser. This, however, can present another inefficiency: neither ad may be delivered to an impression that has a strictly positive match probability with both advertisers. We therefore identify two distinct and interesting tradeoffs in advertising. The need to serve both advertisers leads to the inefficiency of over-supply of ads, sometimes presented as completely irrelevant ads; having no contractual obligation, however, may lead to the inefficiency of no ads (under-supply of ads). Moreover, we find that the contractual obligation weakly decreases the efficiency of ad allocation and, thus, the profits of both the agency and the niche advertiser. However, the mainstream advertiser can be better off with the contractual obligation. Thus, the mainstream advertiser has an incentive to deviate and impose contractual obligation, which, in turn, may make the niche advertiser to adopt the contractual obligation. Hence, we confirm that imposing contractual obligations by both advertisers can arise as an equilibrium, which implies that irrelevant advertising will persist even when we endogenize the contractual obligation choice in equilibrium.

Finally, we consider a situation where there are more available impressions than the agency is obligated to buy. We show that inefficiency in advertising persists, proving the robustness of our findings to the number of available impressions. Even in this case, we show that the contractual restrictions can still influence the agency’s allocation decision, and the same distortion incentive exists for the agency. However, the slack in the number of impressions allows more room for the agency to avoid allocation inefficiency. Thus, the range of the first-period impression receiving the irrelevant ads decreases as the number of impressions increases. Our result suggests that the agency may still allocate an impression to a less relevant advertiser when the available impressions are finite, although the range may be far smaller. Overall, we demonstrate that when the agency serves multiple advertisers, the agency’s strategic consideration due to the contractual restrictions can generate inefficiency in the ad targeting.

2. Literature Review

This paper examines the inefficiency associated with ad agency’s allocation of individual impressions to multiple advertisers. As such, our work contributes to three different streams of research: online advertising, ad targeting, and common agency problem. First, we build on the extensive literature on online advertising investigating new technical developments and their implications. Katona and Sarvary (2010) and Jerath et al. (2011) study the ad auction with a focus on the listing order of advertisers. Several important developments in the auction mechanism have been examined, such as the first-page bid estimates (Amaldoss et al. 2015), keyword matching options (Amaldoss et al. 2016), and exclusive ad placement (Sayedi et al. 2018). Shifting attention to the advertisers’ problem, Desai et al. (2014) examine the branded keyword choice problem, and Shin (2015) studies the advertisers’ bidding strategies under budget constraint and find that in equilibrium, advertisers maximize their bids to run out the competitor’s budget. Recently, Sayedi (2018) studied the implications of real-time bidding (as opposed to traditional reservation contracts) for the advertiser and publisher profits. The extant literature has mostly focused on two main players: advertisers and publishers (or the ad platform). Our paper considers another important player, advertising agency, who can sway the ad auction outcome and examines its role in ad delivery. More importantly, by doing so, we identify a new type of inefficiency in online advertising induced by the contractual restrictions under common agency.

This research is closely related to the literature on targeted advertising (Chen et al. 2001, Iyer et al. 2005). A stream of research shows the effectiveness of using individual customer data such as personal demographic information (Hauser et al. 2009), past browsing or past purchase data (Villas-Boas 1999, Shin and Sudhir 2010), and specific context (Zhang and Katona 2012). Whereas the existing literature has focused on the effectiveness of advertising targeting and the implications of such improved targeting accuracy (Bleier and Eisenbeiss 2015), our work shows that irrelevant advertising will prevail even with the perfect targeting accuracy, suggesting that our finding of agency-induced inefficiency is not caused by imperfect targeting technology but is inherently present in the ad-serving mechanism itself.

Also, the ad allocation inefficiency in this paper arises from the agency problem. The agency problem has been studied mostly in the context of ad pricing schemes, as a direct application of the traditional agency theory (Holmstrom 1979). Asdemir et al. (2012) compare traditional ad pricing scheme (CPM) with a prevalent online ad pricing scheme (CPC), whereas Hu et al. (2016) examine performance-based pricing schemes such as CPC and CPA. Our work is different from these papers in its focus. Departing from the publisher-advertiser relationship, we focus on the conflict of interests between advertisers and the ad agency. Unlike the typical moral hazard problem, which focuses on the bilateral agency relationship, we examine the incentive problem of the ad agency from contracting with multiple advertisers.

In our paper, the main source of the agency’s allocation distortion is its consideration of the future opportunity to serve advertisers. The idea of future consideration has been also examined in the consumption flexibility literature, where consumers’ preference for consumption flexibility has been both theoretically rationalized (Kreps 1979) and empirically identified (Guo 2010). Walsh (1995) and Guo (2006), respectively, study the implications of
consumption flexibility to consumer behavior and firm strategies. In our setting, those opportunity costs arise from either contractual obligations or budget constraints in the context of the agency-advertisers relationship. Thus, the structure of our problem resembles that of a common agency problem, which arises when multiple principals share the same agent (Bernheim and Whinston 1985, Bernheim and Whinston 1986, Dixit et al. 1997). The common agency literature concerns whether self-interested principals can achieve efficiency by inducing the agent’s action to maximize their joint profits. Unlike these existing studies whose main interest lies in deriving the optimal contract, we focus on revealing the deficiency of serving multiple principals. We show that even under the advanced targeting technology, the impression allocation will remain inefficient if the ad agency commits to serving multiple advertisers.

3. Model

We consider an online advertising market where consumers are distributed over a unit line following a uniform distribution $U[0,1]$. There are two advertisers $j \in \{m,n\}$ who are located at the two ends of the line. We denote the advertiser located at point $0$ as advertiser $n$ and the advertiser at point $1$ as advertiser $m$. The advertisers compete with each other.

If a consumer finds an advertised product fits her specific needs, she receives a utility $1$. Here, we model that the match probability of product $j$ with a consumer located at $x$ is a function of the distance from her location to the product location on the line. A consumer located at $x \in [0,1]$ gets utility $u_i(x)$ from buying product $j \in \{m,n\}$, where

$$u_i(x) = \begin{cases} 
1, & \text{with probability } \max\{\alpha_j - |x - x_j|, 0\} \\
0, & \text{otherwise},
\end{cases}$$

(1)

and $\alpha_j$ is the match probability at Product $j$’s location.

The consumer’s expected utility, $E[u_i(x)] = \max\{\alpha_j - |x - x_j|, 0\}$, takes the same form as in the standard Hoteling model. However, unlike the standard Hoteling model, the consumer receives $u_i(x) = 1$ if the product $j$ is a match with the consumer’s interest. Otherwise, the product is not a match, and the consumer receives $u_i(x) = 0$. As the product locates further away from the consumer, her interest is less likely to match with an advertised product.

We consider the following asymmetry between the two advertised products: Product $m$ is a mainstream product that provides a positive match probability for all consumers in the market while product $n$ is a niche product with which some consumers may find no match. To capture this asymmetry, we assume that $\alpha_m = 1$ and $\alpha_n = a$, where $0 \leq a \leq 1$, without loss of generality. Thus, $1 - a$ represents the degree of asymmetry between the two advertisers. As $a$ increases, the asymmetry between two advertisers decreases. Given this asymmetric setting, we can define the match probability of product $j$ with the consumer located at $x$ as

$$\rho_j(x) = \max\{0, \alpha - x\},$$

(2)

$$\rho_m(x) = x.$$  

(3)

One consumer impression arrives at the real-time bidding (RTB) platform in every period. A consumer impression $x$ follows a uniform distribution on $[0,1]$. In this market, a monopolistic advertising agency is contracted to serve both advertisers. The agency allocates each impression with the perfect knowledge of the consumer’s location to one of two advertisers. The agency’s perfect knowledge is not necessary for our results but allows us to highlight that irrelevant advertising is not merely an outcome of firms’ technological capabilities, but of economic incentives. Advertisers receive the same profit margin $\gamma$ from sales. Thus, their expected valuation for an impression is given as $E[\pi_i] = \gamma \cdot \rho_j(x)$.

We consider two periods where each impression arrives sequentially over time. Both advertisers have a budget to show their ads up to one impression to consumers. This setup captures the reality that any single advertiser cannot fully cover all the impressions available in the market. Furthermore, the nature of the relationship between the agency and an advertiser is long-term in practice. We take a simple approach to represent the agency’s long-term perspective by considering a cost-per-action contract, where the agency’s commission is based on the total sales at the constant rate $\theta$, which we normalize to $\theta = 1$ since $\theta$ is not central to our analysis. Therefore, the objective of the agency is to maximize the following expected profit by choosing the allocation of consumer’s impression in each period, $A_i(x) \in \{m,n\}$ ($i = 1, 2$):

$$\max_{A_i} \mathbb{E} [\Pi] = \sum_{i=1}^{2} \sum_{j \in \{m,n\}} (1_{[A_i(x)=j]} \cdot \rho_j(x))$$

$$= \sum_{i=1}^{2} \left[ I_{[A_i(x)=m]} \cdot \rho_m(x) + I_{[A_i(x)=n]} \cdot \rho_n(x) \right],$$

(4)

where $I$ is an indicator function.

Finally, in a typical advertiser-agency contract, the agency is required to deliver a certain number of impressions, which, in our context, is normalized to one impression. Such contractual obligations and advertiser’s budget constraints can affect the agency’s allocation decision. In the next section, we demonstrate the targeting inefficiency arising from these contractual restrictions. We show how the contractual restrictions embedded in the relationship can distort the agency’s impression allocation, leading to unintended targeting inefficiency.

