Technical Appendix: A Model of Two-Sided Costly Communication for Building New Product Category Demand

Michelle Y. Lu and Jiwoong Shin*

November 2016

*Michelle Y. Lu: Assistant Professor of Marketing, McGill University; Jiwoong Shin: Professor of Marketing, Yale School of Management.

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1 Proof of Lemma 4

Lemma 4. *$E\Pi_1^D(d)$ is monotonically decreasing in $d \in [\underline{d}, 1]$ when $k > k^*$.*

Proof. Knowing from the main analysis that $e_1^D(d) = \left(k\pi_f^D(d)\right)^{\frac{1+k}{2}} \left((1-k)\tilde{u}^D(d)\right)^{\frac{1-k}{2}} \cdot \frac{\pi_1^D(d)}{\pi_f^D(d)}$ and $\sigma^{TD}(d) = \left(k\left(\pi_f^D(d)\right)\right)^k \left((1-k)\tilde{u}^D(d)\right)^{1-k}$, we specify the expected payoff under duopoly $E\Pi_1^D(d) = \sigma^{TD}(d)\,\pi_1^D(d) - \frac{1}{2}\left(e_1^D(d)\right)^2 = \left(k\pi_f^D(d)\right)^k \left((1-k)\tilde{u}^D(d)\right)^{1-k} \pi_1^D(d)\left(1 - \frac{k}{2}\frac{\pi_1^D(d)}{\pi_f^D(d)}\right)$. Because $E\Pi_1^D(d) > 0$, the sign of $\frac{\partial E\Pi_1}{\partial d}$ is equivalent to that of its log-transformation, which is $\frac{\partial \log E\Pi_1^D}{\partial d} = \frac{(1-k)}{\tilde{u}^D}\frac{\partial \tilde{u}^D}{\partial d} + \frac{k}{\pi_1^D+\pi_2^D}\frac{\partial\left(\pi_1^D+\pi_2^D\right)}{\partial d} + \frac{1}{\pi_1^D}\frac{\partial \pi_1^D}{\partial d} + \frac{\frac{k}{2}}{1-\frac{k}{2}\frac{\pi_1^D}{\pi_1^D+\pi_2^D}}\frac{\partial}{\partial d}\left(-\frac{\pi_1^D}{\pi_1^D+\pi_2^D}\right)$. We further reduce the above formula to $\frac{\partial \log E\Pi_1^D}{\partial d} = \Psi \cdot G(k,d)$, where $\tilde{m} = (1-d)\,\overline{m}$ and

$$\Psi = \frac{4\overline{m}}{\begin{bmatrix} (2\tilde{m}+3t)\left(4(4-k)\tilde{m}^2 - 12k\tilde{m}t + 9(4-k)t^2\right) \\ \times\left(4\tilde{m}^2+9t^2\right)\left(4\tilde{m}^2 - 36\tilde{m}t + 9t(8v-5t)\right)\end{bmatrix}}$$

$$\begin{aligned} G(k,d) \quad = \quad & k^2 9t\,(2\tilde{m}+3t)^3\left(9t^2 + 4\tilde{m}\,(\tilde{m}+3t-4v)\right) - 32\left(4\tilde{m}^2+9t^2\right)^2\left(\tilde{m}^2 - 6\tilde{m}t + 9t(v-t)\right) + \\ & k\,(2\tilde{m}+3t)\left(32\tilde{m}^2\left(2\tilde{m}^3 + 9t\left(\tfrac{5}{6}\tilde{m}^2 + 2\tilde{m}\,(2t-3v) - 3t^2\right)\right) + 81t^3\left(4\tilde{m}\,(t-12v) - 3t\,(25t-16v)\right)\right) \end{aligned}$$

It is straight that Ψ is positive because $4(4-k)\tilde{m}^2 - 12k\tilde{m}t + 9(4-k)t^2 > 4\tilde{m}^2 - 12\tilde{m}t + 9t^2 = (2\tilde{m}-3t)^2 > 0$ and $4\tilde{m}^2 - 36\tilde{m}t + 9t(8v-5t) = \tilde{u}^D \cdot 72t > 0$ (from the proof of Lemma 1). So $\text{sgn}\left[\frac{\partial \log E\Pi_1^D}{\partial d}\right] = \text{sgn}\left[G(d,k)\right]$. As long as $G(k,d) \le 0$ for all $d \in [\underline{d}, 1]$, $E\Pi_1^D(d)$ monotonically (weakly) decreases in $d \in [\underline{d}, 1]$.

Our task is to find k such that $G(k,d) \le 0$ holds for all $d \in [\underline{d}, 1]$. Notice that given d, $G(k,d)$ is a convex and quadratic function of k and $G(k=1,d) = -6\left(16\tilde{m}^4 + 48\tilde{m}^2t^2 + 27t^4\right) \times \left(4\tilde{m}^2 - 36\tilde{m}t + 9t(8v-5t)\right) < 0$. So there exist two roots $k_1(d)$ and $k_2(d)$ such that $G(k_1,d) = G(k_2,d) = 0$, where $k_1 < 1 < k_2$. And the convexity implies that $G(k,d) \le 0 \Leftrightarrow k \ge k_1(d)$. To have $G(k,d) \le 0$ for every $d \in [\underline{d}, 1]$, k should be greater than $k_1(d)$ for every d, i.e., $k \ge \max_{d\in[\underline{d},1]} k_1(d)$.

We proceed to find $\max_d k_1(d)$. We break down into two cases: the first case is $G(k=0,d) \le 0$ for every $d \in [\underline{d}, 1]$, which means $k_1(d) \le 0$ always holds; whereas the second case pertains $G(k=0,d) > 0$ for some d, which means $k_1(d) > 0$ can happen.

In the first case, as $k \ge 0 \ge \max_{d\in[\underline{d},1]} k_1(d)$, $G(k,d) \le 0$ holds for all d and k. Thus, $E\Pi_1^D(d)$ is monotonically (weakly) decreasing in $d \in [\underline{d}, 1]$ under any k. Then Lemma 4 immediately follows. This case happens when $t \le \frac{4v}{7}$ because $\text{sgn}\left[G(k=0,d)\right] = \text{sgn}\left[9t\,(2t-v) - (3t - \tilde{m}(d))^2\right]$ and $9t\,(2t-v) - (3t-\tilde{m}(d))^2 \le 9t\,(2t-v) - (3t - \tilde{m}(\underline{d}))^2 = \frac{9}{4}t(7t-4v) \le 0$, which reduces to $t \le \frac{4v}{7}$.

When $t > \frac{4v}{7}$, the second case arises such that $k_1(d) > 0$. To find the the value of $\max_{d\in[\underline{d},1]} k_1(d)$, we show that $k_1(d)$ strictly decreases in $d \in [\underline{d}, 1]$. But we defer the proof in the next section. So $k_1(d)$ is maximized at $\underline{d}$, which is $k_1(\underline{d}) = \frac{4v-7t}{3t-4v} \equiv \underline{k}$. Hence, as long as $k \ge \underline{k} \Leftrightarrow G(k,d) \le 0$ holds for all $d \in [\underline{d}, 1]$, which means $E\Pi_1^D$ is monotonically (weakly) decreasing in $d \in [\underline{d}, 1]$. The last step to get Lemma 4 is to verify $k^* > \underline{k}$ when $t > \frac{4v}{7} \Leftrightarrow \frac{v}{t} < \frac{7}{4}$. We denote $\frac{v}{t}$ as $v^{'}$ hereafter and note that $v^{'} \in \left(\frac{3}{2}, \frac{7}{4}\right)$. We prove $k^* > \underline{k}$ for all $v^{'} \in \left(\frac{3}{2}, \frac{7}{4}\right)$ by showing $k^*\left(v^{'}\right) > k^*\left(7/4\right) > \underline{k}\left(3/2\right) > \underline{k}\left(v^{'}\right)$. Recall $k^* = \left(\ln \frac{\tilde{u}^D(\underline{d})}{\tilde{u}^M} - \ln \frac{\pi_1^M}{\pi_1^D(\underline{d})}\right) / \left(\ln \frac{\tilde{u}^D(\underline{d})}{\tilde{u}^M} + \ln \frac{\pi_1^M}{\pi_1^D(\underline{d})}\right) = \left(\ln\left(4\frac{v}{t} - 5\right) - \ln\left(\frac{v}{t} - \frac{1}{2}\right)\right) / \left(\ln\left(4\frac{v}{t} - 5\right) + \ln\left(\frac{v}{t} - \frac{1}{2}\right)\right) = -1 + 2/\left(1 + \ln\left(v^{'} - \frac{1}{2}\right)/\ln\left(4v^{'} - 5\right)\right)$. We find k^* decreases in $v^{'}$ because $\mathcal{L} \equiv \ln\left(v^{'} - \frac{1}{2}\right)/\ln\left(4v^{'} - 5\right)$ increases in $v^{'}$. To see this, we check the derivative w.r.t. $v^{'}$: $\frac{\partial \mathcal{L}}{\partial v^{'}} = \frac{\left(v^{'} - 5/4\right)\ln\left(4v^{'} - 5\right) - \left(v^{'} - 1/2\right)\ln\left(v^{'} - 1/2\right)}{\left(v^{'} - 5/4\right)\left(v^{'} - 1/2\right)\left(\ln\left(4v^{'} - 5\right)\right)^2}$. Due to the positivity of the denominator, $\frac{\partial \mathcal{L}}{\partial v^{'}} > 0 \Leftrightarrow \left(v^{'} - 5/4\right)\ln\left(4v^{'} - 5\right) > \left(v^{'} - 1/2\right)\ln\left(v^{'} - 1/2\right)$. It is straightforward to check the both sides of the above inequality are increasing in $v^{'}$ and $LHS \geq RHS$ holds at the two extremes $v^{'} = 3/2$ and $v^{'} = 7/4$. So $\frac{\partial \mathcal{L}}{\partial v^{'}} > 0$, and k^* decreases in $v^{'}$. Then $k^*\left(v^{'}\right) > k^*\left(7/4\right) = \ln\frac{8}{5}/\ln\frac{5}{2}$. Also notice that $\underline{k} = \frac{4v - 7t}{3t - 4v} = \frac{1 - \frac{4}{7}v^{'}}{\frac{4}{7}v^{'} - \frac{3}{7}}$ monotonically decreases in $v^{'}$. So $\underline{k}\left(v^{'}\right) < \underline{k}\left(3/2\right) = 1/3$. Hence, $k^* > \ln\frac{8}{5}/\ln\frac{5}{2} > 1/3 > \underline{k}$. Then we can conclude $E\Pi_1^D(d)$ indeed monotonically decreases in $d \in [\underline{d}, 1]$ when $k > k^*$. $\square$

