Online Technical Appendix for “Targeted Advertising and Consumer Inferences”

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Contents

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1 Pricing analysis without additional heterogeneity in ϕ_i

This technical online appendix shows the necessity for the additional heterogeneity, introduced in the main paper, "Targeted Advertising and Consumer Inference." In the main paper, we modify the consumer utility to make the pricing analysis more tractable and avoid non-trivial equilibrium results as following:

$$u_{ij} - p_j = \phi_i \left(m_i \cdot v_j \right) - p_j \tag{1}$$

where ϕ_i, drawn from the standard uniform distribution $U[0\,,1]$, is the consumption utility that consumer i receives conditional on having a good match with the product category and the product j (i.e., $m_i \cdot v_j = 1$).

In this note, we show that firm's pricing decision reverts to a trivial two-points case – either one or zero when consumers observe a firm's price after visiting the firm. In particular, we analyze two versions of pricing models. Thus, this note will explain why we used particular modeling choice in our main paper to investigate endogenous pricing. To be more specific, both models in this note are different from the model analyzed in Section 5 of the main paper in that $\phi_i = 1$ for all consumers, and therefore $u_{ij} \in \{0, 1\}$, which is the same as the main model of the paper.

Two models in this note are different from each other in that in the first model firms' prices are not pre-announced, whereas in the second model they are pre-announced. The analysis shows that, in the former, hold-up problem arises, and in equilibrium both firms charge the maximal price $p_A^* = p_B^* = 1$. In the latter, firms engage in a pricing war. Consequently, in equilibrium both firms have incentives to undercut each other until both firms are unable to secure strictly positive profits.

It is also noted that we assume that each firm's realized quality remains unknown to all players, just as we do in our main paper for endogenous pricing. Then, every player of the game has a common prior belief that each firm's expected quality type is 1/2, which is carried throughout the game without any updates.

1.1 Prices are not pre-announced

An equilibrium will consist of each firm j's advertising strategy and pricing strategy (σ_j^*, p_j^*). Consumers observe each firm's price and have rational expectations about each firm's equilibrium advertising strategies. However, they do not observe the total amount of advertising chosen by each firm. Moreover, we assume that without receiving an ad, it is prohibitively costly for consumers to initiate their own search.

Proposition 1 *Suppose that the firms do not announce their prices. Then, in a unique pure-strategy symmetric equilibrium, both firms charge prices* $p_A^* = p_B^* = 1$. *No consumer searches beyond the first firm.*

Proof. Suppose that (p^*, σ^*) is an equilibrium strategy. Consumers do not see the prices until they visit a firm. Given that the prices are symmetric, no satisfied consumers will switch to the second firm.