Moreover, such contractual requirements, by restricting the agency’s impression allocation, may potentially hurt the advertiser. One important issue is whether the advertisers can remove such seemingly detrimental restrictions.
Therefore, we endogize the contractual obligation in the subsequent Section 5, where each advertiser decides whether to impose such a contractual requirement at the outset, considering its own profitability. We examine different scenarios, including when both advertisers impose no contractual obligation and when only one advertiser imposes the contractual obligation. Then, we solve the full game and see when each of these cases (symmetric or asymmetric contractual obligation) can arise in equilibrium. To reflect the reality, our analysis focuses on the budget-constrained advertisers, but we also offer the analysis of the cases where either one or both advertisers are not budget constrained in the online appendix.

4. Analysis

We first examine the realistic situations where the contractual obligations are present in the agency-advertiser relationship. In this section, we analyze the agency’s allocation problem when both advertisers impose the contractual obligation on the agency. To better appreciate the equilibrium allocations, we start by presenting the benchmark allocation.

4.1. Benchmark: First-Best Static Advertising Allocation

Consider a benchmark case where (1) both advertisers have no budget constraint such that any advertiser can cover all the impressions in the market, and (2) the ad agency also has no obligation to serve both advertisers. Recall that we consider a situation where consumer impressions arrive sequentially at the RTB platform. Without any information about a consumer’s characteristics or interest, the agency assigns each impression based on the expected match probability, which is always higher for advertiser m’s product than advertiser n’s product: \( \int_0^{x^0} \rho_n(x)dx = \frac{n}{2} \geq \int_0^{x^0} \rho_m(x)dx = \frac{m}{2} \). Thus, the agency always assigns an impression to advertiser m in every period.

When the platform and advertising agency can use the customer’s individual-level data, the information about each impression and its characteristics affect the agency’s optimal ad allocation rule. The platform now has information about the customer’s characteristics and interest using an unprecedented amount of consumer-level data (Davenport et al. 2001). The information about each individual consumer or impression is far from perfect, and thus, this imperfectness contributes to the inefficiency in online advertising targeting. However, our focus is on the inefficiency arising from the misaligned incentive of the agency from contractual restrictions in the advertising ecosystem. Thus, to abstract away from the issue of targeting accuracy, throughout the paper, we assume that ad agency has the perfect knowledge of the consumer’s location.

The advertising allocation rule \( A_t(x) \in \{m, n\} \) can be simple and efficient. The agency allocates an impression to whoever has the highest match probability. Thus, the optimal allocation rule under the benchmark is as following: For all \( t \in \{1, 2\} \),

\[
A_t^0(x) = \begin{cases} 
    m & \text{if } x_t \geq x^0 \equiv \frac{\alpha_1}{2} \\
    n & \text{otherwise},
\end{cases}
\]

Here, \( x^0 \equiv \frac{\alpha_1}{2} \) is the marginal consumer whose match probability with the two advertisers, the mainstream advertiser (advertiser m) and the niche advertiser (advertiser n), is the same. By definition, for all consumers \( x \leq x^0 \), we have \( \rho_n(x) \geq \rho_m(x) \). Moreover, this marginal consumer is located to the left of the center of the market (i.e., \( \frac{x^0}{2} \)), because advertiser \( m \) has a higher overall match probability than advertiser \( n \). As \( \alpha \) increases (or the asymmetry between two advertisers decreases), the marginal consumer \( x^0 \) converges to \( \frac{x^0}{2} \).

In this case, the expected profit for the agency is

\[
\mathbb{E}[\Pi^0] = 2 \cdot \left( \int_0^{x^0} \rho_n(x)dx + \int_{x^0}^{1} \rho_m(x)dx \right) = 1 + \frac{\alpha_1^2}{2}.
\]

4.2. Strategic Advertising Allocation with Contractual Obligation

We analyze our main model, where we examine how the strategic consideration of ad agency can lead to unintended consequences of advertising inefficiency, especially manifested by the occurrence of sending irrelevant advertising to consumers despite the perfect information about them. Consider a setting where impressions \( x_t \), where \( t = \{1, 2\} \), arrive at the platform sequentially over the two periods. Any single advertiser cannot possibly cover all the impressions, and the advertising agency that serves multiple advertisers has a contractual obligation to provide service for all of them. Thus, the ad agency must allocate one impression to each advertiser.

The advertising agency allocates an impression depending on the match probability with each advertiser. It is different from the benchmark case where the agency assigns each impression to the advertiser with a higher match probability. The agency now considers the fact that it has to assign impressions to both advertisers over the two periods. The ad agency is strategic and forward-looking. It may modify the allocation rule of the first period taking into account this future consideration. There is an opportunity cost of simply allocating an impression to an advertiser with a higher match probability in the first period. The already served advertiser exits the marketplace and will no longer be available in the future. However, the future impression may have an even higher match probability with that advertiser.
Therefore, when an impression arrives in the first period, the agency considers the total expected profit from allocating an impression to each advertiser (i.e., $A_1 = m$ or $A_1 = n$).

\[
\begin{align*}
\mathbb{E}\Pi(A_1 = m) &= \rho_m(x_1) + \int_0^1 \rho_m(x) \, dx \\
\mathbb{E}\Pi(A_1 = n) &= \rho_n(x_1) + \int_0^1 \rho_m(x) \, dx
\end{align*}
\] (7)

Comparing those two expected profits, we get

\[
\mathbb{E}\Pi(A_1 = m) \geq \mathbb{E}\Pi(A_1 = n) \iff \rho_m(x_1) \geq \rho_n(x_1) + \mathcal{C},
\]

where \(\mathcal{C} \equiv \int_0^1 (\rho_m(x) - \rho_n(x)) \, dx = \frac{1 - \alpha^2}{2} \geq 0.\) (9)

Here, the constant \(\mathcal{C} > 0\) is the measure of dynamic adjustment due to strategic consideration for future opportunities. As the asymmetry between two advertisers decreases (i.e., as \(\alpha\) increases), the need for future consideration, which is manifested as \(\mathcal{C}\), lessens: \(\frac{d\mathcal{C}}{d\alpha} < 0\). It will play a critical role in our analysis and understanding of the inefficiency of targeted advertising.

Given this dynamic adjustment constant (\(\mathcal{C}\)), the following proposition formally states the optimal advertising allocation rule for the agency. All the proofs can be found in the Appendix.

**Proposition 1.** When the agency is contractually bound to allocate one impression to each advertiser, the agency allocates the impression \(x_1\) to advertiser \(m\) if \(x_1 \geq \bar{x}\) in the first period; Otherwise, the ad agency allocates it to advertiser \(n\), where the cutoff \(\bar{x}\) is

\[
\bar{x} = \begin{cases} 
\bar{x}' = 1 - \frac{\alpha^2}{2}, & \text{if } \alpha \geq \bar{x} \equiv \sqrt{2} - 1, \\
\bar{x}' = \frac{1 - \alpha^2}{4}, & \text{otherwise } (i.e., \alpha < \bar{x}),
\end{cases}
\]

and the cutoff \(\bar{x}\) is always less than \(\frac{1}{2}\).

Similar to the benchmark, there is a cutoff criterion for the marginal impression, \(\bar{x}\). Clearly, this cutoff under the strategic allocation is always greater than that of the static case: \(x^0 \leq \bar{x}\) for all \(\alpha \in [0, 1]\), because of the agency’s strategic consideration for future opportunity \(\mathcal{C} \geq 0\). Under the strategic allocation, the agency rationally anticipates that the second-period impression could be a better match with advertiser \(m\) (than advertiser \(n\)), and is willing to save the opportunity to serve advertiser \(m\) for the future. Hence, the agency assigns \(x_1\) to advertiser \(n\) even if \(x_1 > x^0\) as long as \(x_1 \leq \bar{x}\). Figure 1 demonstrates this allocation distortion: how the dynamic adjustment \(\mathcal{C}\) shifts the cutoff \(x\) from \(x^0\) to \(\bar{x}\), which is always located to the right of \(x^0\).

This cutoff \(\bar{x}\) takes two different values, \(\bar{x}'\) and \(\bar{x}''\), depending on the level of \(\alpha\). This is because \(\rho_m(x) = \max\{0, \alpha - x\}\) is a piece-wise positive function of \(x\), with one piece exhibiting strictly positive match probability of advertiser \(m\) (i.e., \(\rho_m(x) = \alpha - x\) when \(\alpha - x > 0\), and the other piece representing zero probability (i.e., \(\rho_m(x) = 0\) when \(\alpha - x \leq 0\)). Thus, depending on which piece of this function applies to the sides of the inequality in Equation (9), there can be two different cutoffs as in Proposition 1. Figure 1 separately illustrates these two different cases.