2 Proof of Lemma 5

Lemma 5. *When $t > \frac{4v}{7}$, $k_1(d)$ is strictly decreasing in $d \in [\underline{d}, 1]$. Moreover, $k_1(\underline{d}) = \underline{k} \in (0,1)$ and $k_1(1) < 0$.*

Proof. We suppress function G as $A_1k^2+A_2k+A_3$, where $A_1 = 9t(2\tilde{m}+3t)^3\left(9t^2+4\tilde{m}(\tilde{m}+3t-4v)\right)$, $A_2 = (2\tilde{m}+3t)\left(32\tilde{m}^2\left(2\tilde{m}^3+9t\left(\frac{5}{6}\tilde{m}^2+2\tilde{m}(2t-3v)-3t^2\right)\right)+81t^3\left(4\tilde{m}(t-12v)-3t(25t-16v)\right)\right)$, and $A_3 = -32\left(4\tilde{m}^2+9t^2\right)^2\left(\tilde{m}^2-6\tilde{m}t+9t(v-t)\right)$. Simply, $k_1(d) = -{}^{A_2}/_{(2A_1)} - \sqrt{\left({}^{A_2}/_{(2A_1)}\right)^2 - {}^{A_3}/_{A_1}}$.

We examine how A_1, A_2, and A_3 changes with d when $t > \frac{4v}{7}$. First, $\frac{\partial A_1}{\partial d} = 18\overline{m}t(2\tilde{m}+3t)^2 \cdot a_1(\tilde{m})$, where $a_1(\tilde{m}) \equiv 20\tilde{m}^2+4\tilde{m}(15t-16v)-9t^2-24tv$. Notice that a_1 increases in $\tilde{m} \in \left[0, \frac{3t}{2}\right]$, $\frac{\partial A_1}{\partial d} \leq 18\overline{m}t(2\tilde{m}+3t)^2 \cdot a_1\left(\frac{3t}{2}\right) = 18\overline{m}t(2\tilde{m}+3t)^2 \cdot 6t(21t-20v)$, which is negative under the assumption $v > \frac{3t}{2}$. So A_1 strictly decreases in $d \in [\underline{d}, 1]$.

Second, $\frac{\partial A_2}{\partial d} = 27t^2\left(32\tilde{m}^2(3v-t)+24\tilde{m}t(t+4v)+3t^2(23t+8v)\right) + 96\overline{m}\tilde{m}^3 \cdot a_2(\tilde{m})$, where $a_2(\tilde{m}) = -8\tilde{m}^2 - 35\tilde{m}t + 18t(8v-7t)$. The first argument is obviously positive. We examine the second argument. As $a_2(\tilde{m})$ decreases in $\tilde{m} \in \left[0, \frac{3t}{2}\right]$, $96\overline{m}\tilde{m}^3 \cdot a_2(\tilde{m}) \geq 96\overline{m}\tilde{m}^3 \cdot a_2\left(\frac{3t}{2}\right) = 96\overline{m}\tilde{m}^3 \cdot \frac{3}{2}t(96v-131t) > 0$ under the assumption $v > \frac{3t}{2}$. Hence, $\frac{\partial A_2}{\partial d} > 0$.

Lastly, $\frac{\partial A_3}{\partial d} = 64\overline{m}\left(4\tilde{m}^2+9t^2\right)\left(12\tilde{m}^2\left(\tilde{m}-\frac{3t}{2}\right)+3t \cdot a_3(\tilde{m})\right)$, where $a_3(\tilde{m}) = 2\tilde{m}(12v-7\tilde{m}) - 3t(7\tilde{m}+3t)$. We show $a_3(\tilde{m}) < 0 \Leftrightarrow 12v - 7\tilde{m} < {}^{21t}/_2 + {}^{9t^2}/_{(2\tilde{m})}$. Both $LHS = 12v - 7\tilde{m}$ and $RHS = {}^{21t}/_2 + {}^{9t^2}/_{(2\tilde{m})}$ are decreasing in $\tilde{m} \in \left[0, \frac{3t}{2}\right]$. At the extreme point $\tilde{m} = 0$ (i.e., $d = 1$), $LHS = 12v$ and $RHS \to \infty$. So $RHS > LHS$. At the other extreme point $\tilde{m} = \frac{3t}{2}$ (i.e., $d = \underline{d}$), $LHS = \frac{3}{2}(8v-7t)$ and $RHS = \frac{27t}{2}$. $RHS > LHS$ holds under $t > \frac{4v}{7}$. So $RHS > LHS \Leftrightarrow a_3(\tilde{m}) < 0$. So $12\tilde{m}^2\left(\tilde{m}-\frac{3t}{2}\right)+3t \cdot a_3(\tilde{m}) < 0$. Therefore, $\frac{\partial A_3}{\partial d} < 0$.

Hence, the two arguments $\frac{A_2}{2A_1}$ and $\frac{-A_3}{A_1}$ both strictly increase in $d \in [\underline{d}, 1]$, and thereby k_1 strictly decreases in $d \in [\underline{d}, 1]$.

From the proof of Lemma 4, we already know: (1) $\underline{k} \equiv k_1(\underline{d}) = \frac{4v-7t}{3t-4v} \in (0,1)$ when $t > \frac{4v}{7}$; (2) The statement $k_1(1) < 0$ is equivalent to $G(k, d=1) < 0$. To complete the proof, we check $G(k, d=1) = 729t^5 \cdot (3tk(k-3) + 16(v-t)(k-2)) < 0$ for all $k \in [0,1]$. □

3 Full sequential case – firms move sequentially

In this section, we consider a game where even firms move sequentially (i.e., the innovator chooses effort e_1 first, then the follower chooses its effort e_2 upon observing e_1), and finally consumers choose effort e_c upon observing e_1 and e_2. We call it as "the full sequential case" to contrast the sequential case that we analyze in our main text (i.e., the game in which both firms move simultaneously and then consumers move after observing e_1, e_2), which we call "the partial sequential case." There are two goals that we intend to accomplish here: we want to show: (1) the amplified complementarity effect described in the main text is mitigated when firms are considered to move sequentially; (2) nevertheless, the main result that duopoly leads to larger communication efforts when consumers play important role in communication (small k) still holds;

Proposition A1. *The amplified complementarity effect described in the main text (in Section 5.1.) is mitigated under the full sequential case compared to the partial sequential:* $e_f^{D-full} \leq e_f^{D-partial}$ *and* $e_c^{D-full} \leq e_c^{D-partial}$.

Proof. Consumers maximize their expected utility, $EU = e_f^k \cdot e_c^{1-k} \cdot \tilde{u} - \frac{e_c^2}{2}$ upon observing a given effort e_f $(= e_1 + e_2)$. Hence, consumers' best reply function is given by $e_c^D = \left((1-k)\,\tilde{u} {e_f}^k\right)^{\frac{1}{k+1}}$ (still the same as the simultaneous case). Anticipating the consumers' best response, firm 2 solves the following maximization problem:$\max_{e_2} E\Pi_2 = \sigma\left(e_f, e_c^D\right) \cdot \pi_2^D - \frac{e_2^2}{2}$. So, e_2^D solves the following first-order condition:$\left(\frac{\partial\sigma}{\partial e_2} + \frac{\partial\sigma}{\partial e_c} \cdot \frac{\partial e_c^D}{\partial e_2}\right)\pi_2^D = e_2$, which implicitly determines firm 2's best response to firm 1's effort. Anticipating firm 2's best reply and consumers' best response, firm 1 solves the following maximization problem:$\max_{e_1} E\Pi_1 = \sigma\left(e_1 + e_2^D, e_c^D\right) \cdot \pi_1^D - \frac{e_1^2}{2}$. So, e_1^D solves the following first-order condition:$\left(\frac{\partial\sigma}{\partial e_1} + \frac{\partial\sigma}{\partial e_2}\frac{\partial e_2^D}{\partial e_1} + \frac{\partial\sigma}{\partial e_c} \cdot \frac{\partial e_c^D}{\partial e_1}\right)\pi_1^D = e_1$. Note that $\frac{\partial\sigma}{\partial e_1} = \frac{\partial\sigma}{\partial e_2} = \frac{\partial\sigma}{\partial e_f}$, we can add up the first-order conditions of the two firms and get $e_f\,(= e_1 + e_2)$ in the full sequential communication:

$$\underbrace{\left(\frac{\partial\sigma}{\partial e_f} + \frac{\partial\sigma}{\partial e_c} \cdot \frac{\partial e_c^D}{\partial e_f}\right)\left(\pi_1^D + \pi_2^D\right)}_{\text{partial seq. comm.}} + \frac{\partial\sigma}{\partial e_2}\frac{\partial e_2^D}{\partial e_1}\pi_1^D = e_f \tag{1}$$

