Predictive Analytics and Ship-then-shop Subscription

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Abstract

This paper studies an emerging subscription model called ship-then-shop. Leveraging its predictive analytics and artificial intelligence (AI) capability, the firm curates and ships a product to the consumer, after which the consumer shops (i.e., evaluate product fit and make a purchase decision). The consumer first pays the upfront ship-then-shop subscription fee prior to observing product fit and then pays the product price if she decides to purchase after realizing product fit. We analyze how the firm balances the subscription fee and product price to maximize its profit in the presence of consumers' potential showrooming behaviors. In particular, we focus on how the firm's prediction capability affects its pricing strategies. Our model generates rich insights regarding a firm's prediction capability, search friction, and their interactions on the profitability of the ship-then-shop model.

Keywords: predictive analytics, artificial intelligence, subscription business, ship-then-shop, free-riding, showrooming, ex-ante and ex-post extraction

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1 Introduction

Advances in machine learning techniques and data digitization have catalyzed firms' interest in predictive analytics. Firms are fervently jumping on the predictive analytics bandwagon (eMarketer, 2021b) to optimize operations and performance of their marketing strategies (eMarketer, 2021a).[1] For example, financial service providers invest heavily in AI-powered chatbot services to improve customer relationships management[2], and tech firms deploy data-driven predictive analytics to recommend consumers books to read (Amazon), jobs to apply for (LinkedIn), and friends to contact (Facebook) (FT, 2016). Enhancements in prediction capabilities not only improve the outcomes of firms' pre-existing marketing strategies, such as customer retention and product recommendation, but also motivate firms to qualitatively reinvent their business models. For instance, Agrawal et al. (2018) discuss the potential for predictive analytics and AI technology to transform firms' business models; they predict the emergence of an innovative AI-driven retail strategy called *ship-then-shop* subscription service. In this paper, we investigate this innovative business model that is increasingly gaining traction in practice.

Traditionally, the online shopping process starts with consumer search. The consumer searches for product information, browses various offerings, and evaluates product fit. If the consumer purchases, the firm ships the product and the shopping process terminates. In contrast, under the ship-then-shop model, the shopping process begins with product shipment. The firm leverages the prediction machine to predetermine products that match the consumer's taste and ships the product to her. The consumer then evaluates product fit, and decides whether to purchase or return the product (see Figure 1).

A unique feature of the ship-then-shop model is the separation of payments before

[1] According to eMarketer (2020), "worldwide revenues for big data analytics — including predictive analytics and consumer scoring — is forecast to grow by nearly 450% to reach \$68.09 billion in revenues by 2025."

[2] See `https://go.td.com/3rYkTQr`.

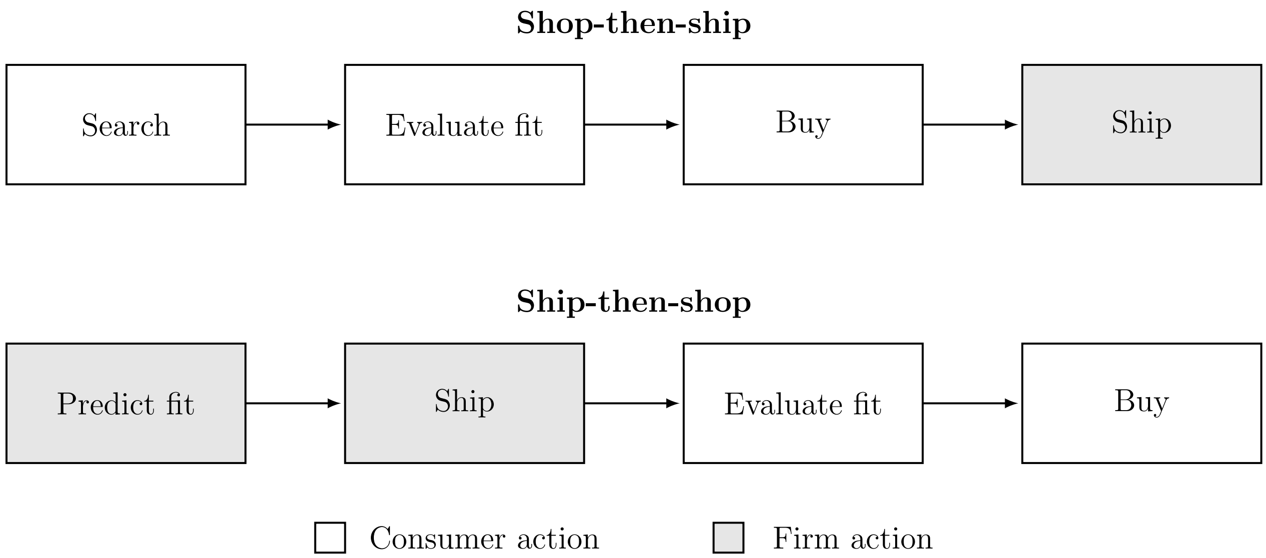

Figure 1: Two Modes of Online Shopping

and after the consumer learns product match. The consumer first pays the upfront service fee[3] prior to observing product fit, and then conditional on subscription, decides product purchase after observing product fit. Therefore, a critical determinant of the ship-then-shop model's success is a sophisticated prediction machine. Only with sufficiently high prediction accuracy would the firm's gains from predictive deliveries outweigh the loss from returns of mismatched products.

Until recently, the idea of predictive shipping has been dismissed by critics as hype (Banker, 2014; DePuy, 2014). However, the emergence of ship-then-shop subscription business models, notably in apparel retail sectors (McKinsey & Co., 2018; Moore, 2020), suggests Agrawal et al. (2018)'s prediction about the emergence of ship-then-shop subscription service is steadily unfolding in reality. The "best exemplar" of the ship-then-shop subscription provider is the apparel company Stitch Fix (Sinha et al., 2016). Stitch Fix leverages its AI algorithm to predict consumers' style preferences and ships personalized clothing items. Consumers try on the clothes and decide whether to purchase or return the products. Trunk

[3]The service fee can be either a subscription fee or a styling fee. For example, Stitch Fix, an online styling service provider, charges $20 styling fee when the curation box is assembled for a consumer (as of March 2022). Even though subscription is not required, a consumer must pay the upfront fee to receive the product and learn its match value.

Club offers a similar "try before you buy" subscription model, whereby the company deploys sophisticated algorithms "to predict the most likely fit for a consumer" and then ships personalized "clothing subscription boxes."[4] Amazon is expanding its "Prime Try Before You Buy" service for its Prime subscribers which uses "a combination of technology innovation and a human touch to curate items." The product category for "Prime Try Before You Buy" ranges from apparel and shoes to accessories and jewelry.[5]

Despite the increasing adoption of ship-then-shop models, little is understood about their economics. Standard two-part tariff solution dictates that firms should choose low product prices to reduce distortion and extract the surplus through high upfront fees; i.e., *high-fee-low-price* strategy (e.g., Clay et al., 1992; Essegaier et al., 2002). Extending this logic to the ship-then-shop model, one may intuit that as AI's predictive capability increases, the firm should raise its subscription fee and lower product price. However, this intuition does not always carry over due to fundamental differences between the ship-then-shop subscription and traditional subscription programs. First, the ship-then-shop subscription that we study focuses on curation subscription rather than replenishment subscription; the subscription value arises primarily from new product discovery. Second, unlike traditional programs wherein product value is fixed, in the ship-then-shop model, the firm's prediction machine helps improve the product match value, which is *ex-ante* unobservable to consumers but materially impacts the firm's strategies. Lastly, ship-then-shop subscription allows the possibility of consumer service free-riding (Shin, 2007), whereby consumers identify the product fit through ship-then-shop service and then purchase the same product elsewhere at a lower price. Such free-riding or showrooming behavior is less feasible for consumers in traditional subscription program where the service and sales are inseparable (e.g., mobile service). Taken together, it is not clear how the firm should balance the subscription fee and product price to maximize its profit, especially in a setting where consumers may showroom. Indeed, we find that the firm adopts the traditional *low-fee-high-price* strategy for either sufficiently low

[4] See `https://bit.ly/3qc2bm4`.

[5] See `https://bit.ly/3FnBYaP`.

or high ranges of prediction capability. Interestingly, however, for intermediate ranges of prediction capability, it is optimal for the firm to adopt *high-fee-low-price* strategy.

The firm balances two revenue channels (ship-then-shop subscription and product sales), jointly optimizing subscription volume vs. upfront fee from all consumers, and product sales volume vs. product margin from the ship-then-shop subscribers. Building on previous research on AI-technology-based recommender systems and content personalization (e.g., Ansari et al., 2018; Dzyabura and Hauser, 2019; Yoganarasimhan, 2020), we develop a parsimonious theoretical framework to elucidate the key economic forces that shape the firm's strategies under the ship-then-shop model. Moreover, we discuss how the firm's optimal strategies and profit vary with advances in AI technology under different market conditions.

The central finding of the paper is that the firm's optimal strategy depends crucially on the trade-off between *ex-ante* vs. *ex-post* surplus extraction. Relative to the traditional shopping approach, ship-then-shop provides two benefits to consumers: superior product match (matching effect) and search cost reduction (convenience effect). In making their subscription decisions, consumers weigh the benefits of the matching effect (which increases consumers' *ex-post* product valuation) and the convenience effect (which increases consumers' *ex-ante* valuation of ship-then-shop program) against the costs of subscription and product price. Moreover, consumers are rational and consider showrooming; i.e., they potentially free-ride off of the ship-then-shop matching service to identify a high-match-value product and then purchase the same product elsewhere at a lower price. The resolution of this trade-off depends on the firm's prediction capability, product price, and the degree of search friction in the market.