Figure 1(a) depicts the case when \(\alpha\) is sufficiently large (i.e., \(\alpha > \bar{x}\)), where the match probability at the cutoff is positive (\(\alpha - \bar{x} > 0\)). In this region, there is little asymmetry between the two advertisers. Therefore, the strategic adjustment for future option \(\mathcal{C}\) is accordingly small (recall \(\frac{d\mathcal{C}}{d\alpha} < 0\)). On the other hand, Figure 1(b) shows that the same strategy adjustment may induce a greater distortion when \(\alpha\) is relatively small (i.e., \(\alpha < \bar{x}\)). First, the smaller \(\alpha\) induces the larger dynamic adjustment \(\mathcal{C}\). Moreover, the marginal impression or cutoff \(\bar{x}''\) is determined at the point where \(\rho_m(x) = \mathcal{C}\). Compared with the scenario where the cutoff point would have been determined by equating \(\rho_m(x)\) with \(\rho_n(x) + \mathcal{C}\), represented by \(\kappa\) in the figure, the agency distorts the allocation even more from the same strategic consideration: \(\kappa < \bar{x}''\).

The first-period impression \(x_1\) that falls within the range of \([x^0, \bar{x}]\) (where \(\bar{x} \in (\bar{x}', \bar{x}'')\)) will receive an ad from the less appealing advertiser, advertiser \(n\), instead of advertiser \(m\) who has a higher match probability. This is the phenomenon we define as irrelevant advertising, that is, to receive an ad from a less relevant advertiser. A more extreme scenario can arise when \(\alpha \leq \bar{x}\). For those on \([\alpha, \bar{x}']\) (the dark area on the line in Figure 1(b)), the ad is not just less relevant, but completely irrelevant. They may receive an ad with a zero match probability. Although the ad agency perfectly knows that there is another advertiser, advertiser \(m\), who has a strictly positive match probability with this particular consumer, the agency still sends an irrelevant ad. This inefficiency, paradoxically, results from the agency’s strategic consideration for future options. In a sense, the ad agency dumps the current impression to a less relevant, sometimes completely irrelevant advertiser. It can spare the mainstream advertiser for the future impression, which may have a higher chance of matching. Thus, the irrelevant ad can be a byproduct of the strategic behavior of the common agency, not necessarily a technological drawback from imperfect targeting. The following corollary, which is one of our main results, summarizes this finding.

**Corollary 1.** When the asymmetry between two advertisers is sufficiently large (i.e., \(\alpha < \bar{x}\)), consumers may receive a completely irrelevant ad from an advertiser with a zero match probability.
Finally, we consider the effect of $\alpha$ on the cutoff $\bar{x}$. It is related to the issues of when and how much the ad agency would distort the ad allocation.

**Proposition 2.** The cutoff $\bar{x}$ first decreases and then increases in $\alpha$ (1) when $\alpha \leq \bar{\alpha}$, the cutoff $\bar{x}$ monotonically decreases in $\alpha$, and (2) when $\alpha > \bar{\alpha}$, it monotonically increases in $\alpha$. Therefore, the cutoff $\bar{x}$ reaches its minimum at $\alpha = \bar{\alpha}$, and its maximum at the two ends of $\alpha$, where $\bar{x}(\alpha = 0) = 0$ and $\bar{x}(\alpha = 1) = \frac{1}{2}$.

The proposition suggests that the ad agency is more likely to allocate an impression to the niche advertiser (i.e., advertiser $n$) when the asymmetry between two advertisers becomes extremely large or small. It is intuitive that as $\alpha$ increases (or the asymmetry between the two advertisers becomes smaller), the agency allocates the impression more tightly. The cutoff $\bar{x}$ converges to the center of the market: that is, $\bar{x} \downarrow \frac{1}{2}$ as $\alpha \to 1$. Moreover, the agency is, paradoxically, more likely to allocate an impression to the niche advertiser when the niche advertiser becomes extremely niche because of the strategic future consideration. As $\alpha$ decreases (or equivalently as the asymmetry becomes larger), the need to consider the future opportunity increases. Thus, $C$ increases as $\alpha$ decreases and $\bar{x}$ increases to $\frac{1}{2}$ that is, $\bar{x} \uparrow \frac{1}{2}$ as $\alpha \to 0$.

Figure 2 illustrates how the cutoff point $\bar{x}$ changes along with $\alpha$: $\bar{x}(\alpha)$ is the upper envelope of two functions, $\bar{x}''(\alpha)$ and $\bar{x}'(\alpha)$. Whereas $\bar{x}''(\alpha)$ is increasing in $\alpha$, $\bar{x}'(\alpha)$ is decreasing in $\alpha$. Therefore, $\bar{x}(\alpha)$ reaches its minimum at $\bar{\alpha}$, and its maximum at both ends of $\alpha$; that is, $\alpha = 0$ and $\alpha = 1$.

### 4.3. Allocation Efficiency

We now evaluate the efficiency of the strategic allocation by comparing the expected profit of the agency under a strategic allocation with contractual obligation ($\mathbb{E}[\Pi^i]$) with the benchmark case ($\mathbb{E}[\Pi^o]$), which is the first-best.

Under a strategic allocation, the ad agency can obtain the following expected profit:

\[
\mathbb{E}[\Pi^i] = \int_0^{\bar{x}} \rho_n(x_1)dx_1 + \bar{x} \cdot \int_0^{1-x} \rho_m(x_2)dx_2
\]

\[
+ \int_{x}^{1} \rho_m(x_1)dx_1 + (1 - \bar{x}) \cdot \int_0^{1-x} \rho_n(x_2)dx_2
\]

\[
= \begin{cases} 
\frac{\alpha^4 - 4\alpha^3 + 10\alpha^2 + 4\alpha + 9}{8} & \text{if } \alpha \geq \bar{\alpha} \\
\frac{\alpha^4 + 6\alpha^2 + 5}{8} & \text{otherwise} 
\end{cases}
\]

(12)

The first two terms represent the scenario when the first period impression falls within the range that the ad agency allocates it to the niche advertiser $n$ in the first period (i.e., $x \leq \bar{x}$). Each term represents the first-period expected profit ($\int_0^{\bar{x}} \rho_n(x_1)dx_1$) and second-period expected profit ($\bar{x} \cdot \int_0^{1-x} \rho_m(x_2)dx_2$), respectively. Similarly, the next two terms represent another scenario when the first-period impression falls within the range that the ad agency allocates it to the mainstream advertiser $m$ in the first period (i.e., $x > \bar{x}$).
From Equations (6) and (12), the difference in expected profit ($\Delta$) can be expressed as

$$\Delta \equiv \mathbb{E} \Pi^0 - \mathbb{E} \Pi^1 = \int_0^1 (\rho_m(x_1) - \rho_n(x_1)) dx_1 + (1 - \bar{\alpha}) \int_0^1 (\rho_m(x_2) - \rho_n(x_2)) dx_2 + \bar{\alpha} \int_0^1 (\rho_m(x_2) - \rho_n(x_2)) dx_2 = \Delta_3 + (1 - \bar{\alpha}) \cdot \Delta_M^3 + \bar{\alpha} \cdot \Delta_N^3. \quad (13)$$

Here, $\Delta_3$ represents the ex ante inefficiency of the first period. When the first-period impression falls in the range $x^0 \leq x_1 < \bar{x}$, the agency allocates it to advertiser $n$, even though its match probability is lower than advertiser $m$. This is because the agency ex ante expects the second-period impression to have a better match with advertiser $m$ than advertiser $n$.

In the second period, two different types of inefficiency arise. First, suppose that the first-period impression is assigned to advertiser $m$ because $x_1 \geq \bar{x}$ (so it can happen with a probability $(1 - \bar{\alpha})$). The second-period impression allocation can be ex post inefficient if the second-period impression falls in the range $x_2 \geq x^0$ where the agency sends a niche product ad to mainstream consumers (i.e., consumers who are more interested in the mainstream product). We thus call it the second-period type-M inefficiency, which corresponds to $\Delta_M^3$. Second, suppose that the agency allocates the first impression to advertiser $n$ (which can arise with a probability $\bar{\alpha}$). The agency then has to allocate the second impression to advertiser $m$, regardless of where the second impression falls. In this case, if $x_2 \leq x^0$, advertiser $n$’s match probability is higher. Because this inefficiency comes from sending a mainstream product ad to niche consumers, we call it the second-period type-N inefficiency, and it is represented by $\Delta_N^3$. Note that $\Delta_M^3$ is always larger than $\Delta_N^3$ because mainstream advertiser has a higher overall match probability. Moreover, as $\alpha$ increases, consumers’ interest in the niche product increases, and thus by definition, $\Delta_M^3$ decreases but $\Delta_N^3$ increases.

We can easily see that $\Delta \equiv \mathbb{E} \Pi^0 - \mathbb{E} \Pi^1 > 0$. By the definition, $\rho_m(x) \geq \rho_n(x)$ for all $x \geq x^0$ and the opposite ($\rho_m(x) < \rho_n(x)$) holds for all $x < x^0$. Hence, all three terms $\Delta_3, \Delta_M^3, \Delta_N^3$ in (13) are positive, leading to $\Delta > 0$. Clearly, the ad allocation is always efficient under no constraint (i.e., $\mathbb{E} \Pi^0$ from the benchmark model), but there exists an efficiency loss from the strategic allocation of the agency in every period due to the contractual constraints. The following proposition presents how this efficiency loss changes as the asymmetry between the two advertisers changes.

