How Tax Efficient Are Equity Styles?

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Abstract

We examine the after-tax returns and tax efficiency of size, value, growth, and momentum equity styles. Despite momentum having five times the turnover of value, they face similar tax rates. Value's tax exposure comes from high dividend income, while momentum's comes primarily from capital gains. While both styles continue to outperform the market on an after-tax basis, tax optimized portfolios can further improve after-tax returns. We examine both optimal tax lot selection and tax trading to improve tax efficiency, finding the latter to be a more powerful tool, especially in a dynamic setting. Optimal trading to manage capital gains incurs less tracking error than avoiding dividend income, resulting in capital gain-heavy styles such as momentum benefitting the most from tax optimization without incurring large tracking error costs.

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Empirical asset pricing studies predominantly focus on expected pre-tax returns. For a taxable investor, however, the after-tax returns of assets are the critical input for investment decisions. We explore the after-tax performance, tax exposure, and tax efficiency of equity style portfolios. First, we decompose the tax exposure of existing investment style portfolios into short and long-term capital gains and income components and compare them across styles. Second, we construct tax optimized versions of the portfolios that seek to minimize taxes while maintaining low tracking error. We examine both tax lot selection and tax trading to improve portfolio tax efficiency and consider both static and dynamic effects. Assessing the relative tax efficiency of various equity styles requires considering portfolios designed to manage taxes, as a rational tax sensitive investor would pursue.

We focus on equity styles based on size, value, growth, and momentum, which pervade the asset pricing literature and are popular investment styles in practice. First, we examine the effective tax rates and relative after-tax performance of available passive style index portfolios from academia and practice that are not designed to be tax aware. We focus primarily on long-only investable indices such as the Russell 1000 and 2000 core market, value, and growth portfolios and AQR Capital Management's U.S. large and small capitalization momentum indices. We also examine the standard Fama and French portfolios going back to 1927. We find that value and momentum have the highest tax exposures, which mute their relative outperformance over the market and growth, though significant after-tax premia remain. We consider two different tax code regimes: the current tax code and historical tax rates matched contemporaneously through time with returns, which are on average more punitive. This initial analysis complements Bergstresser and Pontiff (2012), though we examine different portfolios and calculate tax rates slightly differently.

Further investigating the tax consequences, we find that momentum and value face similar tax rates, despite momentum having five times more turnover than value. The reason is that momentum generates substantial short-term losses, offsetting many of its capital gains, and produces more long-term gains, which makes momentum more tax efficient. Value, on the other hand, has little capital gain exposure, but is exposed to significant dividend income, which is tax inefficient. In addition, the mix of taxation on capital gains versus income varies through time, which has different consequences for different equity styles. When viewed within a broader asset allocation strategy, the effective tax rate on momentum becomes even smaller, due to its realization of short term losses that can offset other gains in a broader portfolio, whereas the effective tax rate for value remains the same since income is not offsetting. Momentum is particularly valuable to a taxable investor in down markets for related reasons—when significant short-term losses can be realized in a down market they are valuable in offsetting capital gains from other less correlated investments within an asset allocation strategy. Portfolios with heavy dividend exposure (e.g., value), however, do not share this asymmetric feature and produce similar tax rates in up and down markets. Momentum, therefore, provides a taxable investor with an implicit hedge in down markets, illustrated vividly during the global financial crisis (GFC), when momentum's performance on an after-tax basis significantly exceeded its pre-tax returns. The tax benefits of momentum's short-term capital losses in a broader portfolio are also illustrated when combining with value, which produces even better after-tax returns from offsetting realized gains and losses within the same portfolio.

Since the portfolios typically studied in the literature (Fama and French (1993), Bergstresser and Pontiff (2012)) are not designed to be tax aware in any way, their analysis greatly understates and can distort the tax efficiency of various equity styles. Hence, the focus of our paper is to analyze how much after-tax returns can be improved across styles through tax optimization, and what the tradeoffs are between tax optimization and tracking error. We construct and analyze tax optimized versions of the equity styles to measure the after-tax performance of these styles with tax awareness and highlight variation in the ability to manage tax exposure across styles.

We consider tax optimized portfolios that seek to minimize taxes subject to low tracking error (to maintain their style characteristics). We construct an after-tax return versus tracking error frontier for each style to assess variation in this tradeoff across styles. We attempt to minimize the tax exposure through two mechanisms. The first is through tax lot choice which determines the cost basis used to compute capital gains. Choosing optimal tax lots incurs no tracking error. However, tax lot optimization only has a small effect on tax savings, even when looking at the portfolio myopically (one period). Moreover, in a dynamic setting, we show tax lot optimization can have an ambiguous effect on taxes, possibly incurring larger taxes than other simple tax lot rules. The second mechanism for reducing taxes is through optimal tax trading, which introduces tracking error to the portfolio. We consider minimizing taxes through optimal tax trading from two dimensions: 1) capital gains and 2) dividend income exposure.