Next, we show that $\frac{\partial e_2^D}{\partial e_1} \leq 0$ such that strategic free-riding effect also gets amplified in the full sequential communication setting. Therefore, from the above equation, we can see this explicit free-riding effect mitigates the amplified complementarity effect in the partial sequential communication, which leads to lower effort level compared to the partial sequential communication setting. To show $\frac{\partial e_2^D}{\partial e_1} \leq 0$, we define a function $F = \left(\frac{\partial\sigma}{\partial e_2} + \frac{\partial\sigma}{\partial e_c} \cdot \frac{\partial e_c^D}{\partial e_2}\right)\pi_2^D - e_2$ and the first-order condition of e_2^D is just $F\left(e_1, e_2^D\left(e_1\right)\right) = 0$. By the Implicit Function Theorem, $\frac{\partial e_2^D}{\partial e_1} = -\frac{\partial F}{\partial e_1} / \frac{\partial F}{e_2}\ |_{e_2 = e_2^D}$. Because $\frac{\partial F}{\partial e_2}$ is the second-order condition, which should be negative at the maximum $e_2 = e_2^D$ according to the optimality principal. Therefore, $\operatorname{sgn}\left[\frac{\partial e_2^D}{\partial e_1}\right] = \operatorname{sgn}\left[\frac{\partial F}{\partial e_1}\ |_{e_2 = e_2^D}\right]$. We specify $\frac{\partial F}{\partial e_1}$ in more details: $\frac{\partial F}{\partial e_1} = \left(\frac{\partial^2\sigma}{\partial e_2 \partial e_1} + \partial\left(\frac{\partial\sigma}{\partial e_c}\cdot\frac{\partial e_c^D}{\partial e_2}\right)/\partial e_1\right)\pi_2^D$. The first argument $\frac{\partial^2\sigma}{\partial e_2\partial e_1} = k\,(k-1)\,e_f^{k-2}e_c^{1-k} \leq 0$. Knowing $\frac{\partial\sigma}{\partial e_c}\cdot\frac{\partial e_c^D}{\partial e_2} = \frac{k(1-k)}{1+k}\cdot((1-k)\,\tilde{u})^{\frac{1-k}{1+k}}\cdot e_f^{\frac{k-1}{k+1}}$, we get the second argument $\partial\left(\frac{\partial\sigma}{\partial e_c}\cdot\frac{\partial e_c^D}{\partial e_2}\right)/\partial e_1 = -\frac{k(1-k)^2}{(1+k)^2}\,((1-k)\,\tilde{u})^{\frac{1-k}{1+k}}\,e_f^{\frac{-2}{k+1}} \leq 0$. Hence, $\operatorname{sgn}\left[\frac{\partial F}{\partial e_1}\right] = \operatorname{sgn}\left[\frac{\partial e_2^D}{\partial e_1}\right] \leq 0$. Denote e_f^{D-full} and $e_f^{D-partial}$ as total firm-side effort level in full and partial sequential communication cases. Therefore, $e_f^{D-full} \leq e_f^{D-partial}$. In both full and partial sequential cases, $e_c^D = \left((1-k)\,\tilde{u}e_f{}^k\right)^{\frac{1}{k+1}}$. Therefore, $e_c^{D-full} \leq e_c^{D-partial}$. □

Proposition A2. *Compared to the simultaneous case, under the full sequential case, the duopoly market leads to higher total firm-side efforts and consumer efforts when k is small.*

Proof. Recall the first-order condition determining the total firm-side effort in the simultaneous communication case $\frac{\partial\sigma}{\partial e_f}\left(\pi_1^D + \pi_2^D\right) = e_f$ and rearrange equation 1 into the following:

$$\underbrace{\frac{\partial\sigma}{\partial e_f}\left(\pi_1^D + \pi_2^D\right)}_{\text{simultaneous comm.}} + \left(\frac{\partial\sigma}{\partial e_c}\underbrace{\frac{\partial e_c^D}{\partial e_f}}_{\geq 0}\left(\pi_1^D + \pi_2^D\right) + \frac{\partial\sigma}{\partial e_2}\underbrace{\frac{\partial e_2^D}{\partial e_1}}_{\leq 0}\pi_1^D\right) = e_f$$

Compared to the simultaneous communication setting, the full sequential communication setting leads to higher effort level when the positive complementarity effect (i.e., $\frac{\partial\sigma}{\partial e_c}\frac{\partial e_c^D}{\partial e_f}\left(\pi_1^D + \pi_2^D\right)$) dominates the negative free-riding effect (i.e., $\frac{\partial\sigma}{\partial e_2}\frac{\partial e_2^D}{\partial e_1}\pi_1^D$). We show one sufficient condition can be $\frac{1-k}{1+k} \geq \frac{\pi_1^D}{\pi_1^D+\pi_2^D}$, which implies that $k \leq \frac{\pi_2^D}{2\pi_1^D+\pi_2^D}$. To show it, we specify the complementarity effect as $\frac{\partial\sigma}{\partial e_c}\frac{\partial e_c^D}{\partial e_f}\left(\pi_1^D + \pi_2^D\right) = \frac{k(1-k)}{1+k}\cdot((1-k)\,\tilde{u})^{\frac{1-k}{1+k}}\cdot e_f^{\frac{k-1}{1+k}}\left(\pi_1^D + \pi_2^D\right)$ and the free-riding effect as $\frac{\partial\sigma}{\partial e_2}\frac{\partial e_2^D}{\partial e_1}\pi_1^D = k\cdot((1-k)\,\tilde{u})^{\frac{1-k}{1+k}}\cdot e_f^{\frac{k-1}{1+k}}\pi_1^D\cdot -\frac{\partial F}{\partial e_1}/\frac{\partial F}{\partial e_2}$, where $F = \left(\frac{\partial\sigma}{\partial e_2} + \frac{\partial\sigma}{\partial e_c}\cdot\frac{\partial e_c^D}{\partial e_2}\right)\pi_2^D - e_2$ is defined in the proof of Proposition A1. Hence, the condition for the non-negative total effect $\frac{\partial\sigma}{\partial e_c}\frac{\partial e_c^D}{\partial e_f}\left(\pi_1^D + \pi_2^D\right) + \frac{\partial\sigma}{\partial e_2}\frac{\partial e_2^D}{\partial e_1}\pi_1^D \geq 0 \Leftrightarrow \frac{1-k}{1+k}\left(\pi_1^D + \pi_2^D\right) - \pi_1^D\cdot\frac{\partial F}{\partial e_1}/\frac{\partial F}{\partial e_2} \geq 0 \Leftrightarrow \frac{1-k}{1+k} \geq \left(\frac{\partial F}{\partial e_1}/\frac{\partial F}{\partial e_2}\right)\cdot\frac{\pi_1^D}{\pi_1^D+\pi_2^D}$. Furthermore, $\frac{\partial F}{\partial e_2} = \left(\frac{\partial^2\sigma}{\partial e_2\partial e_2} + \partial\left(\frac{\partial\sigma}{\partial e_c}\cdot\frac{\partial e_c^D}{\partial e_2}\right)/\partial e_2\right)\pi_2^D - 1$ and $\frac{\partial F}{\partial e_1} = \left(\frac{\partial^2\sigma}{\partial e_2\partial e_1} + \partial\left(\frac{\partial\sigma}{\partial e_c}\cdot\frac{\partial e_c^D}{\partial e_2}\right)/\partial e_1\right)\pi_2^D$, where $\frac{\partial\sigma}{\partial e_c}\cdot\frac{\partial e_c^D}{\partial e_2} = \frac{k(1-k)}{1+k}\cdot((1-k)\,\tilde{u})^{\frac{1-k}{1+k}}\cdot e_f^{\frac{k-1}{k+1}}$. Since both $\sigma = e_f^k e_c^{1-k}$ and $\frac{\partial\sigma}{\partial e_c}\cdot\frac{\partial e_c^D}{\partial e_2}$ are functions of e_f and $e_f = e_1 + e_2$, $\frac{\partial F}{\partial e_2} = \left(\frac{\partial^2\sigma}{\partial e_2\partial e_2} + \partial\left(\frac{\partial\sigma}{\partial e_c}\cdot\frac{\partial e_c^D}{\partial e_2}\right)/\partial e_2\right)\pi_2^D - 1 = \left(\frac{\partial^2\sigma}{\partial e_2\partial e_1} + \partial\left(\frac{\partial\sigma}{\partial e_c}\cdot\frac{\partial e_c^D}{\partial e_2}\right)/\partial e_1\right)\pi_2^D - 1 = \frac{\partial F}{\partial e_1} - 1$. So, $\frac{\partial F}{\partial e_1}/\frac{\partial F}{\partial e_2} = \frac{\partial F}{\partial e_1}/\left(\frac{\partial F}{\partial e_1} - 1\right) = 1 + 1/\left(\frac{\partial F}{\partial e_1} - 1\right) < 1$ as $\frac{\partial F}{\partial e_1} \leq 0$ (from the proof of Proposition A1). Therefore, when $\frac{1-k}{1+k} \geq \frac{\pi_1^D}{\pi_1^D+\pi_2^D} \Leftrightarrow k \leq \frac{\pi_2^D}{2\pi_1^D+\pi_2^D}$, $\frac{1-k}{1+k} \geq \left(\frac{\partial F}{\partial e_1}/\frac{\partial F}{\partial e_2}\right)\cdot\frac{\pi_1^D}{\pi_1^D+\pi_2^D}$ holds and thereby the positive complementarity effect (i.e., $\frac{\partial\sigma}{\partial e_c}\frac{\partial e_c^D}{\partial e_f}\left(\pi_1^D + \pi_2^D\right)$) dominates the negative free-riding effect (i.e., $\frac{\partial\sigma}{\partial e_2}\frac{\partial e_2^D}{\partial e_1}\pi_1^D$), which leads to a higher effort level in the full sequential communication compared to the simultaneous setting.