**Proposition 3.** As $\alpha$ increases, the total inefficiency ($\Delta$) decreases.

The proposition suggests that the inefficiency decreases as the two advertisers become similar. The efficiency loss is the mixture of different sources of efficiency gains and losses from the agency’s strategic behaviors ($\Delta_3, \Delta_M^3, \Delta_N^3$). First, as $\alpha$ increases, the asymmetry between the two advertisers becomes smaller. Thus, there is little reason for the agency to consider the future. Therefore, the first-period inefficiency (i.e., $\Delta_3$) monotonically decreases in $\alpha$. Second, the second-period inefficiency is the weighted average of $\Delta_M^3$ and $\Delta_N^3$: $\Delta_2 \equiv (1 - \bar{\alpha}) \cdot \Delta_M^3 + \bar{\alpha} \cdot \Delta_N^3$. As noted previously, $\Delta_N^3$ is always smaller than $\Delta_M^3$. Moreover, by Proposition 2, the relative weight on $\Delta_N^3$ (i.e., $\bar{\alpha}$) initially decreases but then increases with $\alpha$. Consequently, as $\alpha$ increases, the second-period inefficiency $\Delta_2$ gets larger and then smaller. The overall efficiency loss, as the sum of the first- and second-period inefficiencies, generally follows the first-period inefficiency pattern since the first-period inefficiency changes more drastically than the second-period inefficiency. Thus, the overall inefficiency always decreases in $\alpha$.

### 5. Endogenous Contractual Obligation

When serving advertisers, the ad agency’s decisions are bound by the contract that stipulates the minimum number of impressions the agency needs to deliver to the advertiser. As a result, in our main model, the ad agency serving both advertisers must match one impression with each advertiser over time. In this section, we endogenize the choice of such contractual restriction. We first start the section with a subgame of the partial-obligation case where only one advertiser imposes the contractual obligation. Then, we consider the no-obligation case where both drop the contractual obligation. These analyses help us to examine how the agency’s ad allocation is influenced by the presence or absence of the contractual obligation. After analyzing all the different subgames, we extend the game by allowing the advertisers to choose the contractual requirement at the beginning of the game.

#### 5.1. Subgame with Partial Contractual Obligation

Suppose advertiser $m$ drops the contractual obligation, whereas advertiser $n$ imposes it. In this case, the agency considers the following three options in the first period: serving advertiser $m$, serving advertiser $n$, and serving neither. If the agency serves advertiser $m$, the agency has to serve advertiser $n$ in the second period. If the agency serves advertiser $n$, however, the agency does not have to serve advertiser $m$ in the second period, implying that it may choose one of the three options: $m$, $n$, or $\emptyset$, where $\emptyset$ represents the option of serving neither.
denotes the option of not assigning an impression to either advertiser. First, note that \( \emptyset \) is always dominated by the other two options in the second period. This is so because assigning the impression to any advertiser with a positive match probability earns the agency a nonzero profit. Next, serving advertiser \( n \) again is not possible due to budget constraint. Thus, the agency will choose to allocate the second impression to advertiser \( m \). Finally, if the agency chooses to allocate the first-period impression to neither advertiser (i.e., \( \emptyset \)), it must allocate the second-period impression to advertiser \( n \) due to its obligation for advertiser \( n \). However, this option (i.e., \( A_1 = \emptyset \)) is dominated by \( A_1 = m \), because with \( A_1 = m \), the first-period profit is positive (as opposed to zero) while the second-period profit remains the same due to identical second-period allocation.

This analysis suggests that even in the absence of the contractual obligation of advertiser \( m \), the agency still allocates one impression to each advertiser. Then, the agency’s first-period decision will be governed by the same consideration of the future opportunity cost as discussed in the previous section. Therefore, the agency’s allocation decision will be identical to that in Proposition 1.

Next, suppose advertiser \( n \) drops the obligation, whereas advertiser \( m \) imposes it. By the same logic, it is easy to see that the agency adopts one of the following three options: (1) \( A_1 = m; A_2 = n \), (2) \( A_1 = n; A_2 = m \), and (3) \( A_1 = \emptyset; A_2 = m \). Unlike the previous case, however, option (3) is not completely dominated by option (2), because the agency earns zero profit even with \( A_1 = n \) whenever \( x_1 \geq \alpha \). In this case, the comparison between the first two options remains the same as in Proposition 1 but when \( A_1 = n \) is chosen over \( A_1 = m \), the agency may choose either \( A_1 = n \) or \( A_1 = \emptyset \) if \( \alpha \leq x_1 < \bar{x} \). The next corollary summarizes our discussion thus far.

**Corollary 2.** In the absence of the contractual obligation of one advertiser, the agency’s optimal allocation rule is given as follows:

1. When the contractual obligation is imposed only by advertiser \( n \): \( A_1(x_1) = m \) if \( x \geq \bar{x}; A_1(x_1) = n \) otherwise.
2. When the contractual obligation is imposed only by advertiser \( m \): \( A_1(x_1) = m \) if \( x \geq \bar{x} \); \( A_1(x_1) = n \) if \( x_1 < \min(\alpha, \bar{x}); A_1(x_1) = n \) or \( \emptyset \) if \( \alpha \leq x_1 < \bar{x} \).

The corollary implies that irrelevant advertising persists even in the absence of one advertiser’s contractual obligation. However, completely irrelevant advertising, which exists when \( \alpha \leq x_1 < \bar{x} \), may disappear when it is advertiser \( n \) who drops the contractual obligation.\(^{16}\)

### 5.2. Subgame with No Contractual Obligation

Suppose there is no contractual requirement to serve either advertiser. A strategic agency may choose not to allocate the first-period impression to any advertiser to keep both advertisers available in the second period for better matching with the future impression. Thus, we extend the agency’s strategy space to

\[
A_1(x) \in \{\emptyset, m, n\}.
\]

The advertisers are still budget constrained. The agency can choose the null option (\( \emptyset \)) in the first period to keep both options (of allocating an impression to advertiser \( m \) or \( n \)) open in the second period. This may reduce the allocation efficiency of the first period.

When the first-period impression arrives, the agency considers the total expected profit from these three options: (1) \( A_1 = m \), which automatically leads to \( A_2 = n \) (due to advertiser \( m \)’s budget constraint), (2) \( A_1 = n \), which leads to \( A_2 = m \) (due to advertiser \( n \)’s budget constraint), and (3) \( A_1 = \emptyset \), which can lead to either \( A_2 = m \) or \( A_2 = n \). The expected profits in the first period are given as

\[
\begin{align*}
\mathbb{E}\Pi(A_1 = m) &= \rho_m(x_1) + \int_0^1 \rho_n(x) \, dx, \\
\mathbb{E}\Pi(A_1 = n) &= \rho_n(x_1) + \int_0^1 \rho_m(x) \, dx, \\
\mathbb{E}\Pi(A_1 = \emptyset) &= \max(\rho_m(x), \rho_n(x)) \int_0^1 \, dx.
\end{align*}
\]

By comparing these three expected profits, we can identify conditions for optimal advertising allocation. First, the agency chooses \( A_1 = m \) if and only if the following two conditions are satisfied:

\[
\mathbb{E}\Pi(A_1 = m) \geq \mathbb{E}\Pi(A_1 = n) \iff \rho_m(x_1) \geq \rho_n(x_1) + \mathcal{C},
\]

where \( \mathcal{C} \) was defined in (10) and \( \kappa_m \equiv \int_0^1 \max(\rho_m(x), \rho_n(x)) \, dx = 2 \frac{\mathcal{C}}{m} \). Second, the agency chooses \( A_1 = n \) if and only if the following two conditions are satisfied:

\[
\begin{align*}
\mathbb{E}\Pi(A_1 = m) &\leq \mathbb{E}\Pi(A_1 = n) \iff \rho_m(x_1) \leq \rho_n(x_1) + \mathcal{C}, \\
\mathbb{E}\Pi(A_1 = n) &\geq \mathbb{E}\Pi(A_1 = \emptyset) \iff \rho_n(x_1) \geq \kappa_m,
\end{align*}
\]

where \( \kappa_m = \int_0^1 \max(\rho_m(x), \rho_n(x)) - \rho_m(x) \, dx = \frac{\mathcal{C}}{m} \).

Finally, the agency chooses \( A_1 = \emptyset \) if and only if \( \rho_m(x_1) < \kappa_m \) and \( \rho_n(x_1) < \kappa_n \).

To summarize these findings, we obtain the following optimal allocation rule with the agency’s additional option of not allocating an impression to any advertiser.

**Proposition 4.** Without contractual obligation to serve both advertisers, the agency’s allocation rule is as follows (here, \( \bar{x} \) is defined in Proposition 1; \( \bar{x}_\kappa \equiv \frac{\mathcal{C}}{\beta} \)).

1. Suppose \( \alpha \geq \frac{1}{2} \). We have \( \bar{x}_\kappa < x_1^* \left( \equiv \frac{\alpha}{2} \right) \). Then, if \( x_1 \leq \bar{x}_\kappa \), \( A_1(x_1) = m \). Otherwise, \( A_1(x_1) = n \).
2. Suppose \( \alpha < \frac{1}{2} \). We have \( x_1^* = \bar{x}_\kappa \). Then, if \( x_1 < \bar{x}_\kappa \), \( A_1(x_1) = n \). If \( \bar{x}_\kappa \leq x_1 \leq \bar{x}_\kappa \), \( A_1(x_1) = \emptyset \), that is,
the agency does not assign an impression to any advertiser. If $x_1 \geq \bar{x}_{\kappa m}$, $A_1(x_1) = m$.