We find that minimizing capital gains exposure (and ignoring dividends) improves after-tax returns across all styles without incurring large tracking error or style drifts, where momentum receives the largest after-tax improvements from capital gains optimization. Conversely, we find that dividend yield minimization (ignoring capital gains exposure) has a significant impact on tracking error. The intuition behind these results is that an investor has more discretion on the timing of gain and loss realization than on dividend income. Minimizing capital gains entails shifting more realized gains from short-term to long-term status and realizing more short-term losses, whereas the only way to reduce dividend income is to sell the stock before the ex-date (which triggers both a potential capital gain/loss realization and the wash sale rule, should the investor choose to buy the stock back after the ex-date), which has a much bigger effect on tracking error. Tax optimization is, therefore, less costly—in the sense of tracking error—for strategies whose tax exposure comes mostly from capital gains rather than dividends. In the case of value, which has high dividend exposure, a reduction in dividends is equivalent to a reduction in the value exposure. Value stocks are high dividend paying stocks, so selling or underweighting high dividend paying stocks reduces the alpha of a value strategy. In addition, reducing dividend exposure by selling stocks before the ex-date significantly increases capital gains exposure, which mutes the net improvement on taxes. Momentum, on the other hand, has low dividend exposure and its tax rate is determined predominantly by capital gains. Thus, momentum's tax rate can be reduced more without incurring significant style drift. Tax optimization has the biggest positive impact on value and momentum portfolios, further increasing their positive after-tax premia relative to the market, but with less tracking error for momentum.

The optimizations are myopic in the sense that we minimize taxes period by period. To account for dynamic trading and assess the importance of dynamic optimization for tax efficiency, we conduct a set of dynamic optimizations over various investment horizons that allow our optimizer perfect foresight of the future portfolio. The resultant portfolios provide an upper bound on how much tax efficiency can improve from taking into account future dynamics relative to myopic optimization. We find the improvements from dynamic optimization to be small, even when we have perfect information about future portfolio weights. Myopic optimization appears to capture the bulk of the tax improvements from optimal tax trading, even with perfect foresight of the future portfolio trades. The intuition behind this result is that the first-order effect from tax optimization is from offsetting current realized gains with current losses and deferring short-term gains to long term, which are decided at the current date. Dynamic effects are second order.

Our study is related to Bergstresser and Pontiff (2012), who examine the after-tax returns to individuals, corporations, and broker-dealers who face different tax rates on a set of benchmark portfolios that include the Fama-French size, value, and momentum factors. However, Bergstresser and Pontiff (2012) consider only standard portfolios that are not designed to address taxes in any way. The focus of our study is different in that we examine the tax efficiency of common equity styles and the components of each style that drive tax exposure, using this information to derive tax optimized versions of the equity styles and assess the relative tradeoff between after-tax performance and tracking error across equity styles. Variation in the efficiency of tax minimization across styles, including the assessment of the importance of tax lot determination and tax trading, sheds new light on the tax efficiency and after-tax performance of these styles.

The paper proceeds with Section I describing our data and the after-tax returns and tax exposures of standard equity style portfolios. Section II focuses on optimal tax lot selection to reduce tax exposure. Section III considers optimal tax trading to improve tax efficiency and after-tax returns, separately investigating capital gains and dividend income against the cost of tracking error. Section IV considers tax optimization in a dynamic setting and compares the results to a myopic optimization. Section V concludes.

I. Data and After-Tax Returns

We briefly describe the equity style portfolios we examine and detail our methodology for computing tax exposure and after-tax returns, which slightly modifies Bergstresser and Pontiff (2012) and present the after-tax returns.

A. Equity style portfolios

We focus on the market, value, growth, and momentum equity styles among both large and small cap universes. Research has shown these styles capture much of the cross-sectional variation in returns (Fama and French (1996, 2008)), and are also popular investment styles in the investment management industry.

We focus exclusively on U.S. equity indices and the U.S. tax code.[1] For U.S. small and large cap and value and growth equity styles we use for most of our analysis the Russell 1000, Russell 1000 Value and Russell 1000 Growth indices for our large cap portfolios and use the Russell 2000, Russell 2000 Value and Russell 2000 Growth indices for our small cap portfolios.[2]

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[1] We believe the broader implications addressed in this paper would be similar and could be extended in an international context.