The proposition suggests that the total efforts of firms and consumers can be higher under the full sequential case compared to the simultaneous case when $k$is small. When k is small, the innovator would choose to share the information under the simultaneous case. Then, it must be the case that he also prefers sharing the innovation under the full sequnetial case becasue the market expansion effect through communication is amplified. Thus, the main result that duopoly leads to larger communication efforts when consumers play important role in communication (small k) still holds. $\square$

4 Disclose as entry deterrence

We first analyze the last stage. Upon the reach of the last stage, there are three possible market scenarios. First, if firm 1 discovers an idea at Stage 1 ($I = 1$) and firm 2 does not enter the market at Stage 2 ($\chi = 0$), the outcome is the same as the monopoly situation in the main model where firm 1 is the monopolist and its retail profit is $R_1^M = \sigma^T \cdot \pi_1^M$, where $\pi_1^M = v - \frac{t}{2}$.

Second, if firm 1 fails to discover an idea at Stage 1 ($I = 0$) and firm 2 enters the market at Stage 2 ($\chi = 1$), firm 2 is the monopolist in the market. Similar to the main model, consumer j purchases from firm 2 if the utility $v - t\left(\frac{1}{2} - z_j\right) - p_2 \geq 0 \Rightarrow z_j \geq \frac{2p_2+t-2v}{2t}$. The market demand for firm 2's product is $D_2(v, p_2) = \sigma^T \cdot \max\left\{2 \cdot \frac{v-p_2}{t}, 1\right\}$. The optimal price from F.O.C. is $p_2^{opt} = \arg\max_{p_2} \sigma^T \cdot (p_2 - m_2) \cdot 2 \cdot \frac{v-p_2}{t} = \frac{v+m_2}{2}$. At $p_2^{opt} = \frac{v+m_2}{2}$, $D_2^{opt} = \sigma^T \cdot \frac{v-m_2}{t}$.[1] If $m_2 < v - t$, however, $D_2^{opt} = \sigma^T \cdot \frac{v-m_2}{t} > \sigma^T$, which means even the consumer with furthest location $z_j = 0$ finds the positive utility under this price: $u_j = v - \frac{t}{2} - \frac{v+m_2}{2} = \frac{1}{2}(v - t - m_2) > 0$. Hence, the equilibrium price is determined by making the furthest consumer indifferent between purchase and no purchase: $u_j = v - \frac{t}{2} - p_2 = 0 \Rightarrow p_2^M = v - \frac{t}{2}$, and the market demand is $D_2 = \sigma^T$. Firm 2's monopoly retail profit $R_2^M = \sigma^T \cdot \pi_2^M$, where $\pi_2^M = v - \frac{t}{2} - m_2 > 0$. If $m_2 \geq v - t$, $D_2^{opt} = \sigma^T \cdot \frac{v-m_2}{t} \leq \sigma^T$. So firm 2 optimally charges a price at $p_2^M = \frac{v+m_2}{2}$ and earns a monopoly profit $R_2^M = \sigma^T \cdot \pi_2^M$, where $\pi_2^M = \left(p_2^M - m_2\right) \cdot \frac{v-m_2}{t} = \frac{(v-m_2)^2}{2t}$.

Third, if firm 1 discovers an idea and enters the market at Stage 1 ($I = 1$) and firm 2 enters the market at Stage 2 ($\chi = 1$), similar to the main model, the competitive prices are $p_1^D = \frac{t}{2} + \frac{\tilde{m}_2}{3}$, $p_2^D = \frac{t}{2} + \frac{2\tilde{m}_2}{3}$, where $\tilde{m}_2 = \min\{m_2, \overline{m}(1-d)\}$. For firm 2 to make non-negative retail profit at the competitive prices in the duopoly market, $p_2^D - \tilde{m}_2 \geq 0 \Rightarrow \tilde{m}_2 \leq \frac{3t}{2}$. So, if $\tilde{m}_2 \leq \frac{3t}{2}$, the two firms' retail profits are thereby $R_1^D = \sigma^T \cdot \pi_1^D$ and $R_2^D = \sigma^T \cdot \pi_2^D$, where $\pi_1^D = \frac{(3t+2\tilde{m}_2)^2}{36t}$ and $\pi_2^D = \frac{(3t-2\tilde{m}_2)^2}{36t}$. If $\tilde{m}_2 > \frac{3t}{2}$, firm 2's net profit margin is negative at the competitive prices. To earn a non-negative net margin, firm 2 can only charge a price higher than $\tilde{m}_2$. For any price firm 2 can charge $p_2 \in [\tilde{m}_2, v]$, firm 1's competitive response is $p_1^{BR} = \arg\max \sigma^T \cdot p_1 D_1 = \sigma^T \cdot p_1\left(\frac{1}{2} - \frac{p_1-p_2}{t}\right) = \frac{1}{4}(2p_2 + t)$. However, $D_1\left(p_1^{BR}, p_2\right) = \frac{2p_2+t}{4t} \geq \frac{2\tilde{m}_2+t}{4t} > 1$, which means the furthest consumer $z_j = \frac{1}{2}$ receives positive utility at the price p_1^{BR}: $u_j = v - \frac{t}{2} - \frac{1}{4}(2p_2 + t) \geq v - \frac{t}{2} - \frac{2v+t}{4} = \frac{2v-3t}{4} > 0$. So firm 1 can raise price above p_1^{BR}. Note one thing different from the first scenario lies in that firm 2 has entered the market in this scenario and the lowest price firm 2 can charge is at the marginal cost $\tilde{m}_2$. So firm 1 is unable to charge the same monopoly price as in the first scenario. Instead, the best price firm 1 can charge is the one that makes the furthest consumer $z_j = \frac{1}{2}$ indifferent between purchasing from him and purchasing from firm 2 at the price $p_2 = \tilde{m}_2$, i.e., $v - \frac{1}{2}t - p_1 = v - \tilde{m}_2 \Rightarrow p_1^D = \tilde{m}_2 - \frac{t}{2}$. Therefore, if $\tilde{m}_2 > \frac{3t}{2}$, the equilibrium prices are $p_1^D = \tilde{m}_2 - \frac{t}{2}$ and $p_2^D \in [\tilde{m}_2, v]$. Firm 1 captures all

[1] To ensure firm 2's demand as a monopolist is non-negative, we assume $v > \overline{m}$.

the market demand and earns a profit $R_2^D = \sigma^T \cdot \pi_1^D$, where $\pi_1^D = \tilde{m}_2 - \frac{t}{2}$; whereas firm 2's demand and profit are zero.

In summary, there are three possible market scenarios: (1) Firm 1 is the monopolist firm ($I = 1,\ \chi = 0$). Firm 1's retail profit is

$$R_1^M = \sigma^T \cdot \pi_1^M = \sigma^T \left(v - \frac{t}{2}\right)$$

(2) Firm 2 is the monopolist firm ($I = 0,\ \chi = 1$). Depending on the realization of firm 2's marginal cost, firm 2's retail profit is

$$R_2^M = \sigma^T \cdot \pi_2^M = \begin{cases} \sigma^T \left(v - \frac{t}{2} - m_2\right) & \text{...if } m_2 < v - t \\ \sigma^T \frac{(v-m_2)^2}{2t} & \text{...if } m_2 \geq v - t \end{cases}$$

(3) Both firms enter the market ($I = 1,\ \chi = 1$). Depending on firm 2's actual marginal cost $\tilde{m}_2 = \min\{m_2, \overline{m}(1-d)\}$, the two firms' retail profits are

$$R_1^D = \sigma^T \pi_1^D = \begin{cases} \sigma^T \frac{(3t+2\tilde{m}_2)^2}{36t} & \text{...if } \tilde{m}_2 \leq \frac{3t}{2} \\ \sigma^T \left(\tilde{m}_2 - \frac{t}{2}\right) & \text{...if } \tilde{m}_2 > \frac{3t}{2} \end{cases}$$

$$R_2^D = \sigma^T \pi_2^D = \begin{cases} \sigma^T \frac{(3t-2\tilde{m}_2)^2}{36t} & \text{...if } \tilde{m}_2 \leq \frac{3t}{2} \\ 0 & \text{...if } \tilde{m}_2 > \frac{3t}{2} \end{cases}$$

Next, we analyze stage 2 – the communication stage. Since there is no uncertainty at the start of this stage, the communication subgame unfolds in the same way as in the main model. We directly apply the results from the main model. The total market sizes are given by: $\sigma^T = (k\pi_f)^k ((1-k)\tilde{u})^{1-k}$, where π_f denotes the aggregate average retail profits of the firms who exert positive communication efforts. More specifically, if firm i is the monopolist in the market or firm i is the only firm exerting positive effort in the duopoly market, $\pi_f = \pi_i$; if both firms exert positive efforts in the duopoly market, $\pi_f = \pi_1^D + \pi_2^D$. Given optimal communication efforts, firm i's expected payoff is thereby $E\Pi_i = (k\pi_f)^k ((1-k)\tilde{u})^{1-k} \pi_i \left(1 - \frac{k}{2}\frac{\pi_i}{\pi_f}\right)$.