The optimal allocation rule is similar to Proposition 1 with one more option of no assignment ($A_1 = \emptyset$). The proposition identifies the condition to exercise this third option. When $\alpha \geq \frac{1}{2}$, it is easy to see that $\bar{x}_{\kappa m} \leq \bar{x}_{\kappa n}$, which is depicted in Figure 3(a). In this case, given that $\rho_m(x_1) < \kappa_m$ and $\rho_m(x_1) < \kappa_n$ are equivalent to $\bar{x}_{\kappa m} \leq x_1 < \bar{x}_{\kappa n}$, the agency never chooses the option of no assignment, reverting to the main model. It is so because when $\alpha$ is large, the relative benefit of keeping both advertisers for the future is not greater than the immediate profit from serving one of them. Hence, with a large enough $\alpha$, the agency chooses to allocate the impression to one of the advertisers instead of keeping both options open (i.e., $A_1 = \emptyset$). Here, impressions $x_1 \in [x^0, \bar{x}]$ will receive less relevant advertising from advertiser $n$, instead of advertiser $m$ who has a higher match probability, due to the agent’s allocation distortion.

On the other hand, when $\alpha < \frac{1}{2}$ we have $(x^0 < \bar{x} < \bar{x}_{\kappa n})$, and thus, the agency may choose the option of no assignment as illustrated in Figure 3(b). Interestingly, unlike the main model where the agency needs to serve both advertisers, there is no case of consumers receiving completely irrelevant ads from the advertiser with zero match probability. Without contractual obligation to serve both advertisers, the agency finds it optimal to save both advertisers for the second-period impression instead of dumping one of the advertisers’ ads in which consumers have no interest. Therefore, we find a case where the agency chooses not to assign the impression to any advertiser ($A_1 = \emptyset$) even though their match probability with either product is strictly positive.

**Proposition 5.** Suppose there is no contractual obligation of the ad agency to serve both advertisers. When the asymmetry between two advertisers are sufficiently large ($\alpha < \frac{1}{2}$), there does not exist a case where consumers receive a completely irrelevant advertising from an advertiser with a zero match probability. However, there is always another type of inefficiency of not sending an ad to consumers who have a strictly positive match probability with one of two advertisers.

The proposition identifies an interesting tradeoff of advertising inefficiency based on whether the agency needs to serve both advertisers by the contractual arrangement. Without contractual obligation, the first-period inefficiency of dumping a completely irrelevant ad from an advertiser with a zero match probability does not arise. However, another type of inefficiency occurs where consumers may not receive an ad which can be relevant to them (i.e., under-supply of ads). Here, the agency does not send any ad to potentially interested consumers who have a strictly positive match probability with advertisers. Then, both advertisers and the ad agency bear the cost of inefficiency. Advertisers lose the chance for the transaction while the agency loses the potential profit.

Recall that the first-best allocation is obtained in our benchmark with neither contractual obligation nor budget constraints. Compared with that first-best benchmark, the analysis in this section shows that even in the absence of the contractual obligation, the agency can distort its impression allocation under budget constraint. Intuitively, serving an advertiser in this case still costs the agency the future opportunity because a previously-served advertiser will be no longer available due to budget constraints. On the other hand, suppose the agency has to serve both advertisers by the contract under no budget constraint. In that case, the agency strategically distorts the ad allocation as in our main model because it still has to assign one impression to each advertiser regardless of the budget constraint. These altogether imply that the budget constraint of advertisers in this setting plays a similar role as the contractual obligation.  

Both contractual obligation and budget constraint create an opportunity cost of the future allocation from serving the first-period impression: because of the contract need to serve the other advertiser or the advertiser’s unavailability due to budget. This opportunity cost turns into less relevant ads in the context of common agency. Therefore, the targeting inefficiency associated with irrelevant ads arises due to either contractual obligation or budget constraint in the relationship between an agency and multiple advertisers. However, without contractual obligation, the completely irrelevant advertising does not occur regardless of budget constraint (as evidenced by the benchmark with no budget constraint and Proposition 5 with budget constraint). When there is the budget constraint of advertisers, no-ad inefficiency can
arise only in the absence of the contractual obligation (shown by the comparison of Proposition 5 with no contractual obligation and Corollary 1 with contractual obligation).

5.3. Allocation Efficiency with Partial and No Contractual Obligation

We examine allocation efficiency with partial and no contractual obligation. First, recall that the allocation remains the same as that in Proposition 1, (1) for all \( \alpha \in [0,1] \) under partial contractual obligation and (2) for \( \alpha \geq \frac{1}{2} \) under no contractual obligation. In these cases, the allocation efficiency remains the same as in Section 4.3. Therefore, in our analysis, we focus on the case of \( \alpha < \frac{1}{2} \) under no contractual obligation. In this case, given the optimal allocation in Proposition 4, the ad agency’s expected profit is given as

\[
\mathbb{E}\Pi = \int_0^{\pi} \rho(x) dx + \int_0^1 \rho_m(x) dx + \int_0^{\pi} \rho(x) dx + (1 - \pi_m) \cdot \int_0^1 \rho(x) dx + (\pi_m - \pi_m) \cdot \int_0^1 \max\{\rho_n(x), \rho_m(x)\} dx.
\]

(22)

It is clear that the overall allocation efficiency is lower than the first-best benchmark for all \( \alpha < \frac{1}{2} \). An interesting question is whether the allocation efficiency is higher or lower than that under full contractual obligation. Recall that when \( \alpha < \frac{1}{2} \), both the irrelevant-ad ineffectiveness and the no-ad inefficiency arise under no contractual obligation. Despite these inefficiencies, we find that the allocation under no contractual obligation is always more efficient, that is, \( \mathbb{E}\Pi - \mathbb{E}\Pi^* > 0 \), \( \forall \alpha \in (0, \frac{1}{2}) \). This is because the allocation is more flexible under no contractual obligation. More specifically, without any contractual obligation, the agency can choose the no-allocation option in the first period, which removes restrictions from the second-period allocation and thus makes the second-period allocation efficient. Under no contractual obligation, the agency has a freedom to choose the no-allocation option whenever it is more efficient across the two periods than the option of serving either advertiser. This flexibility makes the overall allocation under no contractual obligation more efficient than under full obligation.

Finally, it is useful to note that the total inefficiency under no contractual obligation also decreases with \( \alpha \) (as in Proposition 3). As \( \alpha \) increases, the relative benefit of keeping both advertisers for the future decreases. Thus, the null option (0) is less likely to be chosen: \( \frac{d(\pi_m - \pi_m)}{d\alpha} < 0 \). Then the no-ad inefficiency decreases, which drives down the overall inefficiency of allocation under no contractual obligation.\(^{20}\)

5.4. Equilibrium of Contractual Obligations

In earlier sections, we have examined the agency’s optimal impression allocations under full, partial, and no contractual obligation. However, it is not a priori clear whether unilaterally removing contractual obligation would help or harm the advertisers. By comparing the advertisers’ profits across different cases, we derive the equilibrium as in the following proposition.

**Proposition 6.** Suppose advertisers decide whether to impose contractual obligations.

1. When \( \alpha \geq \frac{1}{2} \), all of full, partial, and no contractual obligations can emerge as an equilibrium.

2. When \( \frac{\sqrt{3}}{3} \leq \alpha < \frac{1}{2} \), either full or no contractual obligation is observed in equilibrium.

3. When \( \alpha < \frac{\sqrt{3}}{3} \), either full contractual obligation or advertiser \( m \)’s partial contractual obligation is observed in equilibrium.

Moreover, when \( \alpha < \frac{\sqrt{3}}{3} \), at least one advertiser chooses to obligate the agent in equilibrium.

The first part of the proposition is straightforward. When \( \alpha \geq \frac{1}{2} \), the allocation rule does not change across three different regimes (i.e., no, partial, and full contractual obligation). Thus, there is no change in the advertisers’ profits, and they are indifferent about their and their competitors’ decisions, implying any of the four cases may arise as an equilibrium.

However, when \( \alpha < \frac{1}{2} \), the allocation rules differ across three different regimes. To begin, recall that the allocation rule under full and partial contractual obligations remains the same, suggesting that both advertisers are weakly better off by unilaterally imposing the obligation when the other party also does so. Hence, both advertisers’ keeping the obligations can always emerge as an equilibrium for any value of \( \alpha < \frac{1}{2} \).

