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Understanding momentum and reversal☆

Bryan T. Kelly[a,b], Tobias J. Moskowitz[a,b], Seth Pruitt[c,*]

[a] Yale School of Management, Yale University, 165 Whitney Ave., New Haven, CT 06511, USA
[b] AQR Capital Management, Two Greenwich Plaza, Greenwich, CT 06830, USA
[c] W.P. Carey School of Business, Arizona State University, 400 E Lemon St., Tempe, AZ 85287, USA

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ABSTRACT

Stock momentum, long-term reversal, and other past return characteristics that predict future returns also predict future realized betas, suggesting these characteristics capture time-varying risk compensation. We formalize this argument with a conditional factor pricing model. Using instrumented principal components analysis, we estimate latent factors with time-varying factor loadings that depend on observable firm characteristics. We show that factor loadings vary significantly over time, even at short horizons over which the momentum phenomenon operates (one year), and this variation captures reliable conditional risk premia missed by other factor models commonly used in the literature. Our estimates of conditional risk exposure can explain a sizable fraction of momentum and long-term reversal returns and can be used to generate even stronger return predictions.

1. Introduction

Since its introduction by Jegadeesh and Titman (1993), the momentum anomaly has consistently ranked among the most thoroughly researched topics in financial economics. It forms the basis of strategies implemented throughout the asset management industry and underlies a wide range of mutual funds and exchange traded products. Despite its widespread influence on the finance profession, momentum remains a mysterious phenomenon. A variety of positive theories, both behavioral and rational, have been proposed to explain momentum, but none are widely accepted.[1] Momentum also remains one of the few reliable violators of prevailing empirical asset pricing models such as the Fama and French (2015) five-factor model, and research has yet to identify a risk exposure that can explain the cross-sectional return premium associated with recent price performance. Consequently, momentum is often the center piece for debates of market efficiency.

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* Corresponding author.
E-mail addresses: bryan.kelly@yale.edu (B.T. Kelly), tobias.moskowitz@yale.edu (T.J. Moskowitz), seth.pruitt@asu.edu (S. Pruitt).

[1] An incomplete but extensive list includes Barberis et al. (1998), Daniel et al. (1998), Hong and Stein (1999), Gomes et al. (2003), Zhang (2005), Li et al. (2009), Belo (2010), Li and Zhang (2010), Liu and Zhang (2008), Berk et al. (1999), Johnson (2002), Sagi and Seasholes (2007), Liu et al. (2009), Vayanos and Woolley (2013), and Liu and Zhang (2014).

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The objective of this paper is to reevaluate the momentum anomaly through the lens of conditional empirical asset pricing models. How much of the momentum premium can be explained by conditional risk exposure? Our central finding is that variation in stocks' conditional risk premia is strongly linked to momentum through varying factor exposures, which dominate variation in risk premia due to momentum. In our data, the classic momentum strategy (top quintile minus bottom quintile of stocks sorted on past $t-2$ to $t-12$ month raw returns) yields a significant annualized return of 8.3%. However, when we strip out exposure to other priced factors from our conditional model, the residual momentum strategy (that sorts on the residual component of returns unrelated to the conditional factors) generates marginally significant profits of 4.4% per annum. Conversely, sorting stocks on the predicted component of their returns coming from the conditional models yields momentum profits that are three to four times larger. Flipping the analysis around, the bulk of the unconditional momentum effect is well explained by the factors we estimate from our conditional model. Whereas the momentum strategy has a significant annualized alpha of 9.2% (t-stat = 3.0) with respect to the static Fama and French (2015) factors, its alpha with respect to our dynamic factors is negative and insignificant (alpha of −3.2% with t-stat = −1.2).

By carefully constructing factors that more accurately represent the conditional risk-return tradeoff, and with proper specification for time-varying conditional betas on these factors, we show that momentum returns exhibit large exposure to common risks. While we are not the first to try and examine conditional risk models as an explanation for momentum (Conrad and Kaul, 1998; Jegadeesh and Titman, 2002; Grundy and Martin, 2001; Chordia and Shivakumar, 2002) or long-term reversals (Kothari and Shanken, 1992), our methodology for extracting conditional risk premia is substantially more successful at explaining these past return phenomena. We explore and explain how and why our estimates achieve this success.

We first establish a necessary condition for momentum to be explainable with a factor pricing model that momentum captures factor risk exposure. We show that momentum is a strong forecaster of future realized betas on the market return itself, as well as other popular asset pricing factors (e.g., the Fama and French, 2015 five-factor model), over the subsequent year. The predictive power of the momentum characteristic for future betas is strong statistically and economically. For example, the estimated predictive coefficient for market beta indicates that when a stock moves from the 10th percentile of the momentum characteristic to the 90$^{\text{th}}$ percentile, its market beta increases by 0.20. This is a preliminary indication that momentum captures conditional risk exposure. Likewise, long-term reversal, another past return characteristic, has similar predictive power for betas, but short-term reversal has substantially weaker power to predict betas. The link between intermediate and long-term past returns and future betas indicates that premia associated with momentum and long-term reversals may be related, at least in part, to conditional risk exposure. However, the magnitude of the increase in conditional market beta alone is insufficient to explain the spread in returns between high and low momentum (or long-term reversal) stocks. Hence, a conditional version of the Capital Asset Pricing Model (CAPM) is not able to explain these returns.

To rigorously investigate whether beta predictability quantitatively rationalizes the average return patterns associated with price trends, we require an asset pricing model with more than just the market factor. To investigate this idea, we analyze a generic conditional factor pricing model of the form

$$r_{i,t+1} = \beta_{i,t}' f_{t+1} + \epsilon_{i,t+1}, \quad E_t[r_{i,t+1}] = \beta_{i,t}' \lambda_t. \tag{1}$$

In this framework, conditional expected returns $\mu_{i,t}$ are restricted to derive only from exposures ($\beta_{i,t}$) to a set of common risk factors and the associated factor premia ($\lambda_t \equiv E_t[f_{t+1}]$). At a minimum, a successful model will need to explain three facts associated with past return anomalies: (1) a large spread in average returns of around 9% per annum for stocks in the highest quintile of past one year returns over those in the lowest quintile, or in terms of average return per unit of risk, a Sharpe ratio of about 0.50 annually; (2) that a 12-2-month moving average produces better return predictions than alternative moving average windows; and (3) the marginally significant long-term reversal pattern that occurs beyond a year.

Consider for a moment a static version of Eq. (1) so that $\mu_{i,t} = \beta_i' \lambda$ for all t. One condition alone—a sufficiently large spread in β_i—could match momentum's large average return spread. But this condition would also imply that a very long moving average window would provide the best estimate of β_i, since for a static beta the longer horizon provides more data to precisely estimate the beta. This implication suggests that longer windows of past returns should provide the best predictors of future returns, which contradicts the well-known pattern in the data that momentum performs best at intermediate horizons, like the standard 12-month past return applied in most studies. The traditional 12-month momentum implementation prefers to rapidly turnover stock constituents in the high and low quintiles. The empirical probability of a stock transitioning out of the extreme quintiles is roughly 38% per month. This transitioning is not simply noise—longer moving average windows would mechanically reduce turnover, but they also reduce forecasting power. Long-term reversal poses a set of analogous dynamic requirements on beta behavior.

The implication of these facts is that the identities of stocks with the highest and lowest conditional expected returns are changing (rapidly) over time. For a factor model like (1) to match the data, it needs to be a conditional model. Holding the factor premia fixed, dynamic betas ($\beta_{i,t}$) induce variation in the panel of $\mu_{i,t}$s. This variation churns the list of stocks at the top and bottom of the $\mu_{i,t}$ distribution, particularly when there are multiple factors.

While conditional factor models offer a potential conceptual explanation for momentum and other price trend patterns, they pose a difficult estimation challenge. One estimation option is to use observable factors and estimate rolling betas. However, observable factors may be misspecified—especially if reinterpreted as conditional factors when they were originally constructed for use as unconditional factors (e.g., Fama–French factors), and rolling betas may suffer a "staleness bias" as they only slowly incorporate conditioning information. Another option is to estimate monthly realized betas from higher frequency (e.g., daily) data. But with only 20 daily observations per month, realized betas tend to be noisy, and expanding to several months reintroduces potential staleness. Plus, none of this resolves the problem of misspecification in observable factors.

Instead, we follow the conditional factor modeling approach of Kelly et al. (2019), henceforth KPS, who use the method of instrumented principal components analysis (IPCA) to estimate latent factors and factor exposures by parameterizing $\beta_{i,t}$ as a function of observable asset characteristics. By conditioning betas on observable time-varying characteristics, the model can quickly update risk exposures based on characteristic news. And by estimating factors most related to conditional risk exposures, IPCA is freed from using prespecified factors that are prone to misspecification. KPS demonstrate that conditional factor models estimated via IPCA offer significant improvements in describing the cross section of risk and return compared to leading alternatives (such as Fama–French factors with rolling betas).

We find that estimates of $\mu^{IPCA}_{i,t} = \beta'_{i,t}\lambda$ from our conditional factor model offer economically large and statistically significant return forecasting improvements over the traditional momentum effect. We run out-of-sample panel predictive regressions to compare the return forecasting ability of our model-based expected return estimates against the predictive power of momentum and reversal characteristics. For example, the stock-level panel predictive R^2 is 0.02% per month based on momentum, 0.01% based on long-term reversal, and 0.32% based on $\mu^{IPCA}_{i,t}$. Furthermore, the joint R^2 from a bivariate regression including both $\mu^{IPCA}_{i,t}$ and momentum is also 0.32%. In other words, the contribution of momentum to understanding time-varying stock expected returns is negligible after accounting for conditional factor risk compensation. The efficacy of stock momentum and long-term reversal strategies is largely explained by conditional factor exposures, and sorting based on estimates of conditional expected returns produces much stronger return predictability than simple momentum sorts. For example, the baseline long-short quintile spread momentum strategy in our sample is 8.3% per year (with a t-statistic of 3.30 and annualized Sharpe ratio of 0.48). But the long-short strategy coming from conditional factor returns produces average returns of 33.6% per year (with a t-statistic of 14.5). Thus, not only is the model substantially more effective at explaining the behavior of expected returns than ad hoc price trend characteristics, but it also captures compensation for factor exposures that explains a large fraction of the momentum effect and can produce even stronger and more precise return predictability.

Our research question is most closely related to four precursors in the literature on momentum and long-term reversals. First, Conrad and Kaul (1998) suggest that differences in stocks' expected returns can explain momentum profits. Jegadeesh and Titman (2002) argue against this interpretation because it is based on unconditional expectations that they show are not dispersed enough to explain momentum returns. Furthermore, as mentioned above, an explanation based on unconditional expectations will not generate the churn in the list of stocks involved in the momentum strategy nor be able to explain the long-term reversal result. On the other hand, an explanation based on conditional expectations can and does explain these patterns.[2]

Second, Grundy and Martin (2001) decompose returns into a systematic risk component (captured as exposure to the three Fama and French, 1993 factors) and stock-specific residuals and find that the momentum phenomenon is driven entirely by momentum in residual returns. We find that the Grundy and Martin (2001) conclusion is most likely driven by factor model misspecification—in their case due to rolling-window betas on observable factors. Using a model with slow-moving betas and erroneous factors all but ensures that residuals inherit the important variation in expected returns, giving the misleading impression that momentum is a feature of idiosyncratic returns. In contrast, we find weak evidence of residual momentum once we allow for latent factors and conditional beta dynamics. We argue and show that it is this refined model specification that leads to a different conclusion than Grundy and Martin (2001).