4.1 Firm 2's entry decision

At the end of stage 1, firm 2 observes firm 1's disclosure decision (ϕ, d) and its marginal production cost m_2, and makes the entry decision. The entry decision depends on its posterior belief of the existence of firm 1's idea $\mu_2(I|\phi)$ and its actual marginal production cost $\tilde{m}_2 = \min\{m_2, \overline{m}(1-d)\}$. To understand firm 2's entry strategy, we derive the following lemma.

Lemma A1. *(Firm 2's entry strategy)*

- *If* $\mu_2(I=1|\phi) \le \overline{\mu}$ *, firm 2 always prefers to enter the market irrespective of its marginal cost* m_2*, i.e.,* $\chi(\mu_2=\overline{\mu}, m_2) = \chi(\mu_2=0, m_2) = 1$ *for all* $m_2 \in \{\underline{m}, \overline{m}\}$*.*

- *If* $\mu_2(I=1|\phi) = 1$*, firm 2 prefers to enter the market only when its actual marginal cost is lower than* $\frac{3t}{2}$*, i.e.,* $\chi(\mu_2=1, \tilde{m}_2) = 1$ *for* $\tilde{m}_2 \le \frac{3t}{2}$*; otherwise,* $\chi(\mu_2=1, \tilde{m}_2) = 0$ *for* $\tilde{m}_2 > \frac{3t}{2}$*.*

Proof. With belief $\mu_2(I=1|\phi) \le \overline{\mu} < 1$, firm 2 prefers to enter the market even when $m_2 = \overline{m}$ because firm 2's expected payoff by entering the market is $E\Pi_2(\mu_2, m_2) = \mu_2 \cdot 0 + (1-\mu_2) \cdot \left(k\pi_2^M\right)^k \left((1-k)\,\tilde{u}_2^M\right)^{1-k} \pi_2^M \left(1-\frac{k}{2}\right) > f$, where f represents the fixed entry cost that we impose to be zero in the main model. When $m_2 = \underline{m}$, firm 2 expects an even higher payoff because it will earn the positive profit irrespective of firm 1's presence in the market, and thereby enters the market as well.

With belief $\mu_2(I=1|\phi) = 1$, if firm 2's actual marginal cost $\tilde{m}_2 > \frac{3t}{2}$, it is unable to make sales and positive profit by entering the market. So, $\chi(\mu_2, \tilde{m}_2) = 0$. Only when $\tilde{m}_2 \le \frac{3t}{2}$ can firm 2 expect a profitable entry$E\Pi_2^D = \left(k\left(\pi_f^D\right)\right)^k \left((1-k)\,\tilde{u}^D\right)^{1-k} \pi_2^D \left(1-\frac{k}{2}\frac{\pi_2^D}{\pi_f^D}\right) > f$. So, $\chi(\mu_2, \tilde{m}_2) = 1$ when $\tilde{m}_2 \le \frac{3t}{2}$. □

4.2 Firm 1's disclosure strategy

There exist two types of pure strategy equilibrium in the disclosure subgame: (1) the "separating equilibrium" where firm 1 truthfully discloses the existence of idea only when it discovers an idea, i.e., $\phi(I=1) = 1$ and $\phi(I=0) = 0$; and (2) the "pooling strategy" where firm 1 does not disclose the existence of idea irrespective of discovery, i.e., $\phi(I) = 0$, for all $I \in \{0,1\}$. We will first list the conditions under which each strategy can be sustained in equilibrium, and then discuss the parameter space that supports the equilibrium to arise.

4.2.1 Separating Equilibrium

In the separating equilibrium, $\phi^e = I$ and $\mu_2^e(I=1|\phi=1) = 1$ and $\mu_2^e(I=1|\phi=0) = 0$. We use the superscript "e" to denote the equilibrium hereafter. We analyze the entry deterrence equilibrium and the entry invitation equilibrium separately.

Disclosure as Entry Deterrence. Conditional on having an idea $I=1$, firm 1 discloses the idea existence $\phi^e = 1$ truthfully. Upon observing $\phi^e = 1$, firm 2 formulates the posterior belief $\mu_2^e(I=1|\phi=1) = 1$. Under the deterrence motive, $d^e = 0$ and firm 2's actual marginal cost is $m_2 \in \{\underline{m}, \overline{m}\}$. If $m_2 = \overline{m}$, according to Lemma A1, firm 2 chooses not to enter the market knowing that firm 1 already succeeds in idea discovery, which otherwise it would have entered. Hence, firm 1 can deter firm 2 to enter the market by disclosing idea existence and thereby garner monopoly profit. If $m_2 = \underline{m}$, firm 2 is able to produce at the low marginal cost and thereby enter the market. In this case, firm 1 is only able to get the duopoly profit. Hence, conditional on having an idea, firm 1's equilibrium payoff is $E\Pi_1(\phi^e = 1, d^e = 0 | I = 1) = \frac{1}{2}E\Pi_1^M + \frac{1}{2}E\Pi_1^D(\underline{m})$.

$(\phi^e = I, d^e = 0)$ can be supported as an equilibrium strategy if and only if firm 1 has no incentive to deviate to $\left(\phi^e = I, d^{'} = d^{opt}\right)$ or $\left(\phi^{'} = 0, d^e = 0\right)$, where $d^{opt} \in \arg\max_{d\in[\underline{d},1]} E\Pi_1^D(d)$. By deviating to $\left(\phi^e = I, d^{'} = d^{opt}\right)$, firm 1 discloses sufficient information that enables firm 2 to produce a product at a cost of $\overline{m}\left(1 - d^{opt}\right) \leq \frac{3t}{2}$ at the high-cost state $m_2 = \overline{m}$. At the low-cost state $m_2 = \underline{m}$, firm 2 always enters the market. So firm 1's disclosure has no real effect under the assumption $\underline{m} < \overline{m}\left(1 - d^{opt}\right)$. Then firm 1's deviation payoff $E\Pi_1\left(\phi^e = 1, d^{'} = d^{opt} | I = 1\right) = \frac{1}{2}E\Pi_1^D\left(d^{opt}\right) + \frac{1}{2}E\Pi_1^D(\underline{m})$. Firm 1 has no incentive to deviate to $\left(\phi^e = I, d^{'} = d^{opt}\right)$ if and only if

$$E\Pi_1(\phi^e = 1, d^e = 0 | I = 1) > E\Pi_1\left(\phi^e = 1, d^{'} = d^{opt} | I = 1\right)$$
$$\Leftrightarrow \; E\Pi_1^M > E\Pi_1^D\left(d^{opt}\right)$$

Similarly, we analyze another possible deviation $\left(\phi^{'} = 0, d^e = 0\right)$, which means firm 1 does not disclose idea existence even if it has one. Because $\phi^e = I$ is in the equilibrium, upon observing $\phi = 0$, firm 2's posterior belief is $\mu_2^e(I = 1 | \phi = 0) = 0$, and it chooses to enter the market irrespective of its cost realization according to Lemma A1. If $m_2 = \overline{m}$, then $\pi_2^D(\overline{m}) = 0$. In this case, firm 1 is the only firm exerting positive effort in the duopoly market. Hence, $E\Pi_1^D(\overline{m}) = \left(k\pi_1^D(\overline{m})\right)^k \left((1-k)\tilde{u}^D(\overline{m})\right)^{1-k} \pi_1^D(\overline{m})\left(1 - \frac{k}{2}\right)$. If $m_2 = \underline{m}$, as discussed above, firm 1 earns the expected payoff $E\Pi_1^D(\underline{m})$. So, conditional on having an idea, the deviation strategy $\left(\phi^{'} = 0, d^e = 0\right)$ gives firm 1 a payoff $E\Pi_1\left(\phi^{'} = 0, d^e = 0 | I = 1\right) = \frac{1}{2}E\Pi_1^D(\overline{m}) + \frac{1}{2}E\Pi_1^D(\underline{m})$. Firm 1 has no incentive to deviate to $\left(\phi^{'} = 0, d^e = 0\right)$ if and only if

$$E\Pi_1(\phi^e = 1, d^e = 0 | I = 1) > E\Pi_1\left(\phi^{'} = 0, d^e = 0 | I = 1\right)$$
$$\Leftrightarrow \; E\Pi_1^M > E\Pi_1^D(\overline{m})$$

Therefore, disclosure as entry deterrence $(\phi^e = I, \; d^e = 0)$ can be sustained in equilibrium if and only if the two inequalities hold: $E\Pi_1^M > E\Pi_1^D\left(d^{opt}\right)$ and $E\Pi_1^M > E\Pi_1^D(\overline{m})$.