Next, whether the other possibilities can be an equilibrium depends on the relative profit of advertisers under partial versus no contractual obligation. To see this, recall that the agency may choose not to allocate the first-period impression with no contractual obligation. In this case, the advertisers lose the chance to profit from the current impression but may receive a better-matched future impression. The net effect is positive for advertiser \( n \). It is so because the first-period impression that would have been awarded to the advertiser \( n \) when it unilaterally imposes the contractual obligation is not more relevant on average than the second-period impression due to the agency’s allocation distortion. Advertiser \( m \) may also gain from the agency’s no allocation option for the same reason. However, it will experience a loss when the two advertisers are sufficiently asymmetric if advertiser \( m \) unilaterally imposes the contractual obligation. In that case, the agency significantly favors advertiser \( n \) in its first-period allocation. Thus, advertiser \( m \) would have been awarded only highly relevant impressions, which are much more profitable to advertiser \( m \) than the average second-period impression. Therefore, when \( \alpha \) is
sufficiently small (i.e., when $\alpha < \sqrt{10} - \frac{3}{2}$), although advertiser $n$ gets better off, advertiser $m$ gets worse off under no contractual obligation. This implies that advertiser $m$ will deviate and choose to unilaterally impose the obligation even when advertiser $n$ does not. Therefore, in this case, no obligation can never emerge as an equilibrium, but instead, in equilibrium, at least one advertiser will choose to adopt the contractual obligation.\textsuperscript{21}

To summarize our findings thus far, we have shown that an agency serving multiple advertisers may distort its advertising allocation, sometimes delivering completely irrelevant advertising to consumers. This distortion may occur due to either the contractual obligation or the budget constraint, the two manifestations of the agency-induced restrictions. Such restrictions generate inefficiencies associated with irrelevant advertising. Without contractual obligation, however, we find the extreme form of irrelevant advertising inefficiency will disappear. Nevertheless, a different type of inefficiency, in which consumers do not receive potentially relevant advertising, can arise in the first period. Overall, the allocation efficiency weakly improves when the contractual obligation can be removed. However, keeping the contractual obligation for both advertisers can arise as an equilibrium for any value of $\alpha$. In contrast, partial or no contractual obligation can only be observed in equilibrium for a limited set of $\alpha$ values.

6. Three-Period Model: A Case of Excessive Supply of Impressions

Our main model considers two periods where the number of available impressions is not greater than the total number of ads that the agency is obligated to cover. Thus, there is no slack of impressions to bid for in such a case, which makes the contractual obligation to serve both advertisers rigid. This section extends the model to the three-period case where available impressions are abundant with respect to contractual obligations. Thus, the agency will allocate one impression to each advertiser across the three periods, leaving one extra slack impression.

When the first-period impression arrives, the agency may allocate it to $m$, $n$, or neither, considering the total expected profit from each of these options. If it chooses to allocate the first-period impression to either $m$ or $n$, it will allocate one future impression to the unserved advertiser but let the other future impression go unallocated. If it chooses $A_1 = \emptyset$, however, it serves both advertisers with each of the rest two impressions.

Under each case, the second-period allocation goes as follows. First, after $A_1 = m$, the agency decides whether to serve advertiser $n$ with the realized second-period impression or the unknown third-period impression. The former is chosen (i.e., $A_2 = n$) if and only if $\rho_n(x_2) \geq \int_0^1 \rho_n(x_3) \, dx_3$, or equivalently, $x_2 \leq \overline{x}_{(m)2} \equiv \frac{12 \alpha - 3 \alpha^4}{2}$. Then the expected profit from choosing $A_1 = m$ is given as

$$
\mathbb{E} \Pi(A_1 = m) = \rho_m(x_1) + \int_0^1 \rho_n(x_2) \, dx_2 
+ (1 - \overline{x}_{(m)2}) \int_0^1 \rho_n(x_3) \, dx_3.
$$

Similarly, after $A_1 = n$, the agency chooses $A_2 = m$ (over $A_2 = \emptyset$) if and only if $\rho_m(x_2) \geq \int_0^1 \rho_m(x_3) \, dx_3$, or equivalently, $x_2 \geq \overline{x}_{(m)2} \equiv \frac{1}{2}$. Then the expected profit from $A_1 = n$ is given as

$$
\mathbb{E} \Pi(A_1 = n) = \rho_n(x_1) + \int_0^1 \rho_m(x_2) \, dx_2 + \overline{x}_{(m)2} \int_0^1 \rho_n(x_3) \, dx_3.
$$

Finally, after $A_1 = \emptyset$, two impressions are left to be allocated over the two periods. Thus, the second-period allocation problem becomes identical to the two-period allocation problem of our main model. Hence, $A_2 = m$ if $x_2 \leq \overline{x}$ and $A_2 = n$ otherwise. Then the expected profit is

$$
\mathbb{E} \Pi(A_1 = \emptyset) = \int_0^1 \rho_n(x_2) \, dx_2 + \overline{x} \int_0^1 \rho_m(x_3) \, dx_3 
+ \int_0^1 \rho_m(x_2) \, dx_2 + (1 - \overline{x}) \int_0^1 \rho_n(x_3) \, dx_3.
$$

Given these profits, the agency prefers $A_1 = m$ to $A_1 = n$ if and only if $\mathbb{E} \Pi(A_1 = m) \geq \mathbb{E} \Pi(A_1 = n)$. However, the option of no allocation ($\emptyset$) will be chosen if $\mathbb{E} \Pi(A_1 = \emptyset) \geq \mathbb{E} \Pi(A_1 = m)$ and $\mathbb{E} \Pi(A_1 = \emptyset) \geq \mathbb{E} \Pi(A_1 = n)$. Based on this analysis, we can derive the optimal allocation rule of the three impressions (see Section A3 of the online appendix for the detailed derivation).

The first-period allocation rule is given as follows:

$$
A_1(x_1) = \begin{cases} 
 n & \text{if } x_1 \leq \overline{x}_n, \\
 \emptyset & \text{if } \overline{x}_n < x_1 < \overline{x}_m, \\
 m & \text{if } x_1 \geq \overline{x}_m,
\end{cases} \quad \forall \alpha \in [0, 1],
$$

where

$$
\overline{x}_m = \begin{cases} 
\frac{9 + 4 \alpha - 6 \alpha^2 + 4 \alpha^3 - \alpha^4}{5 - 2 \alpha^2 + 4 \alpha^3} & \text{if } \alpha \geq \overline{\alpha} \\
\frac{8}{16} & \text{if } \alpha < \overline{\alpha},
\end{cases}
$$

and

$$
\overline{x}_n = \begin{cases} 
\frac{1 + 12 \alpha - 10 \alpha^2 + 4 \alpha^3 - \alpha^4}{8 \alpha - 6 \alpha^2 - \alpha^4} & \text{if } \alpha \geq \overline{\alpha} \\
\frac{8}{16} & \text{if } \alpha < \overline{\alpha},
\end{cases}
$$

Given the allocation rule, the following proposition examines whether the irrelevant ads persist even under excessive supply of impressions.

**Proposition 7.** When the agency allocates three impressions over three periods, the irrelevant ads exist as long as the two
advertisers are sufficiently asymmetric (i.e., $\alpha$ is sufficiently small).

This proposition highlights the robustness of our main findings. In our main model, where the number of impressions is limited, the irrelevant-ad inefficiency arises due to the opportunity cost of the future impression when the agent serves the first-period impression. Even when impressions are abundant, the same force is in play because advertisers are still budget-constrained. Thus, the agency may spare the mainstream advertiser for the future impression if it is marginally more relevant because its opportunity cost is higher.

However, under the excessive supply of impressions, the slack in the number of impressions allows more room for the agency to avoid unprofitable allocation. When the arrived impression does not show strong interest in either of the products, specifically, when $\tilde{X}_n < x_1 < \tilde{X}_m$, the agency does not allocate the impression to any advertiser. Thus, the allocation distortion and the resulting inefficiency arise only when $x^0 < x_1 \leq \tilde{X}_n$. Such an $x_1$ exists when $\alpha$ is not too large, since a large $\alpha$ implies a significant opportunity cost of serving advertiser $n$, which makes it less profitable to serve advertiser $n$ with any impression $x_1 \geq x^0$. Therefore, as long as the two advertisers are sufficiently asymmetric, irrelevant advertising persists in the three-period model.

Because the slack in the number of impressions implies a smaller opportunity cost, the incentive for the agency to distort the allocation gets weaker as the number of impressions increases. Thus, the first-period impression distortion is less likely to happen in the three-period model than the two-period model of our main analysis. The following corollary confirms this conjecture.

**Corollary 3.** The range of the first-period impression receiving the irrelevant ads shrinks as the number of impressions increases from two to three: $\tilde{X}_n < \tilde{X}$, $\forall \alpha \in [0, 1]$.

One can expect that the range would continue to shrink with greater slack in the number of impressions. Eventually, the first-period irrelevant ads will disappear when the number of impressions goes to infinity. Although we cannot derive the range for irrelevant ads for a general case due to tractability, we confirm this trend by extending our analysis to the four-period model in our online appendix (Section A6). We continue to see that the range of the first-period impression receiving the irrelevant ads shrinks as the number of impressions increases from three to four.

In reality, however, the relevant impressions cannot be infinite. Thus, the number of pertinent impressions can be an important concern for the agency, particularly in a limited contract period. This situation is quite common among many local businesses aiming to gain traction with a local audience with specific targeting criteria, such as restaurants or car dealers targeting nearby residents, or some specialized niche brands targeting a particular set of customer profiles. Indeed, many advertisers reportedly experience the shortage of programmatic ad inventory, and one industry report highlights that online impressions are not unlimited, but it is indeed scarce. Moreover, when there are other advertisers or their agencies competing for a similar set of impressions with the focal agency (although we do not explicitly model them in our paper), some of them may have stochastic cost advantage in a short period of time and win most of the impressions arriving in that period. Our two-period model captures such situations where the impression shortage is a real concern for firms and the contractual restrictions may distort the agency’s allocation decision. However, we also acknowledge that in mass-targeting categories such as general shoes or sports wears, the number of potentially interested consumers can be more than what the agency is obligated to buy. The ad agency is unlikely to experience impression shortage even in a limited contract period in those situations. Nevertheless, our three-period model may represent and shed some insights into such situations. We believe that even in such situations, the need to serve multiple advertisers with contractual restrictions will still influence the agency’s allocation decisions, as the results suggest.