As AI predictive capability increases, such that the expected match value of the shipped product increases, the firm initially lowers the service fee and raises the product price; i.e., it shifts from *ex-ante* to *ex-post* surplus extraction strategy. The intuition revolves around the interplay of the matching effect and convenience effect. If the AI's prediction capability is low, the matching effect is correspondingly low such that the firm sets a high subscription service fee to extract the consumers' *ex-ante* surplus generated by the convenience effect. This low-price-high-fee strategy is qualitatively similar to the standard two-part tariff solution. On the other hand, if the AI's predictive capability increases, the matching effect dominates the convenience effect. The firm lowers the subscription service fee to entice consumers to subscribe, and then through high product price extracts *ex-post* surplus generated by the matching effect. If the AI's prediction capability is sufficiently advanced, the firm can charge an even higher price to extract additional surplus. However, high product price may prompt ship-then-shop subscribers to showroom, which creates interesting dynamics between the sales volume and product margin. The potential to showroom disciplines the firm and exerts downward pressure on product price, such that for high prediction capability, the firm reverts to the low-price-high-fee strategy.

We further characterize the conditions under which the ship-then-shop model is most profitable. We find that the firm's profit increases in *(i)* its AI's prediction capability, *(ii)* the degree of search friction in the market, and *(iii)* the product match potential. Intuitively, consumers' valuations of the matching effect and convenience effect increase in the AI's predictive capability and search friction. Also, greater product match potential increases the upside gain from the ship-then-shop's matching effect such that the firm's profit increases. We further show that the marginal return of AI's predictive capability on the firm's profit decreases in search friction, but increases in the product match potential. The negative interaction between matching and convenience effects provides important managerial insights. For instance, if the firm operates in a market characterized by high search friction, its primary revenue source is the convenience effect, such that improving its AI's predictive capability yields low marginal return. In such cases, the firm should focus more on improving the convenience effect rather than improving the matching effect (e.g., investments in AI technology). On the other hand, if the product match potential is large, it is in the firm's best interest to invest in improving its AI's prediction capability, which yields a higher marginal return than enhancing the convenience effect.

The rest of the paper is organized as follows. The next section discusses the related literature. Section 3 describes the main model, and we present the analysis and main results in Section 4. In Section 5, we demonstrate the robustness of the main insights by analyzing several extensions that relax the price-commitment assumption and consider more general search costs and procurement costs. Section 6 concludes. For ease of exposition, we relegate all proofs and lengthy algebraic expressions to the appendix.

2 Related Literature

This paper lies at the intersection of several research streams: the effect of predictive analytics and economics of AI, recommendation system and targeting, and consumer search. At the core of the ship-then-shop service is the firm's ability make data-driven predictions about consumer preferences. Our model builds on the literature on data-driven services such as recommender system and content personalization. Ansari et al. (2000) explore dynamic recommender systems using a hierarchical Bayesian approach. Lu et al. (2016) and Ansari et al. (2018) address similar issues of recommender system optimization. Previous research also investigate data-driven content personalization. Hauser et al. (2009) demonstrate the value of website personalization based on inferred consumer preferences and cognitive styles. Yoganarasimhan (2020) applies personalization to query-based search and explores the use of machine learning algorithms to rank search results, taking into account users' search and click history. Ning et al. (2021) study the implications of recommendation by the platform and consumer's privacy choice about personal data opt-out. Other studies explore AI-based prediction and its impact (e.g., Cui and Curry, 2005; Huang and Luo, 2016; Schwartz et al., 2017).[6] In line with this strand of literature, our paper studies the strategic implications of AI-based consumer preference prediction under the ship-then-shop subscription program.

Our paper is also closely related to the literature on the effects of targeting accuracy

[6]Agrawal et al. (2019) provide a comprehensive overview of the implications of AI on the practice of economics, and discuss the microeconomic effects of AI adoption on firms' strategies and profits.

on equilibrium outcomes. Several studies focus on the effect of advertising targeting on firm profits (Iyer et al. 2005, Bergemann and Bonatti 2011), and the effect of personalized pricing using customers' past purchase history (Fudenberg and Tirole, 2000; Shin and Sudhir, 2010; Villas-Boas, 1999). Several recent studies investigate the implications of targeting accuracy on the consumer inference and search behaviors (Shin and Yu, 2021), firms' prices and platform revenue (Zhong, 2020), and consumers' data privacy choices (Choi et al., 2022). Similar to these papers, we investigate the effect of targeting accuracy on the firm's equilibrium strategy. However, our focus is on the effect of prediction accuracy on the profitability of ship-then-shop business. We characterize the relationship between firms' prediction capability and the trade-offs between matching effect and convenience effect, which ultimately determine consumers' subscription choices.

Our model considers the possibility of consumers free-riding off of the firm's ship-then-shop service and exhibiting showrooming behavior. In particular, after identifying the high-match product shipped by the firm, consumers may switch from the ship-then-shop firm to the traditional market if the firm charges a sufficiently higher product price than the traditional market.[7] Shin (2007) is the first paper that formally analyzes consumer showrooming and its interaction with retailer competition. Several other studies also investigate the effects of showrooming in the context of competition between offline and online retailers (Jing, 2018; Mehra et al., 2018), between manufacturers and retailers (Kuksov and Liao, 2018), and its impacts on retail formats (Bar-Isaac and Shelegia, 2020). We contribute to this literature by investigating the effect of showrooming on the ship-then-shop service provider's pricing decision.

Finally, the combination of subscription fee and product price in our model resembles a two-part tariff, which has been studied extensively as a tool for price-discrimination among

[7]In most of the showrooming literature, the service-providing firm charges a higher price due to the cost disadvantage arising from selling costs associated with sales service (Shin, 2005). A similar force is at play in our model and induces the ship-then-shop firm to charge a higher price. However, the upward price pressure mainly stems from the AI-based improvement in match value rather than the cost-side effect.

consumers with heterogeneous usage rates (e.g., Clay et al., 1992; Essegaier et al., 2002; Kolay and Shaffer, 2003; Oi, 1971). A common theme in the two-part tariff literature is that firms choose a high upfront fee and low prices: *high-fee-low-price* strategy. Firms then extract the surplus through high fixed fees and induce consumers to purchase larger quantities by reducing price distortion.[8] Our paper is different in that the firm's surplus extraction via subscription fee is *ex-ante* (prior to product match value realization), while extraction via product price is *ex-post* (post product match value realization). In contrast to the standard two-part tariff strategy, we find that the firm adopts *low-fee-high-price* strategy if its prediction capability is intermediate. Also, our model connects the literature on two-part tariff and showrooming, and derives novel insights at their intersection. For instance, we show that improvements in firm's prediction accuracy may increase consumers' incentive to free-ride such that the firm adopts a *high-fee-low-price* strategy when its predictive capability is sufficiently advanced.

3 Model

We consider a monopolist firm and a unit mass of consumers. The firm offers ship-then-shop subscription, whereby it curates a large selection of products from the market and, based on its AI algorithm, predicts and ships best-matching products to its subscribers. Consumers purchase one unit of the product through one of two shopping methods. They can either purchase in the traditional market through their own search efforts, or they can subscribe to the ship-then-shop service. Importantly, the two shopping methods result in different product match qualities which we elaborate below.

[8] Essegaier et al. (2002) show that a monopolist may offer a negative entry fee under limited capacity (also known as "sign up bonus") to discriminate among heavy and light users.

Firm

The firm offers ship-then-shop subscription service and makes two decisions: it sets the subscription fee F and product price p_s, where subscript s denotes *subscription*. We assume that the firm procures products from the market at price p_m, where subscript m denotes *market*. The firm then resells the best-matching products to ship-then-shop subscribers at price p_s, where $(p_s - p_m)$ is the firm's profit margin or premium it can charge for its ship-then-shop service. Thus, the market price p_m effectively serves as the wholesale price. The firm's profit consists of two revenue sources, ship-then-shop subscription and product sales:

$$\mathbb{E}[\pi] = N_s \left(F + D_p \cdot (p_s - p_m) \right), \tag{1}$$

where N_s denotes the number of ship-then-shop service subscribers, F the subscription service fee, and D_p the demand for the shipped product.

Consumers

Consumers make two sequential decisions: subscription and product purchase. After observing the service fee F and product price p_s, consumers decide whether to subscribe to ship-then-shop or search in the traditional market.[9] Depending on their choice of shopping method, consumers face different product match value distributions.

If consumers search in the traditional market, they incur search cost $s \in \{s_L, s_H\}$ to discover the product and realize its match value

$$v_m \sim U[0, V], \tag{2}$$

where V denotes the maximum attainable match value — it can also be interpreted as the

[9] In a model extension, we analyze a scenario in which p_s is unobservable to consumers prior to product receipt. We show that the qualitative insights carry over.

product match potential in a given market. Consumers then decide whether to purchase the product at price $p_m \in [0, V]$.[10,11]

On the other hand, if consumers subscribe to ship-then-shop and receive the shipped product, consumers realize product match value

$$v_s \sim U[\alpha V, V], \tag{3}$$

where $\alpha \in [0, 1]$ captures the firm's prediction capability or matching quality.[12] To illustrate the role of α, if $\alpha = 0$, then the firm's match prediction capability is no better than the consumer's own ability to identify product matches through her search. On the other hand, if $\alpha = 1$, then the firm perfectly identifies and ships the consumer's ideal product, in which case the consumer obtains the maximum match value V. Thus, AI capability shifts the consumers' product match value distribution upwards — we call this the *matching effect* of ship-then-shop.

Upon receiving the ship-then-shop product and realizing match value v_s, consumers choose between three actions: *(i)* purchase the product at price p_s, *(ii)* return the product at hassle cost h, which is not too large,[13] or *(iii)* return the product at hassle cost h and purchase the same product from the traditional market at price p_m. The third action is a form of free-riding or showrooming, in which the ship-then-shop subscriber receives the ship-then-shop firm's product matching service but purchases from a competing channel (Jing, 2018; Mehra et al., 2018; Shin, 2007). As we will discuss later, the potential for consumers to free-ride on the matching service exerts downward pressure on the firm's product price p_s.

While consumers observe F and p_s, the product match values v_s and v_m are *a priori*

[10] Consumers can return products in the traditional market free of charge. However, given that consumers resolve their product match uncertainty through search, they will not return purchased items in equilibrium.

[11] In an extension, we consider the possibility of firms procuring products from the traditional market at lower prices than consumers and show that the qualitative insights carry over.

