AI-Driven Retail Transformation: Shop-then-ship or Ship-then-shop?

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Abstract

This paper studies an emerging subscription model called ship-then-shop. Leveraging its artificial intelligence (AI) prediction machine, the firm curates and ships a product to the consumer, after which the consumer shops (i.e., evaluate product fit and make purchase decision). The consumer first pays the upfront ship-then-shop subscription fee prior to observing product fit, and then pays the product price if she decides to purchase after realizing product fit. We analyze how the firm balances the subscription fee and product price to maximize its profit, and how AI prediction capability affects its strategies and profit. We find that as AI capability increases, the firm's strategy shifts from high-fee-low-price to low-fee-high-price; i.e., its strategy shifts from *ex-ante* surplus extraction through subscription fee to *ex-post* surplus extraction through product price. Our model generates rich insights regarding AI capability, search friction, and their interactions on the profitability of the ship-then-shop model.

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The single most common question corporate executives ask us is: "How will AI affect our business strategy?"

—Agrawal et al. (2018)

1 Introduction

AI (artificial intelligence) is transforming operations in various industries ranging from healthcare and education to transportation and retail (The Economist, 2021). Experts tout AI as the next technological revolution on par with the steam engine (Knowles, 2021; The Economist, 2016). In their thought-provoking book "Prediction Machines," Agrawal et al. (2018) discuss AI's potential to transform firms' business models. The authors posit that reductions in prediction costs will not just enhance productivity, but motivate firms to qualitatively reinvent their core business strategy. Retail is one of the several industries at such inflection point. For example, Agrawal et al. (2018) present an interesting thought experiment in which they predict the emergence of an innovative AI-driven retail strategy called *ship-then-shop* subscription service.

Traditionally, the shopping process starts with consumer search. The consumer searches for product information, browses offerings, and evaluates product fit. If the consumer purchases, the firm ships the product and the shopping process terminates. In contrast, under the novel ship-then-shop model, the shopping process begins with product shipment. The firm leverages the prediction machine to predetermine products that match the consumer's taste and ships the product to her. The consumer then evaluates product fit, and decides whether to purchase or return the product (see Figure 1). The critical determinant of the ship-then-shop model's success is a sophisticated prediction machine. Only with sufficiently high prediction accuracy would the firm's gains from predictive deliveries outweigh the loss from returns of mismatched products.

Until recently, the idea of predictive shipping has been dismissed by critics as hype

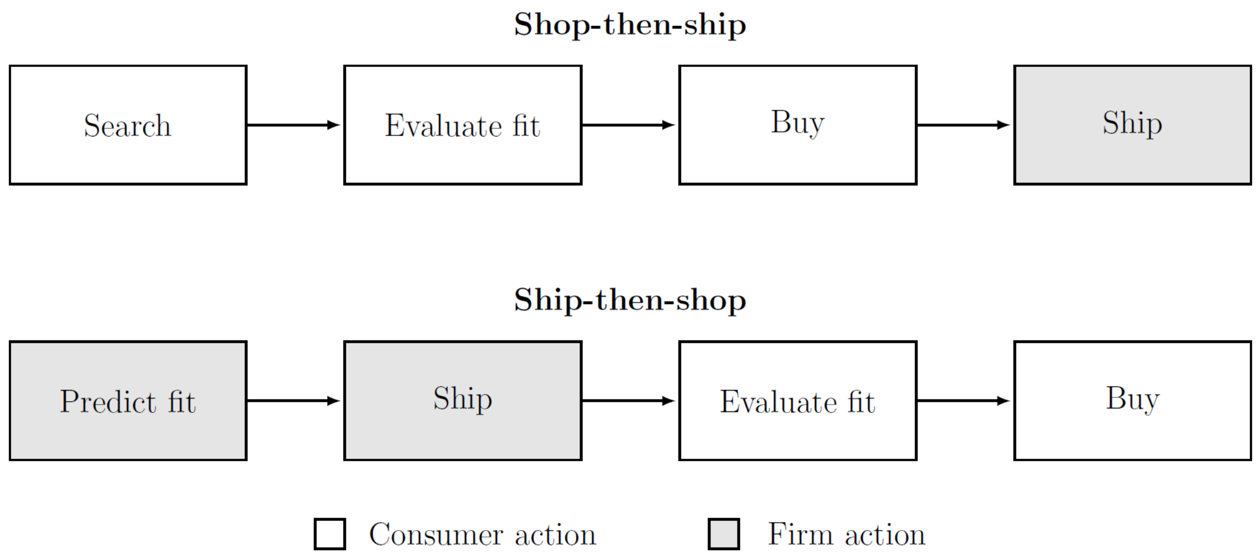

Figure 1: Shop-then-ship vs. Ship-then-shop

(Banker, 2014; DePuy, 2014). However, the emergence of ship-then-shop subscription business models, notably in the beauty, food, and apparel retail sectors (Chen, 2019; McKinsey & Co., 2018; Moore, 2020), suggests Agrawal et al. (2016)'s prediction is steadily unfolding in reality. The "best exemplar" of the ship-then-shop subscription provider is the apparel company Stitch Fix (Sinha et al., 2016). Stitch Fix leverages AI algorithms to predict consumers' style preferences and ships personalized clothing items. Consumers try on the clothes and decide whether to purchase or return the products. In the food category, HelloFresh offers a similar subscription model, whereby the company analyzes consumers' culinary preferences and curates cooking subscription boxes (Hallgrimsdottir, 2018).

Despite the increasing adoption of ship-then-shop models, little is understood about their economics. A unique feature of ship-then-shop models is the separation of payments before and after the consumer learns product match. The consumer first pays the upfront subscription fee prior to observing product fit, and then conditional on subscription, makes product purchase decision after observing product fit. Relative to the traditional shop-then-ship approach, ship-then-shop thus provides two benefits to consumers: superior product match (matching effect) and search cost reduction (convenience effect). In making their subscription decision, consumers weigh the benefits of the matching effect (which increases consumers' *ex-post* product valuation) and the convenience effect (which increases consumers' *ex-ante* valuation of ship-then-shop program) against the costs of the subscription fee and product price. The firm jointly optimizes two revenue channels: subscription to ship-then-shop and product sales; it balances *ex-ante* and *ex-post* surplus extraction opportunities to maximize its profit. This paper develops a parsimonious theoretical framework to elucidate the key economic forces that shape the firm's strategies under the ship-then-shop model. Moreover, we discuss how the firm's optimal strategies and profit vary with advances in AI technology under different market conditions.

The central finding of the paper is that the firm's optimal strategy depends crucially on the trade-off between *ex-ante* vs. *ex-post* surplus extraction. As AI capability increases, such that the expected product match value increases, the firm lowers the subscription fee and raises the product price; i.e., it shifts from *ex-ante* to *ex-post* surplus extraction strategy. The intuition revolves around the interplay of the two benefits of ship-then-shop subscription: matching effect and convenience effect. If the AI capability is low, the matching effect is correspondingly low such that the firm sets a high subscription fee to extract the consumers' *ex-ante* surplus generated by the convenience effect. This low-price-high-fee strategy is qualitatively similar to the standard two-part tariff solution. On the other hand, if the AI capability is high, the matching effect dominates the convenience effect. The firm lowers the subscription fee to entice consumers to subscribe, and then through high price extracts *ex-post* surplus generated by the matching effect. Interestingly, if the AI capability is sufficiently advanced, the firm offers a negative fee (i.e., "sign up bonus"), which motivates consumers with low search cost to subscribe.

We characterize the conditions under which the ship-then-shop model is most profitable. We find that the firm's profit increases in *(i)* its AI capability, *(ii)* the degree of search friction in the market, and *(iii)* the consumers' product valuation heterogeneity. Intuitively, consumers' valuations of the matching effect and convenience effect increase in the AI capability and search friction. Also, greater product match heterogeneity increases the upside potential for the ship-then-shop's matching effect such that the firm's profit increases. We further show that the marginal return of AI capability on the firm's profit decreases in search friction, but increases in the heterogeneity in consumers' valuation. The negative interaction between matching and convenience effects provides important managerial insights. For instance, if the firm operates in a market characterized by high search friction, its primary revenue source is the convenience effect, such that improving its AI capability yields low marginal return. In such cases, the firm should focus more on improving the convenience effect rather than improving the matching effect through investments in AI tools. On the other hand, if consumers' product match heterogeneity is large, it is in the firm's best interest to invest in improving its AI capability, which yields a higher marginal return than enhancing the convenience effect.