Third, Chordia and Shivakumar (2002) decompose stock returns into a component that is forecastable with macroeconomic predictor variables and an unforecastable shock. They conclude that momentum returns are best captured through the conditional expected returns predicted by macroeconomic variables rather than through the residual. They conjecture that the predictable component proxies for dynamic factor risk premia (in contrast to the conclusions of Grundy and Martin, 2001). However, they leave this conjecture untested. In addition, prespecified macro factors also provide scope for misspecification and may not capture conditional risk exposure. Our paper provides the explicit missing link between momentum returns and factor risk exposures.

The rest of the paper is organized as follows. Section 2 establishes evidence that price trends predict risk exposures, which lays the groundwork for our model. Section 3 describes our conditional factor model specification and the IPCA estimation approach. Section 4 presents our central finding that the variation in expected returns driven by conditional risk exposures swamps that due to momentum and reversal effects. For completeness, we also show that conditional betas explain part of short-term reversal, though we view short-term reversal as a largely separate and liquidity-driven phenomenon. Section 5 explores the robustness of our results along several dimensions, and Section 6 offers conclusions from our analysis.

2 We show that different specifications of the conditioning information lead to varying degrees of portfolio turnover while retaining significant profitability.

2. Past returns predict betas

In this section we show the robust stylized fact that a stock's recent past returns forecast its future realized betas on aggregate risk factors.

2.1. Data

Our data set is the one studied in KPS, composed of stock returns and 36 characteristics from Freyberger et al. (2020). That sample spans 1966 to 2014, restricts attention to stock-month observations for which all 36 characteristics are nonmissing, and ultimately includes 12,813 unique stocks and 1,403,544 stock-month observations. To deal with outliers, each characteristic is cross-sectionally ranked, then these ranks are divided by the number of stocks in that cross section, and then they are cross-sectionally demeaned so that they live in the $[-0.5, 0.5]$ interval.

2.2. Market beta prediction results

We begin by examining whether characteristics associated with various trading strategies that produce positive abnormal returns, especially those associated with past returns, have predictability for changes in future market betas. A vast literature shows these characteristics generate anomalous return predictability not captured by the static CAPM, implying that variation in unconditional market beta associated with these characteristics is insufficient to explain their returns. In many cases, long/short portfolios based on these characteristics sorts have an unconditional market beta close to zero. However, if risk exposures change, these characteristics could be related to future betas, and hence a conditional risk model may better capture their returns. As a simple starting point, we examine whether these characteristics predict future market betas and then proceed to examine a multifactor model with time-varying risk exposure.

We construct stock-level monthly realized Ordinary Least Squares (OLS) betas on the market return using daily returns data within month t. We also construct quarterly betas from daily data in months t to $t+2$, semiannual betas from daily data in months t to $t+5$, and annual betas from daily data in months t to t+11. We then explore the predictability of realized betas using recent stock momentum. We regress betas during months t to $t+h$ (h =0, 2, 5 or 11) on cumulative returns from month $t-12$ to $t-2$ (the standard momentum measure from Asness, 1994; Fama and French, 1996; Asness et al., 2013) in a stock-month panel, along with other characteristics (clustering standard errors by stock and by month).

$$\beta_{t,t+h} = a + b_h r_{t-12,t-2} + C_h' X_{t-1} + \epsilon_{t,t+h}, \quad (2)$$

where X_{t-1} is a $L \times N$ matrix of L characteristics across N stocks at time $t-1$ and C_h are the estimated regression coefficients relating these characteristics to future market betas. The momentum characteristic, $r_{t-12,t-2}$, is broken out separately here to highlight its relation to future market betas.

The results in Table 1 show that stock momentum (highlighted in bold) is a powerful predictor of future realized market betas. The predictive coefficient for one-month realized market beta is 0.23 (t=11.88). This coefficient implies that as a stock transitions from the 10th to the 90th momentum percentile, its market beta increases by 0.18 (0.23×0.8), or equivalently, an 18% increase in its market risk premium. Results are similar across the realized betas calculated from 3, 6, and 12 months of future daily returns.

While the market beta predictability is strong, the economic magnitude of 0.18 beta difference between the top and bottom deciles of momentum stocks is insufficient to explain the difference in their average returns. Even with a market risk premium of 6%, the 0.18 beta difference only explains about 1.1 percentage points of the 8%-9% momentum premium. Hence, the conditional CAPM, which only has time-varying market beta, is not able to capture momentum profits. For this reason, we explore below a richer conditional multifactor model.

Momentum is not unique in its ability to forecast future realized betas. Long-term reversals, another past return characteristic, based on prior three- to five-year returns also significantly forecasts future market betas, though the predictive coefficients are half those for momentum, as shown in Table 1. Many other commonly studied characteristics also significantly forecast realized betas. Naturally, one of the strongest beta predictors is past realized beta (market beta), along with firm size (market capitalization). However, as the table shows in a multifactor regression, even after accounting for these characteristics, momentum and long-term reversal characteristics remain highly significant for forecasting market beta.

Finally, short-term reversal, another past return characteristic based on the past one-month return, has statistically significant, but economically small, predictive content for future market betas. This weak result is consistent with short-term reversals being more tied to liquidity effects (e.g. Nagel, 2012) than time-varying risk.

2.3. Multifactor beta prediction results

Since we will consider multifactor models below, it is useful to assess how past returns predict the betas with respect to other factors commonly used to proxy for aggregate risk. Table A.2 in the appendix reports results from running regression Eq. (2) using annual betas on the Fama and French (2015) factors Small Minus Big (SMB), High Minus Low (HML), Robust Minus Weak (RMW), and Conservative Minus Aggressive (CMA) in place of the market beta, where betas are computed using daily returns in the same manner. Other than SMB, momentum strongly predicts beta exposure to each of HML, RMW, and CMA, even when controlling for book-to-market, gross profitability, and investment, which are the characteristics HML, RMW, and CMA are based on, respectively. Long-term reversal also shows significant predictability for future betas on all four factors. Finally, short-term reversal has small and insignificant predictability for future betas, consistent with its lack of predictability for market betas and its interpretation as primarily a liquidity-driven phenomenon.

Table 1
Predicting realized market betas with characteristics.

We construct stock-level monthly realized OLS betas on the market return using daily returns data within month t. We also construct quarterly betas from daily data in months t to $t+2$, semiannual betas from daily data in months t to $t+5$, and annual betas from daily data in months t to t+11. We regress betas during months t to $t+h$ (h =0, 2, 5 or 11) on cumulative returns from month $t-12$ to $t-2$ in a stock-month panel, along with other characteristics. Results are reported below and slope coefficients in regressions using ranks are multiplied by 100 for readability. Standard errors are clustered by month and firm. We use *** to denote statistical significance at the 0.1% level, ** denotes significance at the 1% level, and * denotes significance at the 5% level.

Characteristic	Realized beta			
	One-month	Three-month	Six-month	Twelve-month
Assets	−0.28***	−0.31***	−0.33***	−0.32***
Assets-to-market	0.16***	0.14***	0.14***	0.14***
Bid-ask spread	0.08***	0.05**	0.03*	0.02
Book-to-market	−0.02	−0.02	0.01	0.03
Capital intensity	0.02	0.01	0.01	0.01
Capital turnover	0.21***	0.18***	0.14***	0.11***
Cash-flow-to-book	−0.01	−0.01	−0.01	−0.01
Cash-to-short-term-inv.	0.01	0.01	0.02	0.02
Earnings-to-price	0.02	0.01	−0.00	−0.01
FF3 Idio. vol.	0.07***	0.08***	0.08***	0.07***
Fixed costs-of-sales	0.12***	0.11***	0.11***	0.11***
Gross profitability	−0.05*	−0.05**	−0.05*	−0.04
Intermed. mom	−0.02	−0.02	−0.02*	−0.03**
Investment	−0.01	0.00	0.01	0.02*
Leverage	0.03	0.04*	0.04**	0.05***
Long-term reversal	0.11***	0.11***	0.10***	0.08***
Market beta	0.77***	0.76***	0.75***	0.72***
Market cap.	0.83***	0.81***	0.81***	0.80***
Momentum	0.23***	0.23***	0.23***	0.20***
Net operating assets	−0.03*	−0.03*	−0.02	−0.02
Operating accruals	0.01	0.01*	0.01*	0.02**
Operating leverage	−0.17***	−0.16***	−0.13***	−0.10***
PPE-chg-to-Assets-chg	−0.03**	−0.02**	−0.02*	−0.01
Price rel. 52wk high	−0.20***	−0.18***	−0.16***	−0.14***
Price-to-cost-margin	−0.01	−0.01	−0.01	−0.01
Profit margin	0.01	0.01	0.01	−0.00
Return on NOA	−0.04**	−0.05***	−0.05***	−0.05***
Return on assets	−0.13***	−0.16***	−0.17***	−0.17***
Return on equity	0.05*	0.08***	0.09***	0.10***
SGA-to-sales	−0.13***	−0.11***	−0.10***	−0.12***
Sales-to-assets	0.03*	0.04**	0.04**	0.04**
Sales-to-price	0.06	0.06	0.07*	0.08*
Short-term reversal	0.03**	0.04***	0.05***	0.04***
Tobin's Q	0.01	0.01	0.03	0.05
Turnover	0.42***	0.41***	0.40***	0.38***
Unexplained volume	0.01	0.01	0.00	−0.00
Constant	0.77***	0.78***	0.79***	0.78***
R^2 (%)	7.78	17.53	22.76	25.55

The conclusion from these beta forecasting results is that many "anomaly" characteristics, specifically those based on past returns such momentum and long-term reversal, are potent indicators of future realized risk, even when only looking at risk through the lens of the simple, one-dimensional CAPM or through multifactor models such as the Fama and French (2015) five-factor model. Factor betas are significantly time-varying, and stock characteristics appear useful for tracking that beta variation. This finding hints at a route to reconciling the average return patterns associated with stock characteristics, namely those based on past returns, within a dynamic conditional model of risk and return. We pursue this route next by looking at dynamic multifactor models.

3. Models

We postulate a conditional factor pricing model for individual stocks and investigate the model's viability for explaining observed momentum and long-term reversal effects for expected returns. Conditionality enters into our model via factor loadings, which we specify to be a function of observable firm characteristics. As a firm's characteristics evolve, so do its conditional risk exposures and thus its model-implied expected returns. From the model's point of view, any relative persistence in stock performance—such as that captured by a momentum strategy—must originate from relative persistence in risk exposures.

The primary empirical challenge is estimating a conditional asset pricing model. We use firm characteristics as instruments to help identify firms' otherwise hard-to-measure dynamic factor exposures. Within this "instrumented loading" specification, we explore two model variants. The first uses IPCA, which treats factors as latent and estimates the factors that best associate with characteristics-based risk exposures. The second and more restrictive model relies on observable prespecified factors such as the Fama–French factors.