[12] We use the term AI broadly to refer to the AI's prediction capability. Thus, we use the terms "prediction capability" and "AI capability" interchangeably

[13] We assume that $h < p_m$.

unknown. Consumers observe v_s under ship-then-shop subscription only upon receiving the product, and v_m under the traditional shopping only after product search.

Consumer utility consists of two components: product consumption utility and product match value. Product consumption utility is common across all products in the same category, whereas product match value depends on the specific product that consumers find through either their own search or ship-then-shop recommendation. Thus, the consumer's utility is

$$u = u_0 + v, \tag{4}$$

where u_0 is the product consumption utility, and v the product match value. We normalize u_0 to zero without loss of generality. The consumer's product match value v depends on her choice of shopping method. If she subscribes to ship-then-shop after paying fee F, her product match value v_s is drawn from $U[\alpha V, V]$. If she searches in the traditional market, her product match value v_m is drawn from $U[0, V]$.

The consumer's net utility from subscribing to ship-then-shop is

$$u_s = -F + \delta \cdot \begin{cases} v_s - p_s & \text{if purchase from ship-then-shop firm,} \\ -h & \text{if return without purchase,} \\ -h + v_s - p_m & \text{if return and purchase from traditional channel,} \end{cases} \tag{5}$$

where $\delta \in (0, 1)$ is the discount factor, capturing the delayed product consumption under ship-then-shop, and $s \in \{s_L, s_H\}$ denotes the consumer's search cost.[14] For ease of exposition, we hereafter set $\delta \to 1$.

[14]We treat search costs primarily as costs associated with product discovery. Thus, we assume that when consumers return the ship-then-shop product and purchase from traditional channel, they do not incur search costs.

On the other hand, the consumer's net utility in the traditional market is

$$u_m = -s + \begin{cases} v_m - p_m & \text{if purchase,} \\ 0 & \text{if not purchase,} \end{cases} \tag{6}$$

where $s \in \{s_L, s_H\}$ is the heterogeneous search cost. Consumers are low-type ($s = s_L$) or high-type ($s = s_H$) with equal probability. We normalize s_L to zero without loss of generality; i.e., $0 = s_L < s_H$. Moreover, to focus on the more interesting case where all consumers may search in the traditional market, we assume that $s_H \leq (V - p_m)^2 / 2V$.[15] It is important to note that if the consumer purchases from the ship-then-shop firm, she does not incur the search cost s (see (5)), whereas if she buys from the traditional market, she does (see (6)). That is, ship-then-shop subscription facilitates shopping by saving consumers' search cost. We call this the *convenience effect* of ship-then-shop.

Overall, in making their subscription decisions, consumers weigh the potential gains from the matching effect (which increases consumers' *ex-post* product valuation) and the convenience effect (which increases consumers' *ex-ante* valuation of ship-then-shop program) against the cost of subscription and product price. The game sequence is summarized in Figure 2.

4 Analysis

We solve for subgame perfect Nash equilibrium using backward induction.

[15] If $s_H > (V - p_m)^2 / 2V$, the game degenerates to a trivial case where the high-type consumers do not consider buying in the traditional market.

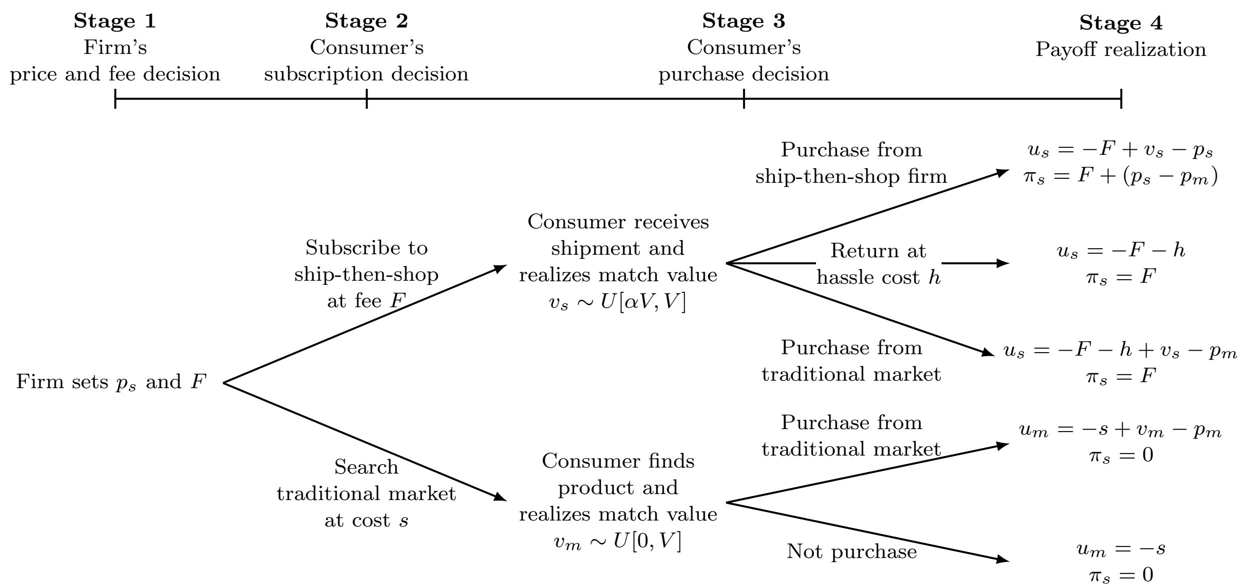

Figure 2: Game Sequence

4.1 Consumer Decision

Consumers' decisions are two-fold: ship-then-shop subscription and product purchase. Conditional on subscribing to ship-then-shop, consumers either *(i)* purchase from the ship-then-shop firm, which yields utility $v_s - p_s$, *(ii)* return the product at hassle cost h, which yields utility $-h$ or *(iii)* return the product at hassle cost h and then purchase the same product from the traditional market at price p_m, which yields utility $-h + v_s - p_m$.

As we later demonstrate, the firm will always set product price

$$p_s \leq p_m + h, \tag{7}$$

such that consumers do not have incentive to free-ride. This price cap can be viewed as a form of "price matching" (Jing, 2018; Mehra et al., 2018). Consumers purchase the shipped product if and only if the realized match value net of price exceeds the disutility they incur from returning the product: $v_s - p_s \geq -h$. Note that if $p_s - h < \alpha V$, ship-then-shop subscribers always buy. Therefore, the marginal consumer who purchases is

$$\bar{v} \equiv \max\{\alpha V, p_s - h\}. \tag{8}$$

Consumers also decide whether to subscribe to ship-then-shop or search in the traditional market. Consumers' expected utility from subscribing to ship-then-shop is

$$\mathbb{E}[u_s] \;\; = \;\; -F + \left(\int_{\alpha V}^{\bar{v}} \frac{-h}{V(1-\alpha)} dv_s + \int_{\bar{v}}^{V} \frac{v_s - p_s}{V(1-\alpha)} dv_s \right) \tag{9}$$

The first term denotes the subscription service fee, and the second term in brackets the expected utility from either returning or purchasing the shipped product.

If consumers search in the traditional market at search cost $s \in \{0, s_H\}$, their expected utility is

$$\mathbb{E}[u_m] = -s + \int_{p_m}^{V} \frac{v_m - p_m}{V} dv_m = -s + \frac{(V - p_m)^2}{2V}. \tag{10}$$

Consumers compare their expected utility from ship-then-shop subscription in (9) and that from traditional market in (10). Let $\bar{s}$ denote the search cost for which consumers are indifferent between the two shopping options. Solving $\mathbb{E}[u_s] = \mathbb{E}[u_m]$ yields

$$\bar{s} = F - \left(\frac{(V(2\alpha h + V) - \bar{v}(2h + \bar{v}) - 2p_s(V - \bar{v}))}{2V(1-\alpha)} - \frac{(V - p_m)^2}{2V} \right). \tag{11}$$

While consumers with search cost $s > \bar{s}$ subscribe to ship-then-shop, those with search cost $s \leq \bar{s}$ search in the traditional market.

Specifically, *(i)* if $\bar{s} \leq 0$, then all consumers subscribe; *(ii)* if $0 < \bar{s} \leq s_H$, then high-type consumers ($s = s_H$) subscribe, while low-type consumers ($s = 0$) choose the traditional market; and *(iii)* if $s_H < \bar{s}$, then all consumers choose the traditional market.

4.2 Firm Decision

The firm sets product price and service fee in anticipation of consumers' subscription and purchase decisions. The firm's expected profit is

$$\mathbb{E}[\pi(p_s, F)] = N_s(p_s, F)\Big(\underbrace{F}_{\substack{\textit{ex-ante} \\ \text{surplus}}} + \underbrace{\frac{V - \bar{v}}{V(1-\alpha)}(p_s - p_m)}_{\substack{\textit{ex-post} \\ \text{surplus}}} \Big), \tag{12}$$

where $N_s(p_s, F)$ denotes the number of ship-then-shop subscribers, F the service fee, and the last term the expected margin from product sales. $\frac{V-\bar{v}}{V(1-\alpha)}$ is the probability that a ship-then-shop subscriber purchases the shipped product. The firm procures the product at exogenous market price p_m and then resells it at price p_s.

The firm's profit in (12) reveals two channels through which the firm extracts consumer surplus: *ex-ante* surplus extraction through service fee F (i.e., expected consumer surplus prior to product match value realization), and *ex-post* surplus extraction through product price p_s (i.e., consumer surplus associated with the realized product match value). Throughout the paper, we use the following terminology: whenever the firm, in response to some change in market characteristic, places more weight on the fee F (vs. the product price p_s) for surplus extraction, we say that the firm adopts *ex-ante* surplus extraction strategy. Conversely, if it places more weight on the price p_s, we say it adopts *ex-post* surplus extraction strategy.