The literature on the economics of AI explores the impact of AI on a wide range of areas such as the labor market, innovation and economic policy (e.g., Acemoglu and Restrepo, 2020; Alekseeva et al., 2021; Furman and Seamans, 2019; Korinek and Stiglitz, 2020).[1] The marketing literature largely focuses on marketing applications of AI technology such as recommender system and content personalization (e.g., Ansari et al., 2018; Hauser et al., 2009; Yoganarasimhan, 2020). In line with this strand of literature, our paper studies the strategic implications of AI-based product recommendation under the ship-then-shop subscription program.

2 Model

There are two players: a monopolist firm and a unit mass of consumers. The firm offers ship-then-shop subscription, whereby it leverages AI technology to predict consumers' product preferences and ships the product to the consumers. Consumers purchase one unit of the

[1] Goldfarb et al. (2019) provide a comprehensive overview of the implications of AI on the practice of economics, and discuss the microeconomic impacts of AI adoption on firms' strategies and profits.

product through one of two shopping methods. They can either purchase in the traditional market through their own search efforts, or they can subscribe to the ship-then-shop service. Importantly, the two shopping methods result in different product match qualities which we elaborate below.

Firm

The firm offers ship-then-shop subscription service and makes two decisions: it sets the subscription fee F and product price p_s, where subscript s denotes *subscription*. We assume that the firm procures products from the market at price p_m, where subscript m denotes *market*. The firm then resells the best-matching products to ship-then-shop subscribers at price $p_s = p_m + \kappa$, where κ is the firm's profit margin or premium it can charge for its ship-then-shop service. Therefore, the firm's profit consists of two revenue sources, ship-then-shop subscription and product sales:

$$\mathbb{E}[\pi] = N_s \left(F + D_p \kappa\right), \tag{1}$$

where N_s denotes the demand for ship-then-shop subscription, F the subscription fee, D_p the demand for the shipped product, and κ the product sales margin.

Consumers

Consumers make two sequential decisions: subscription and product purchase. After observing the subscription fee F and product price p_s, consumers decide whether to subscribe to ship-then-shop or search in the traditional market.[2] Depending on their choice of shopping method, consumers face different product match value distributions.

If consumers search in the traditional market, they incur search cost $s \in \{s_L, s_H\}$ and

[2] In Section A1 of the Online Appendix, we analyze a scenario in which p_s is unobservable to consumers before they receive a product. We show that the qualitative insights carry over.

find a product with match value

$$v_m \sim U[0, V], \tag{2}$$

where V denotes the maximum attainable match value — it can also be interpreted as the product match heterogeneity in a given market. Consumers then decide whether to purchase the product at price $p_m \in [0, V]$.

On the other hand, if consumers subscribe to ship-then-shop and receive the shipped product, consumers realize product match value

$$v_s \sim U[\alpha V, V], \tag{3}$$

where $\alpha \in [0, 1]$ captures the firm's AI capability or matching quality. To illustrate the role of α, if $\alpha = 0$, then the firm's match prediction capability is no better than the consumer's own ability to identify product matches through her own search. On the other hand, if $\alpha = 1$, then the firm perfectly identifies and ships the consumer's ideal product, in which case the consumer obtains the maximum match value V. Thus, AI capability shifts the consumers' product match value distribution upwards — we call this the *matching effect* of ship-then-shop. Consumers then decide whether to purchase the product at price p_s or return it at hassle cost h, which is not too large.[3]

While consumers observe F and p_s, the product match values v_s and v_m are *a priori* unknown. Consumers observe v_s under ship-then-shop subscription only upon receiving the product, and v_m under the traditional shopping only after product search.

Consumer utility consists of two components: product consumption utility and product match value. Product consumption utility is common across all products in the same category, whereas product matching value captures heterogeneous consumer preferences in

[3]We assume that $h < p_m (1 - p_m/V)$, which ensures the equilibrium fee is not always negative.

a given product category. In total, the consumer's utility is

$$u = u_0 + v, \tag{4}$$

where u_0 is the product consumption utility, and v the product match value. We normalize u_0 to zero without loss of generality. The consumer's product match value v depends on her choice of shopping method. If she subscribes to ship-then-shop after paying fee F, her product match value v_s is drawn from $U[\alpha V, V]$. If she searches in the traditional market, her product match value v_m is drawn from $U[0, V]$.

The consumer's net utility from subscribing to ship-then-shop is

$$u_s = -F + \delta \cdot \begin{cases} v_s - p_s & \text{if purchase,} \\ 0 - h & \text{if return,} \end{cases} \tag{5}$$

where $\delta \in (0, 1)$ is the discount factor, capturing the delayed product consumption under ship-then-shop. For ease of exposition, we hereafter set $\delta \uparrow 1$.

On the other hand, the consumer's net utility in the traditional market is

$$u_m = -s + \begin{cases} v_m - p_m & \text{if purchase,} \\ 0 & \text{if not purchase,} \end{cases} \tag{6}$$

where $s \in \{s_L, s_H\}$ is the heterogeneous search cost. Consumers are low-type ($s = s_H$) or high-type ($s = s_H$) with equal probability. We normalize s_L to zero without loss of generality; i.e., $0 = s_L < s_H$. Moreover, to focus on the more interesting case where all consumers may search in the traditional market, we assume that $s_H \leq (V - p_m)^2 / 2V$.[4] It is important to note that consumer's utility from ship-then-shop subscription in (5) does

[4]If $s_H > (V - p_m)^2 / 2V$, the game degenerates to a trivial case where the high-type consumers do not consider buying in the traditional market.

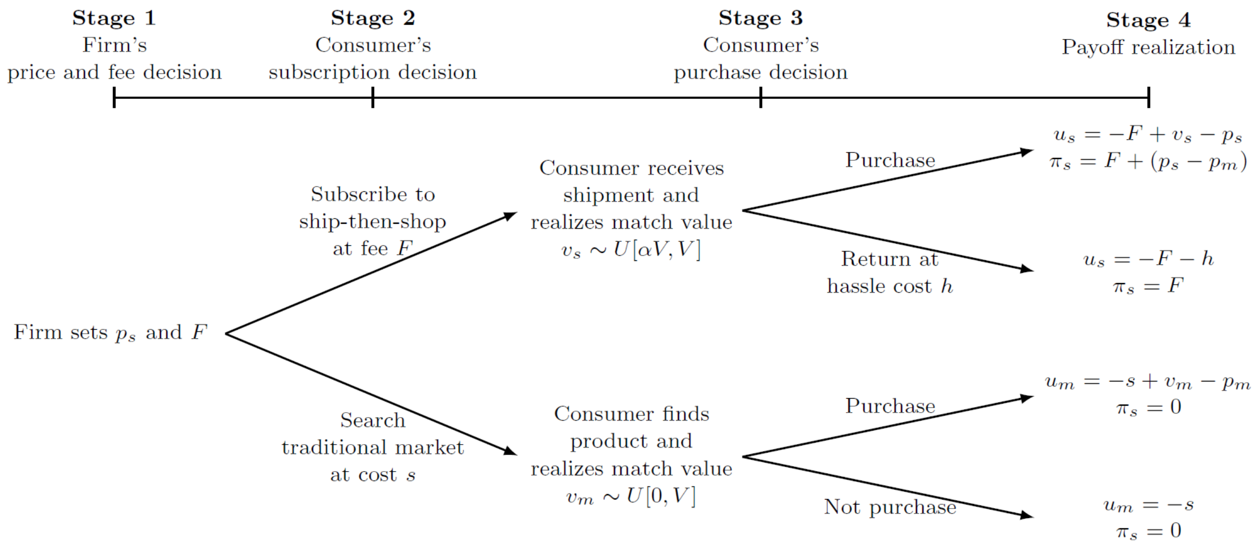

Figure 2: Game Sequence

not contain search cost s, while her utility from the traditional market in (6) does. That is, ship-then-shop subscription facilitates shopping by saving consumers' search cost. We call this the *convenience effect* of ship-then-shop.

Overall, in making their subscription decision, consumers weigh the potential gains from the matching effect (which increases consumers' *ex-post* product valuation) and the convenience effect (which increases consumers' *ex-ante* valuation of ship-then-shop program) against the cost of the subscription fee. The game sequence is summarized in Figure 2.

3 Analysis

We solve for subgame perfect Nash equilibrium using backward induction.

3.1 Consumer Decision

Consumers' decisions are two-fold: ship-then-shop subscription and product purchase. Conditional on subscribing to ship-then-shop, consumers purchase the shipped product if and only if the realized match value net of price exceeds the disutility they incur from returning the product: $v_s - p_s \geq -h$. Note that if $p_s - h < \alpha V$, ship-then-shop subscribers always buy. Therefore, the marginal consumer who purchases is

$$\bar{v} \equiv \max\left[\alpha V, p_s - h\right]. \tag{7}$$

Consumers also decide whether to subscribe to ship-then-shop or search in the traditional market. Consumers' expected utility from subscribing to ship-then-shop is

$$\mathbb{E}[u_s] \quad = \quad -F + \left(\int_{\alpha V}^{\bar{v}} \frac{-h}{V(1-\alpha)} dv + \int_{\bar{v}}^{V} \frac{v - p_s}{V(1-\alpha)} dv \right) \tag{8}$$

The first term denotes the subscription fee, and the second term in brackets the expected utility from either returning or purchasing the shipped product.