This extension shows that advertising inefficiency of delivering less relevant advertising to consumers still persists even when the number of available impressions is greater than what the agency is obligated to buy, although the range may be far smaller. It thus provides the robustness of our findings to the number of available impressions.

### 7. Conclusion

The last decade has witnessed a rapid advancement of online ad targeting, which had altered the way ads are served. Although the great promise of the advanced targeting technology was the highly relevant ads, in reality, a significant number of consumers still experience irrelevant ads. The objective of this paper was to offer an economic explanation for this paradoxical situation, based on the online ad-serving market mechanism. We focus on the often-ignored ad agency’s role in the context of the real-time bidding platform, and we have identified agency-induced targeting inefficiencies, which can provide important managerial and policy implications.

We first find that the ad agency serving multiple advertisers, out of its self-interest, distorts the ad allocation due to strategic consideration. The agency expects future impressions to be better matched with the mainstream
advertiser. However, it has to provide both advertisers with a minimum number of impressions as required by the contract. Therefore, it may dump the current impression to a less-matched niche advertiser to save a future and potentially more-relevant impression for the mainstream advertiser. In an extreme case, an impression is allocated to the niche advertiser with zero match probability, which results in completely irrelevant advertising. Consumers receive irrelevant ads not because of technological imperfection but because of the incentive problem arising from contractual obligations inherent to the agency that serves multiple advertisers.

Without being contractually bound, a strategic agency may choose not to allocate an impression by forgoing the current mediocre opportunity for a better future opportunity in the presence of the shortage of relevant impressions. Although the inefficiency associated with completely irrelevant ads will disappear in this case, the agency’s strategic consideration distorts the allocation, which forces the agency to deliver less relevant ads to consumers. Moreover, we endogenize the contract terms: whether and when the advertisers would impose contractual obligation to the ad agency. We find that full contractual obligations from both advertisers can emerge in equilibrium. Also, we show that irrelevant advertising still persists even when the number of available impressions is greater than what the agency is obligated to buy, proving the robustness of our findings to the number of available impressions when the available impressions are finite.

The goal of our research is to establish the existence of online targeting inefficiencies due to ad agency, which has been overlooked in the literature. Exploring the optimal contract that can alleviate those inefficiencies can be an important next move. Also, we consider a single agency and thus a constant ad price in this paper to zoom in on the agency-advertiser relationship. However, in reality, multiple ad agencies representing distinct advertisers participate in the real-time bidding platform to win the impression for their clients. Although considering agency competition can quickly complicate the analysis, it may uncover a new layer of strategic interaction that could lead to some new insights. Finally, we do not take into account the consumers’ response to irrelevant ads. It would be worthwhile to examine whether attribute their discomfort to the advertiser or the platform and what the implications of this disutility of consumers will be. These are all exciting avenues for further investigation, which we leave for future research.

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Appendix
A.1. Proof of Proposition 1
We have the result directly from Equation (9), by solving $\rho_m(x) = \rho_n(x) + C$. Because $\rho_n(x) = \max(0, a - x)$ is a piecwise but continuous function, this yields two different solutions of $\tau$ as given in the proposition. Here note that $a > \tilde{x}$ is obtained from $\tau < a$. Finally, it is obvious that $\tau' = \frac{1}{2} - \frac{(1 - a_0)}{4} < \frac{1}{2}$ and that $\tau'' = \frac{1}{2} - \frac{(1 - a_0)}{4} < \frac{1}{2}$.

A.2. Proof of Corollary 1
When $a \leq \tilde{x}$, by Proposition 1, the agency allocates the first-period impression to advertiser $n$ if and only if $x_1 < \tau''$. Also by definition, whenever the first-period impression $x_1$ falls in the range $[a, \tau']$, we have $\rho_n(x_1) = 0$. Therefore, consumer $x_1 \in [a, \tau']$ receives an ad from advertiser $n$ whose match probability is zero.

A.3. Proof of Proposition 2
\[
\frac{\partial \Delta}{\partial \alpha} = \begin{cases} 
\frac{1 - \alpha}{2} & \text{if } \alpha > \tilde{x} \\
-\frac{\alpha}{2} & \text{if } \alpha \leq \tilde{x}
\end{cases} \quad (A.1)
\]
This proves the first part of the proposition. This, together with $\tau'(\alpha = \tilde{x}) = \tau'(\tilde{x} = 1) = \tau'(\alpha = 0) = \frac{1}{2}$ implies that $\tau$ achieves its minimum at $\alpha = \tilde{x}$ but its maximum at $\alpha = 0$ and $\alpha = 1$. This completes the proof.

A.4. Proof of Proposition 3
Given Equation (13), the total inefficiency is given as
\[
\Delta = \begin{cases} 
-a^4 + 4a^3 - 2a^2 - 4a + 7 & \text{if } \alpha > \tilde{x} \\
-a^4 + 2a^3 + 16 & \text{otherwise}
\end{cases} \quad (A.2)
\]
Thus,
\[
\frac{\partial \Delta}{\partial \alpha} = \begin{cases} 
\frac{(1-a)(1+2a-a^2)}{4} & \text{if } \alpha > \tilde{x} \\
\frac{a(1+a)^2}{2} & \text{otherwise}
\end{cases} < 0. \quad (A.3)
\]

A.5. Proof of Proposition 4
First, note that $\rho_m(x) < \kappa_m$ is equivalent to $x_1 < x_{\kappa_m} \equiv \frac{-a}{2}$ and that $\rho_n(x) < \kappa_n$ is equivalent to $x_1 > x_{\kappa_n} \equiv \frac{a(a+1)}{2}$. Then it follows that $A_1 = m$ if $x_1 \geq \max\{x_{\kappa_m}, x_{\kappa_n}\}$; $A_1 = n$ if $x_1 \leq \min\{x_{\kappa_m}, x_{\kappa_n}\}$; and $A_1 = 0$ if $x_{\kappa_n} < x_1 < x_{\kappa_m}$. However, given the definitions of $x_{\kappa_m}$ and $x_{\kappa_n}$, we have $x_{\kappa_n} \geq x_{\kappa_m}$ if $\alpha \geq \frac{1}{2}$ but $x_{\kappa_n} < x_{\kappa_m}$ otherwise. Therefore, when $\alpha \geq \frac{1}{2}$, $A_1 = 0$ can never be observed and $A_1 = m$ if $x_1 \geq \tilde{x}$ but $A_1 = n$ otherwise. When $\alpha < \frac{1}{2}$, $A_1 = m$ if $x_1 \geq \tilde{x}$; $A_1 = n$ if $x_1 \leq \tilde{x}$; and $A_1 = 0$ if $x_{\kappa_n} < x_1 < x_{\kappa_m}$. \□
A.6. Proof of Proposition 5
To prove the first part, note that \( \rho_m(x) > 0 \), \( \forall x < a \) but \( \Sigma_m = \frac{\alpha(4-\alpha)}{2} < a \). This implies \( \rho_m(x) > 0 \), \( \forall x < \Sigma_m \). Thus, the match probability is always positive whenever the impression is allocated to advertiser \( m \). Therefore, there is no completely irrelevant-ad ineffectiveness. For the second part, recall from Proposition 4 that when \( a < \frac{1}{2} \), we have \( A_1 = \emptyset \) for \( \Sigma_m < x_1 < \Sigma_m \). Because \( \rho_m(x) > 0 \), \( \forall x \in (0,1] \), the first-best allocation allows the agency to earn at least \( \int_{x_m}^{1} \rho_m(x)dx > 0 \), whereas the dynamic allocation leads to zero profit for the agency. Hence, there arises an ineffectiveness due to no-ad allocation. This completes the proof. \( \square \)

A.7. Proof of Claims in Section 5.3
We prove the following two claims for \( a < \frac{1}{2} \): (1) \( \mathcal{EIT}_m > \mathcal{EIT}_m^* \) and (2) \( \mathcal{EIT}_m - \mathcal{EIT}_m^* < 0 \). First, 
\[
\mathcal{EIT}_m - \mathcal{EIT}_m^* = \int_{x_m}^{1} \rho_m(x)dx + \int_{\Sigma_m}^{x_m} \rho_m(x)dx + \int_{0}^{x_m} \rho_m(x)dx \ 
\]

This then, when \( a < \frac{1}{2} \) because the profits remain the same, both advertisers are indifferent across all three regimes. Therefore, all of full, partial, and no contractual obligation can emerge as an equilibrium.