Disclosure as Entry Invitation. By adopting the disclosure strategy such as $\phi^e = I$ and $d^e = d^{opt}$ conditional on $\phi = 1$, firm 1 can lower firm 2's marginal cost to $\overline{m}\left(1 - d^{opt}\right)$ and enable it to enter the market at the high cost state $m_2 = \overline{m}$. At the low cost state $m_2 = \underline{m}$, as discussed previously, firm 1's disclosures have no real impact. Hence, conditional on having an idea, firm 1's equilibrium payoff is $E\Pi_1\left(\phi^e=1,d^e=d^{opt}|I=1\right)=\frac{1}{2}E\Pi_1^D\left(d^{opt}\right)+\frac{1}{2}E\Pi_1^D\left(\underline{m}\right)$. $\left(\phi^e=I,d^e=d^{opt}\right)$ can be supported as an equilibrium strategy if and only if firm 1 has no incentive to deviate to $\left(\phi^e=I,d^{'}=0\right)$ or $\left(\phi^{'}=0,d^{'}=0\right)$. That is,

$$E\Pi_1\left(\phi^e=1,d^e=d^{opt}|I=1\right)\geq E\Pi_1\left(\phi^e=1,d^{'}=0|I=1\right)$$
$$\Leftrightarrow\ E\Pi_1^D\left(d^{opt}\right)\geq E\Pi_1^M$$

and

$$E\Pi_1\left(\phi^e=1,d^e=d^{opt}|I=1\right)\geq E\Pi_1\left(\phi^{'}=0,d^{'}=0|I=1\right)$$
$$\Leftrightarrow\ E\Pi_1^D\left(d^{opt}\right)\geq E\Pi_1^D\left(\overline{m}\right)$$

Hence, disclosure as entry invitation ($\phi^e=I$ and $d^e=d^{opt}$ conditional on $\phi^e=1$) can arise in equilibrium when $E\Pi_1^D\left(d^{opt}\right)\geq E\Pi_1^M$ and $E\Pi_1^D\left(d^{opt}\right)\geq E\Pi_1^D\left(\overline{m}\right)$.

4.2.2 Pooling Equilibrium.

In the pooling equilibrium, $\phi^e=0$ for all $I\in\{0,1\}$ and $\mu_2^e\left(I=1|\phi=0\right)=\overline{\mu}$. Similarly, we need to check whether firm 1 can earn a higher deviation payoff than the equilibrium payoff. Conditional on firm 1's success in idea discovery, the deviation strategy firm 1 can choose is to disclose the idea existence. So the deviation strategy include $\left(\phi^{'}=I,d^e=0\right)$ or $\left(\phi^{'}=I,d^{'}=d^{opt}\right)$. Due to the nature of verifiability of the idea existence, we consider consumer's off-the-equilibrium belief upon seeing $\phi^{'}=1$ is $\mu_2^{'}\left(I=1|\phi^{'}=1\right)=1$. Then the deviation payoffs in the pooling equilibrium correspond to the equilibrium payoffs in the entry deterrence equilibrium and the entry invitation equilibrium, i.e., $E\Pi_1\left(\phi^{'}=1,d^e=0|I=1\right)=\frac{1}{2}E\Pi_1^M+\frac{1}{2}E\Pi_1^D\left(\underline{m}\right)$ and $E\Pi_1\left(\phi^{'}=1,d^{'}=d^{opt}|I=1\right)=\frac{1}{2}E\Pi_1^D\left(d^{opt}\right)+\frac{1}{2}E\Pi_1^D\left(\underline{m}\right)$. Hence, firm 1 has no incentive to deviate from $(\phi^e=0,d^e=0)$ if and only if

$$E\Pi_1\left(\phi^e=0,d^e=0|I=1\right)>E\Pi_1\left(\phi^{'}=1,d^e=0|I=1\right)$$
$$\Leftrightarrow\ E\Pi_1^D\left(\overline{m}\right)>E\Pi_1^M$$

and

$$E\Pi_1\left(\phi^e=0,d^e=0|I=1\right)>E\Pi_1\left(\phi^{'}=1,d^{'}=d^{opt}|I=1\right)$$
$$\Leftrightarrow\ E\Pi_1^D\left(\overline{m}\right)>E\Pi_1^D\left(d^{opt}\right)$$

Hence, no disclosure ($\phi^e = 0$ and $d^e = 0$) can arise in equilibrium when $E\Pi_1^D(\overline{m}) \geq E\Pi_1^M$ and $E\Pi_1^D(\overline{m}) > E\Pi_1^D(d^{opt})$.

As we can see, the preceding analysis boils down firm 1's equilibrium disclosure strategy to the search for the three cutoffs at which the conditions $E\Pi_1^M \gtreqless E\Pi_1^D(d^{opt})$, $E\Pi_1^M \gtreqless E\Pi_1^D(\overline{m})$, and $E\Pi_1^D(\overline{m}) \gtreqless E\Pi_1^D(d^{opt})$ hold, respectively.

First, according to Proposition 4 in the main model, we know that $E\Pi_1^M > E\Pi_1^D(d^{opt})$ can hold only when $k > k^*$. And when $k > k^*$, $d^{opt} = \underline{d} \equiv 1 - \frac{3t}{2\overline{m}}$, and thereby $\overline{m}(1 - d^{opt}) = \frac{3t}{2}$. When $k \leq k^*$, $E\Pi_1^D(d^{opt}) \geq E\Pi_1^M$.

Second, $E\Pi_1^M > E\Pi_1^D(\overline{m}) \Leftrightarrow (\pi_1^M)^k (\tilde{u}^M)^{1-k} \pi_1^M > (\pi_1^D(\overline{m}))^k (\tilde{u}^D(\overline{m}))^{1-k} \pi_1^D(\overline{m}) \Leftrightarrow k > \dot{k} \equiv \left(\ln \frac{\tilde{u}^D(\overline{m})}{\tilde{u}^M} - \ln \frac{\pi_1^M}{\pi_1^D(\overline{m})}\right) / \left(\ln \frac{\tilde{u}^D(\overline{m})}{\tilde{u}^M} + \ln \frac{\pi_1^M}{\pi_1^D(\overline{m})}\right) = \left(\ln\left(4\frac{v}{t}+1-4\frac{\overline{m}}{t}\right) - \ln\left(\frac{2\frac{v}{t}-1}{2\frac{\overline{m}}{t}-1}\right)\right) / \left(\ln\left(4\frac{v}{t}+1-4\frac{\overline{m}}{t}\right) + \ln\left(\frac{2\frac{v}{t}-1}{2\frac{\overline{m}}{t}-1}\right)\right)$. Conversely, when $k \leq \dot{k}$, $E\Pi_1^M \leq E\Pi_1^D(\overline{m})$. Moreover, the cutoff value $\dot{k}$ exists in the range of $(0, 1)$ because $4\frac{v}{t} + 1 - 4\frac{\overline{m}}{t} > \frac{2\frac{v}{t}-1}{2\frac{\overline{m}}{t}-1} > 1$ for all $v > \frac{3t}{2} > 0$ and $\frac{3t}{2} < \overline{m} < v$.

Third, $E\Pi_1^D(\overline{m}) > E\Pi_1^D(d^{opt}) \Leftrightarrow \left(\frac{\pi_1^D(\overline{m})}{\pi_f^D(d^{opt})} \frac{\tilde{u}^D(d^{opt})}{\tilde{u}^D(\overline{m})}\right)^k > \left(\frac{\pi_1^D(d^{opt})\tilde{u}^D(d^{opt})}{\pi_1^D(\overline{m})\tilde{u}^D(\overline{m})}\right)^{\frac{1-\frac{k}{2}\frac{\pi_1^D(d^{opt})}{\pi_f^D(d^{opt})}}{1-\frac{k}{2}}}$. Nevertheless, it is trickier to get the cutoff value $\ddot{k}$ at which $E\Pi_1^D(\overline{m}) = E\Pi_1^D(d^{opt})$ because d^{opt} also depends on the value k according to Proposition 6 in the main model. In the next lemma, we will show the existence and the uniqueness of the cutoff $\ddot{k}$ such that $k > \ddot{k} \Leftrightarrow E\Pi_1^D(\overline{m}) > E\Pi_1^D(d^{opt})$, and the relation of the three cutoffs k^*, $\dot{k}$, and $\ddot{k}$.