If consumers search in the traditional market at search cost $s \in \{0, s_H\}$, their expected utility is

$$\mathbb{E}[u_m] = -s + \int_{p_m}^{V} \frac{v - p_m}{V} dv = -s + \frac{(V - p_m)^2}{2V}. \tag{9}$$

Consumers compare their expected utility from ship-then-shop subscription in (8) and that from traditional market in (9). Let $\bar{s}$ denote the search cost for which consumers are indifferent between the two shopping options. Solving $\mathbb{E}[u_s] = \mathbb{E}[u_m]$ yields

$$\bar{s} = F - \left(\frac{(V(2\alpha h + V) - \bar{v}(2h + \bar{v}) - 2p_s(V - \bar{v}))}{2V(1-\alpha)} - \frac{(V - p_m)^2}{2V} \right). \tag{10}$$

While consumers with search cost $s > \bar{s}$ subscribe to ship-then-shop, those with search cost $s \leq \bar{s}$ search in the traditional market. Specifically,

1. if $\bar{s} \leq 0$, then all consumers subscribe;
2. if $0 < \bar{s} \leq s_H$, then high-type consumers ($s = s_H$) subscribe, while low-type consumers ($s = 0$) choose the traditional market; and

3. if $s_H < \bar{s}$, then all consumers choose the traditional market.

3.2 Firm Decision

The firm sets product price and subscription fee in anticipation of consumers' subscription and purchase decisions. The firm's expected profit is

$$\mathbb{E}[\pi(p_s, F)] = N_s(p_s, F)\Big(\underbrace{F}_{\substack{\textit{ex-ante}\\ \text{surplus}}} + \underbrace{\frac{V - \bar{v}}{V(1-\alpha)}(p_s - p_m)}_{\substack{\textit{ex-post}\\ \text{surplus}}}\Big), \tag{11}$$

where $N_s(p_s, F)$ denotes the demand for ship-then-shop subscription, F the subscription fee, and the last term the expected margin from product sales. $\frac{V-\bar{v}}{V(1-\alpha)}$ is the probability that a ship-then-shop subscriber purchases the shipped product. The firm procures the product at market price p_m and then resells it at price p_s.

The firm's profit in (11) reveals two channels through which the firm extracts consumer surplus. The firm balances *ex-ante* surplus extraction through subscription fee F, and *ex-post* surplus extraction through product price p_s.

The firm determines $N_s(p_s, F)$ by adjusting p_s and F. Given the binary search cost space (i.e., $s \in \{0, s_H\}$), the firm considers two demand regimes: partial coverage and full coverage. Under partial coverage, the firm induces only the high-type consumers ($s = s_H$) to subscribe to ship-then-shop: $N_s(p_s, F) = 1/2$. Under full coverage, it induces both the high- and low-type consumers ($s = 0$) to subscribe: $N_s(p_s, F) = 1$.

Under partial coverage, only the high-type consumers subscribe to ship-then-shop such that $N_s(p_s, F) = 1/2$. The firm's problem is

$$
\begin{aligned}
\max_{p_s, F} \quad & \mathbb{E}[\pi_{\text{part}}] = \frac{1}{2}\left(F + \frac{V-\bar{v}}{V(1-\alpha)}(p_s - p_m)\right) \\
\text{subject to} \quad & 0 < \bar{s}(p_s, F) \leq s_H,
\end{aligned}
\tag{12}
$$

where $\bar{s}$ is defined in (10) and $\bar{v}$ is the marginal consumer who purchases the product as defined in (7). The (IC) constraint $0 < \bar{s}(p_s, F) \leq s_H$ ensures only the high-type consumers subscribe to ship-then-shop. The firm sets F such that the (IC) constraint binds. Solving this yields

$$
F^*_{\text{part}}(p_s) = s_H + \frac{(V(2\alpha h + V) - \bar{v}(2h + \bar{v}) - 2p_s(V - \bar{v}))}{2V(1-\alpha)} - \frac{(V - p_m)^2}{2V}, \tag{13}
$$

$$
p_s^* = \max\left[\alpha V + h, p_m\right]. \tag{14}
$$

Under full coverage, both consumer types subscribe and $N_s(p_s, F) = 1$. The firm's problem is

$$
\begin{aligned}
\max_{p_s, F} \quad & \mathbb{E}[\pi_{\text{full}}] = F + \frac{V-\bar{v}}{V(1-\alpha)}(p_s - p_m) \\
\text{subject to} \quad & \bar{s}(p_s, F) \leq 0.
\end{aligned}
\tag{15}
$$

Following the reasoning above, we obtain $F^*_{\text{full}}(p_s) = F^*_{\text{part}}(p_s) - s_H$, such that the low-type consumers' (IC) constraint binds. The following lemma summarizes the optimal product price and subscription fee under each regime.

Lemma 1. *The firm's optimal product price under both partial and full coverage is* $p_s^* = \max\left[\alpha V + h, p_m\right]$. *The optimal subscription fees under partial and full coverage, respectively, are* F^*_{part} *and* $F^*_{full} = F^*_{part} - s_H$.

Before solving for the optimal coverage choice, we highlight an important relationship between product price and subscription fee. Observe that under either the partial or full coverage, the firm's 'best-response' fee $F^*(p_s)$ is decreasing in p_s:

$$\frac{\partial F^*(p_s)}{\partial p_s} = -\min\left[1, \frac{V+h-p_s}{V(1-\alpha)}\right] \leq 0.^5 \tag{16}$$

This implies that the optimal subscription fee and optimal product price are strategic substitutes. In optimizing the product price, the firm not only trades off the usual margin versus sales, but also considers whether to extract *ex-ante* surplus through F or to extract *ex-post* surplus through p_s. The latter trade-off constitutes one of the key forces of the model. As we later demonstrate, whether the firm opts for *ex-ante* or *ex-post* surplus extraction depends crucially on the firm's AI capability (α) and search friction (s_H).

Proposition 1. *The firm's product price and subscription fee are strategic substitutes:* $\frac{\partial F^*(p_s)}{\partial p_s} \leq 0$.

Next, we determine the firm's optimal coverage, and thereby characterize the optimal subscription fee. The firm compares the optimal profits under partial and full coverage, which are, respectively,

$$\mathbb{E}[\pi^*_{\text{part}}] = \begin{cases} \frac{1}{4}\left(\alpha V + 2s_H - \frac{p_m^2}{V}\right) & \text{if } p_m \leq \alpha V + h, \\ \frac{\alpha V(V+2h) - 2p_m(\alpha V + h) + h^2 + \alpha p_m^2}{4(1-\alpha)V} & \text{if } p_m > \alpha V + h, \end{cases} \tag{17}$$

and

$$\mathbb{E}[\pi^*_{\text{full}}] = \begin{cases} \frac{\alpha V^2 - p_m^2}{2V} & \text{if } p_m \leq \alpha V + h, \\ \frac{\alpha(p_m - V)^2 - 2h(p_m - \alpha V) + h^2}{2(1-\alpha)V} & \text{if } p_m > \alpha V + h. \end{cases} \tag{18}$$

The following proposition characterizes the firm's optimal coverage choice.

Proposition 2. *The firm chooses full coverage if α is sufficiently large such that $\alpha > \tilde{\alpha}$ (or equivalently, s_H is sufficiently small such that $s_H \leq \tilde{s}$). Otherwise, it chooses partial coverage.*[6]

[5]Note that the firm never sets $p_s > V + h$.

[6]The thresholds $\tilde{\alpha}$ and $\tilde{s}$ are characterized in the proof.

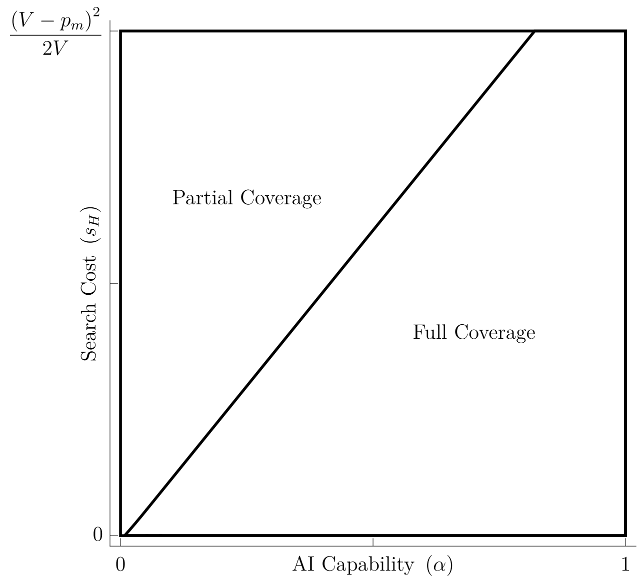

Figure 3: Optimal Coverage

Proposition 2 shows that the firm chooses full coverage if its AI capability is sufficiently sophisticated or the search friction in the market is mild (see Figure 3). Intuitively, advanced AI capability (i.e., large α) increases the ship-then-shop service's matching effect. This implies that even the low-type consumers, who can search in the traditional market at low search cost, have high valuation for the ship-then-shop service. The increased valuation motivates the firm to cover the whole market.