The firm determines $N_s(p_s, F)$ by adjusting p_s and F. Given the binary search cost space (i.e., $s \in \{0, s_H\}$), the firm considers two demand regimes: partial coverage and full coverage. Under partial coverage, the firm induces only the high-type consumers ($s = s_H$) to subscribe to ship-then-shop: $N_s(p_s, F) = 1/2$. Under full coverage, it induces both the high- and low-type consumers ($s = 0$) to subscribe: $N_s(p_s, F) = 1$.

Under partial coverage ($N_s(p_s, F) = 1/2$), the firm's problem is

$$\begin{aligned} \max_{p_s, F} \quad & \mathbb{E}[\pi_{\text{part}}] = \frac{1}{2}\left(F + \frac{V - \bar{v}}{V(1-\alpha)}(p_s - p_m)\right) \\ \text{subject to} \quad & 0 < \bar{s}(p_s, F) \leq s_H \text{ and } p_s \leq p_m + h \end{aligned} \tag{13}$$

where $\bar{v}$ is the marginal consumer who purchases the product as defined in (8) and $\bar{s}$ is defined in (11). The (IC) constraint $0 < \bar{s}(p_s, F) \leq s_H$ ensures only the high-type consumers subscribe to ship-then-shop. The condition $p_s \leq p_m + h$ prevents showrooming: if $p_s > p_m + h$, then the s_H-consumers who subscribe to ship-then-shop switch to the traditional market after receiving the ship-then-shop service, such that the firm's product sales is 0. Therefore, under partial coverage, the firm's optimal product price satisfies $p_s \leq p_m + h$. Solving (13) yields

$$F^*_{\text{part}}(p_s) = s_H + \frac{(V(2\alpha h + V) - \bar{v}(2h + \bar{v}) - 2p_s(V - \bar{v}))}{2V(1-\alpha)} - \frac{(V - p_m)^2}{2V}, \tag{14}$$

$$p^*_s = \min\left\{\max\left\{\alpha V + h, p_m\right\}, p_m + h\right\}. \tag{15}$$

Under full coverage ($N_s(p_s, F) = 1$), the firm's problem is

$$\begin{aligned} \max_{p_s, F} \quad & \mathbb{E}[\pi_{\text{full}}] = F + \frac{V - \bar{v}}{V(1-\alpha)}(p_s - p_m) \\ \text{subject to} \quad & \bar{s}(p_s, F) \leq 0 \text{ and } p_s \leq p_m + h, \end{aligned} \tag{16}$$

where the (IC) constraint $\bar{s}(p_s, F) \leq 0$ ensures both consumer types subscribe to ship-then-shop. Similar to the partial coverage case, the constraint $p_s \leq p_m + h$ prevents subscribers from showrooming. Following the reasoning above, we obtain that the optimal price under full coverage coincides with that under partial coverage, while the optimal fee under full coverage is $F^*_{\text{full}}(p_s) = F^*_{\text{part}}(p_s) - s_H$, such that the low-type consumers' (IC) constraint binds. The following lemma summarizes the optimal product price and subscription fee under each regime.

Lemma 1. *The firm's optimal product price under both partial and full coverage is*

$p_s^* = \min\{\max\{\alpha V + h, p_m\}, p_m + h\}$. *The optimal subscription fees under partial and full coverage, respectively, are* F^*_{part} *and* $F^*_{full} = F^*_{part} - s_H$.

Before solving for the optimal coverage choice, we highlight an important relationship between product price and service fee. Observe that under either the partial or full coverage, the firm's *best-response* fee $F^*(p_s)$ is decreasing in p_s:

$$\frac{\partial F^*(p_s)}{\partial p_s} = -\min\left\{1, \frac{V + h - p_s}{V(1-\alpha)}\right\} \leq 0.^{16} \tag{17}$$

This implies that the optimal service fee and optimal product price are strategic substitutes. In optimizing the product price, the firm not only trades off the usual margin versus sales, but also considers whether to extract *ex-ante* surplus through F or to extract *ex-post* surplus through p_s. The latter trade-off constitutes one of the key forces of the model. As we later demonstrate, whether the firm opts for *ex-ante* or *ex-post* surplus extraction depends crucially on the firm's AI capability (α) and search friction (s_H).

Proposition 1. *The firm's product price and service fee are strategic substitutes:* $\frac{\partial F^*(p_s)}{\partial p_s} \leq 0$.

Next, we determine the firm's optimal coverage, and thereby characterize the optimal service fee. The firm compares the optimal profits under partial and full coverage, which are, respectively,

$$\mathbb{E}[\pi^*_{\text{part}}] = \begin{cases} \frac{1}{4}\left(\alpha V + 2s_H - \frac{p_m^2}{V}\right) & \text{if } p_m \leq \alpha V + h, \\ \frac{\alpha\left(V^2 - 2V(p_m + s_H - h) + p_m^2\right) + h(h - 2p_m) + 2s_H V}{4V(1-\alpha)} & \text{if } p_m > \alpha V + h, \end{cases} \tag{18}$$

and

$$\mathbb{E}[\pi^*_{\text{full}}] = \begin{cases} \frac{\alpha V^2 - p_m^2}{2V} & \text{if } p_m \leq \alpha V + h, \\ \frac{h^2 - 2h(p_m - \alpha V) + \alpha(p_m - V)^2}{2V(1-\alpha)} & \text{if } p_m > \alpha V + h. \end{cases} \tag{19}$$

[16] Note that the firm never sets $p_s > V + h$.

Proposition 2. *The firm chooses full coverage if α is sufficiently large such that $\alpha > \tilde{\alpha}$ (or equivalently, s_H is sufficiently small such that $s_H \leq \tilde{s}$). Otherwise, it chooses partial coverage.*[17]

Proposition 2 shows that the firm chooses full coverage if its AI capability is sufficiently sophisticated or the search friction in the market is mild. Intuitively, advanced AI capability (i.e., large α) increases the ship-then-shop service's matching effect. This implies that even the low-type consumers, who can search in the traditional market at low search cost, have high valuation for the ship-then-shop service. The increased valuation motivates the firm to cover the whole market.

In terms of search friction, the firm covers the whole market if search friction is mild (i.e., small s_H). Recall that the second benefit of ship-then-shop is the convenience effect: ship-then-shop facilitates consumer shopping by reducing their search costs. Therefore, if search friction is severe, the convenience effect becomes more valuable to consumers such that the firm can extract large consumer surplus. In this case, the firm charges a high service fee and extracts the high-type consumers' surplus, at the expense of forgoing subscription from low-type consumers.

Based on the firm's optimal coverage, we obtain the optimal price and fee:

$$p_s^* = \min\left\{\max\left\{\alpha V + h, p_m\right\}, p_m + h\right\}, \text{ and } F^* = F_{\text{part}}^*\left(p_s^*\right) - \begin{cases} 0 & \text{if } \alpha \leq \tilde{\alpha}, \\ s_H & \text{if } \alpha > \tilde{\alpha}. \end{cases} \tag{20}$$

In the following proposition, we present the main result about how the firm's equilibrium strategy (p_s^*, F^*) interacts with *(i)* the firm's AI prediction capability, *(ii)* the degree of search friction in the market, and *(iii)* the consumer hassle costs.

Proposition 3. *The firm's equilibrium strategy (p_s^*, F^*) varies as follows.*

[17] The thresholds $\tilde{\alpha}$ and $\tilde{s}$ are characterized in the proof.

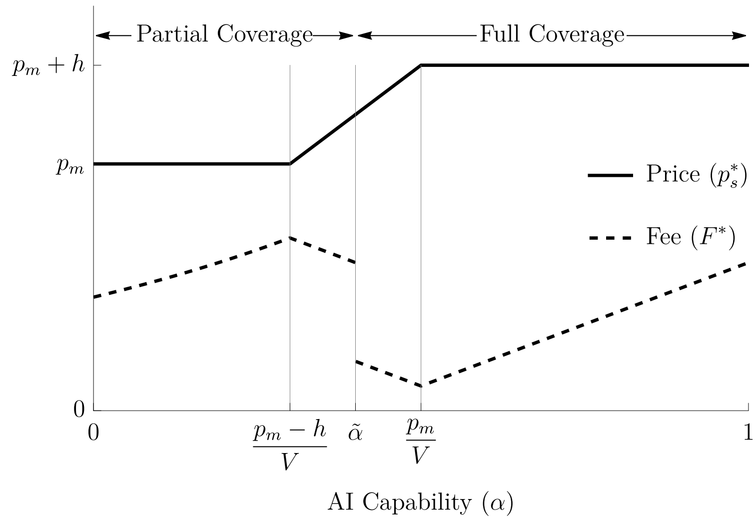

Figure 3: Impact of AI Capability on Price and Fee ($V = 1, p_m = 0.5, h = s_H = 0.1$)

(i) With respect to α: if $\frac{p_m-h}{V} < \alpha \leq \frac{p_m}{V}$, p_s^* increases in α; otherwise, it is constant in α. F^* varies non-monotonically in α with a discontinuous drop at $\tilde{\alpha}$.[18] Specifically,

– when α is either sufficiently low or high ($\alpha \leq \frac{p_m-h}{V}$ or $\frac{p_m}{V} < \alpha$), F^* increases in α;

– when α is in the intermediate range ($\frac{p_m-h}{V} < \alpha \leq \frac{p_m}{V}$), F^* decreases in α.

(ii) With respect to s_H: p_s^* is constant in s_H, while F^* weakly increases in s_H.

(iii) With respect to h: p_s^* weakly increases in h, while F^* decreases in h.

If the firm's prediction accuracy is low (i.e., $\alpha \leq \frac{p_m-h}{V}$), the expected match quality is poor, which dampens consumers' valuation for ship-then-shop. In this range, the primary source of customer benefits is the convenience effect. Therefore, if α is small, the firm adopts *ex-ante* surplus extraction strategy. For intermediate level of prediction accuracy, the product match quality is sufficiently high that the primary source of customer benefit switches from the convenience effect to the matching effect. Thus, the firm changes its pricing strategy from *ex-ante* to *ex-post* surplus extraction; i.e., it raises the product price and lowers the subscription fee (see changes in price and fee patterns as α increases in the interval $[0, p_m/V]$ in Figure 3).