3.1. Instrumented principal component analysis

KPS provide a detailed analysis of the IPCA model, which we summarize here. They model the $N \times T$ panel of excess returns as a conditional pricing model of the form

$$r_{i,t+1} = \underbrace{(z'_{i,t}\Gamma)}_{\beta_{i,t}} f_{t+1} + \epsilon_{i,t+1}. \quad (3)$$

Assets are exposed to a set of K unobservable factors, which are denoted f_{t+1}. The strong association between stock-level characteristics and future market betas (Table 1) motivates the assumption that characteristics are good instruments for conditional risk. IPCA directly embeds this feature within the specification of $\beta_{i,t}$. In particular, assets' dynamic conditional factor loadings may depend on observable asset characteristics contained in the $L \times 1$ instrument vector $z_{i,t}$ (which includes a constant). The $L \times K$ matrix Γ defines the mapping between a potentially large number of characteristics and a small number of risk factor exposures. In particular, Γ is the set of linear combinations of candidate characteristics that best predicts betas.[3] Latent factors and loadings in model (3) are estimated by minimizing the sum of squared model errors. KPS provide an efficient computational algorithm for estimation, and Kelly et al. (2020) analyze its asymptotic properties.

The IPCA mapping between characteristics and loadings provides a formal statistical bridge between characteristics and factor betas. The "restricted" form of the model shown in Eq. (1) also imposes that characteristics influence expected returns only because they determine betas. This form of the model fixes the factor model intercept (alpha) to zero, imposing the economic restriction that risk premia solely reflect compensation for covariance risk exposure. With this restriction, the model can only accommodate a momentum effect *if* it represents factor risk, defined as covariance with the latent factors recovered by this estimation procedure. Whether common risk generates momentum (and long-term reversals) is an empirical question. If momentum or other past return effects are better described as anomaly alphas generated from firm-specific returns, the restricted model above will not generate momentum and will be outperformed by an alternative model. We investigate these empirical tests below.

3.2. Instrumented Fama–French model

The best overall conditional model studied by KPS uses latent factors. However, they also find that standard models with observable factors, such as the Fama–French five-factor model, are dramatically improved when their loadings are instrumented with observable characteristics.[4] Including characteristics in the specification of Fama–French loadings puts an observable factor model on the same informational footing as IPCA.

Specifically, our second variation on model (3) replaces f_{t+1} with the five Fama–French factors rather than treating f_{t+1} as latent. In this case, it is convenient to rewrite model (3):

$$r_{i,t+1} = \text{vec}(\Gamma)'(f_{t+1} \otimes z_{i,t}) + \epsilon_{i,t+1}. \quad (4)$$

The term $f_{t+1} \otimes z_{i,t}$ is the $KL \times 1$ vector of each factor interacted with each characteristic. Because the factors are observable, an OLS regression of returns onto the factor/characteristic interactions recovers Γ and in turn recovers the conditional loadings, $\beta_{i,t}$. Thus, Γ can be determined by a panel OLS regression of individual stock returns on aggregate factors interacted with lagged characteristics.[5] In robustness analyses, we also consider the traditional ad hoc dynamic beta estimation from the literature that uses rolling 60-month time series regressions to estimate conditional betas.

Our main IPCA analyses use a five-factor IPCA specification. This choice is motivated by the analysis in KPS and makes our model comparable with the Fama–French five-factor specification that we study.

3.3. Models of momentum

To investigate the ability of an asset pricing model to capture the momentum effect, we analyze three competing return predictors, each of which proxies for the true conditional expected asset return, $E_t[r_{i,t+1}]$.

The first predictor is the traditional momentum signal. This uses each asset's recent past return performance—defined as a moving average of prior returns—as a signal for conditional expected future returns. This predictor is agnostic of the model. Our primary momentum construction estimates $E_t[r_{i,t+1}]$ with $\bar{r}_{i,t} = \sum_{j=2}^{12} r_{i,t-j}$, which is the standard 2–12 momentum filter originally proposed by Jegadeesh and Titman (1993) and Asness (1994) and popularized by Fama and French (1996) and Carhart (1997).

The second predictor is the model-based predictor, defined as the conditional expectation of the factor component of returns, $\beta'_{i,t}\lambda_t$, where $\lambda_t = E_t[f_{t+1}]$. To focus squarely on the role of time-varying risk exposures, our analysis treats the expected factor return as constant: $E_t(f_{t+1}) = \lambda$. Hence, time-varying risk premia on assets only occur through their time-varying factor loadings or betas. Our estimate of this static price of risk is the simple average of the factor realizations in the estimation sample: for in-sample calculations, it is $\widehat{\lambda} = \frac{1}{T}\sum_{t=1}^{T} f_t$. For out-of-sample calculations, it is $\widehat{\lambda}_t = \frac{1}{t}\sum_{\tau=1}^{t} f_t$, which technically

[3] The model accommodates a potentially large set of conditioning instruments by performing a dimension reduction in the characteristic space. If there are many characteristics that provide noisy but informative signals about a stock's risk exposures, then aggregating characteristics into linear combinations isolates the signal and averages out the noise. It also easily handles unbalanced panels, as KPS describe.

[4] The characteristics provide information about the conditional Fama–French factor loadings, similar in spirit to Ferson and Harvey (1991), who use macroeconomic variables to estimate time-varying market betas. Our approach uses individual asset characteristics to help estimate time-varying betas on the five Fama–French factors using IPCA methodology with observable factors.

[5] Write $\tilde{z}_{i,t+1} \equiv f_{t+1} \otimes z_{i,t}$, and then $\text{vec}(\hat{\Gamma})$ is given by $(\sum_{i,t} \tilde{z}_{i,t+1}\tilde{z}'_{i,t+1})^{-1} \sum_{i,t} \tilde{z}_{i,t+1} r_{i,t+1}$.

sees time variation but only due to the varying estimation sample.

The third predictor is recent unexplained stock performance—defined as a moving average of model residuals $\epsilon_{i,t}$ defined in (1). We use residual stock returns after controlling for the conditional factor model and therefore define residual momentum as $\bar{\epsilon}_{i,t} = \sum_{j=2}^{12} \epsilon_{i,t-j}$.

We compare the return predictability of these three variables to evaluate the success or failure of the asset pricing model in describing the momentum anomaly. The predictive content of $\bar{r}_{i,t}$ establishes the baseline momentum effect in our sample. If an asset pricing model successfully explains the momentum effect, then all of the predictive content from raw momentum should be attributable to $\beta'_{i,t}\lambda_t$, and $\bar{\epsilon}_{i,t}$ will have zero incremental predictive power. If instead the model does a poor job of capturing momentum, then the signal content of stock momentum will be inherited by residuals, and as a result residual momentum will enter as a significant predictor defining a profitable strategy. It is also possible that residual momentum could exceed raw momentum if controlling for $\beta'_{i,t}\lambda_t$ hedges out factors negatively correlated to momentum that have positive expected returns.

We conduct prediction tests in a variety of ways. First, we run panel predictive regressions of $r_{i,t+1}$ on $\bar{r}_{i,t}$, $\beta'_{i,t}\lambda_t$, and $\bar{\epsilon}_{i,t}$. We also consider predictive regressions that instead use the cross-sectional ranks of each of these predictors, which is a closer counterpart to the often studied cross-sectional sorts in the literature. Likewise, to easily compare our findings with the prior literature, we analyze trading strategies that sort stocks into portfolios based on each signal to track and compare signal performance.

4. Can a conditional model explain momentum and reversals?

We evaluate IPCA's ability to explain momentum in a host of specifications and data sets, both in sample and out of sample. We also investigate how our findings differ when using a range of alternative formation windows other than 2–12 and find that our model-based conditional expectations explain a large portion of momentum and long-term reversal phenomena. We also show that short-term reversal is a distinct phenomenon not captured by IPCA.

4.1. IPCA model

Table 2 reports our main empirical result, and it reports results for the performance of the IPCA model in explaining the momentum effect.

Panel A of Table 2 reports univariate panel predictive regressions of the form

$$r_{i,t+1} = c_0 + c_1 s_{i,t} + e_{i,t+1},$$

where $s_{i,t}$ represents either 2–12 return momentum ($\bar{r}$), the model-based expected return ($\beta'\lambda$), or momentum in model residuals ($\bar{\epsilon}$) (we drop the inessential t subscripts from here on). In all regressions, we cluster standard errors by month to account for cross-sectional correlation in returns. A perfect proxy for the true conditional expected return would have an intercept of zero, a slope of one, and a comparatively high R^2 (Mincer and Zarnowitz, 1969). While this is possible for the raw predictors, it will not be the case for ranked predictors due to their scaling.

The "$\bar{r}$" columns of Panel A establish the baseline behavior of stock momentum in the primary sample. It is interesting to note that the raw return momentum signal has no predictive power for future returns. Both its predictive slope coefficient and panel R^2 are almost exactly zero. Only after it is converted into a cross-sectional rank (fourth column) does return momentum predict future returns, in which case the monthly panel R^2 on past momentum rank is 0.02%, with a significantly positive slope coefficient of 0.72 (t-stat = 2.5). The interpretation of this slope coefficient is that the highest momentum stock is predicted to have a monthly return that is 0.72% greater than the lowest momentum stock. This low predictive R^2 nonetheless translates into potent trading strategy performance, as indicated in the portfolio sorts of Panel B. Sorting on $\bar{r}$ produces an annualized return spread between the two most extreme quintile-sorted portfolios, Q5–Q1, of 8.3%, on average (t-stat = 3.3) with a Sharpe ratio of 0.48 (t-stat = 3.3). Sharpe ratio t-statistics are based on the asymptotic standard error formula of Lo (2002).

In comparison with return momentum, the model-based conditional expected return ("$\beta'\lambda$" columns of Panel A) has much stronger predictive power. In its raw form, the predictive slope is 0.99 (t-stat = 14). While this slope is significantly different from 0 at the 0.1% level, it is insignificantly different from 1.0, and the intercept is indistinguishable from 0. Thus, we cannot reject the hypothesis that the model gives unbiased estimates of conditional expected stock returns. The corresponding monthly panel R^2 is 0.37%, or 18 times higher than the ranked version of return momentum. Unlike momentum, converting $\beta'\lambda$ to a cross-sectional rank slightly weakens its predictive signal, as measured by the R^2 (which is 0.32%). Nonetheless, the slope coefficient implies that the highest $\beta'\lambda$ stock is predicted to have a monthly return that is 3.24% greater than the lowest $\beta'\lambda$ stock. Forming a Q5 - Q1 portfolio spread based on the model's (in-sample) conditional expectation estimate produces an annualized average return of 33.6% (t-stat = 16.5) and a Sharpe ratio of 2.39 (t-stat = 14.8).

Finally, we consider momentum in IPCA prediction residuals. In univariate analysis, the predictive power of residual momentum is weaker than that of total return momentum. The univariate slope coefficient on the ranked signal falls to an insignificant 0.30 (t-stat = 1.3), and the Q5–Q1 Sharpe ratio falls to a marginally significant 0.30 (t-stat = 1.9). Turning to Panel B, we find that our factor-model-based strategy is far more profitable than raw momentum, while the residual momentum strategy produces insignificant profits. This latter result is the opposite of what Grundy and Martin (2001) find, a point that we investigate in further detail in Section 5.