Next, when \( a < \frac{1}{2} \), given the previous profits, observe the following:
\[
\mathcal{EIT}_m - \mathcal{EIT}_m^* = \left\{ \begin{array}{ll}
\frac{-(1-2a)^2}{32} & \text{if } a \geq \sqrt{2} - 1 \\
- \frac{a^2(3a^2 + 4a - 2)}{32} & \text{otherwise}
\end{array} \right.
\]

\[
< 0 \text{ if and only if } a < \frac{\sqrt{10} - 2}{3}.
\]

\[
\mathcal{EIT}_m - \mathcal{EIT}_m^* = \left\{ \begin{array}{ll}
\frac{-(1-2a)^2}{32} & \text{if } a \geq \sqrt{2} - 1 \\
- \frac{5a^2 + 12a - 6}{32} & \text{otherwise}
\end{array} \right.
\]

\[
< 0, \forall a \in \left[ 0, \frac{1}{2} \right].
\]

Thus, together with full contractual obligation, partial contractual obligation of advertiser \( m \) is an equilibrium when \( a < \frac{\sqrt{10} - 2}{3} \) but no contractual obligation is an equilibrium otherwise. \( \square \)

A.8. Proof of Proposition 6
First, the advertisers’ profits under full contractual obligation are given as 
\[
\mathcal{EIT}_m = \int_{0}^{x_m} \rho_m(x)dx + \int_{\Sigma_m}^{x_m} \rho_m(x)dx + \int_{\Sigma_m}^{1} \rho_m(x)dx
\]

This calls for the less relevant advertiser (i.e., advertiser \( n \)) if \( x_1 > x_0 \) is equivalent to \( \rho_m(x_1) \geq \rho_m(x_1) \). Thus, the agency allocates \( x_1 \) to the less relevant advertiser \( n \). Such an \( x_1 \) exists if and only if \( \Sigma_m > x_0 \), or equivalently, \( a < 0.7058 \). This completes the proof. \( \square \)

A.9. Proof of Proposition 7
First, according to (26), \( A_1 = \emptyset \) if and only if \( x_1 > \Sigma_m \). Moreover, \( x_1 > x_0 \) is equivalent to \( \rho_m(x_1) \geq \rho_m(x_1) \). Thus, the agency allocates \( x_1 \) to the less relevant advertiser (i.e., advertiser \( m \)) if \( x_0 < x_1 < \Sigma_m \). Such an \( x_1 \) exists if and only if \( \Sigma_m > x_0 \), or equivalently, \( a < 0.7058 \). This completes the proof. \( \square \)

A.10. Proof of Corollary 3
The result follows from the following inequality:
\[
\frac{3 - 4a + 6a^2 - 4a^3 + a^4}{16} > 0 \text{ (since } a \leq 1) \quad \text{ if } a > 0
\]

\[
\frac{4 - 8a + 2a^2 + a^3}{8} > 0 \text{ (since } a \leq 0.5) \text{ if } a \leq 0.5.
\]
Endnotes

1 In Akami’s 2018 Consumer Attitudes Toward Data Privacy and Security Survey, only 18% of consumers said ads “often” seemed to understand their needs. 47% report that ads seem to understand their needs at least “sometimes,” 26% said online ads “hardly ever” understand them, and 9% said they “never” understand them (https://www.akami.com/us/en/multimedia/documents/report/akamai-research-consumer-attitudes-toward-data-privacy.pdf).

2 In a recent survey by MediaPost.com, almost 82% of consumers report that they often receive irrelevant ads (https://www.mediapost.com/publications/article/326515/data-still-makes-ads-irrelevant-consumers-say.html).


5 In a typical agency‑advertiser contract, each campaign is run by the so-called “insertion order” that stipulates a specific campaign period and the number of impressions to deliver, meeting certain targeting criteria specified by the client. Generally, the advertiser specifies its target number of impressions, targeting criteria, and the budget for the campaign in the insertion order. For example, a local Subaru dealer may set forth in the insertion order that the agency must deliver 100,000 impressions, who are interested in cars in greater New York areas, for the next three weeks. Based on these terms, the agency bids for its clients (including this Subaru dealer) irrespective of ad spot as long as impressions meet those targeting criteria.

6 Most ad platforms offer tools for ad agencies to work for multiple advertisers, such as Google’s Authorized Buyers account (https://support.google.com/authorizedbuyers/answer/6138000?hl=en) and Twitter’s multi-use login (https://business.twitter.com/en/blog/agencies-manage-multiple-twitter-ads-accounts.html). Moreover, how to handle multiple clients with potentially overlapping target consumers is a common subject among ad agency messaging boards and forums (e.g., https://www.linkedin.com/pulse/how-can-your-agency-handle-multiple-clients-same-industry-forster/).

7 In programmatic bidding, an advertising agency automates online ad purchasing on behalf of the advertisers, using individual-level data and online targeting methods. It continuously monitors several different campaign performances. The realizations (or performances) of those different ad campaigns would systematically affect the agency’s programmatic bidding priorities over time. Suppose a niche brand ad campaign is behind schedule for delivering its contracted number of impressions. Then, the agency may divert some impressions that may better fit another brand to a niche brand, which may have difficulty finding appropriate impressions. For example, suppose an ad agency needs to provide Subaru with 100,000 impressions that are interested in cars in greater New York areas for a month. If the agency could only deliver 50,000 impressions before the last week of its campaign, it may urgently change its automated bidding program priority to meet Subaru’s contractual requirement of 100,000 impressions. Therefore, even if a relatively less relevant impression arrives, the agency may deliver it to Subaru.

8 Gordon et al. (2021) provide a comprehensive review of inefficiencies in digital advertising, especially organizational inefficiencies arising from the conflicts of interests between advertisers and their self-interested agencies.

9 Villas-Boas (1994) studies the benefits and costs of hiring the same advertising agency as competitors by focusing on information transfer across competing firms. Although the setting is similar, our focus is different. We study the ad agency’s strategic distortion of impression allocation over time in the context of real-time bidding.

10 Although this specific formulation of the match probability presents a clear distinction between the mainstream and the niche advertiser, our results do not depend on this specific match probability formulation. In Section A1 of the online appendix, we show the robustness of our main results by analyzing the general match probability.

11 To focus on the ad allocation problem of the ad agency, we abstract away from the bidding game in the ad exchange. This is a limitation of the current research, and future research may examine the implications of the strategic interaction of the ad exchange with the agency’s allocation incentive.

12 The advertising cost (or its budget constraint) includes not only the commission to the agency for their service but also the advertising price that is paid to the publisher (through the agency), which typically occurs per impression. As we assume away the bidding game, we consider a constant ad price, which should be incurred for every impression regardless of its relevance.

13 In practice, the agency and advertisers agree on a general contract to start a relationship of service agreement. After establishing their relationship, they again negotiate about the individual insertion order (which specifies the agency commission and the target number of impressions) on a campaign-by-campaign basis. In this repeated relationship, the agency has an incentive to use its client’s budget in the client’s best interest. Our approach is a reduced-form way to capture this spirit without modeling the repeated interactions between them. Many popular press articles recognize such an aligned incentive between the agency and the advertiser (e.g., https://www.adexchanger.com/data-driven-thinking/programmatic-a-series-of-cascading-interconnected-contracts/). Under this contractual arrangement, we analyze a rational behavior by the market participants.

14 We assume that the advertisers have more bargaining power in determining the contract terms in agency-advertisers relationship. We abstract away the negotiation process to determine the actual contract terms between the agency and advertisers. However, this reflects a typical relationship in practice in the advertising industry.

15 The accuracy of information depends on the type and amount of customer data (Shin and Yu 2021). For example, even gender is usually only 75% accurate, and the precision of data in most platforms can be anywhere between 10% and 20% (Forbes, “How Accurate is Marketing Data?” July 5, 2017).

16 The similarity of allocation under partial and full contractual obligation as shown in Corollary 2 is obtained due to the budget constraint of advertisers. Thus, as long as the budget constraint remains the same as in the main model, consideration of any greater number of periods will yield the same result. However, if the budgets are larger (e.g., budgets to cover two periods in a three-period model), the allocation under partial contractual obligation may be different from that under full contractual obligation.

17 The previous conditions can be translated into the cutoff criteria \( \tau_{\alpha}, \tau_{\alpha'}, \) and \( \tau_{\alpha''} \): \( \rho_{\alpha}(x) > \rho_{\alpha'}(x) + \varepsilon \) is equivalent to \( x_1 > \tau_{\alpha}; \rho_{\alpha}(x) > \varepsilon \) is equivalent to \( x_1 > \tau_{\alpha'}; \) and \( \rho_{\alpha}(x) > \varepsilon \) is equivalent to \( x_1 < \tau_{\alpha''} \).

18 To summarize, (1) the case under contractual obligation reverts to the main model both with and without budget constraints; and (2) under no contractual obligation, the case with budget constraint is analyzed in Section 5.2, and the case without budget constraint reverts to the benchmark model.

19 Because of the different way each of these restrictions creates the future opportunity cost, they may induce a different allocation outcome when asymmetrically imposed on the two advertisers. The online appendix presents such a case when one or both advertisers’ budget is unconstrained (see Section A2). However, when both are budget constrained, as shown in Section 5.1, the agency’s optimal allocation under asymmetrically imposed contractual obligations remains identical to that of Proposition 1.
The claims in this section are formally proved in the Appendix.

Our objective in this section is to demonstrate that contractual obligation can always emerge as an equilibrium outcome and not to claim the uniqueness of the equilibrium. As such, our discussion focuses on the existence of equilibria where at least one advertiser imposes the contractual obligation.

In our online appendix, we also consider a case without contractual obligation (Section A4) and a case without budget constraint (Section A5) in the three-period model. In both cases, the irrelevant advertising continues to exist. Moreover, the completely irrelevant advertising persists in the latter case, confirming our main finding that contractual obligation can generate extreme case of allocation inefficiency, completely irrelevant advertising.


References