However, as the prediction accuracy increases further, the firm reverts to *ex-ante* surplus

[18] $\tilde{\alpha}$ is the threshold value of α at which the firm switches from partial to full coverage (see Proposition 2).

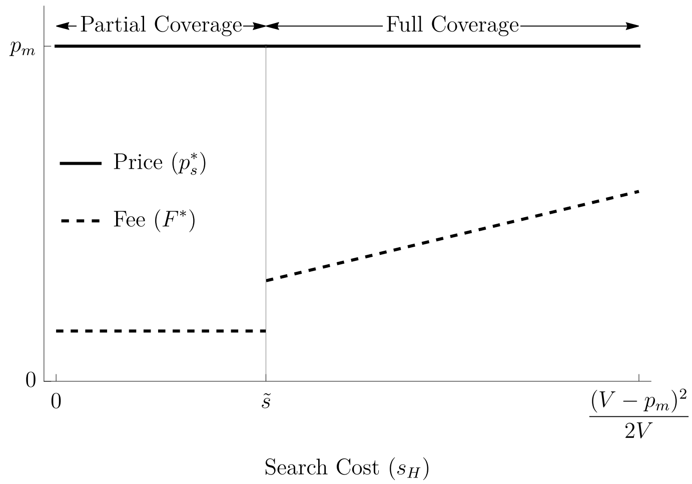

Figure 4: Impact of Search Cost on Price and Fee ($V = 1, p_m = 0.5, h = 0.1, \alpha = 0.25$)

extraction strategy, even though the matching effect outweighs the convenience effect. In particular, for all $\alpha \in [p_m/V, 1]$, p_s^* remains constant while F^* increases in α. The rationale behind this reversion is that consumers' incentives to free-ride increase as the firm's product price increases. Consumers receive high-quality matching services from the ship-then-shop firm but purchase from the traditional market. To prevent sales loss from such free-riding, the firm caps the product price and raises the fee instead. In sum, the firm's extraction strategy follows a non-monotonic pattern with respect to the prediction accuracy: it adopts *ex-post* extraction for intermediate ranges of α, and *ex-ante* extraction for extreme ranges of α. This result sheds light on a novel strategic benefit — i.e., limiting consumer free-riding — of the high-fee-low-price strategy.

Also, the fee pattern with respect to s_H reflects the convenience effect, the second benefit of ship-then-shop subscription. Intuitively, the value of shopping without searching increases as searching in the traditional market becomes more costly. This lifts consumers' valuation of ship-then-shop, which allows the firm to charge higher service fee (see Figure 4).

Finally, we also find that $\frac{\partial p_s^*}{\partial h} \geq 0$ and $\frac{\partial F^*}{\partial h} < 0$. Higher hassle cost of returning unwanted products has a lock-in effect, which allows the firm to raise the product price. Thus, the firm extracts greater *ex-post* surplus by charging a high price, while it lowers its upfront service fee to compensate for the risk of product mismatch.

4.3 Profitability of Ship-then-shop

Next, we examine the profitability of ship-then-shop by exploring the comparative statics of the firm's equilibrium profit with respect to firm-specific factor (e.g., AI's prediction capability) and market-specific factors (e.g., the severity of search friction and the product match potential).

Proposition 4. *The firm's expected profit increases in α, s_H, and V; i.e.,* $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} > 0$, $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} \geq 0$, *and* $\frac{\partial \mathbb{E}[\pi^*]}{\partial V} > 0$.

Intuitively, the firm's profit increases in both α and s_H. Equipped with higher AI capability, the firm provides a higher-quality match to consumers. Also, higher search friction in the traditional market increases the consumers' comparative valuations for ship-then-shop. Both enlarge the total surplus, which the firm extracts through the optimal fee-price combination in (20). Finally, as the maximum attainable match value V increases, the upside potential of the matching effect under ship-then-shop rises, increasing the firm's profit.

Given that the profitability of ship-then-shop increases in AI capability and market search friction, lay intuition suggests that the firm should invest in enhancing both the matching and convenience effect. However, due to the linkage between *ex-ante* and *ex-post* consumer surplus, we find that the two effects are substitutes; i.e., returns from one effect diminishes the returns from the other.

Proposition 5. *The interaction effect of AI capability and search friction on the firm's profit is negative:* $\frac{\partial}{\partial s_H}\left(\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}\right) \leq 0$. *Moreover, the marginal return of α (s_H) on the firm's expected profit increases (decreases) in V;* $\frac{\partial}{\partial V}\left(\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}\right) \geq 0$ *and* $\frac{\partial}{\partial V}\left(\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}\right) \leq 0$.

While the firm's profit increases in α and s_H, the interaction effect of AI capability and search friction on the firm's profit is negative: $\frac{\partial}{\partial s_H}\left(\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}\right) \leq 0$. This suggests that for the firm

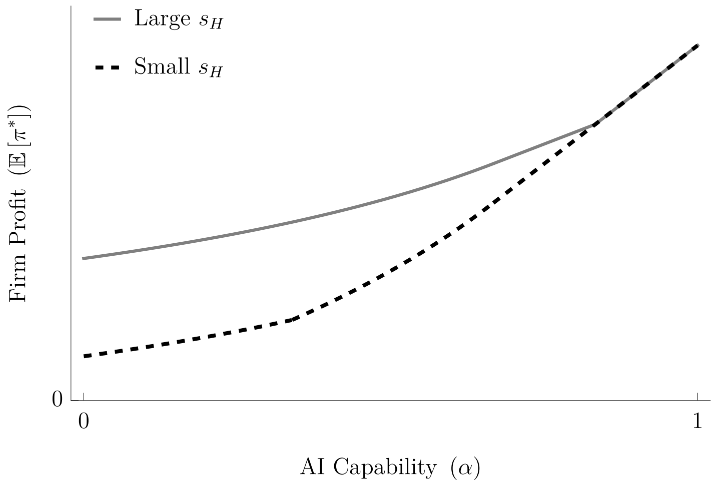

Figure 5: Search Cost and Marginal Return

offering ship-then-shop service, matching effect and convenience effect are substitutes. For instance, if the search friction in the market is severe such that the convenience effect is large, the marginal effect of AI capability on firm's profit diminishes (see Figure 5). Intuitively, the firm capitalizes on the convenience effect by charging high service fees, thereby extracting the high-type consumers' surplus at the expense of foregoing demand from low-type consumers. As fewer consumers subscribe to ship-then-shop, the total returns from the matching effect decreases.

Finally, Proposition 5 reveals an important insight regarding product match potential. The marginal return from the matching effect increases in product match potential, whereas the marginal return from the convenience effect decreases in product match potential (see Figure 6). If V is small, then consumers face little product match uncertainty such that the matching effect under ship-then-shop adds little value. In this case, the ship-then-shop's value derives primarily from the convenience effect; therefore, the return from convenience effect dominates that from the matching effect. On the other hand, if V is large, the relationship reverses. The greater the product match potential, the greater the return from AI prediction capability because the firm can more effectively harvest the upside gains from the matching effect. In this case, the primary source of ship-then-shop's value is the matching effect. Therefore, the marginal return from the matching effect dominates that from the

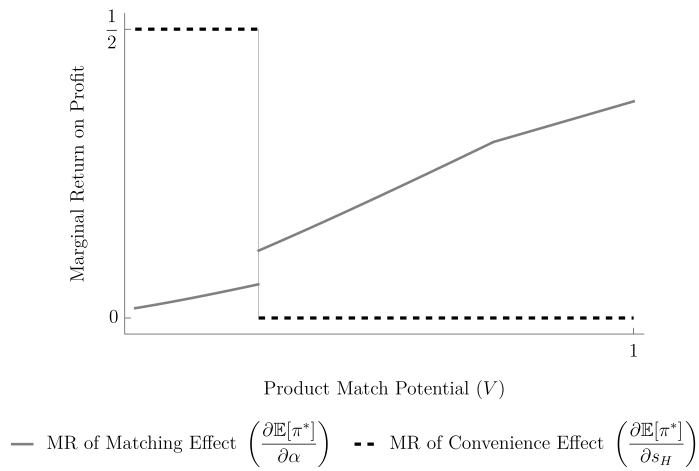

Figure 6: Marginal Returns of Matching and Convenience Effects

convenience effect.

The insights from Propositions 4 and 5 help explain the emergence of ship-then-shop business models in certain markets. Our analysis suggests ship-then-shop models are likely to be profitable if the firm's AI capability is advanced, the search friction in the target market is severe, or the consumers' product match potential is large. By and large, these findings are consistent with real-world observations that the emergence of ship-then-shop businesses has been concentrated in markets characterized by high search friction and product match potential; apparel categories (e.g., Stitch Fix, Trunk Club, and Prime Try Before You Buy) or accessories like glasses and shoes (Warby Parker, SneakerTub), whose fashion trends evolve quickly such that consumers entail high search costs or high match uncertainty. These firms ship products either with minimal consumer input during the curation process, lowering consumers' time and effort required for preference estimation (i.e., enhancing the convenience effect) or even with substantial consumer input in the curation process (e.g., uploading clothing images, completing extensive preference questionnaires, and communicating with "personal shoppers"), providing high matching benefit capitalized on their AI capability.

Our analysis also informs managerial decision-making under ship-then-shop subscription model. A ship-then-shop firm that considers investing in AI capability should be mindful of the source of marginal return on investment. For instance, it should exercise caution before rushing to improve its AI capability (e.g., hiring data scientists), especially in markets characterized by high search friction: the firm should consider focusing on improving the convenience of the consumers' purchase process (e.g., by reducing the time and effort required for gathering consumer information for preference estimation). On the other hand, in product categories with large product match potential, it is in the firm's best interest to invest in AI capability improvements, which yield higher marginal returns than enhancing the convenience effect.

5 Extensions

In this section, we relax several assumptions imposed in our main model to show the generality and robustness of our main results. Specifically, we consider *(i)* unobservable product price case by relaxing the price-commitment assumption, *(ii)* a general $s_L > 0$ case by relaxing the normalization assumption of $s_L = 0$, and *(iii)* a more plausible case of retail procurement where a firm can procure products from the traditional market at a lower price than consumers do.