In terms of search friction, the firm covers the whole market if search friction is mild (i.e., small s_H). Recall that the second benefit of ship-then-shop is the convenience effect: ship-then-shop facilitates consumer shopping by reducing their search costs. Therefore, if search friction is severe, the convenience effect becomes more valuable to consumers such that the firm can extract large consumer surplus. In this case, the firm charges a high subscription fee and extracts the high-type consumers' surplus, at the expense of forgoing subscription from low-type consumers.

Based on the firm's optimal coverage, we obtain the optimal price and fee:

$$p_s^* = \max\left[\alpha V + h, p_m\right], \text{ and } F^* = F_{\text{part}}^*\left(p_s^*\right) - \begin{cases} 0 & \text{if } \alpha \leq \tilde{\alpha}, \\ s_H & \text{if } \alpha > \tilde{\alpha}. \end{cases} \tag{19}$$

We present the main result in the following proposition. We describe how the firm's equilibrium strategy (p_s^*, F^*) interacts with *(i)* the firm's AI capability and *(ii)* the degree of search friction in the market.

Proposition 3. *The firm's equilibrium strategy (p_s^*, F^*) varies as follows.*

(i) With respect to α: p_s^* weakly increases in α, while F^* varies non-monotonically in α.

- *If $\alpha \leq \frac{p_m - h}{V}$, then F^* increases in α, with a discontinuous drop at $\tilde{\alpha}$.*[7]
- *If $\alpha > \frac{p_m - h}{V}$, then F^* decreases in α.*

(ii) With respect to s_H: p_s^* is independent of s_H, while F^* weakly increases in s_H.

The price pattern with respect to the firm's AI capability (α) reflects the matching effect, the first benefit of ship-then-shop service. Advances in AI capability increase the product match quality such that the firm is more confident consumers will like the shipped products. Therefore, if the the firm's prediction accuracy is high (i.e., $\alpha > \frac{p_m - h}{V}$), as α increases, the firm shifts from *ex-ante* to *ex-post* surplus extraction; i.e., it lowers the subscription fee and raises the product price (see Figure 4).

If the firm's prediction accuracy is low (i.e., $\alpha \leq \frac{p_m - h}{V}$), the expected match quality is poor, which dampens consumers' valuation for ship-then-shop. In this range, the primary source of customer benefits is the convenience effect. Consumers' anticipation of a low-quality product match induces the firm to fix product price at marginal cost p_m. Since the firm's product price is constrained by the marginal cost p_m in this range, it raises the subscription fee instead in response to advances in AI capability. Taken together, for small α (i.e., $\alpha \leq \frac{p_m - h}{V}$), the firm harvests the matching effect through F, whereas for large α

[7] $\tilde{\alpha}$ is the threshold value of α at which the firm switches from partial to full coverage (see Proposition 2).

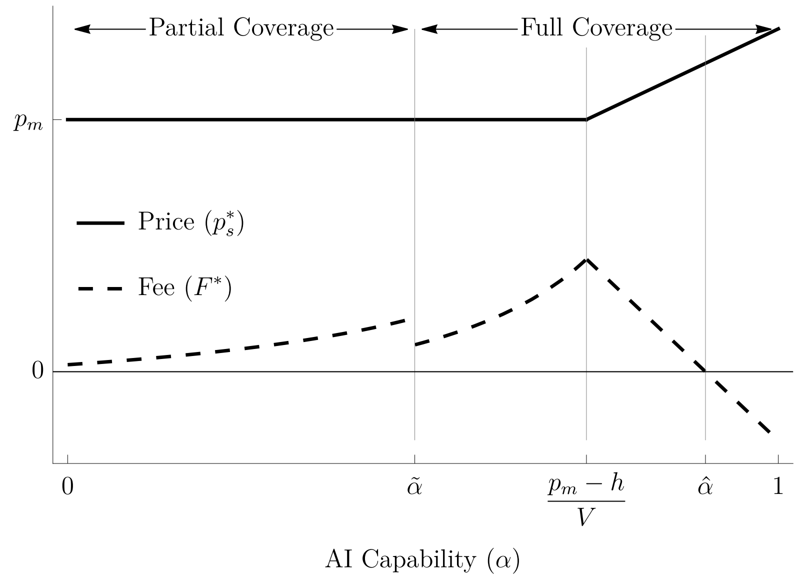

Figure 4: Impact of AI Capability on Price and Fee ($V = 1, p_m = 0.75, h = s_H = 0.02$)

(i.e., $\alpha > \frac{p_m - h}{V}$), it does so through p_s. This is because the primary source of customer benefits switches from the convenience effect to the matching effect. Thus, for large α (i.e., $\alpha > \frac{p_m - h}{V}$), the product price increases while the subscription fee decreases because these two are strategic substitutes (see Proposition 1).

Interestingly, if the firm's prediction accuracy is sufficiently high (i.e., $\alpha > \hat{\alpha}$), the subscription fee becomes negative. The firm subsidizes consumers to compensate for their *ex-ante* uncertainty by offering a "sign-up bonus." Then, the firm extracts the surplus through high product price p_s once the customers resolve their uncertainty through realized high matching values. The following lemma characterizes the condition under which the firm offers negative subscription fee.

Lemma 2. *The firm's optimal subscription fee F^* is negative if $\alpha > \hat{\alpha}$, where $\hat{\alpha} \in \left[\frac{p_m - h}{V}, 1\right)$.*

The fee pattern with respect to s_H reflects the convenience effect, the second benefit of ship-then-shop subscription. Intuitively, the value of shopping without searching increases as searching in the traditional market becomes more costly. This lifts consumers' valuation of ship-then-shop, which allows the firm to charge higher subscription fee (see Figure 5).

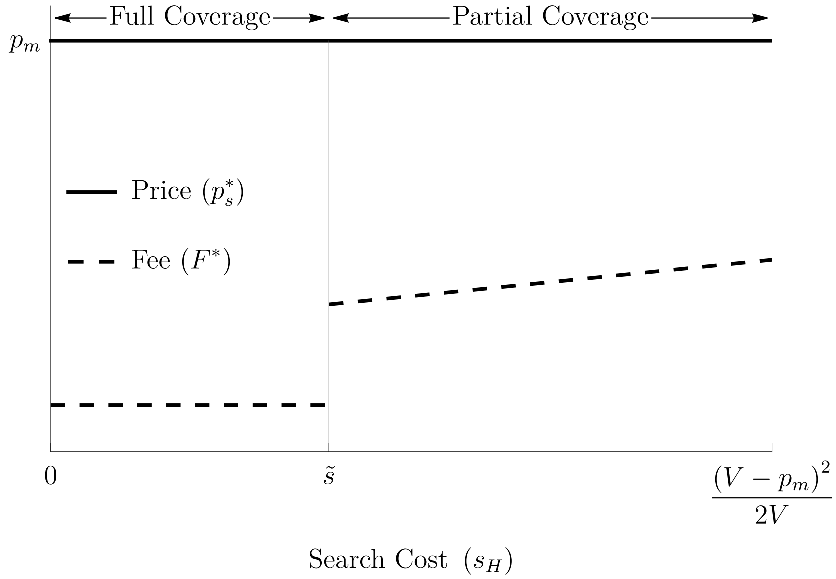

Figure 5: Impact of Search Cost on Price and Fee ($V = 1, p_m = 0.75, h = 0.02, \alpha = 0.5$)

Finally, we find that $\frac{\partial p_s^*}{\partial h} \geq 0$ and $\frac{\partial F^*}{\partial h} < 0$. Higher hassle cost of returning unwanted products has a lock-in effect, which allows the firm to raise the product price. Thus, the firm extracts greater *ex-post* surplus by charging a high price, while it lowers its upfront subscription fee to compensate for the risk of product mismatch.

3.3 Profitability of Ship-then-Shop

In this section, we examine the effects of firm-specific factors (e.g., AI capability) and market-specific factors (e.g., the severity of search friction and the extent of product match heterogeneity among consumers) on the profitability of ship-then-shop. The following proposition summarizes the comparative statics of the firm's equilibrium profit with respect to the market primitives.