A firm's advertising strategy is the fraction of perceived good-types and perceived bad-types to be covered with advertisement, i.e., $\sigma_j = (\sigma_j^g, \sigma_j^b)$. Each firm chooses the amount of advertising given the expected strategy chosen by its competitor. Firm A's direct demand given the firm's chosen level of advertisement $\widetilde{\sigma}_j^g$ and $\widetilde{\sigma}_j^b$ is $D_A^{Dir}(p_A, \widetilde{\sigma}_A, ; p^*, \sigma^*) = \frac{\mu_0}{2} \cdot \left[\left(\alpha + (1-\alpha)\mu_0\right) \cdot \widetilde{\sigma}_A^g \cdot (1 - \frac{\sigma_B^{g*}}{2}) + (1-\alpha)(1-\mu_0) \cdot \widetilde{\sigma}_A^b \cdot (1 - \frac{\sigma_B^{b*}}{2})\right] \cdot (1 - p_A)$. Also, the consumer's search decision when she is not satisfied with the first firm's product (i.e.$u_{iB} = 0$) under the endogenous pricing is following: Some consumers may visit firm B first and subsequently decide whether to search for firm A. If $u_{iB} = 0$, then the consumer does not buy the product, and she searches the other firm B if $\Pr[m_i = 1|\theta, u_{iB} = 0] \cdot \frac{1}{2} \max\{0, \phi_i - p_A^*\} - t \geq 0 \Leftrightarrow \phi_i \geq p_A^* + \frac{2t}{\Pr[m_i=1|\theta, u_{iB}=0]}$. On the other hand, if $u_{iB} = 1$, then the consumer buys the product without searching for firm B if $1 - p_B^* \geq \frac{1}{2}(1 - p_A^*) - t \Leftrightarrow p_A^* \geq 2p_B^* - 1 - 2t$. In a symmetric equilibrium with $p_A^* = p_B^*$ this condition does not hold, and therefore consumers who are satisfied with the first firm do not search for the second firm. Therefore, firm A's indirect demand from those who visit firm B first and then search for firm A subsequently is $D_A^{Ind}(p_A; p^*, \sigma^*) = \frac{\mu_0}{2} \cdot \frac{1}{2} \cdot \Big[\left(\alpha + (1-\alpha)\mu_0\right) \cdot \big((1 - \widetilde{\sigma}_A^g)\sigma_B^{g*} \cdot (1 - p_A^* - \frac{2t}{\Pr[m_i=1|\theta^{0,1}, u_{iB}=0]}) + \frac{\widetilde{\sigma}_A^g \cdot \sigma_B^{g*}}{2} \cdot (1 - p_A^* - \frac{2t}{\Pr[m_i=1|\theta^{1,1}, u_{iB}=0]})\big) + (1-\alpha)(1-\mu_0) \cdot \big((1 - \widetilde{\sigma}_A^b)\sigma_B^{b*} \cdot (1 - p_A^* - \frac{2t}{\Pr[m_i=1|\theta^{0,1}, u_{iB}=0]}) + \frac{\widetilde{\sigma}_A^b \cdot \sigma_B^{b*}}{2} \cdot (1 - p_A^* - \frac{2t}{\Pr[m_i=1|\theta^{1,1}, u_{iB}=0]})\big)\Big]$ if $p_A < p_A^* + \frac{2t}{\Pr[m_i=1|\theta, u_{iB}=0]}$, which must hold in equilibrium where $p_A = p_A^*$. However, if $p_A \geq p_A^* + \frac{2t}{\Pr[m_i=1|\theta^{1,1}, u_{iB}=0]}$, then $D_A^{Ind}(p_A; p^*, \sigma^*) = \frac{\mu_0}{2} \cdot \frac{1}{2} \cdot \Big[\left(\alpha + (1-\alpha)\mu_0\right) \cdot \big((1 - \widetilde{\sigma}_A^g)\sigma_B^{g*} + \frac{\widetilde{\sigma}_A^g \cdot \sigma_B^{g*}}{2}\big) + (1-\alpha)(1-\mu_0) \cdot \big((1 - \widetilde{\sigma}_A^b)\sigma_B^{b*} + \frac{\widetilde{\sigma}_A^b \cdot \sigma_B^{b*}}{2}\big)\Big] \cdot (1 - p_A)$.

Firm A's expected direct and indirect demands are computed as following: The demand from consumers who visit firm A first, is $D_A^{dir}(p_A, \tilde{\sigma}_A; p_A^*, \sigma_A^*) =$

$$\mu \cdot \frac{1}{2} \cdot \Bigg(\big(\alpha + (1-\alpha)\mu\big) \cdot \tilde{\sigma}_A^g \cdot (1 - \frac{\sigma_B^{g*}}{2}) + (1-\alpha)(1-\mu) \cdot \tilde{\sigma}_A^b \cdot (1 - \frac{\sigma_B^{b*}}{2}) \Bigg) \cdot \Bigg(\Pr(t_i > p_A - \frac{p_B^* + 1}{2}) + \Pr(t_i \leq p_A - \frac{p_B^* + 1}{2}) \cdot (\frac{1}{2} + \frac{1}{2} \cdot \mathbf{1}(p_A < p_B^*) + \frac{1}{4} \cdot \mathbf{1}(p_A = p_B^*)) \Bigg)$$

If $p_A < p_B^*$, then $t_i > p_A - \frac{p_B^*+1}{2}$ for all $t_i \in [T - \Delta, T + \Delta]$, so the expected direct demand simplies to

$$\mu \cdot \frac{1}{2} \cdot \Bigg(\big(\alpha + (1-\alpha)\mu\big) \cdot \tilde{\sigma}_A^g \cdot (1 - \frac{\sigma_B^{g*}}{2}) + (1-\alpha)(1-\mu) \cdot \tilde{\sigma}_A^b \cdot (1 - \frac{\sigma_B^{b*}}{2}) \Bigg).$$

Firm A's indirect demand from those who visit firm B first and then search for firm A subsequently is $D_A^{ind}(p_A, \tilde{\sigma}_A; \mathbf{p}^*, \sigma^*) =$

$$\mu \Bigg(\big(\alpha + (1-\alpha)\mu\big) \cdot \frac{1}{2} \cdot (1 - \frac{\tilde{\sigma}_A^g}{2}) \sigma_B^{g*} + (1-\alpha)(1-\mu) \cdot \frac{1}{2} \cdot (1 - \frac{\tilde{\sigma}_A^b}{2}) \sigma_B^{b*} \Bigg) \cdot \Bigg(\frac{E(m_i | u_{iB} = 0, a_B) \cdot \frac{1}{2} \cdot (1 - p_A^*) - (T - \Delta)}{2\Delta} + \frac{\frac{1}{2} \cdot (1 - p_A^*) - (1 - p_B^*) - (T - \Delta)}{2\Delta} \Bigg) \cdot \frac{1}{2}.$$