5.1 Unobservable Product Price

The main model assumes that the firm can commit to a product price that consumers can observe prior to their subscription decision (Jing, 2018; Mehra et al., 2018; Shin, 2007). While this is consistent with how a number of firms set prices in practice, there are cases in which firms do not price-commit. For example, firms that offer ship-then-shop may first collect subscription fee and then decide product price as they ship the products to their subscribers (e.g., through hidden fees, surcharges, etc.). In this section, we assess the robustness of our main insights to relaxing the price-commitment assumption. Specifically, we delay the firm's product price decision from Stage 1 to Stage 3, which is when consumers receive the product and decide whether to purchase the shipped product. All other model specifications remain unchanged.

Similar to the main model, consumers make subscription decisions based on subscription fee and product price. The key difference is that consumers cannot observe the *actual* price; instead, they consider the *expected* product price p_s^e. In Stage 3, the firm decides p_s taking into account p_s^e. Note that once consumers subscribe to ship-then-shop, their expected price is immaterial to the firm's profit. Conditional on consumer subscription, the firm's product pricing problem in Stage 3 is

$$\max_{p_s \le p_m + h} \frac{V - \bar{v}}{V(1-\alpha)}(p_s - p_m),$$

where $\bar{v} = \max\{\alpha V, p_s - h\}$ as in (8), and the constraint $p_s \le p_m + h$ prevents consumer showrooming. This yields

$$\tilde{p}_s^* = \min\left\{\max\left\{\alpha V + h, \frac{V + h + p_m}{2}\right\}, p_m + h\right\}. \tag{21}$$

In equilibrium, consumers' expectations align with the firm's optimal price. Therefore, $\tilde{p}_s^*$ in (21) is the equilibrium price in the scenario without price-commitment. Barring a minor linear transformation, $\tilde{p}_s^*$ is identical to $p_s^* = \min\{\max\{\alpha V + h, p_m\}, p_m + h\}$, the optimal price with price-commitment in the main model. In particular, all the comparative statics with respect to α, V, h, and p_m qualitatively carry over from the main model.

Furthermore, since the remaining model features are unchanged, the qualitative insights pertaining to consumers' subscription decisions and the firm's optimal fee sustain as well. For instance, the qualitative patterns of equilibrium price and fee patterns with respect to AI capability (α) are preserved. Figure 7 shows the impact of AI capability on price p_s and fee F without price commitment (compare with Figure 4).

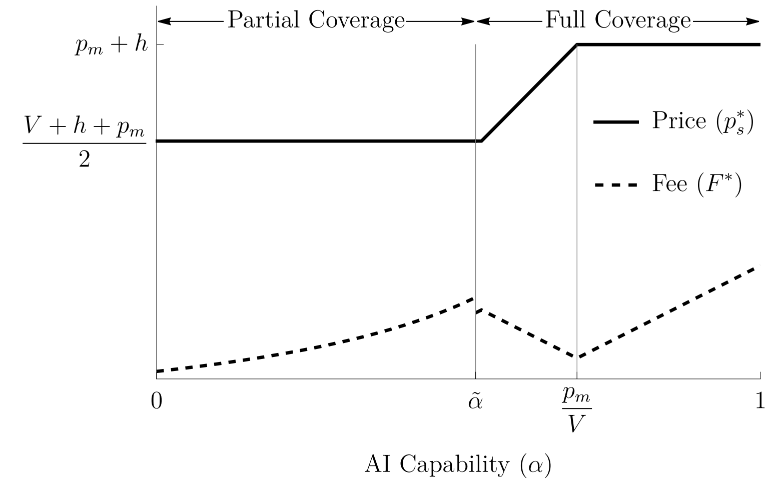

Figure 7: Impact of AI Capability on Price and Fee ($V = 1, p_m = 0.75, h = 0.5, s_H = 0.02$)

5.2 General $s_L \in (0, s_H)$

In this section, we show that normalizing s_L to 0 is indeed without loss of generality. It suffices to show that the insights under general $s_L \in (0, s_H)$ is qualitatively the same as that under $s_L = 0$ and some scaled $s_H > 0$. To that end, suppose $s_L \in (0, s_H)$.

First, from equation (13) and the fact that the optimal price p_s^* is independent of consumers' search costs, we obtain that $F^*_{\text{part}} = s_H + \xi_0$ and $F^*_{\text{full}} = s_L + \xi_0$ for some ξ_0 which is independent of s_L and s_H. Therefore, only departure from the normalized main model is that the optimal fee under full coverage is shifted upward by s_L. More generally, it follows that the qualitative insights pertaining to the dynamics of the optimal subscription fee and price with respect to the model primitives, in particular α, are preserved.

Second, since p_s^* is independent of coverage, the ship-then-shop firm's optimal profit can be compactly written as

$$\mathbb{E}[\pi^*] = \max\left\{\frac{1}{2}\left(s_H + \xi_1\right), s_L + \xi_1\right\},$$

for some ξ_1 which is independent of s_L and s_H. Therefore, the qualitative insights under general s_L can be mapped to the normalized main model as follows:

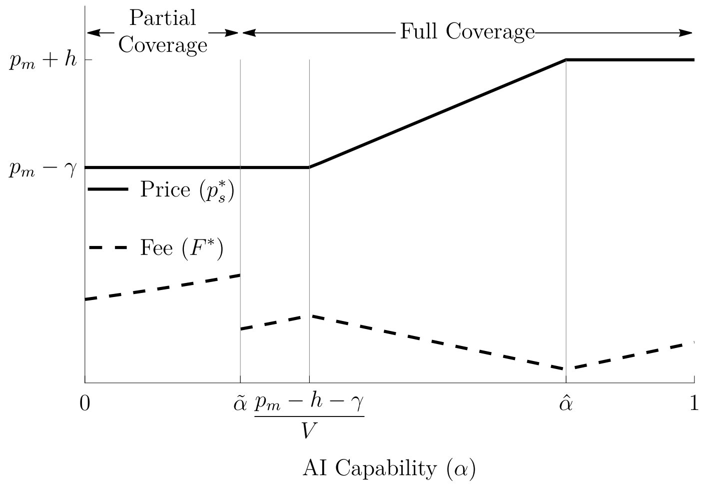

Figure 8: Price and Fee with Wholesale Price Discount ($V = 1, p_m = 0.5, h = s_H = \gamma = 0.1$)

1. if $0 < s_L < \frac{1}{2}s_H$, then re-define search costs $s'_L = 0$ and $s'_H = s_H - 2s_L$; and
2. if $\frac{1}{2}s_H < s_L < s_H$, then full coverage dominates partial coverage (because $\frac{1}{2}(s_H + \xi_1) < s_L + \xi_1$); this is qualitatively the same as the case with $s'_L = 0$ and $s'_H = \epsilon$ for some small $\epsilon > 0$.

5.3 Wholesale Price Discount

In the main model, we assumed that both the ship-then-shop firm and consumers face the same price at the traditional market. We relax this assumption and consider the possibility that the ship-then-shop firm may procure products from the traditional market at a lower price than do consumers (e.g., due to power in the distribution channel, volume discounts, etc.). To that end, suppose the firm can procure products at a price $p_m - \gamma$, for some $\gamma \in [0, p_m)$, whereas consumers purchase at price p_m.

Our analysis shows that the qualitative insights from the main model are robust to settings with lower wholesale prices (see Figure 8). An interesting quantitative departure from the main model is the change in subscription coverage. Specifically, the parametric region for which the firm induces both consumer segments to subscribe becomes larger as the wholesale cost decreases. The intuition is simple: lower wholesale cost implies higher margin from product sales, which in turn increases the returns from the matching effect. As a result, for larger γ, the firm shifts from partial to full coverage for smaller α threshold in order to capitalize on the matching effect from the subscribers.

6 Conclusion

Advances in AI technology, driven by machine learning techniques and data digitization, are fundamentally reshaping the business landscape. As improvements in AI algorithms enable firms to predict consumer preferences with greater accuracy, firms are adapting by reinventing their core business strategy. A notable example of such business transformation gaining traction in the retail sector is the ship-then-shop program. Unlike the traditional shopping model, which begins with consumer search and ends with product shipment, under the ship-then-shop model, the firm leverages its AI capability to predict consumers' preferences and ships the product to them; consumers then evaluate product fit and decide whether to purchase or return the product. In this paper, we develop a parsimonious game theory model that unveils nuanced economic forces underlying the ship-then-shop subscription model in the presence of consumer showrooming behaviors.

We show that the firm's optimal service fee and product price depend crucially on the trade-off between *ex-ante* and *ex-post* surplus extraction strategies. If AI prediction capability is sufficiently low, the firm capitalizes on the convenience effect (stemming from the reduction in consumer search cost): it raises the fee and lowers the price. This strategy emphasizes *ex-ante* surplus extraction. As the firm's AI capability improves and in the intermediate range, it starts exploiting the matching effect (stemming from superior product fit) by adopting the *ex-post* surplus extraction strategies of lowering the fee and raising the price. However, when the AI's predictive capability becomes sufficiently advanced, the firm reverts to *ex-ante* surplus extraction where the firm can extract surplus more from the higher fee rather than the product price due to the consumers' free-riding behaviors. The potential for consumers to showroom disciplines the firm by exerting downward pressure on price.

Moreover, we find that the ship-then-shop subscription model is most profitable *(i)* when AI capability is advanced, *(ii)* when the search friction in the market is severe, or *(iii)* when the product match potential is large. We also show that the marginal return of AI capability on the firm's profit decreases in search friction but increases in the product match potential. These insights provide important guidance for managers implementing the innovative subscription model. For example, investing in AI capability is more fruitful when the product match potential is large, as it enables the firm to better reap the upside gains from the matching effect under ship-then-shop.