Proposition 4. *The firm's expected profit increases in α, s_H, and V; i.e.,* $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} > 0$, $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} \geq 0$, *and* $\frac{\partial \mathbb{E}[\pi^*]}{\partial V} > 0$.

The rationale behind Proposition 4 revolves around the two benefits of ship-then-shop: matching effect and convenience effect. The firm's profit increases in both α and s_H due

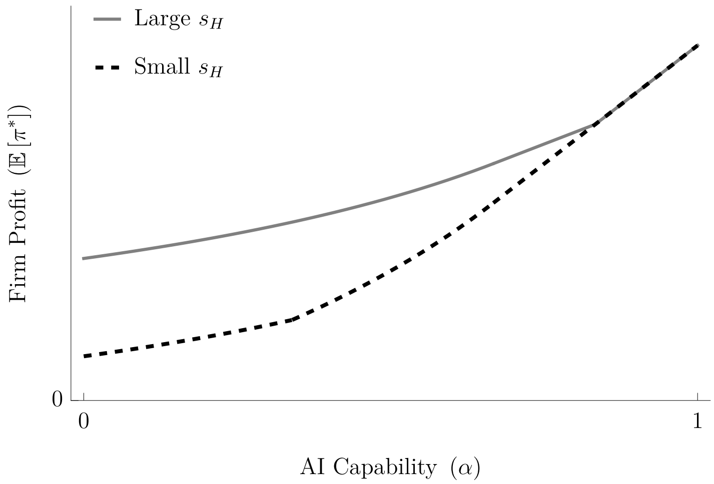

Figure 6: Search Cost and Marginal Return

to these two effects. Equipped with higher AI capability, the firm provides higher-quality match to consumers. Also, higher search friction in the traditional market increases the consumers' comparative valuations for ship-then-shop. Both enlarge the total surplus, which the firm extracts through the optimal fee-price combination in (19). Finally, as the maximum attainable match value V increases, the upside potential of the matching effect under ship-then-shop increases, which in turn increases the firm's profit.

Given that the profitability of ship-then-shop increases in AI capability and market search friction, lay intuition suggests that the firm should invest in enhancing both the matching and convenience effect. However, due to the linkage between *ex-ante* and *ex-post* consumer surplus, we find that the two effects are substitutes; i.e., returns from one effect diminishes the returns from the other.

Proposition 5. *The interaction effect of AI capability and search friction on the firm's profit is negative:* $\frac{\partial}{\partial s_H}\left(\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}\right) \leq 0$. *Moreover, the marginal return of* α *(*s_H*) on the firm's expected profit increases (decreases) in* V*;* $\frac{\partial}{\partial V}\left(\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}\right) \geq 0$ *and* $\frac{\partial}{\partial V}\left(\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}\right) \leq 0$.

While the firm's profit increases in α and s_H, the interaction effect of AI capability and search friction on the firm's profit is negative: $\frac{\partial}{\partial s_H}\left(\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}\right) \leq 0$. This suggests that for the firm offering ship-then-shop service, matching effect and convenience effect are substitutes.

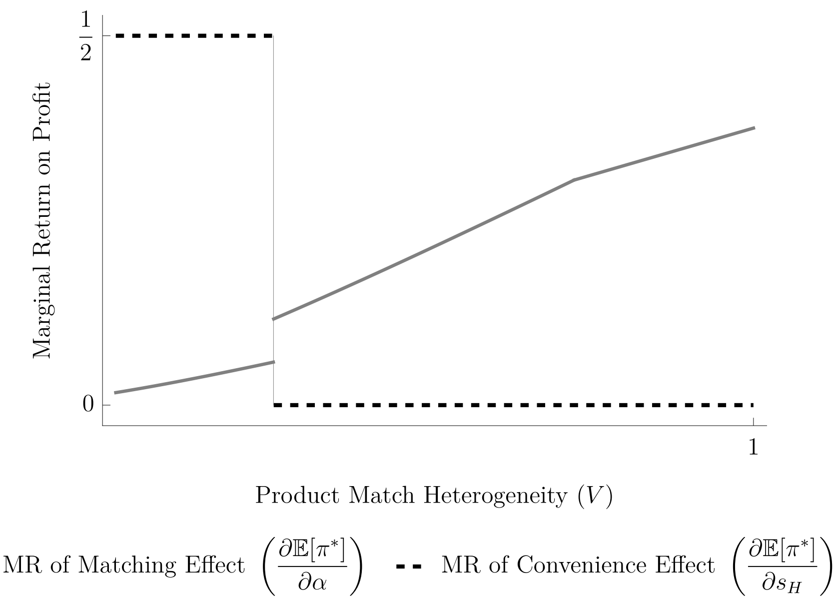

Figure 7: Marginal Returns of Matching and Convenience Effects

For instance, if the search friction in the market is severe such that the convenience effect is large, then the the marginal effect of AI capability on firm's profit diminishes (see Figure 6). Intuitively, the firm capitalizes on the convenience effect by charging high subscription fees, thereby extracting the high-type consumers' surplus at the expense of foregoing demand from low-type consumers. As fewer consumers subscribe to ship-then-shop, the total returns from the matching effect decreases.

Finally, Proposition 5 reveals an important insight regarding product match heterogeneity. The marginal return from the matching effect increases in product match heterogeneity, whereas the marginal return from the convenience effect decreases in product match heterogeneity (see Figure 7). If V is small, then consumers face little product match uncertainty such that the matching effect under ship-then-shop adds little value. In this case, the ship-then-shop's value derives primarily from the convenience effect; therefore, the return from convenience effect dominates that from the matching effect. On the other hand, if V is large, the relationship reverses. The more dispersed consumers' product match values are, the greater the return from AI capability because the firm can harvest more effectively the upside potential of the matching effect. In this case, the primary source of ship-then-shop's value is the matching effect. Therefore, the marginal return from the matching effect dominates that from the convenience effect.

The insights from Propositions 4 and 5 help explain the emergence of ship-then-shop business models in certain markets. Our analysis suggests ship-then-shop models are likely to be profitable if the firm's AI capability is advanced, the search friction in the target market is severe, or the consumers' product match heterogeneity is large. By and large, our findings are consistent with real-world observations that the emergence of ship-then-shop businesses has been concentrated in markets characterized by high search friction and large preference heterogeneity, such as apparel (e.g., Stitch Fix, Trunk Club and Wantable Fitness) and food delivery (e.g., HelloFresh, Sunbasket, and Blue Apron).

The propositions also inform managerial decision-making. A ship-then-shop firm that considers investing in AI capability should be mindful of the source of marginal return on investment. For instance, it should exercise caution before rushing to improve its AI capability (e.g., hiring data scientists), especially in markets characterized by high search friction: the firm should consider focusing on improving the convenience of the consumers' purchase process (e.g., by extending the clothing line to categories whose fashion trends evolve quickly such that consumers entail high search costs). On the other hand, in product categories with large product match heterogeneity among consumers, it is in the firm's best interest to invest in AI capability improvements, which yield higher marginal returns than enhancing the convenience effect.

4 Conclusion

Advances in AI technology, driven by deep learning techniques and data digitization, are fundamentally reshaping the business landscape. As improvements in AI algorithms enable firms to predict consumer preferences with greater accuracy, firms are adapting by reinventing their core business strategy. A notable example of such business transformation gaining traction in the retail sector is the ship-then-shop program. Unlike the traditional shop-then-ship model, which begins with consumer search and ends with product shipment, under the ship-then-shop model, the firm leverages its AI capability to predict consumers' preferences and ships the product to them; consumers then evaluate product fit and decide whether to purchase or return the product. In this paper, we develop a parsimonious game theory model that unveils nuanced economic forces underlying the ship-then-shop subscription model.

We show that the firm's optimal subscription fee and product price depend crucially on the trade-off between *ex-ante* and *ex-post* surplus extraction strategies. If AI capability is low, the firm capitalizes on the convenience effect (stemming from the reduction in consumer search cost): it raises the fee and lowers the price. This strategy emphasizes *ex-ante* surplus extraction. On the other hand, if the firm's AI capability is advanced, it exploits the matching effect (stemming from superior product fit) by lowering the fee and raising the price. Furthermore, if the prediction machine is sufficiently accurate, the firm offers a negative fee, effectively subsidizing consumers to join ship-then-shop.

Moreover, we find that the ship-then-shop subscription model is most profitable *(i)* when AI capability is advanced, *(ii)* when the search friction in the market is severe, or *(iii)* when the customer heterogeneity in matching value is large. We also show that the marginal return of AI capability on the firm's profit decreases in search friction but increases in consumers' valuation heterogeneity. These insights provide important guidance for managers implementing the innovative subscription model. For example, investing in AI capability is more fruitful when there is sufficient product match heterogeneity among customers, as it enables the firm to better reap the upside potential of the matching effect under ship-then-shop.