] The expected profit of firm A is defined similarly in (10). The first order conditions are:

$$\frac{\partial \pi_A(p_A, \tilde{\sigma}_A; p^*, \sigma^*)}{\partial p_A} \Bigg|_{p_A = p^*} = 0, \quad \frac{\partial \pi_A(p_A, \tilde{\sigma}_A; p^*, \sigma^*)}{\partial \tilde{\sigma}_A} \Bigg|_{\tilde{\sigma}_A = \sigma^*} = 0.$$

The former condition holds if and only if $D_A^{dir} + D_A^{ind} + p^* \cdot (\frac{\partial D_A^{dir}}{\partial p_A} + \frac{\partial D_A^{ind}}{\partial p_A}) = D_A^{dir} + D_A^{ind} > 0$. The equality is because of the fact that both the direct and indirect demand only depend on firm A's expected price, and not on the actual price charged by the firm. Therefore, firm A has an incentive to deviate and charge a greater price. Therefore, firm A has a profitable deviation to a higher price, and any price $p^* < 1$ cannot be an equilibrium.

So, in a unique equilibrium, both firms charge the price $p^* = 1$. As the entire consumer surplus is extracted, no consumer will search beyond the first firm. ∎

1.2 Prices are pre-announced

If both firms engage in advertising, then there is no equilibrium in which both firms charge the same price.

Proposition 2 *Suppose that the prices are announced. Then, there is no symmetric equilibrium in which firms can obtain positive profits.*

Proof. In order to generate revenue, firms must choose positive level of advertising. Then, consumers who observe both firms' advertisements will choose between two firms based on the price. So, given symmetric prices, each firm has an incentive to deviate to a marginally lower price. This continues until prices reach 0. However, given the positive advertising costs, both firms are not able to obtain positive profits. ■

This is because each firm has a profitable deviation to undercut the other firm and have all consumers who receive ads from both firms visit the firm first. So, we can consider an equilibrium in which both firms engage in advertising and charge different prices. We can also consider an equilibrium in which only one firm engages in advertising.

2 Proposition 3: Comparative statics of λ^{non}

In this section, we explore the comparative results of λ^{non}, which we omit in the main paper for brevity. More precisely, the exact value of λ^{non} depends on the model primitives of k, T and p.

Proposition 3 *Suppose* $k > \max\{\frac{1}{24}, \frac{p\mu_0}{2}(\frac{3}{4} + \frac{T-\Delta}{24\Delta})\}$*. Then,* $\frac{\partial \lambda}{\partial k} \leq 0$*,* $\frac{\partial \lambda}{\partial T} \geq 0$*, and* $\frac{\partial \lambda}{\partial p} \geq 0$*.*

Proof. From the proof of Proposition 4 in the paper, (1) $k > \frac{p\mu_0}{2}(\frac{3}{4} + \frac{T-\Delta}{24\Delta})$ is a sufficient condition for the unique existence of $\lambda^{non} \in (0,1)$, and (2) λ^{non} satisfies the first-order condition: $\Gamma^{non}(\lambda) = 0$, where in Case II,

$$\Gamma^{non}(\lambda) = p\mu_0\left[1 - \frac{\lambda}{4} - \frac{\lambda}{24\Delta}\left(\frac{\mu_0}{3-2\mu_0}\frac{2(1-\lambda)}{3(2-\lambda)}(1-p) - (T-\Delta)\right)\right] - 2k\lambda.$$

For the rest of the proof, we simply denote λ^{non} by λ if there is no confusion.

First, $\frac{d\Gamma^{non}(\lambda)}{dk} = 0$, i.e., $-2\lambda = \xi^{non} \cdot \frac{\partial \lambda}{\partial k}$, where $\xi^{non} := 2k + \frac{1}{4} + \frac{1}{24\Delta}\left(\frac{2\mu_0(1-p)}{3(3-2\mu_0)}\frac{(2-\lambda)^2-2}{(2-\lambda)^2} - (T-\Delta)\right)$. Note that for $\lambda \in [0,1]$, $\frac{(2-\lambda)^2-2}{(2-\lambda)^2} \in [-1, \frac{1}{2}]$. So, $\xi^{non} \geq 0$ if $2k + \frac{1}{4} - \frac{1}{6} > 0$, or equivalently, $k > \frac{1}{24}$. Then, because -2λ is negative and ξ^{non} is positive, this proves $\frac{\partial \lambda}{\partial k} \leq 0$.