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Appendix: Proofs

Lemma 1. Let consumers discount future payoffs by δ. Substituting $F^*_{\text{part}}(p_s)$ into (13) and differentiating with respect to p_s yields

$$\frac{\partial}{\partial p_s}\mathbb{E}[\pi_{\text{part}}] = \begin{cases} \frac{1-\delta}{2} & \text{if } p_s < \alpha V + h, \\ \frac{(1-\delta)h + p_m - (2-\delta)p_s + (1-\delta)V}{2V(1-\alpha)} & \text{if } \alpha V + h \leq p_s. \end{cases}$$

FOC, combined with the showroom-prevention constraint $p_s \leq p_m + h$, implies

$$p_s^* = \min\left\{p_m + h, \max\left\{\alpha V + h, V + h - \frac{V + h - p_m}{2-\delta}\right\}\right\}. \tag{22}$$

Setting $\delta \uparrow 1$ yields $p_s^* = \min\{p_m + h, \max\{\alpha V + h, p_m\}\}$. Therefore,

$$F^*_{\text{part}} = s_H + \begin{cases} \frac{1}{2}\left(\alpha V - \frac{p_m^2}{V}\right) - h & \text{if } p_m \leq \alpha V, \\ p_m - \frac{p_m^2}{2V} - \frac{\alpha V}{2} - h & \text{if } \alpha V < p_m \leq \alpha V + h, \\ \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)} & \text{if } \alpha V + h < p_m. \end{cases}$$

Under full coverage, F is lowered by s_H such that low-type consumers' IC constraint bind. ■

Proposition 1. It suffices to show $\frac{\partial}{\partial p_s}F^*_{\text{part}}(p_s) < 0$. If $p_s \leq V + h$,

$$\frac{\partial}{\partial p_s}F^*_{\text{part}}(p_s) = \begin{cases} -1, & p_s \leq \alpha V + h, \\ -\frac{V + h - p_s}{V(1-\alpha)}, & p_s > \alpha V + h \end{cases} < 0.$$

■

Proposition 2 . First, the difference $\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}]$ is increasing in α because

$$\frac{\partial\left(\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}]\right)}{\partial \alpha} = \frac{1}{4} \cdot \begin{cases} \frac{(V+h-p_m)^2}{4V(1-\alpha)^2} & \text{if } \alpha \leq \frac{p_m - h}{V} \\ V & \text{if } \frac{p_m-h}{V} < \alpha \end{cases} > 0. \tag{23}$$

Second, $\left(\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}]\right)|_{\alpha=0} < 0$ and $\lim_{\alpha \uparrow 1}\left(\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}]\right) > 0$, because

$$\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}]|_{\alpha=0} = \frac{h^2 - 2hp_m - 2s_H V}{4V} \leq \frac{h^2 - 2hp_m - 2s_H V}{4V}|_{h=0} = -\frac{s_H}{2} < 0$$

and

$$\lim_{\alpha \uparrow 1} \mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] = \frac{V^2 - 2s_H V - p_m^2}{4V} \geq \frac{V^2 - 2s_H V - p_m^2}{4V}\Big|_{s_H = \frac{(V-p_m)^2}{2V}} = \frac{(V-p_m)p_m}{2V} > 0.$$

Finally, Intermediate Value Theorem (IVT) ensures unique existence of $\tilde{\alpha} \equiv \{\alpha \in (0,1) : \mathbb{E}[\pi_{\text{full}}] = \mathbb{E}[\pi_{\text{part}}]\}$, such that $\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] \geq 0 \Leftrightarrow \alpha \geq \tilde{\alpha}$. Next, we show $\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] \geq 0$ is equivalent to s_H being small. First, algebraic manipulations yield

$$\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] \geq 0 \Leftrightarrow \frac{1}{2}\left(F^*_{\text{part}}(p^*_s) + \int_{\bar{v}}^{V} \frac{p^*_s - p_m}{V(1-\alpha)} dv_s\right) - s_H \geq 0.$$

Note

$$\frac{\partial}{\partial s_H}\left(\frac{1}{2}\left(F^*_{\text{part}}(p^*_s) + \int_{\bar{v}}^{V} \frac{p^*_s - p_m}{V(1-\alpha)} dv_s\right) - s_H\right) = -\frac{1}{2}.$$

Second, if $s_H \downarrow 0$, then $\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] \geq 0$ because increasing F infinitesimally and excluding s_L-consumers is unprofitable. Let

$$\xi \equiv \frac{1}{2}\left(F^*_{\text{part}}(p^*_s) + \int_{\bar{v}}^{V} \frac{p^*_s - p_m}{V(1-\alpha)dv}\right) - s_H\Big|_{s_H = \frac{(V-p_m)^2}{2V}}.$$

If $\xi \geq 0$, full coverage dominates partial coverage for all s_H. If $\xi < 0$, IVT ensures unique existence of $\hat{s} \in \left(0, \frac{(V-p_m)^2}{2V}\right)$ such that $\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] \geq 0$ iff $s_H \leq \hat{s}$. For general ξ, we have

$$\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] \geq 0 \Leftrightarrow \tilde{s} \equiv \min\left\{\hat{s}, \frac{(V-p_m)^2}{2V}\right\}.$$

■

Proposition 3. Comparative statics for p_s^* is trivial and omitted. Note

$$\frac{\partial F^*}{\partial \alpha} = \begin{cases} \frac{(V+h-p_m)^2}{2V(1-\alpha)^2} & \text{if } \alpha \leq \frac{p_m-h}{V}, \\ -\frac{V}{2} & \text{if } \frac{p_m-h}{V} < \alpha \leq \frac{p_m}{V}, \\ \frac{V}{2} & \text{if } \frac{p_m}{V} < \alpha. \end{cases}$$

At $\alpha = \tilde{\alpha}$, coverage shifts from partial to full, such that F^* decreases by magnitude s_H.

F^* is independent of s_H under full coverage. Under partial coverage, $F^* = s_H + \zeta$, where ζ is independent of s_H. Proposition 2 implies full coverage for $s_H \leq \tilde{s}$ and partial coverage for $s_H > \tilde{s}$. Therefore, F^* is independent of s_H for $s_H \leq \tilde{s}$, increases by s_H at $s = \tilde{s}$, and increases for $s_H > \tilde{s}$.

With respect to h, we have

$$\frac{\partial F^*}{\partial h} = \begin{cases} -\frac{p_m-h-\alpha V}{V(1-\alpha)} & \text{if } \alpha \leq \frac{p_m-h}{V}, \\ -1 & \text{if } \frac{p_m-h}{V} < \alpha \end{cases} < 0.$$

■

Proposition 4. The profit change with respect to α is as follows. If $\alpha \leq \frac{p_m-h}{V}$, then

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} = \frac{(V+h-p_m)^2}{2V(1-\alpha)^2} \cdot \begin{cases} 1, & s_H \leq \frac{h^2-2h(p_m-\alpha V)+\alpha(V-p_m)^2}{2V(1-\alpha)}, \\ \frac{1}{2}, & s_H > \frac{h^2-2h(p_m-\alpha V)+\alpha(V-p_m)^2}{2V(1-\alpha)} \end{cases} > 0; \tag{24}$$

and if $\frac{p_m-h}{V} < \alpha$, then

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} = \frac{V}{2} \cdot \begin{cases} 1 & \text{if } s_H \leq \frac{\alpha V^2 - p_m^2}{2V}, \\ \frac{1}{2} & \text{if } s_H > \frac{\alpha V^2 - p_m^2}{2V} \end{cases} > 0. \tag{25}$$

Since $\mathbb{E}[\pi^*]$ is continuous in α, this suffices to establish that $\mathbb{E}[\pi^*]$ increases in α.

The profit change with respect to s_H is as follows. If $\alpha \leq \frac{p_m-h}{V}$, then

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \begin{cases} 0 & \text{if } s_H \leq \frac{h^2-2h(p_m-\alpha V)+\alpha(V-p_m)^2}{2V(1-\alpha)}, \\ \frac{1}{2} & \text{if } s_H > \frac{h^2-2h(p_m-\alpha V)+\alpha(V-p_m)^2}{2V(1-\alpha)} \end{cases} \geq 0; \tag{26}$$

and if $\frac{p_m-h}{V} < \alpha$, then

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \begin{cases} 0 & \text{if } s_H \leq \frac{\alpha V^2 - p_m^2}{2V}, \\ \frac{1}{2} & \text{if } s_H > \frac{\alpha V^2 - p_m^2}{2V} \end{cases} \geq 0. \tag{27}$$

Since $\mathbb{E}[\pi^*]$ is continuous in s_H, this suffices to establish that $\mathbb{E}[\pi^*]$ increases in s_H.

The profit change with respect to V is as follows. If $\alpha \leq \frac{p_m-h}{V}$, then

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial V} = \frac{2hp_m - h^2 + \alpha(V - p_m^2)}{2V^2(1-\alpha)} \cdot \begin{cases} \frac{1}{2} & \text{if } s_H \leq \frac{h^2-2h(p_m-\alpha V)+\alpha(p_m-V)^2}{2V(1-\alpha)}, \\ 1 & \text{if } s_H > \frac{h^2-2h(p_m-\alpha V)+\alpha(p_m-V)^2}{2V(1-\alpha)}. \end{cases}$$

Note that the numerator $2hp_m - h^2 + \alpha(V - p_m^2)$ is positive because

$$\frac{\partial}{\partial h}\left(2hp_m - h^2 + \alpha(V - p_m^2)\right) = 2(p_m - h) > 0,$$

which implies that

$$2hp_m - h^2 + \alpha(V - p_m^2) \geq 2hp_m - h^2 + \alpha(V - p_m^2)|_{h=0} = \alpha(V - p_m^2) > 0.$$

Therefore, $\frac{\partial \mathbb{E}[\pi^*]}{\partial V} > 0$.