Appendix: Proofs

Lemma 1. Simplifying (12) yields

$$\mathbb{E}[\pi_{\text{part}}] = \frac{1}{2} \cdot \begin{cases} -\frac{p_m^2}{2V} + s_H + \frac{\alpha V}{2}, & p_s \leq \alpha V + h, \\ \frac{h^2 + 2p_m(p_s - (\alpha V + h)) + \alpha V(V + 2(h - s_H)) - (1-\alpha)p_m^2 - p_s^2 + 2s_H V}{2V(1-\alpha)}, & p_s > \alpha V + h. \end{cases}$$

First, if $p_s \leq \alpha V + h$, firm profit is independent of p_s. Therefore, there exists a continuum of optima. To fix a unique equilibrium, we consider consumers discounting future payoffs by $\delta < 1$. $\mathbb{E}[\pi_{\text{part}}] = (1-\delta)p_s - \frac{p_m^2}{2V} + s_H - \frac{1}{2}V(1-(1+\alpha)\delta)$, which increases in p_s. Therefore, $p_s = \alpha V + h$. Second, if $p_s > \alpha V + h$, $\frac{\partial^2 \mathbb{E}[\pi_{\text{part}}]}{\partial p_s^2} = -\frac{1}{V(1-\alpha)} < 0$. FOC implies $p_s = \max\left[\alpha V + h, p_m\right]$. By continuity of firm profit w.r.t. p_s,

$$p_s^* = \max\left[\alpha V + h, p_m\right].$$

Therefore,

$$F^*_{\text{part}} = \begin{cases} \frac{h^2 - 2p_m(h + \alpha V) + \alpha V(V + 2(h - s_H)) + \alpha p_m^2 + 2s_H V}{2V(1-\alpha)}, & \alpha \leq \frac{p_m - h}{V}, \\ s_H - h + p_m - \frac{p_m^2}{2V} - \frac{\alpha V}{2}, & \alpha > \frac{p_m - h}{V}. \end{cases}$$

Under full coverage, F is lowered by s_H such that low-type consumers' IC constraint bind. ■

Proposition 1. It suffices to show $\frac{\partial}{\partial p_s} F^*_{\text{part}}(p_s) < 0$. If $p_s \leq V + h$,

$$\frac{\partial}{\partial p_s} F^*_{\text{part}}(p_s) = \begin{cases} -1, & p_s \leq \alpha V + h, \\ -\frac{V + h - p_s}{V(1-\alpha)}, & p_s > \alpha V + h \end{cases} < 0.$$

■

Proposition 2 . First,

$$\frac{\partial\left(\mathbb{E}[\pi_{\text{full}}]-\mathbb{E}[\pi_{\text{part}}]\right)}{\partial\alpha}=\frac{1}{4}\cdot\begin{cases}\frac{(V+h-p_m)^2}{V(1-\alpha)^2}, & \alpha\leq\frac{p_m-h}{V},\\ V, & \alpha>\frac{p_m-h}{V},\end{cases}>0.$$

Second, $\left(\mathbb{E}[\pi_{\text{full}}]-\mathbb{E}[\pi_{\text{part}}]\right)|_{\alpha=0}<0$ and $\lim_{\alpha\uparrow 1}\left(\mathbb{E}[\pi_{\text{full}}]-\mathbb{E}[\pi_{\text{part}}]\right)>0$:

$$\mathbb{E}[\pi_{\text{full}}]-\mathbb{E}[\pi_{\text{part}}]|_{\alpha=0}=\frac{h^2-2hp_m-2s_HV}{4V}\leq\frac{h^2-2hp_m-2s_HV}{4V}|_{h=0}=-\frac{s_H}{2}<0,$$

and

$$\lim_{\alpha\uparrow 1}\mathbb{E}[\pi_{\text{full}}]-\mathbb{E}[\pi_{\text{part}}]=\frac{V^2-2s_HV-p_m^2}{4V}\geq\frac{V^2-2s_HV-p_m^2}{4V}|_{s_H=\frac{(V-p_m)^2}{2V}}=\frac{(V-p_m)p_m}{2V}>0.$$

Finally, Intermediate Value Theorem (IVT) ensures unique existence of $\tilde{\alpha}\equiv\{\alpha\in(0,1):\mathbb{E}[\pi_{\text{full}}]=\mathbb{E}[\pi_{\text{part}}]\}$, such that $\mathbb{E}[\pi_{\text{full}}]-\mathbb{E}[\pi_{\text{part}}]\geq 0\Leftrightarrow\alpha\geq\tilde{\alpha}$.

Next, we show $\mathbb{E}[\pi_{\text{full}}]-\mathbb{E}[\pi_{\text{part}}]\geq 0$ is equivalent to s_H being small. First, algebraic manipulations yield

$$\frac{1}{2}\left(F^*_{\text{part}}(p^*_s)+\int_{\bar{v}}^{V}\frac{p^*_s-p_m}{V(1-\alpha)dv}\right)-s_H\geq 0.$$

Note

$$\frac{\partial}{\partial s_H}\left(\frac{1}{2}\left(F^*_{\text{part}}(p^*_s)+\int_{\bar{v}}^{V}\frac{p^*_s-p_m}{V(1-\alpha)dv}\right)-s_H\right)=-\frac{1}{2}.$$

Second, if $s_H\downarrow 0$, full coverage dominates partial coverage because increasing F infinitesimally and excluding s_L-consumers is unprofitable. Finally, define

$$\xi\equiv\frac{1}{2}\left(F^*_{\text{part}}(p^*_s)+\int_{\bar{v}}^{V}\frac{p^*_s-p_m}{V(1-\alpha)dv}\right)-s_H|_{s_H=\frac{(V-p_m)^2}{2V}}.$$

If $\xi \geq 0$, full coverage dominates partial coverage for all s_H. If $\xi < 0$, IVT ensures unique existence of $\hat{s} \in \left(0, \frac{(V-p_m)^2}{2V}\right)$ such that $\mathbb{E}[\pi_{\text{full}}] - \mathbb{E}[\pi_{\text{part}}] \geq 0$ if and only if $s_H \leq \hat{s}$, where

$$\tilde{s} \equiv \min\left[\hat{s}, \frac{(V-p_m)^2}{2V}\right].$$

■

Proposition 3. Comparative statics for p_s^* is trivial and omitted. Note

$$\frac{\partial F^*}{\partial \alpha} = \begin{cases} \frac{(V+h-p_m)^2}{2V(1-\alpha)^2}, & \alpha \leq \frac{p_m-h}{V}, \\ -\frac{V}{2}, & \alpha > \frac{p_m-h}{V}. \end{cases}$$

At $\alpha = \tilde{\alpha}$, coverage shifts from partial to full, such that F^* decreases by s_H.

For comparative statics w.r.t. s_H, note F^* is independent of s_H under full coverage (because marginal consumer is low-type). Under partial coverage, $F^* = s_H + \zeta$, where ζ is independent of s_H. Proposition 2 implies full coverage for $s_H \leq \tilde{s}$ and partial coverage for $s_H > \tilde{s}$. Therefore, F^* is independent of s_H for $s_H \leq \tilde{s}$, increases by s_H at $s = \tilde{s}$, and increases for $s_H > \tilde{s}$. ■

Lemma 2. Suppose $\alpha > \max\left[\tilde{\alpha}, \frac{p_m-h}{V}\right]$ such that $p^* = \alpha V + h$ and $\mathbb{E}[\pi_{\text{full}}] > \mathbb{E}[\pi_{\text{part}}]$. Note $\frac{\partial F^*}{\partial \alpha} = -\frac{V}{2} < 0$. At $\alpha = \frac{p_m-h}{V}$, F^* under full coverage attains maximum value $\frac{V(p_m-h)-p_m^2}{2V} > 0$ for all $h < p_m(1-p_m/V)$; and $\lim_{\alpha \uparrow 1} F^* = -h - \frac{(V-p_m)^2}{2V} < 0$. Therefore, $F^* < 0$ for all $\alpha > \frac{2(V(p_m-h))-p_m^2}{V^2}$, and $\tilde{\alpha} > \frac{2(V(p_m-h))-p_m^2}{V^2}$ implies $F^* < 0$ for all $\alpha > \tilde{\alpha}$, where $\hat{\alpha} \equiv \max\left[\tilde{\alpha}, \frac{2(V(p_m-h))-p_m^2}{V^2}\right]$. ■

Proposition 4. If $p_m \leq \alpha V + h$,

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} = \frac{V}{2} \cdot \begin{cases} 1, & s_H \leq \frac{\alpha V^2 - p_m^2}{2V}, \\ \frac{1}{2}, & s_H > \frac{\alpha V^2 - p_m^2}{2V} \end{cases} > 0. \tag{20}$$