Univariate tests indicate that the model produces much more potent return predictions than simple momentum signals. A more direct test of the conditional model's ability to explain momentum is to conduct joint predictive regressions controlling for momentum and the model-based expected return simultaneously. These bivariate regressions

Table 2
Momentum and the IPCA model.
Panels A and C report coefficient estimates, t-statistics, and R^2 (in percentage) of univariate and bivariate panel regressions, respectively, of the next month's excess stock returns on the current month's signal (left three columns) or signal rank (right three columns), where t-statistics use standard errors clustered by month. Panel B reports annualized average returns (left three columns) and Sharpe ratios (right three columns) of equal-weighted quintile portfolios (Q1, Q2, etc.) sorted on each of the three signals. The row labeled Q5–Q1 denotes the spread portfolio that is long Q5 and short Q1, and t-statistics are provided for these averages and Sharpe ratios. Average returns are reported in annualized percentage. Slope coefficients in regressions using ranks are multiplied by 100 for readability.

A. Univariate regressions	Raw signal			Rank signal		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Constant	0.01	0.00	0.01	0.01	0.01	0.01
(t-stat)	(3.59)	(0.13)	(3.70)	(3.70)	(3.70)	(3.70)
Coeff	−0.00	0.99	−0.00	0.72	3.24	0.30
(t-stat)	(−0.29)	(14.02)	(−0.18)	(2.52)	(13.91)	(1.26)
R^2 (%)	0.00	0.37	0.00	0.02	0.32	0.00
B. Portfolio sorts	Average return			Sharpe ratio		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Q1	7.96	−4.59	10.17	0.30	−0.22	0.38
Q2	8.59	5.59	10.14	0.43	0.29	0.51
Q3	10.26	9.76	10.21	0.55	0.49	0.55
Q4	12.64	15.93	10.61	0.67	0.75	0.57
Q5	16.25	29.01	14.57	0.69	1.14	0.64
Q5–Q1	8.29	33.59	4.39	0.48	2.39	0.30
(t-stat)	(3.30)	(16.47)	(2.09)	(3.29)	(14.81)	(1.88)
C. Bivariate regressions	Raw signal			Rank signal		
	1	2	3	4	5	6
Constant	0.00	−0.00	0.01	0.01	0.01	0.01
(t-stat)	(0.11)	(−0.03)	(3.24)	(3.70)	(3.70)	(3.70)
$\bar{r}$	−0.01		−0.00	−0.14		3.10
(t-stat)	(−1.45)		(−0.29)	(−0.42)		(3.13)
$\beta'\lambda$	1.06	1.03		3.27	3.30	
(t-stat)	(11.83)	(12.96)		(11.88)	(12.62)	
$\bar{\epsilon}$		−0.00	0.00		−0.34	−2.57
(t-stat)		(−2.40)	(0.28)		(−1.29)	(−2.90)
R^2 (%)	0.40	0.39	0.00	0.32	0.32	0.05

are shown in Panel C. We find that the predictive content of momentum is mostly subsumed by the model-based expected return. Controlling for the conditional model, the momentum signal loses significance and even switches sign. This fact holds whether we compare raw or cross-sectionally ranked momentum signals (columns 1 and 4). The model-based expected returns appear to subsume all of the predictive information that the momentum signal provides. These results indicate that conditional expected returns, derived from dynamic factor exposures, are primarily responsible for the predictability embodied by momentum. We develop this interpretation further in the subsequent analysis.

4.2. IPCA out of sample

The timing of our model ensures that conditioning characteristics entering into $\beta_{i,t}$ are fully known to market participants at or before time t. The estimates of the static parameter matrix Γ, however, use information from the entire sample. So, while there is no look-ahead bias in Table 2, the model-based results are in-sample predictions.

In Table 3, we conduct the same comparative predictive analysis using entirely out-of-sample model-based prediction. In particular, when forecasting realized return $r_{i,t+1}$, we estimate Γ using data only through date t. Our initial estimation window uses data from January 1966 to June 1971 to forecast the July 1971 return (i.e., we require at least a five-year estimation window). In each subsequent month, we recursively reestimate the model and construct forecasts using an expanding, backward-looking sample. Small differences in performance of total return momentum between Tables 2 and 3 are due to the difference between full sample (1966–2014) and out-of-sample (1971–2014) evaluation periods.

The out-of-sample prediction power of IPCA's conditional expectations is very similar to the in-sample findings. The univariate out-of-sample R^2 is 0.28% for the raw and ranked versions of $\beta'\lambda$. The univariate intercept is insignificant, but the slope coefficient drops below one to 0.77, indicating an attenuation bias in out-of-sample $\beta'\lambda$. The slightly lower predictive power could be poorer out-of-sample performance or simply due to less data being used to construct each forecast. At the same time, residual momentum offers insignificant predictive slopes in both

Table 3
Momentum and the IPCA model, out of sample
We estimate the IPCA model recursively out of sample with an initial training sample of 1966–1971. For more table details, see Table 2's notes.

A. Univariate regressions

	Raw signal			Rank signal		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Constant	0.01	0.00	0.01	0.01	0.01	0.01
(t-stat)	(3.73)	(1.42)	(3.81)	(3.82)	(3.82)	(3.82)
Coeff	−0.00	0.77	−0.00	0.69	3.03	0.35
(t-stat)	(−0.49)	(12.15)	(−0.00)	(2.37)	(13.46)	(1.45)
R^2 (%)	0.00	0.28	0.00	0.01	0.28	0.00

B. Portfolio sorts

	Average return			Sharpe ratio		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Q1	9.24	−2.90	10.63	0.34	−0.14	0.40
Q2	9.19	7.03	10.48	0.46	0.37	0.53
Q3	11.12	10.85	11.25	0.61	0.56	0.62
Q4	13.35	16.48	11.73	0.72	0.79	0.64
Q5	16.55	27.99	15.36	0.71	1.13	0.69
Q5–Q1	7.30	30.89	4.74	0.42	2.29	0.33
(t-stat)	(2.79)	(15.17)	(2.16)	(2.78)	(13.76)	(1.96)

C. Bivariate regressions

	Raw signal			Rank signal		
	1	2	3	4	5	6
Constant	0.00	0.00	0.01	0.01	0.01	0.01
(t-stat)	(1.45)	(1.33)	(3.46)	(3.82)	(3.82)	(3.82)
$\bar{r}$	−0.01		−0.01	−0.08		2.74
(t-stat)	(−1.56)		(−0.62)	(−0.24)		(2.66)
$\beta'\lambda$	0.84	0.80		3.05	3.07	
(t-stat)	(11.26)	(11.45)		(11.60)	(12.19)	
$\bar{\epsilon}$		−0.00	0.01		−0.24	−2.20
(t-stat)		(−1.97)	(0.66)		(−0.91)	(−2.40)
R^2 (%)	0.32	0.30	0.01	0.28	0.28	0.03

the raw and ranked cases (t-stat = 0.0 and t-stat = 1.5, respectively).

Panel B shows that the trading strategy performance of model-based predictions is not an artifact of in-sample overfitting. While there is minor attenuation of the out-of-sample Q5–Q1 annualized average return of 30.9% and Sharpe ratio of 2.29 (compared to an in-sample 33.6% and 2.39, respectively), it remains a more than fourfold improvement on simple return momentum. The residual momentum strategy delivers marginally significant profitability with an annualized Sharpe ratio of 0.33 (t-stat = 1.96).

As in Panel C of Table 2, the predictive coefficients on $\bar{r}$ and $\bar{\epsilon}$ are small and insignificant in either the raw or ranked signal cases. These results indicate that, even on an out-of-sample basis, the model-based conditional expected returns subsume the predictive information in momentum. The close similarity between Tables 2 and 3 indicate that our findings are not driven by in-sample biases.

4.3. Interpretation

Before moving on to the predictability of other past return horizons, it is important to understand what we mean by saying the IPCA model "explains" momentum. Our results imply that the momentum characteristic (past 12-month return) picks up time-varying exposure to latent factor risk, defined through the covariance matrix of returns. Sorting on past 12-month returns is a noisy measure of sorting on conditional beta exposure to priced factors in the economy. Hence, controlling for an accurate measure of this time-varying exposure captures the bulk of momentum's premium. Past returns are a simple, ad hoc way of capturing covariance risk to priced factors. Economically, these results imply that past return anomalies are exposed to conditional covariances of priced factors, but we do not yet know why those factors are priced. Put differently, our model uses characteristics known to be related to average returns from a vast empirical literature to construct latent factor exposure whose dynamics happen to be picked up reasonably well by the simple moving average of returns. But our framework (and the literature) remains silent on why those characteristics are related to average returns in the market or what economic risks or state variables they represent. That question remains elusive and is beyond the scope of this paper. However, what is important to take away from our results is that past return sorts, such as momentum, inherently pick up this time-varying risk exposure, which in part leads to a significant return for these strategies.

4.4. Other formation windows

Thus far, we have focused on IPCA's ability to capture the momentum effect via a conditional factor model. Of the other various price trend phenomena shown in the literature (including short-term and long-term re-

Table 4
Other formation windows. Regression results using alternative windows to form $\bar{r}$, capturing long- and short-term reversals.

Formation		Rank signal regressions				
		Univariate		Bivariate		
		$\bar{r}$	R^2 (%)	$\bar{r}$	$\beta'\lambda$	R^2 (%)
2	12	0.72	0.02	−0.14	3.27	0.32
		(2.52)		(−0.42)	(11.88)	
13	24	−0.67	0.01	−0.30	3.12	0.32
		(−3.34)		(−1.54)	(13.91)	
25	36	−0.42	0.01	0.00	3.08	0.32
		(−2.52)		(0.01)	(14.13)	
1	1	−2.04	0.12	−1.02	4.08	0.34
		(−7.08)		(-2.76)	(14.61)	

versal), 2–12 momentum has received the most attention due to its strength, robustness, and its survival to trading costs (Asness et al., 2014; Frazzini et al., 2018). Asness et al. (2013) find that long-term reversal exists across many asset classes. Moreover, KPS find that long-term reversal is a statistically significant instrument in these data, indicating that it provided useful conditioning information even in the presence of a host of other characteristics.

In Table 4, we revisit other commonly studied price trend predictors vis-a-vis the IPCA conditional factor model. In addition to the traditional momentum strategy based on returns over 2–12 months prior to portfolio formation, we consider signals based on returns over the 13–24 and 25–36 months prior to portfolio formation (long-term reversal of DeBondt and Thaler, 1985) and returns over the most recent one month (short-term reversal of Jegadeesh, 1990). All of these price trend signals are only predictive when they are cross-sectionally ranked, therefore we focus our comparisons on panel predictive regressions using ranked signals. For each signal, we report the univariate panel regression as well as the bivariate regression that controls for the model-estimated conditional expected return. The estimated $\beta'\lambda$ is the same in all cases (the model is estimated once and used for all comparisons), and we investigate whether the model's expected returns help explain the performance of each past return signal.