Lemma A2. *(a) There exists the unique cutoff $\ddot{k}$ such that $E\Pi_1^D(d^{opt}) \geq E\Pi_1^D(\overline{m})$ when $k \leq \ddot{k}$ and $E\Pi_1^D(\overline{m}) > E\Pi_1^D(d^{opt})$ when $k > \ddot{k}$; (b) The three key cutoffs $\ddot{k}$, k^*, and $\dot{k}$ satisfy the relation: $\ddot{k} < k^* < \dot{k}$.*

Proof. To show the two statements, we break down into fours parts and show them in a chronological order: (1) $\dot{k} > k^*$; (2) the existence of $\ddot{k}$ in the range $(0, k^*)$; (3) the uniqueness of $\ddot{k}$; (4) $E\Pi_1^D(d^{opt}) \geq E\Pi_1^D(\overline{m})$ when $k \leq \ddot{k}$ and $E\Pi_1^D(\overline{m}) > E\Pi_1^D(d^{opt})$ when $k > \ddot{k}$.

(1) We show $\dot{k} > k^*$. Rewrite $\dot{k} = -1 + 2/\left(1 + \ln\left(\frac{2\frac{v}{t}-1}{2\frac{\overline{m}}{t}-1}\right)/\ln\left(4\frac{v}{t}+1-4\frac{\overline{m}}{t}\right)\right)$. We find $\dot{k}$ increases with $\overline{m}$ by showing $\ln\left(\frac{2\frac{v}{t}-1}{2\frac{\overline{m}}{t}-1}\right)/\ln\left(4\frac{v}{t}+1-4\frac{\overline{m}}{t}\right)$ decreases with $\overline{m}$. This is easily to verify because its derivative w.r.t. $\overline{m}$ is $\frac{2\left(2\frac{\overline{m}}{t}-1\right)\ln\left(\frac{2v-t}{2\overline{m}-t}\right) - \left(4\frac{v}{t}+1-4\frac{\overline{m}}{t}\right)\ln\left(4\frac{v}{t}+1-4\frac{\overline{m}}{t}\right)}{\frac{t}{2}\left(4\frac{v}{t}+1-4\frac{\overline{m}}{t}\right)\left(2\frac{\overline{m}}{t}-1\right)\left(\ln\left(4\frac{v}{t}+1-4\frac{\overline{m}}{t}\right)\right)^2}$. Since the denominator is positive, then its sign equals the sign of the numerator. We show in the next that the numerator is negative. This is so because the derivative of the numerator w.r.t. $\overline{m}$ is $\frac{4}{t}\left(\ln\left(\frac{2v-t}{2\overline{m}-t}\right) + \ln\left(4\frac{v}{t} + 1 - 4\frac{\overline{m}}{t}\right)\right) > 0$. Because $\overline{m} < v$, the numerator is smaller than $2\left(2\frac{v}{t} - 1\right)\ln\left(\frac{2v-t}{2v-t}\right) - \left(4\frac{v}{t} + 1 - 4\frac{v}{t}\right)\ln\left(4\frac{v}{t} + 1 - 4\frac{v}{t}\right) = 0$. Therefore, $\frac{\partial \ln\left(\frac{2\frac{v}{t}-1}{2\frac{\overline{m}}{t}-1}\right)/\ln\left(4\frac{v}{t}+1-4\frac{\overline{m}}{t}\right)}{\partial \overline{m}} < 0$, and thereby $\dot{k}$ increases with $\overline{m}$. Because $\overline{m} > \frac{3t}{2}$, $\dot{k}(\overline{m}) > \dot{k}\left(\frac{3t}{2}\right) = \left(\ln\left(4\frac{v}{t}-5\right) - \ln\left(\frac{v}{t}-\frac{1}{2}\right)\right)/\ln\left(4\frac{v}{t}-5\right) + \ln\left(\frac{v}{t}-\frac{1}{2}\right) = k^*$. So we get the first relation of the cutoffs, $\dot{k} > k^*$.

(2) We show the existence of the cutoff $\ddot{k}$ at which $E\Pi_1^D(\overline{m}) = E\Pi_1^D(d^{opt})$ in the range $(0, k^*)$. First, we show at the point $k = k^*$, $E\Pi_1^D(\overline{m}) > E\Pi_1^D(d^{opt})$. Recall that $E\Pi_1^D(\overline{m}) > E\Pi_1^M$ when $k < \dot{k}$ and $E\Pi_1^M \geq E\Pi_1^D(d^{opt})$ when $k \geq k^*$. Since $\dot{k} > k^*$ as we have already show in part (1), when at $k = k^*$, the relation $E\Pi_1^D(\overline{m}) > E\Pi_1^M = E\Pi_1^D(d^{opt})$.

Next, we will show at the point $k = 0$, $E\Pi_1^D(\overline{m}) < E\Pi_1^D(d^{opt})$. At $k = 0$, $E\Pi_1^D(\overline{m}) = \tilde{u}^D(\overline{m})\pi_1^D(\overline{m})$ and $E\Pi_1^D(d^{opt}) = \tilde{u}^D(d^{opt})\pi_1^D(d^{opt})$. We first get d^{opt} and then compare $E\Pi_1^D(\overline{m})$ and $E\Pi_1^D(d^{opt})$. $E\Pi_1^D(d) = \tilde{u}^D(d)\pi_1^D(d) = \left(\left(4(1-d)^2\overline{m}^2 - 36(1-d)\overline{m}t + 9t(8v-5t)\right)/72t\right) \cdot \left((2(1-d)\overline{m}+3t)^2/36t\right)$. The optimal disclosure amount from F.O.C. is $1 + \frac{3\left(\sqrt{t(2t-v)}-t\right)}{\overline{m}}$, which is smaller than 1 because $\sqrt{t(2t-v)} - t < 0$ for all $0 < t < \frac{2v}{3}$. $1 + \frac{3\left(\sqrt{t(2t-v)}-t\right)}{\overline{m}}$ can be re-written to $\underline{d} + \frac{3\left(2\sqrt{t(2t-v)}-t\right)}{2\overline{m}}$, where $\underline{d} = 1 - \frac{3t}{2\overline{m}}$. Hence, $d^{opt} = \max\left\{\underline{d}, \underline{d} + \frac{3\left(2\sqrt{t(2t-v)}-t\right)}{2\overline{m}}\right\}$. If $2\sqrt{t(2t-v)} - t > 0 \Leftrightarrow t > \frac{4v}{7}$, $d^{opt} > \underline{d}$ (also stated in Proposition 5 in the main model); It $t \leq \frac{4v}{7}$, $d^{opt} = \underline{d}$. By the optimality principal, $E\Pi_1^D(d^{opt}) \geq E\Pi_1^D(\underline{d}) = \left(v - \frac{5}{4}t\right)t$. It is easy to check that $E\Pi_1^D(\overline{m}) = \left(\overline{m} - \frac{t}{2}\right)\left(v + \frac{t}{4} - \overline{m}\right)$ decreases with $\overline{m} \in \left(\frac{3t}{2}, v\right)$ because the profit-maximizing $\overline{m} = \frac{1}{8(3t+4v)} < \frac{3t}{2}$ for all $t \in [\frac{v}{2}, \frac{2v}{3})$. So $E\Pi_1^D(\overline{m}) < E\Pi_1^D\left(\frac{3t}{2}\right) = \left(v - \frac{5}{4}t\right)t = E\Pi_1^D(\underline{d}) \leq E\Pi_1^D(d^{opt})$. Therefore, $E\Pi_1^D(d^{opt}) > E\Pi_1^D(\overline{m})$ when $k = 0$.

In summary, given the facts that $E\Pi_1^D(\overline{m}) > E\Pi_1^D(d^{opt})$ at $k = k^*$ and $E\Pi_1^D(\overline{m}) < E\Pi_1^D(d^{opt})$ at $k = 0$, there exists at least one cutoff $\ddot{k}$ at which $E\Pi_1^D(\overline{m}) = E\Pi_1^D(d^{opt})$ in the range $(0, k^*)$.

(3) We first show that the expected payoff functions $E\Pi_1^D(\overline{m})$ and $E\Pi_1^D(d^{opt})$ are unimodal in k. Together with the facts that $E\Pi_1^D(\overline{m}) > E\Pi_1^D(d^{opt})$ at $k = k^*$ and $E\Pi_1^D(\overline{m}) < E\Pi_1^D(d^{opt})$ at $k = 0$, we can reach the conclusion that the cutoff $\ddot{k}$ is unique in the range $(0, k^*)$.