If $p_m > \alpha V + h$,

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} = \frac{(V+h-p_m)^2}{2V(1-\alpha)^2} \cdot \begin{cases} 1, & s_H \leq \frac{h^2-2h(p_m-\alpha V)+\alpha(p_m-V)^2}{2V(1-\alpha)}, \\ \frac{1}{2}, & s_H > \frac{h^2-2h(p_m-\alpha V)+\alpha(p_m-V)^2}{2V(1-\alpha)} \end{cases} > 0. \tag{21}$$

Since $\mathbb{E}[\pi^*]$ is continuous in α, $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} > 0$. If $p_m \leq \alpha V + h$,

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \begin{cases} 0, & s_H \leq \frac{\alpha V^2 - p_m^2}{2V}, \\ \frac{1}{2}, & s_H > \frac{\alpha V^2 - p_m^2}{2V} \end{cases} \leq 0. \tag{22}$$

If $p_m > \alpha V + h$,

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \begin{cases} 0, & s_H \leq \frac{h^2-2h(p_m-\alpha V)+\alpha(V-p_m)^2}{2V(1-\alpha)}, \\ \frac{1}{2}, & s_H > \frac{h^2-2h(p_m-\alpha V)+\alpha(V-p_m)^2}{2V(1-\alpha)} \end{cases} \leq 0. \tag{23}$$

Since $\mathbb{E}[\pi^*]$ is continuous in s_H, $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} \geq 0$. If $p_m \leq \alpha V + h$,

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial V} = \frac{1}{2}\left(\alpha + \frac{p_m^2}{V^2}\right) \cdot \begin{cases} \frac{1}{2}, & s_H \leq \frac{\alpha V^2 - p_m^2}{2V}, \\ 1, & s_H > \frac{\alpha V^2 - p_m^2}{2V} \end{cases} > 0.$$

If $p_m > \alpha V + h$, then

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial V} = \frac{2hp_m - h^2 + \alpha(V - p_m^2)}{2V^2(1-\alpha)} \cdot \begin{cases} \frac{1}{2}, & s_H \leq \frac{h^2-2h(p_m-\alpha V)+\alpha(p_m-V)^2}{2V(1-\alpha)}, \\ 1, & s_H > \frac{h^2-2h(p_m-\alpha V)+\alpha(p_m-V)^2}{2V(1-\alpha)}. \end{cases}$$

Note $\frac{\partial}{\partial h}\left(2hp_m - h^2 + \alpha(V - p_m^2)\right) > 0 \Rightarrow 2hp_m - h^2 + \alpha(V - p_m^2) \geq 2hp_m - h^2 + \alpha(V - p_m^2)|_{h=0} = \alpha(V - p_m^2) > 0 \Rightarrow \frac{\partial \mathbb{E}[\pi^*]}{\partial V} > 0$. Since $\mathbb{E}[\pi^*]$ is continuous in V, $\frac{\partial \mathbb{E}[\pi^*]}{\partial V} > 0$. ■

Proposition 5. Cross-derivatives follow from (20) and (21). Also, $\frac{\partial}{\partial s_H}\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} \leq 0$ because value of $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ for s_H greater than thresholds in (20) and (21) is half that for s_H less than thresholds. Finally, $\frac{\partial}{\partial V}\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha} > 0$ because, from (20) and (21), $\frac{V}{2}$ and $\frac{(V+h-p_m)^2}{2V(1-\alpha)^2}$ both increase in V. Moreover, $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ increases at $p_m = \alpha V + h$ due to Claim 1 (see Section A2 of the Online Appendix).

From (22) and (23), excluding discontinuities, $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}$ is independent of V. Finally, $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}$ decreases at discontinuities due to Claim 2 (see Section A2 of the Online Appendix).

■

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Online Appendix

A1 Extension: Unobservable Product Price

The main model assumes that the firm can commit to a product price that consumers can observe prior to their subscription decision. While this is consistent with how a number of firms set prices in practice, there are cases in which firms do not price-commit. For example, firms that offer ship-then-shop may first collect subscription fee and then decide product price as they ship the products to their subscribers (e.g., through hidden fees, surcharges, service fees, etc.). In this section, we assess the robustness of our main insights to relaxing the price-commitment assumption. Specifically, we delay the firm's product price decision from Stage 1 to Stage 3, which is when consumers receive the product and decide whether to purchase the shipped product. All other model specifications remain unchanged.

Similar to the main model, consumers make subscription decisions based on subscription fee and product price. The key difference is that consumers cannot observe the *actual* price; instead, they consider the *expected* product price p_s^e. In Stage 3, the firm decides p_s taking into account p_s^e. Note that once consumers subscribe to ship-then-shop, their expected price is immaterial to the firm's profit. Conditional on consumer subscription, the firm's product pricing problem in Stage 3 is

$$\max_{p_s} \frac{V-\bar{v}}{V(1-\alpha)}(p_s-p_m),$$

where $\bar{v}=\max\left[\alpha V, p_s-h\right]$ as in equation (7). This yields

$$\tilde{p}_s^* = \max\left[\alpha V + h, \frac{V+h+p_m}{2}\right]. \tag{A1}$$

In equilibrium, consumers' expectations align with the firm's optimal price. Therefore,

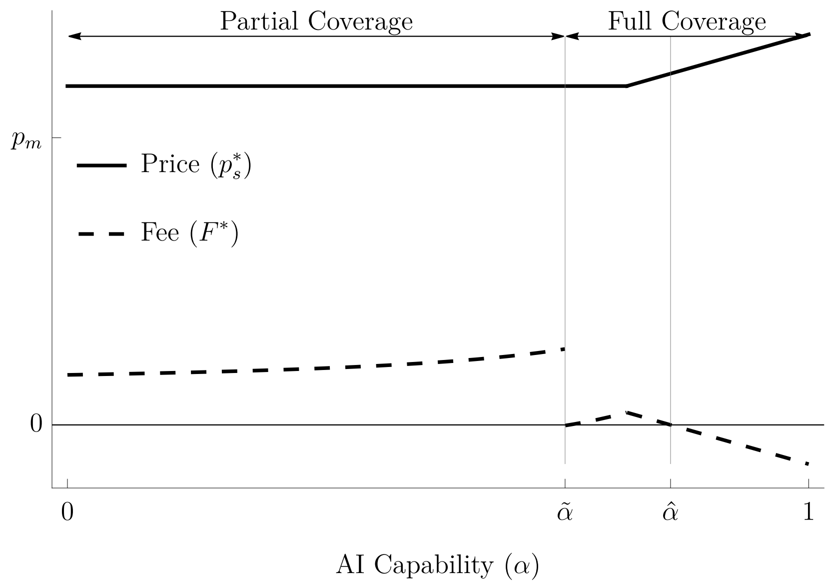

Figure A1: Impact of AI Capability on Price and Fee ($V = 1, p_m = 0.75, h = 0.02, s_H = 0.1$)

$\tilde{p}_s^*$ in equation (A1) is the equilibrium price in the scenario without price-commitment. Barring a minor linear transformation, $\tilde{p}_s^*$ is identical to $p_s^* = \max[\alpha V + h, p_m]$, the optimal price with price-commitment in the main model. In particular, all the comparative statics with respect to α, V, h, and p_m are qualitatively the same as the main model.

Furthermore, since the remaining model features are unchanged, the qualitative insights pertaining to consumers' subscription decisions and the firm's optimal fee carry over as well. For instance, the qualitative patterns of equilibrium price and fee patterns with respect to AI capability (α) are preserved. Figure A1 shows the impact of AI capability on price p_s and fee F without price commitment (compare this to Figures 4).

A2 Statements and Proofs of Claims

Claim 1. $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ *is non-decreasing in* V *at* $V = \frac{p_m - h}{\alpha}$.

Proof of Claim 1. At $V = \frac{p_m - h}{\alpha}$, we have $\frac{V}{2} = \frac{(V+h-p_m)^2}{2V(1-\alpha)^2}$. Now, we obtain from (20) and (21) that $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ is decreasing in V at $V = \frac{p_m - h}{\alpha}$ only if $s_H \leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)}$ for $V < \frac{p_m - h}{\alpha}$

and $s_H > \frac{\alpha V^2 - p_m^2}{2V}$ for $V > \frac{p_m - h}{\alpha}$. Due to Claim 3, these conditions imply the conditions (A2) and (A3). However, the proof of Case (iii) in Claim 2 (see below) shows that (A2) and (A3) cannot jointly hold. This proves that $\frac{\partial \mathbb{E}[\pi^*]}{\partial \alpha}$ cannot be decreasing in V at $V = \frac{p_m - h}{\alpha}$. ■

Claim 2. $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}$ *decreases at discontinuities with respect to* V*.*

Proof of Claim 2. From (22) and (23), there are three discontinuities to consider: V at which (i) $s_H = \frac{\alpha V^2 - p_m^2}{2V}$, (ii) $s_H = \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)}$, and (iii) $p_m \leq \alpha V + h$.