Confirming well-known results from the literature, long-term reversal (defined as performance either two or three years prior to portfolio formation) and short-term reversal enter with significant negative predictive coefficients (momentum's coefficients are repeated here from Table 2 for convenient comparison). The amount of predictability coming from momentum (0.02%) is greater than what long-term reversal provides (0.01%) but much less than what short-term reversal provides (0.12%), consistent with the literature (Grinblatt and Moskowitz, 2004).

The bivariate regression shows that model-based conditional expected returns subsume the predictive effects of every longer-term price trend signal. Momentum and both versions of long-term reversal become statistically insignificant and in some cases switch sign. Furthermore, the bivariate R^2 is essentially equal to the univariate R^2 for $\beta'\lambda$ from Table 2, suggesting no incremental predictive power for past returns once we control for conditional expected returns from the model.[6]

On the other hand, the magnitude of the short-term reversal's coefficient is cut in half from −2.0 (t-stat = -7.1) to −1.0 (t-stat = -2.8), but it remains statistically significant. High-frequency reversal is at least partially driven by illiquidity of small firms (Asness et al., 2014; Nagel, 2012; Lo and MacKinlay, 1990), and therefore part of the effect does not stem from systematic risk compensation, so it is perhaps unsurprising that it survives in a model that abstracts from trading costs and liquidity issues.

5. Robustness and further analysis

In this section we consider the robustness of our results along several dimensions. First, we exclude momentum itself as an instrument and analyze its comparative importance given the richness of the remaining instrumental information. Second, we repeat this exercise not only removing momentum but also removing any characteristic based on past returns. Third, we estimate the instrumented Fama–French model to explore whether conditional risk exposures continue to explain momentum returns when using prespecified risk factors (none of which contain momentum either). Fourth, we conduct spanning tests of momentum and reversal portfolios versus IPCA factors and show that resulting alphas are generally insignificant.

After these robustness checks, we turn to daily data to support our interpretation with novel out-of-sample analysis that echoes our initial findings in Section 2: our dynamic beta estimates are excellent predictors of future returns' covariance with aggregate factors. Finally, we show that our model-based $\beta'\lambda$ strategy avoids crashes in the momentum strategy documented by Daniel and Moskowitz (2016) and more generally exhibits less higher-moment risk than using ad hoc past return signals for momentum portfolios.

5.1. Robustness

In this subsection, we show that our conclusions are robust to dropping price-trend characteristics (including momentum itself) from the model, and are qualitatively unchanged when reporting strategies alpha with respect to the five (Fama and French, 2015) factors. Moreover, we discuss the turnover inherent in our benchmark model and show how it can be reduced quite easily.

[6] Our referee points out that long-term reversal and momentum experience significant January effects. Jegadeesh and Titman (1993) and Yao (2012) suggest that the long-term reversal effect is almost entirely earned in the month of January and likely reflects size effects. In our data set we also find that long-term reversal is insignificant outside of the month of January. The fact that our model accounts for long-term reversal even during the month of January reveals that stock-level conditioning information such as size helps explain the profitability of long-term reversal more broadly. We also note that momentum loses money about three-fourths of the time in January, a higher proportion than other months (it loses money about one-third of the time overall). Interestingly, our model-based strategy loses money about one-sixth of the time and never in January.

Table 5
Robustness.
For the first panel (return momentum), the alpha is with respect to the Fama and French (2015) five-factor model. For remaining panels, the alpha is with respect to the estimated IPCA factors. We omit the alpha calculation for Panels E and F as they are identical to the first panel's. Alpha is reported in annualized percentage.

	Univariate regression		Average return	Q5-Q1 Sharpe ratio	FF 5-factor
	$\bar{r}$		R^2 (%)		Alpha
Return momentum ($\bar{r}$)					
Estimate	0.72	0.02	8.29	0.48	9.18
t-stat	(2.52)		(3.30)	(3.29)	(2.95)

	Bivariate regression			Average return		Q5-Q1 Sharpe ratio		IPCA 5-factor
	$\beta'\lambda$	$\bar{r}$	R^2 (%)	$\beta'\lambda$	$\bar{\epsilon}$	$\beta'\lambda$	$\bar{\epsilon}$	Alpha
Panel A: Main								
Estimate	3.27	−0.14	0.32	33.60	4.40	2.39	0.30	−3.20
t-stat	(11.88)	(0.42)		(16.48)	(2.10)	(14.82)	(1.89)	(−1.23)
Panel B: Main OOS								
Estimate	3.05	−0.08	0.28	30.88	4.75	2.29	0.33	2.00
t-stat	(11.60)	(−0.24)		(15.17)	(2.17)	(13.75)	(1.97)	(0.48)
Panel C: Excluding momentum and reversal OOS								
Estimate	2.85	0.51	0.26	29.76	4.51	2.16	0.31	5.09
t-stat	(11.82)	(1.69)		(14.31)	(2.06)	(13.11)	(1.89)	(1.25)
Panel D: Excluding all return variables OOS								
Estimate	1.54	0.78	0.09	14.97	3.87	0.93	0.27	0.91
t-stat	(6.04)	(2.66)		(6.13)	(1.76)	(6.03)	(1.73)	(0.23)
Panel E: Fama–French instrumented								
Estimate	1.83	0.45	0.12	19.82	7.46	1.56	0.54	
t-stat	(8.79)	(1.52)		(10.70)	(3.73)	(10.21)	(3.56)	
Panel F: Fama–French rolling								
Estimate	0.09	0.93	0.03	2.34	8.02	0.23	0.70	
t-stat	(0.54)	(3.68)		(1.58)	(4.84)	(1.58)	(4.84)	

5.1.1. *Alpha tests*

We report alphas from regressions of the form

$$mom_t = \alpha + \delta' f_t + error_t,$$

where f_t are a set of candidate asset pricing factors as well as Newey and West (1987) t-statistics with three lags. As a baseline for comparison, the top panel of Table 5 reports results for the basic momentum strategy. Momentum's average return is significantly positive, as is its alpha with respect to the five (Fama and French, 2015) factors.

Panels A and B report results corresponding to our main in-sample and out-of-sample analyses (Tables 2 and 3). The main analysis in this table is the final column, which reports the unconditional momentum alpha versus the corresponding IPCA model for each sample. The alpha in both cases is economically small and statistically insignificant. Panel A shows that the in-sample IPCA factors (exactly those estimated in KPS) are associated with a negative momentum alpha of −3.2% per year (t-stat = - 1.2). Panel B shows that alpha is also insignificant when IPCA factors are estimated on an out-of-sample basis (alpha of 2.0% with t-stat = 0.5).

5.1.2. *Excluding momentum and long-term reversal from IPCA*

In our IPCA specifications thus far, momentum itself is an instrument in the beta specification. Importantly, this does not mechanically imply that the conditional asset pricing model can explain momentum, because the model tightly restricts the use of characteristics. Specifically, characteristics can only enter as a means of capturing conditional covariances among returns. The evidence of Section 2 indicates that momentum is a powerful predictor of factor exposures, justifying its role as an instrument for beta. Nevertheless, one may wonder if the IPCA model's ability to explain the momentum effect is driven by including momentum as a beta instrument. To investigate this, Panel C drops all momentum and long-term reversal variables from the set of IPCA instruments and uses the out-of-sample estimates of IPCA factors. Despite removing these characteristics, the estimated IPCA model continues to explain a large part of the momentum effect. Momentum's alpha estimate is moderate in this case (5.1% per annum) but insignificant (t-stat = 1.25). The $\bar{r}$ bivariate predictive coefficient remains insignificant in the presence of $\beta'\lambda$. Furthermore, the residual momentum strategy based on this model continues to have a comparatively small Sharpe ratio of 0.31 (t-stat = 1.89).

5.1.3. *Excluding all past return characteristics from IPCA*

Panel D repeats the analysis of Panel C, excluding not only momentum and long-term reversal characteristics but also all characteristics based on past returns: price relative to 52-week high and short-term reversal. Again, the analysis is based on the out-of-sample estimates of IPCA factors. Removing these characteristics as well has little effect on the results. The estimated IPCA model continues to explain a large part of the momentum effect. Momentum's alpha estimate is now a miniscule 0.91% per annum and is statistically no different from zero (t-stat = 0.23), and the residual momentum strategy based on this model has a Sharpe ratio of 0.27 (t-stat = 1.73).

Based on this evidence, conditional expected returns measured by the IPCA factor model appear to capture a sizable chunk of past return anomalous trading strategies such as momentum and long-term reversals, even when excluding past return variables as instruments. The residual momentum that remains after accounting for these factors is either nonexistent or unreliable. The most optimistic results for residual momentum produce an economically significant alpha of 1–5% but with substantial noise. Even under this optimistic light, however, conditional expected returns from our model capture two-thirds of the momentum returns, and with a lot more precision. It is perfectly reasonable, too, that the momentum phenomenon may be a function of both common risk premia and idiosyncratic return predictability—it is just that the former are more substantial.

5.1.4. Instrumented Fama–French model

Our analysis has centered on conditional expected returns from a latent factor model. The basic insight of using stock characteristics to instrument for conditional factor loadings can likewise be applied to prespecified observable factors. As described in Eq. (4), we can estimate conditional loadings on Fama–French factors in direct analogy with the conditional loading in IPCA. In this simple model, returns are regressed onto factors interacted with firm characteristics, transparently illustrating that beta instruments are only helpful insofar as they predict risk exposure.

Panel E reports results using this conditional version of the Fama–French model to estimate expected returns. This model takes the five Fama and French (2015) factors as given but uses all of the characteristics listed in Table A.1 in the appendix, including the momentum characteristics themselves, to measure time-varying betas. The only difference between this model and IPCA is that here we do not estimate the latent factors but use those specified by Fama and French (2015): the betas on those factors are instrumented with all available characteristics.

In the bivariate regression in Panel E, the Fama–French model-based $\beta'\lambda$ captures much of the predictive information in momentum, where the past return, controlling for $\beta'\lambda$, has an insignificant and reduced slope coefficient of 0.45 (t-stat = 1.52). However, residual momentum from this model remains potent with an annualized average return of 7.5% (t-stat = 3.7) and a Sharpe ratio of 0.54 (t-stat = 3.6).[7] In other words, like its static counterpart, the instrumented Fama–French model is also unable to explain the momentum effect. This failure occurs even though momentum has been included as one of the characteristics instrumenting the Fama–French factors' betas.

Finally, Panel F shows the performance of momentum vis-a-vis the Fama–French model when betas are estimated in rolling 60-month regressions. Comparison of Panels E and F shows that while instrumented Fama–French betas do not explain momentum, they do offer some improvement over the rolling-window regression approach adopted by Grundy and Martin (2001). Panel F is essentially an update of the Grundy and Martin (2001) result, and it shows that model-based expected returns are insignificant predictors of future returns. It also shows that the residual momentum strategy delivers a higher Sharpe ratio (0.70, t-stat = 4.8) than raw momentum (0.48, t-stat = 3.3), consistent with the findings of Grundy and Martin (2001). From these results, Grundy and Martin (2001) conclude that momentum is an idiosyncratic return phenomenon and an anomaly with respect to models of aggregate risk. Our IPCA-based analysis, however, suggests that the rolling Fama–French model is misspecified, and the misspecification assigns too much of the interesting structure in returns to model residuals. A more flexible specification of the risk-return tradeoff that accounts for dynamic betas and latent factors demonstrates that aggregate risk can explain much of the momentum effect in the data and residual momentum is a comparatively small component of expected return dynamics.