To show unimodality, we specify $E\Pi_1^D(\overline{m}) = \left(k\pi_1^D(\overline{m})\right)^k \left((1-k)\tilde{u}^D(\overline{m})\right)^{1-k} \pi_1^D(\overline{m})\left(1 - \frac{k}{2}\right)$ and the derivative w.r.t. k is $\frac{\partial E\Pi_1^D(\overline{m})}{\partial k} = \iota \cdot \left(-\frac{1}{2-k} - \ln\frac{(1-k)\tilde{u}^D(\overline{m})}{k\pi_1^D(\overline{m})}\right)$, where $\iota = \frac{(2-k)}{2}\pi_1^D(\overline{m})\left(k\pi_1^D(\overline{m})\right)^k \cdot \left((1-k)\tilde{u}^D(\overline{m})\right)^{1-k} > 0$. So, $\frac{\partial E\Pi_1^D(\overline{m})}{\partial k} \lesseqgtr 0 \Leftrightarrow -\frac{1}{2-k} \lesseqgtr \ln\frac{(1-k)\tilde{u}^D(\overline{m})}{k\pi_1^D(\overline{m})}$. Notice that both sides of the inequality are monotonically decreasing in k. Moreover, at $k = 0$, $-\frac{1}{2-k} = -\frac{1}{2}$ and $\ln\frac{(1-k)\tilde{u}^D(\overline{m})}{k\pi_1^D(\overline{m})} \to \infty$. Thus, $-\frac{1}{2-k} < \ln\frac{(1-k)\tilde{u}^D(\overline{m})}{k\pi_1^D(\overline{m})}$ and $\frac{\partial E\Pi_1^D(\overline{m})}{\partial k} < 0$. When at $k = 1$, $-\frac{1}{2-k} = -1$ and $\ln\frac{(1-k)\tilde{u}^D(\overline{m})}{k\pi_1^D(\overline{m})} \to -\infty$. Thus, $-\frac{1}{2-k} > \ln\frac{(1-k)\tilde{u}^D(\overline{m})}{k\pi_1^D(\overline{m})}$ and $\frac{\partial E\Pi_1^D(\overline{m})}{\partial k} > 0$. Hence, there exists a unique $\hat{k} \in [0, 1]$ s.t. $\frac{\partial E\Pi_1^D(\overline{m})}{\partial k} \leq 0$ when $k \leq \hat{k}$ and $\frac{\partial E\Pi_1^D(\overline{m})}{\partial k} \geq 0$ when $k \geq \hat{k}$. So, $E\Pi_1^D(\overline{m})$ qualifies the definition of a unimodal function.

Similarly, we specify $E\Pi_1^D(d^{opt}) = \left(k\pi_f^D(d^{opt})\right)^k \left((1-k)\tilde{u}^D(d^{opt})\right)^{1-k} \pi_1^D(d^{opt})\left(1 - \frac{k}{2}\frac{\pi_1^D(d^{opt})}{\pi_f^D(d^{opt})}\right)$. Then $\frac{\partial E\Pi_1^D(d^{opt})}{\partial k} = \frac{\partial E\Pi_1^D(d)}{\partial k}\Big|_{d=d^{opt}} = \kappa \cdot \left(-\frac{\pi_1^D(d^{opt})}{2\pi_f^D(d^{opt}) - k\pi_1^D(d^{opt})} - \ln\frac{(1-k)\tilde{u}^D(d^{opt})}{k\pi_f^D(d^{opt})}\right)$, where $\kappa = \left(\pi_1^D(d^{opt})\left(2\pi_f^D(d^{opt}) - k\pi_1^D(d^{opt})\right)/2\pi_f^D(d^{opt})\right)\left(k\pi_f^D(d^{opt})\right)^k \left((1-k)\tilde{u}^D(d^{opt})\right)^{1-k} > 0$. So, $\frac{\partial E\Pi_1^D(d^{opt})}{\partial k} \lesseqgtr 0 \Leftrightarrow -\frac{\pi_1^D(d^{opt})}{2\pi_f^D(d^{opt}) - k\pi_1^D(d^{opt})} \lesseqgtr \ln\frac{(1-k)\tilde{u}^D(d^{opt})}{k\pi_f^D(d^{opt})}$. For simplicity, we define $LHS = -\frac{\pi_1^D(d^{opt})}{2\pi_f^D(d^{opt}) - k\pi_1^D(d^{opt})}$ and $RHS = \ln\frac{(1-k)\tilde{u}^D(d^{opt})}{k\pi_f^D(d^{opt})}$. It is straightforward that both LHS and RHS are monotonic decreasing functions of k. Moreover, at $k = 0$, $LHS = -\frac{\pi_1^D(d^{opt})}{2\pi_f^D(d^{opt})}$ and $RHS \to \infty$. So $LHS < RHS$ and $\frac{\partial E\Pi_1^D(d^{opt})}{\partial k} < 0$. At $k = 1$, $LHS = -\frac{\pi_1^D(d^{opt})}{2\pi_f^D(d^{opt})-\pi_1^D(d^{opt})}$ and $RHS \to -\infty$. So, $LHS > RHS$ and $\frac{\partial E\Pi_1^D(d^{opt})}{\partial k} > 0$. Hence, there must exist a unique $\tilde{k} \in [0, 1]$ s.t. $\frac{\partial E\Pi_1^D(d^{opt})}{\partial k} \leq 0$ when $k \leq \tilde{k}$ and $\frac{\partial E\Pi_1^D(d^{opt})}{\partial k} \geq 0$ when $k \geq \tilde{k}$. So, $E\Pi_1^D\left(d^{opt}\right)$ also qualifies the definition of a unimodal function.

(4) Lastly, given the uniqueness of the cutoff $\ddot{k} \in (0, k^*)$ at which $E\Pi_1^D(\overline{m}) = E\Pi_1^D\left(d^{opt}\right)$ and the facts that $E\Pi_1^D(\overline{m}) > E\Pi_1^D\left(d^{opt}\right)$ at $k = k^*$ and $E\Pi_1^D(\overline{m}) < E\Pi_1^D\left(d^{opt}\right)$ at $k = 0$, we immediately know that $E\Pi_1^D\left(d^{opt}\right) \geq E\Pi_1^D(\overline{m})$ when $k \leq \ddot{k}$ and $E\Pi_1^D(\overline{m}) > E\Pi_1^D\left(d^{opt}\right)$ when $k > \ddot{k}$. □

Proposition 9. *(Equilibrium)*

1. *When k is sufficiently small ($k \leq \ddot{k}$) or large ($k > \dot{k}$), the innovator discloses the product existence truthfully whenever it discovers an idea ("separating equilibrium"). In particular,*
   (a) *When k is sufficiently large ($k > \dot{k}$), entry deterrence occurs. Firm 1 never reveals any key information ($d^e = 0$), which deters firm 2 from entering the market when $m_2 = \overline{m}$.*
      i. *When k is sufficiently small ($k \leq \ddot{k}$), entry invitation occurs. Firm 1 reveals sufficient key information ($d^e > \underline{d}$), which encourages firm 2 to enter the market.*
   (b) *When k is intermediate ($\ddot{k} < k \leq \dot{k}$), the innovator does not disclose the existence of the idea irrespective of the true state ("pooling equilibrium"). Firm 2 always enters the market.*

Proof. The proof immediately follows from Lemma A1 and Lemma A2. Disclosure as entry deterrence ($\phi^e = I,\ d^e = 0$) can be sustained in equilibrium if and only if $E\Pi_1^M > E\Pi_1^D\left(d^{opt}\right)$ and $E\Pi_1^M > E\Pi_1^D(\overline{m})$. $E\Pi_1^M > E\Pi_1^D\left(d^{opt}\right) \Leftrightarrow k > k^*$ and $E\Pi_1^M > E\Pi_1^D(\overline{m}) \Leftrightarrow k > \dot{k}$. From Lemma A2 that $\dot{k} > k^*$, the two conditions hold together when $k > \dot{k}$. Upon seeing $\phi^e = 1$ and $d^e = 0$, firm 2's belief $\mu_2(I = 1|\phi = 1) = 1$. According to Lemma A1, firm 2 enters the market only when $m_2 = \underline{m}$. Upon seeing $\phi = 0$ and $d = 0$, firm 2's belief $\mu_2(I = 1|\phi = 1) = 0$ and firm 2 always enters the market (by Lemma A1).

Disclosure as entry invitation ($\phi^e = I$ and $d^e = d^{opt}$ conditional on $\phi^e = 1$) can arise in equilibrium when $E\Pi_1^D\left(d^{opt}\right) \geq E\Pi_1^M$ and $E\Pi_1^D\left(d^{opt}\right) \geq E\Pi_1^D(\overline{m})$. $E\Pi_1^D\left(d^{opt}\right) \geq E\Pi_1^M \Leftrightarrow k \leq k^*$ and $E\Pi_1^D\left(d^{opt}\right) \geq E\Pi_1^D(\overline{m}) \Leftrightarrow k \leq \ddot{k}$. From Lemma A2 that $\ddot{k} \leq k^*$, the two conditions hold together when $k \leq \ddot{k}$. Upon seeing $\phi^e = 1$ and $d^e = d^{opt}$, firm 2's belief $\mu_2(I = 1|\phi = 1) = 1$ and its actual marginal cost is $\min\left\{\underline{m}, \overline{m}\left(1 - d^{opt}\right)\right\} \leq \frac{3t}{2}$. According to Lemma A1, firm 2 enters the market. Upon seeing $\phi = 0$ and $d = 0$, firm 2's belief $\mu_2(I = 1|\phi = 1) = 0$ and firm 2 always enters the market (by Lemma A1).

Lastly, no disclosure ($\phi^e = 0$ and $d^e = 0$) can arise in equilibrium when $E\Pi_1^D(\overline{m}) \geq E\Pi_1^M$ and $E\Pi_1^D(\overline{m}) > E\Pi_1^D\left(d^{opt}\right)$. $E\Pi_1^D(\overline{m}) \geq E\Pi_1^M \Leftrightarrow k \leq \dot{k}$ and $E\Pi_1^D(\overline{m}) > E\Pi_1^D\left(d^{opt}\right) \Leftrightarrow k > \ddot{k}$.

From Lemma A2 that $\ddot{k} < \dot{k}$, the two conditions old when $\ddot{k} < k \leq \dot{k}$. In this case, firm 2's belief $\mu_2 (I = 1|\phi = 1) = \mu$. According to Lemma A1, firm 2 enters the market irrespective of the realization of the cost. □