Consider Case (i), which applies to $p_m \leq \alpha V + h$, or equivalently $V \geq \frac{p_m - h}{\alpha}$. Due to Claim 3 (see below), if $s_H \leq \frac{\alpha V^2 - p_m^2}{2V}$, then $V \geq \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}$. Therefore, if $p_m \leq \alpha V + h$, or equivalently $V \geq \frac{p_m - h}{\alpha}$,

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \begin{cases} 0, & V \geq \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}, \\ \frac{1}{2}, & V < \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}. \end{cases}$$

Consider Case (ii), which applies to $p_m > \alpha V + h$, or equivalently $V < \frac{p_m - h}{\alpha}$. Due to Claim 3 (see below)

$$\begin{aligned} s_H &\leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)} \\ &\Leftrightarrow V \\ &\geq \frac{s - \alpha(h - p_m + s) + \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h - 2p_m + s)\right)}}{\alpha}. \end{aligned}$$

Therefore, if $p_m > \alpha V + h$, or if $V < \frac{p_m - h}{\alpha}$, then

$$\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \begin{cases} 0, & V \geq \frac{s - \alpha(h - p_m + s) + \sqrt{(1-\alpha)(s^2 - \alpha(h+s)(h - 2p_m + s))}}{\alpha}, \\ \frac{1}{2}, & V < \frac{s - \alpha(h - p_m + s) + \sqrt{(1-\alpha)(s^2 - \alpha(h+s)(h - 2p_m + s))}}{\alpha}. \end{cases}$$

Consider Case (iii). $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H}$ increases at discontinuity only if $\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = 0$ for $p_m > \alpha V + h$ and

$\frac{\partial \mathbb{E}[\pi^*]}{\partial s_H} = \frac{1}{2}$ for $p_m < \alpha V + h$. This requires:

$$\frac{s - \alpha(h - p_m + s) + \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h-2p_m+s)\right)s}}{\alpha} < \frac{p_m - h}{\alpha} \tag{A2}$$

and

$$\frac{p_m - h}{\alpha} < \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}, \tag{A3}$$

which simplify to

$$\frac{h(h+2s)}{2(h+s)} < p_m < \frac{h+s}{2} \text{ and } \alpha > \frac{s^2}{(h+s)(h-2p_m+s)}. \tag{A4}$$

Since $\frac{\partial}{\partial p_m}\frac{s^2}{(h+s)(h-2p_m+s)} > 0$, (A4) holds only if $\alpha > \frac{s^2}{(h+s)(h-2p_m+s)}$ holds for smallest value of p_m, which under condition $\frac{h(h+2s)}{2(h+s)} < p_m < \frac{h+s}{2}$ is $p_m = \frac{h(h+2s)}{2(h+s)}$. But

$$\frac{s^2}{(h+s)(h - 2\frac{h(h+2s)}{2(h+s)} + s)} = 1,$$

which contradicts $\alpha < 1$. ■

Claim 3. *$p_m \leq V$ implies the following two equivalences:*

$$s_H \leq \frac{\alpha V^2 - p_m^2}{2V} \Leftrightarrow V \geq V_1 \textit{ and } s_H \leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)} \Leftrightarrow V \geq V_2,$$

where

$$V_1 \equiv \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha},$$
$$\textit{and } V_2 \equiv \frac{s - \alpha(h - p_m + s) + \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h-2p_m+s)\right)}}{\alpha}.$$

Proof of Claim 3. For the first equivalence, note that $s_H \leq \frac{\alpha V^2 - p_m^2}{2V}$ can be rearranged in terms of V to the condition that either $V \leq \frac{s - \sqrt{s^2 - \alpha p_m^2}}{\alpha}$ or $V \geq \frac{s + \sqrt{s^2 - \alpha p_m^2}}{\alpha}$. We show that the first condition cannot hold if $p_m \leq V$. Since $p_m \leq V$, we have

$$V \leq \frac{s - \sqrt{s^2 - \alpha p_m^2}}{\alpha} \Rightarrow p_m \leq \frac{s - \sqrt{s^2 - \alpha p_m^2}}{\alpha} \Rightarrow -2s_H \geq p_m(1-\alpha),$$

which is impossible since s_H and p_m are positive and $\alpha < 1$.

For the second equivalence, note that $s_H \leq \frac{h^2 - 2h(p_m - \alpha V) + \alpha(V - p_m)^2}{2V(1-\alpha)}$ can be rearranged in terms of V to the condition that either $V \leq \frac{s - \alpha(h - p_m + s) - \sqrt{(1-\alpha)(s^2 - \alpha(h+s)(h - 2p_m + s))}}{\alpha}$ or $V \geq \frac{s - \alpha(h - p_m + s) + \sqrt{(1-\alpha)(s^2 - \alpha(h+s)(h - 2p_m + s))}}{\alpha}$. We show that the first condition cannot hold if $p_m \leq V$. Since $p_m \leq V$, we have $V \leq \frac{s - \alpha(h - p_m + s) - \sqrt{(1-\alpha)(s^2 - \alpha(h+s)(h - 2p_m + s))}}{\alpha}$ implies

$$p_m \leq \frac{s - \alpha(h - p_m + s) - \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h - 2p_m + s)\right)}}{\alpha},$$

which simplifies to

$$s(1-\alpha) - \alpha h - \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h - 2p_m + s)\right)} \geq 0. \tag{A5}$$

The left-hand side of (A5) is decreasing in h due to Claim 4 (see below); therefore, the inequality (A5) must hold at $h = 0$. But at $h = 0$, the left-hand side of (A5) reduces to

$$(1-\alpha)s - \sqrt{(1-\alpha)s(2\alpha p_m + (1-\alpha)s)},$$

which is less than 0, because

$$\sqrt{(1-\alpha)s(2\alpha p_m + (1-\alpha)s)} \geq \sqrt{(1-\alpha)s(0 + (1-\alpha)s)} = (1-\alpha)s.$$

Therefore, (A5) does not hold for any $h \geq 0$. ■

Claim 4. *If $\alpha \leq 1$, $0 \leq h \leq p_m$ and $s \geq 0$, then*

$$\frac{\partial}{\partial h}\left(s(1-\alpha) - \alpha h - \sqrt{(1-\alpha)\left(s^2 - \alpha(h+s)(h - 2p_m + s)\right)}\right) \leq 0. \tag{A6}$$

Proof of Claim 4. Writing out the derivative and simplifying yields

$$\text{(A6)} \Leftrightarrow \sqrt{(1-\alpha)\left(\alpha(h+s)(2p_m-h-s)+s^2\right)} > (1-\alpha)(-(p_m-h-s)).$$

First, suppose $p_m \geq h+s$ such that $(1-\alpha)(-(p_m-h-s))$ is negative. Since

$$\sqrt{(1-\alpha)\left(\alpha(h+s)(2p_m-h-s)+s^2\right)} > 0,$$

the inequality holds. Second, suppose $p_m < h+s$. Then

$$\frac{\partial(h+s)(2p_m-h-s)}{\partial h} = -2(h+s-p_m) < 0 \text{ and } \frac{\partial(1-\alpha)(-(p_m-h-s))}{\partial h} = 1-\alpha > 0,$$

such that $\sqrt{(1-\alpha)\left(\alpha(h+s)(2p_m-h-s)+s^2\right)}$ is decreasing in h while $(1-\alpha)(-(p_m-h-s))$ is increasing in h. Therefore, to show that $\sqrt{(1-\alpha)\left(\alpha(h+s)(2p_m-h-s)+s^2\right)} > (1-\alpha)(-(p_m-h-s))$, it suffices to show that the inequality holds for the largest value of h in the interval $[0, p_m(1-p_m/V)]$ (see Footnote 3). In fact, we can show that the inequality holds for a value greater than the upperbound: $h = p_m$. Substituting $h = p_m$ into the inequality and simplifying yields

$$\sqrt{(1-\alpha)\left(\alpha(p_m-s)(p_m+s)+s^2\right)} > (1-\alpha)s.$$

Since $\sqrt{(1-\alpha)\left(\alpha(p_m-s)(p_m+s)+s^2\right)}$ is increasing in p_m, we obtain

$$\sqrt{(1-\alpha)\left(\alpha(p_m-s)(p_m+s)+s^2\right)} > \sqrt{(1-\alpha)\left(\alpha(0-s)(0+s)+s^2\right)} = (1-\alpha)s.$$

This completes the proof. ■