5.1.5. Turnover

Above, we show an impressive Sharpe ratio for the optimal trading strategy implied by our model. This indicates that the Stochastic Discount Factor (SDF) implied by our model lies much closer to the mean-variance-efficient frontier than does the standard momentum portfolio. From a practical investment perspective, trading costs and other frictions may prevent investors from achieving the attractive risk-return tradeoff of the model-based optimal strategy. To explore this practical consideration, we show turnover of the model-based strategy and compare it to turnover of the traditional momentum strategy.

We find that our main model-based strategy requires three times as much turnover as raw momentum. Hence, the gross performance we have reported is unlikely to be realized net of trading costs. We find that even the instrumented Fama and French model-based strategy involves 1.4 times as much turnover as momentum. Evidently, the same instrumenting information that helps us approach the mean-variance frontier also leads the strategy to rebalance frequently. This is an interesting economic point linked to our earlier findings that conditional betas exhibit more time-variation than typically captured by rolling time series regressions.

We also note that the model-based strategy continues to achieve large Sharpe ratio improvements over momentum even after we constrain its turnover. For example, by dropping all return variables, we have turnover that is 10% lower than that of the momentum strategy. Yet this version of the model-based optimal strategy (shown in Table 5) still returns, on average, 15% per annum with a Sharpe ratio of 0.9—much greater than that afforded by momentum. In short, there is scope for optimizing the three-way tradeoff between risk, return, and trading costs that makes the IPCA-implied strategy attractive for practical investment strategies.

[7] Because the factors in Panel E are identical to those used in the top panel, we refrain from simply repeating the top panel's alpha statistics here and in Panel F.

5.2. Predicting realized betas with conditional betas

Section 2 provides strong evidence that characteristics forecast realized market betas and realized betas on the Fama and French (2015) factors. IPCA prescribes a specific linear combination of characteristics, given by $z'_{i,t}\Gamma$, as the best predictors of stock i's beta on each IPCA factor. In this section, we return to our original motivation of realized beta prediction and test whether the characteristic combination dictated by IPCA is indeed a superior predictor for exposures to IPCA factors on an out-of-sample basis.

To do so, we first construct daily realizations of our estimated factors.[8] We estimate the model out of sample on the monthly primary sample. We then hold fixed the $\hat{\Gamma}_{t-1}$ estimate (using monthly data before month t) and construct out-of-sample daily IPCA factor returns $\hat{f}_{\tau\in t}$ using data on realized stock returns (for each day τ in month t) and characteristics as of month $t-1$.[9] This is an out-of-sample exercise because no daily data are ever used in the estimation of $\hat{\Gamma}_{t-1}$, and furthermore the month t data were not used. We then calculate the realized out-of-sample beta of each stock on each IPCA factor using daily returns within month t, which we denote the one-month $RealBeta_t^{OOS}$—to construct three-month, six-month, and twelve-month $RealBeta_t^{OOS}$ we use daily returns within months t-to-$t+2$, t-to-$t+5$, and t-to-$t+11$, respectively. To test the predictive power of IPCA betas for out-of-sample realized betas, we regress $RealBeta_t^{OOS}$ on the conditional beta ($\hat{\beta}_{i,t-1} = z_{i,t-1}\hat{\Gamma}$) estimated by IPCA based on characteristics in month $t-1$, prior to the daily returns used to construct $RealBeta_t^{OOS}$.

Table 6 reports the results. IPCA conditional betas are powerful and unbiased predictors of future realized factor exposures. The IPCA $\beta^k_{i,t-1}$ receives a slope coefficient that is very nearly one in all cases, both economically and statistically.[10] While the constant is usually statistically significant, its economic magnitude is always tiny. The R^2 ranges between 1% and 8% for monthly realized betas, between 4% and 26% for 3-month realized betas, between 4% and 26% for 6-month realized betas, and between 7% and 44% for 12-month realized betas. We conclude that the estimated conditional betas are efficient predictors of out-of-sample realized betas, providing strong support for our claim that accurate expected returns are driven by accurate risk exposures.

5.3. Momentum crashes

Daniel and Moskowitz (2016) investigate momentum "crashes"—periods during which the momentum strategy loses at least half its value. Of the two biggest crashes they focus on, the 2009 crash is in our post-1966 data. Over the three-month period from March to May 2009, the $\bar{r}$ momentum strategy lost a cumulative 55.5%, shown in Fig. 1 in the narrow shaded region. According to the decomposition in the figure, the crash is due almost entirely to residual momentum. The $\bar{\epsilon}$ strategy (the dotted line) loses a cumulative 48.7% from March to May 2009, virtually identical to total return momentum. The momentum crash, on the other hand, is essentially absent from the $\beta'\lambda$ strategy. From March to May 2009, the IPCA spread portfolio loses only 6.8%.

The largest ever three-month loss for the $\beta'\lambda$ strategy is 11.5% and occurs around the unwinding of the tech boom in early 2000. This and a variety of other summary statistics are reported in Table 7, using overlapping three-month returns from the underlying monthly strategies.[11] The $\beta'\lambda$ strategy is superior to the $\bar{r}$ momentum strategy according to every risk metric analyzed. This result is not surprising given the very high Sharpe ratio of the $\beta'\lambda$ strategy but need not have been the case for tail events. Interestingly, the standard deviation of the Q5–Q1 strategy based on $\beta'\lambda$ is the same as that for residual momentum, but skewness is positive for $\beta'\lambda$ and negative for residual momentum.

Why does the model-based strategy avoid these left tail events, such as momentum's fate in 2009? The answer and the reason our model-based momentum strategy largely avoids these crashes is that IPCA aggregates information across a wide range of conditioning characteristics, only one of which is momentum. For example, strategies based on other characteristics fared far better than momentum in the first half of 2009. A primary example being a market beta strategy (long high beta stocks, short low beta stocks) that was up 49% from March to May 2009. Daniel and Moskowitz (2016) in fact show that the 2009 momentum crash largely comes from the large market rebound coupled with the fact that at that time, the momentum strategy was doing the opposite—shorting high market beta stocks and going long low beta stocks. They show that momentum's up-market beta is more than double its down-market beta during bear markets but not reliably during other times.

IPCA intrinsically accounts for such beta dynamics. Its tangency portfolio takes optimal positions based on the distributions of the factors themselves rather than on an individual signal for individual stocks as in the case of momentum. Knowledge of stock-level dynamic betas is rolled into the model's estimated conditional mean-variance-efficient factors. A momentum strategy, be it raw or residual, does not have this flexibility. The direction of the strategy is preordained so that even if the beta of the momentum strategy switches sign as shown in Daniel and Moskowitz (2016), the momentum trade pays no heed. The correct asset pricing model, however, will use conditional covariance dynamics to explicitly reformulate the conditionally mean-variance-efficient factor, and will therefore explicitly trade out of the momentum position prior to the crash exactly because of the sign flip in its ex ante betas.

Fig. 2 plots the correlation between the momentum and market beta characteristics, with the 2009 crash period shaded. Throughout 2008, there was a modest negative correlation, which led these two managed portfo-

[8] We order the factors by the explained variance.

[9] The appendix describes this in detail.

[10] This is not a mechanical result. In diversified portfolios, portfolio weights do not map one-for-one to realized betas—otherwise it would be the case that large stocks would always have the largest market betas, which of course is not what one sees in the data.

[11] Monthly returns tell the same story—see Table A.3 in the appendix.

Table 6
Predicting realized beta with conditional beta.
Standard errors are clustered by month and firm. Usual t-statistics (of the null that the parameter equals zero) are reported in parentheses. For slope coefficients, we also report in rows labeled "[$t:\beta=1$]" t-statistics of the null that the parameter equals one.

	Factor				
	1	2	3	4	5
A: One-month *RealBeta*^{OOS}					
Constant	−0.00	−0.01	0.00	−0.00	−0.00
(t-stat)	(−0.61)	(−3.14)	(0.42)	(−0.21)	(−1.03)
Slope	1.00	1.01	1.00	1.00	1.00
(t-stat)	(479.12)	(183.32)	(158.53)	(144.74)	(143.95)
[$t:\beta=1$]	[0.26]	[1.77]	[0.18]	[−0.03]	[0.44]
R^2 (%)	7.88	2.11	1.62	2.16	1.29
B: Three-month *RealBeta*^{OOS}					
Constant	0.00	−0.01	0.00	−0.00	−0.00
(t-stat)	(0.45)	(−2.90)	(1.19)	(−0.04)	(−1.04)
Slope	1.00	1.02	1.01	1.00	1.01
(t-stat)	(388.46)	(134.51)	(133.23)	(136.79)	(122.62)
[$t:\beta=1$]	[−0.25]	[2.20]	[1.22]	[0.16]	[1.03]
R^2 (%)	25.86	7.15	4.97	7.25	4.38
C: Six-month *RealBeta*^{OOS}					
Constant	0.00	−0.01	0.00	−0.00	−0.00
(t-stat)	(1.35)	(−3.81)	(1.82)	(−0.12)	(−1.25)
Slope	1.00	1.03	1.02	1.00	1.02
(t-stat)	(366.51)	(118.06)	(120.31)	(128.50)	(111.91)
[$t:\beta=1$]	[−0.73]	[3.49]	[2.86]	[0.41]	[1.77]
R^2 (%)	37.02	9.22	7.64	11.00	6.39
D: Twelve-month *RealBeta*^{OOS}					
Constant	0.00	−0.01	0.01	−0.00	−0.01
(t-stat)	(2.70)	(−5.00)	(2.20)	(−0.40)	(−1.86)
Slope	1.00	1.05	1.05	1.01	1.03
(t-stat)	(339.69)	(91.38)	(99.47)	(119.90)	(90.77)
[$t:\beta=1$]	[−1.55]	[4.56]	[4.65]	[0.67]	[2.63]
R^2 (%)	43.73	9.21	9.56	13.95	6.99

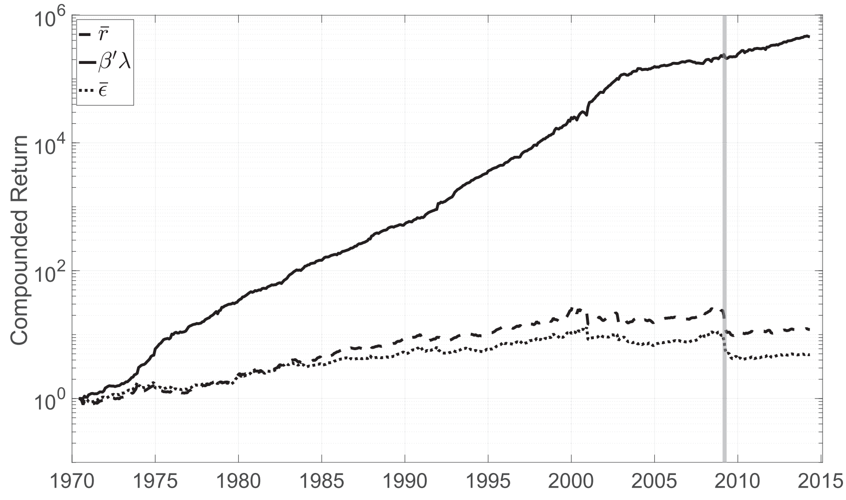

Fig. 1. Cumulative momentum strategy returns. This figure shows cumulative strategy returns on log (base ten) scale. The dashed line shows the quintile spread strategy for total return momentum $\bar{r}$, the solid line shows that based on IPCA $\beta'\lambda$, and the dotted line shows residual momentum $\bar{\epsilon}$. The narrow shaded region is the 2009 momentum crash. The analysis is based on the main out-of-sample estimates, using data from 1966 to 2014 with 1966–1971 as the initial training window.

lios to modestly hedge each other's risk. As market turbulence came in the wake of Lehman Brother's failure in September 2008, this negative correlation increases: IPCA identifies that these portfolios' hedging property is becoming stronger. Just before the momentum crash, stocks with low momentum are more likely than usual to have high market beta. As the crash unfolds, this hedging property becomes stronger still. IPCA rolls all of this information into its low dimensional factor structure, as it attempts to identify mean-variance-efficient pricing factors. Hence,

Table 7
Momentum strategy summary statistics.
The table shows summary statistics from the Q5, Q1, and Q5-Q1 spread portfolios quarterly returns, constructed using return momentum $\bar{r}$, expected returns $\beta'\lambda$, or residual momentum $\bar{\epsilon}$. Numbers are nonannualized. $\beta'\lambda$ comes from out-of-sample IPCA estimated on the primary sample.