Second, $\frac{d\Gamma^{non}(\lambda^{non})}{dT} = 0$, i.e., $\frac{\lambda}{24\Delta} = \xi^{non} \cdot \frac{\partial \lambda}{\partial T}$, and therefore if $k > \frac{1}{24}$, then $\frac{\partial \lambda}{\partial T} \geq 0$.

Third, $\frac{d\Gamma^{non}(\lambda^{non})}{dp} = 0$, i.e., $\mu_0\left[1 - \frac{\lambda}{4} - \frac{\lambda}{24\Delta}\left(\frac{\mu_0}{3-2\mu_0}\frac{2(1-\lambda)}{3(2-\lambda)}(1-p) - (T-\Delta)\right)\right] + p\mu_0\frac{\lambda}{24\Delta}\frac{\mu_0}{3-2\mu_0}\frac{2(1-\lambda)}{3(2-\lambda)} = \xi^{non} \cdot \frac{\partial \lambda}{\partial p}$. Because of the first-order condition, the left-hand side is equivalent to $\frac{2k\lambda}{p} + p\mu_0\frac{\lambda}{24\Delta}\frac{\mu_0}{3-2\mu_0}\frac{2(1-\lambda)}{3(2-\lambda)}$, which is clearly positive. Therefore, $\frac{\partial \lambda}{\partial p} \geq 0$. ■

3 Corollary 1 for all $T \geq \Delta$

Corollary 1 *For all T, if k is sufficiently large and $\mu_0 \geq \frac{1}{2}$, as α becomes high, $\mathbb{E}\Pi^{tar*}(q) \geq \mathbb{E}\Pi^{non*}(q)$.*

Proof. From now on, superscripts *non* and *tar* are shortened by n and t, respectively. The equilibrium profit under non-targeted advertising is $\mathbb{E}\Pi^{n*}(q_A) = p \cdot \mu_0 \cdot q_A \cdot \big[\lambda^n \cdot q_A(1-\mathbb{E}[\frac{\lambda^n q}{2}]) + (1-\lambda^n \cdot q_A) \cdot \mathbb{E}[\lambda^n q(1-q)] \cdot X^n + \frac{\lambda^n \cdot q_A}{2} \cdot \mathbb{E}[\lambda^n q(1-q)] \cdot Y^n\big] - k(\lambda^n q_A)^2$, where $X^n = \max\{\frac{\frac{\mu_0}{3-2\mu_0} \cdot \frac{3-2\lambda^n}{3(2-\lambda^n)}(1-p)-(T-\Delta)}{2\Delta}, 0\}$, $Y^n = \max\{\frac{\frac{\mu_0}{3-2\mu_0} \cdot \frac{2}{3}(1-p)-(T-\Delta)}{2\Delta}, 0\}$, $\mathbb{E}[\frac{\lambda^n q}{2}] = \frac{\lambda^n}{4}$ and $\mathbb{E}[\lambda^n q(1-q)] = \frac{\lambda^n}{6}$. Under targeted advertising, the equilibrium profit is $\mathbb{E}\Pi^{t*}(q_A) = p \cdot \mu_0(\alpha + (1-\alpha)\mu_0) \cdot q_A \cdot \big[\lambda^t \cdot q_A(1-\mathbb{E}[\frac{\lambda^t q}{2}]) + (1-\lambda^t \cdot q_A) \cdot \mathbb{E}[\lambda^t q(1-q)] \cdot X^t + \frac{\lambda^t \cdot q_A}{2} \cdot \mathbb{E}[\lambda^t q(1-q)] \cdot Y^t\big] - k(\mu_0 \lambda^t q_A)^2$, where $X^t = \max\{\frac{\zeta(1-p) \cdot \frac{3-2\lambda^t}{3(2-\lambda^t)} - (T-\Delta)}{2\Delta}, 0\}$ and $Y^t = \max\{\frac{\zeta(1-p) \cdot \frac{2}{3} - (T-\Delta)}{2\Delta}, 0\}$. Note that X^n, Y^n, X^t, $Y^t \in [0,1]$ are fraction of consumers who search beyond the first firm. Furthermore, because of consumers' positive inferences upon being targeted, $X^t \geq X^n$ and $Y^t \geq Y^n$, where equality holds when $\alpha = 0$.