Finally, if $\frac{p_m - h}{V} < \alpha$,

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial V} = \frac{1}{2}\left(\alpha + \frac{p_m^2}{V^2}\right) \cdot \begin{cases} \frac{1}{2} & \text{if } s_H \le \frac{\alpha V^2 - p_m^2}{2V}, \\ 1 & \text{if } s_H > \frac{\alpha V^2 - p_m^2}{2V} \end{cases} > 0.$$

∎

Proposition 5. From (24) and (25), we obtain that the value of $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ if s_H is greater than the respective thresholds in (24) and (25) is half that if s_H is less than the thresholds. Therefore, the derivative $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ decreases in s_H. Next, $\frac{\partial}{\partial V}\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} > 0$ because $\frac{V}{2}$ and $\frac{(V+h-p_m)^2}{2V(1-\alpha)^2}$ both increase in V. Moreover, $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ increases at $V = \frac{p_m - h}{\alpha}$ due to Claim 1. Excluding discontinuities, $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}$ is independent of V. Finally, $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}$ decreases at discontinuities due to Claim 2. ∎

Online Appendix: Statements and Proofs of Claims

Claim 1. $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ *is non-decreasing in V at $V = \frac{p_m - h}{\alpha}$.*

Proof of Claim 1. At $V = \frac{p_m - h}{\alpha}$, we have $\frac{V}{2} = \frac{(V+h-p_m)^2}{2V(1-\alpha)^2}$. Now, we obtain from equations (24) and (25) in the main text that $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ is decreasing in V at $V = \frac{p_m - h}{\alpha}$ only if $s_H \leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)}$ for $V < \frac{p_m - h}{\alpha}$ and $s_H > \frac{\alpha V^2 - p_m^2}{2V}$ for $V > \frac{p_m - h}{\alpha}$. Due to Claim 3 (see below), these conditions imply the conditions (A1) and (A2) (see below). However, the proof of Case (iii) in Claim 2 (see below) shows that (A1) and (A2) cannot jointly hold. This proves that $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ cannot decrease in V at $V = \frac{p_m - h}{\alpha}$. ■

Claim 2. $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}$ *decreases at discontinuities with respect to V.*

Proof of Claim 2. From equations (26) and (27) in the main text, there are three discontinuities to consider: V at which (i) $s_H = \frac{\alpha V^2 - p_m^2}{2V}$, (ii) $s_H = \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)}$, and (iii) $p_m \leq \alpha V + h$.

Consider Case (i), which applies to $p_m \leq \alpha V + h$, or equivalently $V \geq \frac{p_m - h}{\alpha}$. Due to Claim 3 (see below), if $s_H \leq \frac{\alpha V^2 - p_m^2}{2V}$, then $V \geq \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}$. Therefore, if $p_m \leq \alpha V + h$, or equivalently $V \geq \frac{p_m - h}{\alpha}$, then

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \begin{cases} 0 & \text{if } V \geq \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}, \\ \frac{1}{2} & \text{if } V < \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}. \end{cases}$$

Consider Case (ii), which applies to $p_m > \alpha V + h$, or equivalently $V < \frac{p_m - h}{\alpha}$. Due to Claim 3 (see below), the inequality $s_H \leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)}$ is equivalent to

$$V \geq \frac{s - \alpha(h - p_m + s) + \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h - 2p_m + s)\right)}}{\alpha}.$$

Therefore, if $p_m > \alpha V + h$, or if $V < \frac{p_m - h}{\alpha}$, then

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \begin{cases} 0 & \text{if } V \geq \frac{s-\alpha(h-p_m+s)+\sqrt{(1-\alpha)(s^2-\alpha(h+s)(h-2p_m+s))}}{\alpha}, \\ \frac{1}{2} & \text{if } V < \frac{s-\alpha(h-p_m+s)+\sqrt{(1-\alpha)(s^2-\alpha(h+s)(h-2p_m+s))}}{\alpha}. \end{cases}$$

Consider Case (iii). $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}$ increases at discontinuity only if $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = 0$ for $p_m > \alpha V + h$ and $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \frac{1}{2}$ for $p_m < \alpha V + h$. This requires:

$$\frac{s - \alpha(h - p_m + s) + \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h - 2p_m + s)\right)}s}{\alpha} < \frac{p_m - h}{\alpha} \tag{A1}$$

and

$$\frac{p_m - h}{\alpha} < \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}, \tag{A2}$$

which simplify to

$$\frac{h(h+2s)}{2(h+s)} < p_m < \frac{h+s}{2} \text{ and } \alpha > \frac{s^2}{(h+s)(h-2p_m+s)}. \tag{A3}$$

Since $\frac{\partial}{\partial p_m} \frac{s^2}{(h+s)(h-2p_m+s)} > 0$, (A3) holds only if the inequality $\alpha > \frac{s^2}{(h+s)(h-2p_m+s)}$ holds for smallest value of p_m, which under condition $\frac{h(h+2s)}{2(h+s)} < p_m < \frac{h+s}{2}$ is $p_m = \frac{h(h+2s)}{2(h+s)}$. But

$$\frac{s^2}{(h+s)(h - 2\frac{h(h+2s)}{2(h+s)} + s)} = 1,$$

which contradicts $\alpha < 1$. ■

Claim 3. $p_m \leq V$ *implies the following two equivalences:*

$$s_H \leq \frac{\alpha V^2 - p_m^2}{2V} \Leftrightarrow V \geq V_1 \text{ and } s_H \leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)} \Leftrightarrow V \geq V_2,$$

where

$$V_1 \equiv \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}$$

and

$$V_2 \equiv \frac{s - \alpha(h - p_m + s) + \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h - 2p_m + s)\right)}}{\alpha}.$$

Proof of Claim 3. For the first equivalence, note that $s_H \leq \frac{\alpha V^2 - p_m^2}{2V}$ can be rearranged in terms of V to the condition that either $V \leq \frac{s - \sqrt{s^2 - \alpha p_m^2}}{\alpha}$ or $V \geq \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}$. We show that the first condition cannot hold if $p_m \leq V$. Since $p_m \leq V$, we have

$$V \leq \frac{s - \sqrt{s^2 - \alpha p_m^2}}{\alpha} \Rightarrow p_m \leq \frac{s - \sqrt{s^2 - \alpha p_m^2}}{\alpha} \Rightarrow -2s_H \geq p_m(1-\alpha),$$

which is impossible since s_H and p_m are positive and $\alpha < 1$.

For the second equivalence, note that $s_H \leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)}$ can be rearranged in terms of V to the condition that either $V \leq \frac{s - \alpha(h - p_m + s) - \sqrt{(1-\alpha)(s^2 - \alpha(h+s)(h-2p_m+s))}}{\alpha}$ or $V \geq \frac{s - \alpha(h - p_m + s) + \sqrt{(1-\alpha)(s^2 - \alpha(h+s)(h-2p_m+s))}}{\alpha}$. We show that the first condition cannot hold if $p_m \leq V$. Since $p_m \leq V$, we have $V \leq \frac{s - \alpha(h - p_m + s) - \sqrt{(1-\alpha)(s^2 - \alpha(h+s)(h-2p_m+s))}}{\alpha}$ implies

$$p_m \leq \frac{s - \alpha(h - p_m + s) - \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h - 2p_m + s)\right)}}{\alpha},$$

which simplifies to

$$s(1-\alpha) - \alpha h - \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h - 2p_m + s)\right)} \geq 0. \tag{A4}$$

The left-hand side of (A4) is decreasing in h due to Claim 4 (see below); therefore, the inequality (A4) must hold at $h = 0$. But at $h = 0$, the left-hand side of (A4) reduces to

$$(1-\alpha)s - \sqrt{(1-\alpha)s(2\alpha p_m + (1-\alpha)s)},$$

which is less than 0, because

$$\sqrt{(1-\alpha)s(2\alpha p_m + (1-\alpha)s)} \geq \sqrt{(1-\alpha)s(0 + (1-\alpha)s)} = (1-\alpha)s.$$

Therefore, (A4) does not hold for any $h \geq 0$. ■

Claim 4. *If $\alpha \leq 1$, $0 \leq h \leq p_m$ and $s \geq 0$, then*

$$\frac{\partial}{\partial h}\left(s(1-\alpha) - \alpha h - \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h-2p_m+s)\right)}\right) \leq 0. \tag{A5}$$

Proof of Claim 4. Writing out the derivative and simplifying yields

$$\text{(A5)} \Leftrightarrow \sqrt{(1-\alpha)\left(\alpha(h+s)(2p_m-h-s)+s^2\right)} > (1-\alpha)(-(p_m-h-s)).$$

First, suppose $p_m \geq h+s$ such that $(1-\alpha)(-(p_m-h-s))$ is negative. Since

$$\sqrt{(1-\alpha)\left(\alpha(h+s)(2p_m-h-s)+s^2\right)} > 0,$$

the inequality holds. Second, suppose $p_m < h+s$. Then

$$\frac{\partial(h+s)(2p_m-h-s)}{\partial h} = -2(h+s-p_m) < 0 \text{ and } \frac{\partial(1-\alpha)(-(p_m-h-s))}{\partial h} = 1-\alpha > 0,$$

such that $\sqrt{(1-\alpha)\left(\alpha(h+s)(2p_m-h-s)+s^2\right)}$ is decreasing in h while $(1-\alpha)(-(p_m - h-s))$ is increasing in h. Therefore, to show that $\sqrt{(1-\alpha)\left(\alpha(h+s)(2p_m-h-s)+s^2\right)} > (1-\alpha)(-(p_m-h-s))$, it suffices to show that the inequality holds for the largest value of h in the interval $[0, p_m(1-p_m/V)]$ (see Footnote 6). In fact, we can show that the inequality holds for a value greater than the upperbound: $h = p_m$. Substituting $h = p_m$ into the inequality and simplifying yields

$$\sqrt{(1-\alpha)\left(\alpha(p_m - s)(p_m + s) + s^2\right)} > (1-\alpha)s.$$

Since $\sqrt{(1-\alpha)\left(\alpha(p_m - s)(p_m + s) + s^2\right)}$ is increasing in p_m, we obtain

$$\sqrt{(1-\alpha)\left(\alpha(p_m - s)(p_m + s) + s^2\right)} > \sqrt{(1-\alpha)\left(\alpha(0 - s)(0 + s) + s^2\right)} = (1-\alpha)s.$$

This completes the proof. ■