	Q5			Q1			Q5-Q1		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
Mean	4.35	7.27	4.01	2.52	−0.62	2.88	1.90	7.97	1.23
Standard deviation	13.18	13.80	12.55	15.46	11.49	15.44	8.96	7.37	7.39
Skewness	0.04	0.34	−0.12	1.24	0.13	1.01	−1.32	1.68	−1.69
Excess kurtosis	0.87	1.19	0.40	6.90	2.20	4.76	7.79	7.58	8.46
Minimum	−39.38	−41.32	−40.68	−46.57	−40.02	−44.82	−55.54	−11.54	−48.66
1st percentile	−30.92	−26.47	−29.67	−32.48	−31.10	−31.13	−30.41	−5.88	−28.52
Median	4.42	6.37	4.11	1.17	−0.30	1.69	2.59	6.88	1.88
99th percentile	37.99	42.82	33.90	50.83	28.63	48.04	24.10	32.89	17.54
Maximum	60.15	73.89	43.18	115.97	59.48	106.78	37.28	61.20	26.29

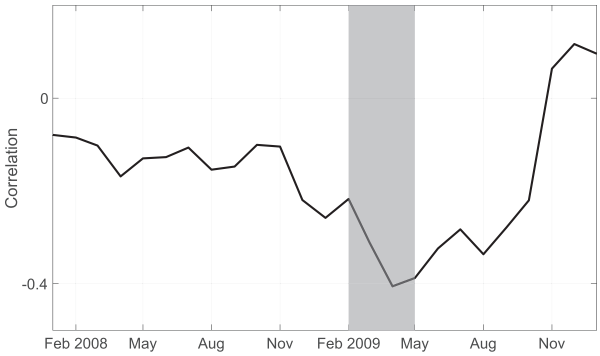

Fig. 2. Correlation between the momentum and market beta characteristics. The shaded region is the 2009 momentum crash. The cross-sectional correlation is computed for each month from every firm in our data.

the IPCA strategy's performance is only mildly affected by the momentum crash because it combines many strategies, some of which performed exceedingly well during the crash, and leverages their dynamic hedging behavior.[12] Importantly, the IPCA framework is able to capture these dynamics quickly, whereas simple ad hoc moving averages of past returns do not, leaving momentum portfolios exposed to these crash events.[13]

6. Conclusion

Momentum and reversal are well-known predictors of future market returns. We show that these past return characteristics are strongly predictive of a stock's realized exposures to common risk factors. This fact is direct evidence that price trend strategies are in part explainable as compensation for common factor exposure. Motivated by this observation, we examine the role of price trends as conditioning information in a full-fledged conditional factor pricing model. Using IPCA to estimate latent risk factors and assets' time-varying betas on these factors, we show that the momentum and long-term reversal effects are capturing conditional risk premia largely explained by conditional betas in our no-arbitrage factor pricing model. The results offer a new risk-based interpretation to some of the return predictability from momentum and reversal strategies and provide insights into how to improve return predictability using our model. Future research may pin down what economic risks or state variables our latent factors capture, but past return sorts appear to be good predictors of future factor exposure that is priced in equilibrium.

Appendix A

Estimating IPCA at the monthly frequency expresses the idea that betas may vary from month to month. When we construct daily factor realizations, we use the Γ estimated from monthly frequency characteristics and returns, along with those monthly frequency characteristics. This embodies the assumption that betas may change between months but are constant within each month.

Recall (cf. KPS) that the IPCA factor estimate in month t is given by a cross-sectional regression of stock returns realized in month t on betas realized in month $t-1$:

$$f_t = \left(\beta_{t-1}'\beta_{t-1}\right)^{-1}\beta_{t-1}'r_t,$$

where β_{t-1} is the $(N \times K)$ matrix of the K factor exposures for each of the N stocks, whose returns are in the $N-$vector r_t.[14] The IPCA model assumes that

$$\beta_{t-1} = Z_{t-1}\Gamma$$

for a $(N \times L)$ matrix of observable characteristics Z_{t-1} that are mapped to risk exposures via the asset- and time-invariant mapping Γ. Therefore the factor estimate can be

[12] This illustrates the impact of IPCA's explicit consideration of time-varying characteristic correlation. A static combination of characteristic-managed portfolios, for instance, obtained by simply running PCA on the portfolios, would miss this information.

[13] Daniel and Moskowitz (2016) also show that rolling regression estimates of market beta also fail to capture these dynamics quickly enough, and in fact, when using these estimates to hedge market exposure to mitigate momentum crash, risk can make things even worse. IPCA, by using changing characteristics to update risk exposures, is better able to adjust exposure to avoid these crashes.

[14] This is a common expression. If betas were static, this is also the factor estimate of (asymptotic) principal components in Chamberlain and Rothschild (1983) and Connor and Korajczyk (1986). Furthermore, to the extent that portfolio sorts can also be cast as a regression on indicator-like regressors, this is also how sorted portfolios a la Fama and French (1993) are constructed (although in multiple univariate regressions instead of a multivariate regression).

Table A.1
Characteristics.

- Assets
- Assets-to-market
- Bid-ask spread
- Book-to-market
- Capital intensity
- Capital turnover
- Cash-flow-to-book
- Cash-to-short-term-inv.
- Earnings-to-price
- FF3 Idio. vol.
- Fixed costs-of-sales
- Gross profitability
- Intermed. mom. (*)
- Investment
- Leverage
- Long-term reversal (*)
- Market beta
- Market cap.
- Momentum (*)
- Net operating assets
- Operating accruals
- Operating leverage
- PPE-chg-to-assets
- Price rel. 52wk high
- Price-to-cost-margin
- Profit margin
- Return on NOA
- Return on assets
- Return on equity
- SGA-to-sales
- Sales-to-assets
- Sales-to-price
- Short-term reversal
- Tobin's Q
- Turnover
- Unexplained volume

Notes: Assets are total assets. Assets-to-market are total assets over market capitalization as of the end December. Bid-ask spread is the average daily bid-ask spread in the previous month. Book-to-market is book equity over market equity. Capital intensity is the ratio of depreciation and amortization to total assets. Capital turnover is net sales to lagged total assets. Cash-flow-to-book is the ratio of net income to book equity. Cash-to-short-term-inv. is the ratio of cash and short-term investments to total assets. Earnings-to-price is the ratio of income before extraordinary items to market capitalization. FF3 Idio. vol. is the standard deviation of residuals from the Fama–French three-factor model using daily data in that month. Fixed costs-of-sales is selling, general, and administrative expenses plus research and development expenses plus advertising expenses, divided by net sales. Gross profitability is gross profits divided by book equity. Intermed. mom. is the 7-to-12-months-ago sum of returns. Investment is the year-on-year growth of total assets. Leverage is the ratio of long-term debt and debt in current liabilities to long-term debt plus debt in current liabilities plus shareholders' equity. Long-term reversal is the 13-to-36-months-ago sum of returns. Market beta is correlation between stock and market excess returns times the ratio of their volatilities: the former from daily returns over the past year and the latter from overlapping three-day returns over the past five years. Market cap. is the total market capitalization. Momentum is the 2-to-12-months-ago sum of returns. Net operating assets is operating assets minus operating liabilities, scaled by total assets. Operating accruals are changes in noncash working capital minus depreciation, scaled by total assets. Operating leverage is the sum of cost of goods sold and selling, general, and administrative expenses over total assets. PPE-chg-to-assets is the property, plants, and equipment changes over lagged total assets. Price rel. 52wk high is the ratio of last month's price to its highest weekly price during the previous 52 weeks. Price-to-cost-margin is the difference between sales and cost of goods sold, divided by net sales. Profit margin is operating income after depreciation over net sales. Return on NOA is ratio of operating income after depreciation divided by lagged net operating assets. Return on assets is income before extraordinary items divided by total assets. Return on equity is income before extraordinary items divided by book equity. SGA-to-sales is the ratio of selling, general, and administrative expenses to net sales. Sales-to-assets is the ratio of net sales to total assets. Sales-to-price is the ratio of net sales to market capitalization. Short-term reversal is the lagged one-month return. Tobin's Q is total assets plus market equity minus cash and short-term investments minus deferred taxed, divided by total assets. Turnover is the month's volume traded of shares outstanding. Unexplained volume is the difference between actual volume and that predicted from the prior month using a regression of daily volume on a constant and absolute returns. Characteristics denoted (*) are those dropped when excluding momentum and reversal.

written

$$f_t = \left(\Gamma' Z'_{t-1} Z_{t-1} \Gamma\right)^{-1} \Gamma' Z'_{t-1} r_t$$
$$= \left(\Gamma' Z'_{t-1} Z_{t-1} \Gamma\right)^{-1} \Gamma' x_t,$$

where x_t is the $L-$vector of characteristic-managed portfolio realizations given by the cross product of the month $(t-1)$ characteristics with the month t stock returns.

To create instead daily factor realizations within month t, all we must do is change the characteristic-managed portfolio realizations to be at the daily frequency. Let $\tau(t)$ be a day within month t and $r_{\tau(t)}$ the $N-$vector of stock returns realized that day. Then

$$x_{\tau(t)} = Z'_{t-1} r_{\tau(t)}$$

are the returns on the L characteristic-managed portfolios on day τ of month t. The daily factor estimate is then the

Table A.2
Predicting multifactor betas with characteristics.

We construct stock-level monthly realized OLS annual betas on the nonmarket Fama-French (2015) factor returns return using daily data in months t to t+11. We regress betas during months t to $t+11$ on cumulative returns from month $t-12$ to $t-2$ in a stock-month panel, along with other characteristics. Results are reported below. Standard errors are clustered by month and firm. We use *** to denote statistical significance at the 0.1% level, ** denotes significance at the 1% level, and * denotes significance at the 5% level.