First, suppose $\alpha \to 1$. A sufficient condition for $\mathbb{E}\Pi^{t*}(q_A)|_{\alpha=1} - \mathbb{E}\Pi^{n*}(q_A) \geq 0$ is obtained as follows. The difference can be expressed as $\mathbb{E}\Pi^{t*}(q_A)|_{\alpha=1} - \mathbb{E}\Pi^{n*}(q_A) \geq$ (by replacing X^n and Y^n with bigger terms X^t and Y^t, respectively)

$$
\begin{aligned}
&= -k(q_A)^2(\mu_0\lambda^t - \lambda^n)(\mu_0\lambda^t + \lambda^n) + p\mu_0 q_A(\lambda^t - \lambda^n)\Big[\big(1 - \frac{\lambda^t + \lambda^n}{4}\big)q_A + \big(1 - q_A(\lambda^t + \lambda^n)\big)\frac{X^t}{6} + \big(\lambda^t + \lambda^n\big)\frac{q_A Y^t}{12}\Big] \\
&\geq (\lambda^t - \lambda^n)\Big(-k(q_A)^2(\mu_0\lambda^t + \lambda^n) + p\mu_0 q_A\Big[\big(1 - \frac{\lambda^t + \lambda^n}{4}\big)q_A + \big(1 - q_A(\lambda^t + \lambda^n)\big)\frac{X^t}{6} + \big(\lambda^t + \lambda^n\big)\frac{q_A Y^t}{12}\Big]\Big) \\
&= (\lambda^t - \lambda^n)(\Sigma^t + \Sigma^n),
\end{aligned}
$$

where $\Sigma^t = q_A^2\big(-k\mu_0\lambda^t + p\mu_0\big[1 - \frac{\lambda^t}{4} - \frac{\lambda^t \cdot X^t}{6} + \frac{\lambda^t \cdot Y^t}{12}\big]\big) + p\mu_0 q_A \cdot \frac{X^t}{6}$ and $\Sigma^n = q_A^2\big(-k\lambda^n + p\mu_0\big[1 - \frac{\lambda^n}{4} + \frac{\lambda^n \cdot X^t}{6} + \frac{\lambda^n \cdot Y^t}{12}\big]\big) + p\mu_0 q_A \cdot \frac{X^t}{6}$. The second inequality is obtained by replacing $(\mu_0\lambda^t - \lambda^n)$ in the beginning of the first line with a bigger term $(\lambda^t - \lambda^n)$. Next, sufficient conditions for Σ^t and Σ^n are non-negative are identified. Further note that from Proposition **??**, if k is sufficiently large, then $\lambda^t - \lambda^n > 0$ for $\alpha = 1$, which will show that $\mathbb{E}\Pi^{t*}(q_A)|_{\alpha=1} - \mathbb{E}\Pi^{non*}(q_A) \geq 0$.

The expression $\Sigma^t = q_A^2\big(-k\mu_0\lambda^t + 2k\mu_0^2\lambda^t + \big\{-2k\mu_0^2\lambda^t + p\mu_0\big[1 - \frac{\lambda^t}{4} - \frac{\lambda^t}{6} \cdot X^t + \frac{\lambda^t}{12} \cdot Y^t\big]\big\}\big) + p\mu_0 q_A \cdot \frac{X^t}{6}$, where the expression inside the brackets $\{\cdot\}$ vanishes because of the first-order condition. Note that

X^t is a probability, so $X^t \in [0,1]$. Therefore, by plugging in $X^t = 0$, $\Sigma^t \geq -k\mu_0\lambda^t + 2k\mu_0^2\lambda^t \geq 0$, i.e., $\mu_0 \geq \frac{1}{2}$.

Similarly, $\Sigma^n = q_A^2\big(-k\lambda^n + 2k\lambda^n - 2k\lambda^n + p\mu_0\big[1 - \frac{\lambda^n}{4} + \frac{\lambda^n \cdot X^t}{6} + \frac{\lambda^n \cdot Y^t}{12}\big]\big) + p\mu_0 q_A \cdot \frac{X^t}{6} \geq q_A^2 \cdot k\lambda^n \geq 0$, where the inequality holds by first replacing X^t and Y^t by smaller terms X^n and Y^n, then applying the first-order condition, and finally setting $X^n = 0$ for the very last term outside the parentheses. In particular, if α is sufficiently close to 1 and $\mu_0 \geq \frac{1}{2}$, then $\mathbb{E}\Pi^{t*}(q_A) \geq \mathbb{E}\Pi^{non*}(q_A)$. ■