Characteristic	Realized beta			
	SMB	HML	RMW	CMA
Assets	−0.97***	0.13	0.81***	0.33**
Assets-to-market	0.52***	0.35***	−0.87***	0.04
Bid-ask spread	−0.56***	−0.20***	0.92***	−0.23***
Book-to-market	−0.15*	0.03	0.32***	−0.12
Capital intensity	−0.07**	−0.05	0.04	−0.00
Capital turnover	0.12*	−0.22**	−0.35***	0.30***
Cash-flow-to-book	0.09***	0.05*	−0.07**	0.08***
Cash-to-short-term-inv.	0.07*	−0.08**	−0.18***	0.08**
Constant	0.45***	−0.51***	−0.60***	−0.53***
Earnings-to-price	0.24***	0.31***	−0.29***	0.37***
FF3 Idio. vol.	0.17***	−0.17***	−0.48***	−0.14***
Fixed costs-of-sales	−0.29***	−0.47***	−0.03	−0.16*
Gross profitability	−0.02	0.21***	0.12	0.00
Intermed. mom	−0.03	−0.02	0.04	−0.05
Investment	−0.03	−0.13***	0.08*	−0.41***
Leverage	0.02	−0.04	0.06	−0.05
Long-term reversal	−0.12***	−0.23***	0.23***	−0.51***
Market beta	0.46***	−0.41***	−0.55***	−0.46***
Market cap.	−0.01	−0.57***	−0.31**	−0.54***
Momentum	−0.02	−0.34***	0.21***	−0.38***
Net operating assets	0.12***	0.11**	−0.16***	0.13***
Operating accruals	0.01	−0.03*	−0.03	−0.00
Operating leverage	−0.07	0.39***	0.33***	−0.14
PPE-chg-to-assets-chg	−0.02	0.02	0.14***	−0.08**
Price rel. 52wk high	−0.06**	0.05*	0.20***	0.06
Price-to-cost-margin	0.14**	0.15**	0.01	−0.10
Profit margin	−0.23***	−0.21***	0.39***	−0.11*
Return on NOA	−0.01	0.10**	0.07	0.09**
Return on assets	−0.06	0.25***	−0.06	−0.08
Return on equity	−0.22***	−0.35***	0.61***	−0.30***
SGA-to-sales	0.02	−0.10	0.09	0.25***
Sales-to-assets	0.03	−0.17***	−0.10*	−0.12**
Sales-to-price	−0.16*	−0.25**	0.30**	−0.27**
Short-term reversal	0.01	−0.04*	0.06**	−0.05
Tobin's Q	0.08	−0.19*	−0.14	−0.06
Turnover	0.11***	−0.45***	−0.11**	−0.49***
Unexplained volume	0.05***	0.06***	−0.02*	0.06***
R^2 (%)	4.78	8.77	3.77	5.55

Table A.3
Momentum strategy summary statistics, monthly returns.

Notes: The table shows summary statistics for the Q5, Q1, and Q5-Q1 spread portfolios monthly returns, constructed using return momentum $\bar{r}$, expected returns $\beta'\lambda$, or residual momentum $\bar{\epsilon}$. Numbers are nonannualized. $\beta'\lambda$ comes from out-of-sample IPCA estimated on the primary data sample.

	Q5			Q1			Q5-Q1		
	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$	$\bar{r}$	$\beta'\lambda$	$\bar{\epsilon}$
					Monthly returns				
Mean	1.38	2.33	1.28	0.77	−0.24	0.89	0.61	2.57	0.40
Standard deviation	6.70	7.15	6.47	7.79	5.97	7.77	5.01	3.89	4.19
Skewness	−0.46	0.64	−0.58	1.06	−0.29	0.96	−1.99	2.31	−2.44
Excess kurtosis	2.76	6.41	2.08	6.81	3.18	6.26	13.35	14.31	16.24
Minimum	−31.59	−29.18	−31.29	−28.85	−31.75	−29.39	−36.48	−10.41	−35.05
1st percentile	−20.68	−17.53	−18.08	−19.33	−18.29	−18.62	−16.31	−4.89	−13.06
Median	1.63	2.19	1.52	0.51	0.06	0.49	1.14	2.13	0.93
99th percentile	17.01	21.60	15.23	23.03	12.72	24.19	11.32	15.37	9.73
Maximum	28.81	45.70	22.74	50.77	30.19	50.42	21.62	35.12	12.35

linear combination of these portfolios' daily returns given by

$$f_{\tau(t)} = \left(\Gamma' Z'_{t-1} Z_{t-1} \Gamma\right)^{-1} \Gamma' x_{\tau(t)}.$$

References

Asness, C., Frazzini, A., Israel, R., Moskowitz, T., 2014. Fact, fiction, and momentum investing. J. Portf. Manag. 40 (5), 75–92.
Asness, C. S., 1994. Variables that explain stock returns. Dissertation. University of Chicago.
Asness, C.S., Moskowitz, T.J., Pedersen, L.H., 2013. Value and momentum everywhere. J. Financ. 68 (3), 929–985.
Barberis, N., Shleifer, A., Vishny, R., 1998. A model of investor sentiment. J. Financ. Econ. 49 (3), 307–343.
Belo, F., 2010. Production-based measures of risk for asset pricing. J. Monet. Econ. 57, 146–163.
Berk, J.B., Green, R.C., Naik, V., 1999. Optimal investment, growth options, and security returns. J. Financ. 54 (5), 1553–1607.
Carhart, M.M., 1997. On persistence in mutual fund performance. J. Financ. 52 (1), 57–82.
Chamberlain, G., Rothschild, M., 1983. Arbitrage, factor structure, and mean-variance analysis on large asset markets. Econometrica 51 (5), 1281–1304.
Chordia, T., Shivakumar, L., 2002. Momentum, business cycle, and time–varying expected returns. J. Financ. 57 (2), 985–1019. 01096.
Connor, G., Korajczyk, R.A., 1986. Performance measurement with the arbitrage pricing theory: a new framework for analysis. J. Financ. Econ. 15 (3), 373–394.
Conrad, J., Kaul, G., 1998. An anatomy of trading strategies. Rev. Financ. Stud. 11 (3), 489–519.
Daniel, K., Hirshleifer, D., Subrahmanyam, A., 1998. Investor psychology and security market under- and overreactions. J. Financ. 53 (6), 1839–1885.
Daniel, K., Moskowitz, T.J., 2016. Momentum crashes. J. Financ. Econ. 122 (2), 221–247.
DeBondt, W.F.M., Thaler, R., 1985. Does the stock market overreact? J. Financ. 40 (3), 793–805.
Fama, E.F., French, K.R., 1993. Common risk factors in the returns on stocks and bonds. J. Financ. Econ. 33 (1), 3–56.
Fama, E.F., French, K.R., 1996. Multifactor explanations of asset pricing anomalies. J. Financ. 51 (1), 55–84.
Fama, E.F., French, K.R., 2015. A five-factor asset pricing model. J. Financ. Econ. 116 (1), 1–22.
Ferson, W.E., Harvey, C.R., 1991. The variation of economic risk premiums. J. Polit. Econ. 99 (2), 385–415.
Frazzini, A., Israel, R., Moskowitz, T.J., 2018. Trading costs. Technical Report. Unpublished working paper. National Bureau of Economic Research.
Freyberger, J., Neuhierl, A., Weber, M., 2020. Dissecting characteristics nonparametrically. Rev. Financ. Stud. 33 (5), 2326–2377.
Gomes, J.F., Kogan, L., Zhang, L., 2003. Equilibrium cross section of returns. J. Polit. Econ. 111, 693–732.
Grinblatt, M., Moskowitz, T.J., 2004. Predicting stock price movements from past returns: the role of consistency and tax-loss selling. J. Financ. Econ. 71 (3), 541–579.
Grundy, B.D., Martin, J.S.M., 2001. Understanding the nature of the risks and the source of the rewards to momentum investing. Rev. Financ. Stud. 14 (1), 29–78. 01043.
Hong, H., Stein, J.C., 1999. A unified theory of underreaction, momentum trading, and overreaction in asset markets. J. Financ. 54 (6), 2143–2184.
Jegadeesh, N., 1990. Evidence of predictable behavior of security returns. J. Financ. 45 (3), 881–898.
Jegadeesh, N., Titman, S., 1993. Returns to buying winners and selling losers: implications for stock market efficiency. J. Financ. 48 (1), 65–91.
Jegadeesh, N., Titman, S., 2002. Cross-sectional and time-series determinants of momentum returns. Rev. Financ. Stud. 15 (1), 143–157.
Johnson, T.C., 2002. Rational momentum effects. J. Financ. 57 (2), 585–608.
Kelly, B.T., Pruitt, S., Su, Y., 2019. Characteristics are covariances: a unified model of risk and return. J. Financ. Econ. 134 (3), 501–524.
Kelly, B.T., Pruitt, S., Su, Y., 2020. Instrumented principal components analysis. Technical Report.
Kothari, S.P., Shanken, J., 1992. Stock return variation and expected dividends: a time-series and cross-sectional analysis. J. Financ. Econ. 31 (2), 177–210.
Li, D., Zhang, L., 2010. Does the q-theory with investment frictions explain anomalies in the cross-section of returns? J. Financ. Econ. 98, 297–314.
Li, E.X.N., Livdan, D., Zhang, L., 2009. Anomalies. Rev. Financ. Stud. 22 (11), 4301–4334.
Liu, L.X., Whited, T.M., Zhang, L., 2009. Investment-based expected stock returns. J. Polit. Econ. 117, 1105–1139.
Liu, L.X., Zhang, L., 2008. Momentum profits, factor pricing, and macroeconomic risk. Rev. Financ. Stud. 21.
Liu, L.X., Zhang, L., 2014. A neoclassical interpretation of momentum. J. Monet. Econ. 67, 109–128.
Lo, A.W., 2002. The statistics of sharpe ratios. Financ. Anal. J. 58 (4), 36–52.
Lo, A.W., MacKinlay, A.C., 1990. When are contrarian profits due to stock market overreaction? Rev. Financ. Stud. 3 (2), 175–205.
Mincer, J.A., Zarnowitz, V., 1969. The evaluation of economic forecasts. In: Economic Forecasts and Expectations: Analysis of Forecasting Behavior and Performance. National Bureau of Economic Research, Inc, pp. 3–46. NBER. Chapters
Nagel, S., 2012. Evaporating liquidity. Rev. Financ. Stud. 25 (7), 2005–2039.
Newey, W.K., West, K.D., 1987. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica 55 (3), 703–708.
Sagi, J.S., Seasholes, M.S., 2007. Firm-specific attributes and the cross-section of momentum. J. Financ. Econ. 84 (2), 389–434.
Vayanos, D., Woolley, P., 2013. An institutional theory of momentum and reversal. Rev. Financ. Stud. 26 (5), 1087–1145.
Yao, Y., 2012. Momentum, contrarian, and the january seasonality. J. Bank. Financ. 36 (10), 2757–2769.
Zhang, L., 2005. The value premium. J. Financ. 60 (1), 67–103.