[2] The Russell 1000 is a cap-weighted portfolio of the 1,000 largest stocks traded on the NYSE, AMEX, and NASDAQ as of June of each year. The Russell 1000 Value Index is comprised of the top 35-50% of stocks among the Russell 1000 that have the highest value characteristics as determined by the highest book-to-price ratios and the lowest I/B/E/S forecast long-term growth means. The Russell 1000 Growth Index is comprised of the 35-50% of stocks with the lowest book-to-price ratios and the highest I/B/E/S growth forecasts. Russell applies a non-linear probability algorithm to the distribution of stocks based on these two variables that typically identifies about the top 35% as value stocks, the bottom 35% as growth stocks and then weights the middle 30% of stocks as both value and growth. The Russell 2000 index is a value-weighted portfolio of the next 2,000 largest stocks in the U.S., and the Russell 2000 value and growth indices are defined as above among those 2,000 stocks. Russell excludes stocks trading below $1 per share, pink sheet and bulletin board stocks, closed-end funds, limited partnerships, royalty trusts, foreign stocks, and ADRs. Reconstitution occurs annually in June of each year, where stocks deleted in between reconstitution dates are not replaced, and spin-offs and IPOs are the only additions allowed in between reconstitution dates. Prior to 1987, they reconstituted on a quarterly basis and then changed to semi-annual from 1987 to June 30, 1989. Dividends are reinvested on the ex-date.

For the momentum style, we use the AQR Capital Momentum indices, which are constructed in a manner similar to the Russell methodology. The large cap momentum index takes the 1,000 largest stocks in the U.S. based on market capitalization (e.g., the Russell 1000) and ranks each stock based on its past year return (from *t-12* to *t-2*), following the convention in the literature of skipping the most recent month's return to avoid microstructure issues and high frequency and liquidity trades (see Jegadeesh and Titman (1993), Asness (1994), Fama and French (1996, 2008), and Grinblatt and Moskowitz (2004)). The top third of stocks based on momentum are then selected and value-weighted to form the momentum index. The same process is repeated for the next largest 2,000 stocks to form the small cap momentum index. The indices are reconstituted quarterly on the last day of each quarter.[3]

Our use of real-world index portfolios serves three purposes. First, the index portfolios are designed to passively capture a particular equity style, so there are no information issues associated with the portfolios' trading. Second, since the portfolios themselves are investable, they provide a set of style portfolios and returns that investors could actually obtain, providing a measure of the real after-tax return to each style that is achievable in practice. Finally, since other real-world issues, such as transactions costs, rebalancing frequency, and other implementation problems, are not explicitly modeled in this paper, use of real-world portfolios that presumably already address these issues allows us to focus on tax costs alone.[4] However, we also report a set of results for academic portfolios commonly used in the literature, notably those from Ken French's Data Library, to show that our conclusions are robust to other style implementations.

Stock return data comes from the Center for Research in Security Prices (CRSP) over the period July 1974 to June 2010. NASDAQ stocks first enter the CRSP universe in 1973, so 1974 is the earliest year we can start the sample while maintaining stock universe consistency, since the momentum indices require a year's worth of return history. The returns to the indices above are available for most of the sample period, from July 1979 for the Russell indices and from January 1980 for the AQR indices, and then we extend the series back to July 1974 by replicating the index using the CRSP universe of stocks and following, as closely as data availability allows, the official methodologies outlined for each index. As a check of our replication methodology, we compute the return correlation of each replicated index versus its actual index over their overlapping periods in

[3] The same security exclusions, security changes between reconstitutions, and treatment of dividends used by Russell are applied.

[4] For example, academic versions of momentum portfolios typically rebalance monthly, but the momentum index we use is rebalanced quarterly due to practical limitations on transactions costs.

Table A1 of the Internet Appendix. The return correlations are consistently above 0.98 for the large cap portfolios and between 0.92 and 0.99 for the small cap portfolios. We also examine an equal-weighted combination of value and momentum to further examine the observed benefits of combining value with momentum (Asness (1997) and Asness, Moskowitz, and Pedersen (2012)) on an after-tax basis.

In addition to examining investable indices, we also look at portfolios created and commonly used in academic studies, notably those of Fama and French (1993, 1996 and 2008). We focus exclusively on long-only portfolios and do not address the tax consequences of shorting or the efficacy of after-tax returns for long-short style portfolios. We use the CRSP value-weighted index as the market portfolio. For value, we use the large and small cap high book-to-market portfolios, which comprise the long side of Fama and French's HML factor. For growth, we use the large and small cap versions of Fama and French's low book-to-market portfolios, which comprise the short-side of HML. For momentum, we use the large and small cap versions of Fama and French's high momentum portfolio, which comprises the long side of their momentum factor UMD. A benefit of using these portfolios is that they provide returns going back to 1927. A drawback is they are not investable portfolios.

B. Tax Calculations and Assumptions

To calculate the tax exposure and after-tax returns of the equity style portfolios, we follow Bergstresser and Pontiff (2012), with some slight modifications. Specifically, we follow the U.S. tax code and detail below the assumptions we make about the income percentile of the investor, which affects tax rates, the choice of tax lots for the cost basis, and the netting of gains and losses and ability to carry forward excess losses according to the tax code.[5]

Tax rates. Tax rates are obtained from two sources: the Federal Individual Income Tax Rates History 1913 - 2009 from the Tax Foundation in Washington, D.C. and historical capital gains rates from the Department of the Treasury, Office of Tax Analysis (November 3, 2008). Tax rates on capital gains and dividend income vary by household income percentile. Table A2 in the Internet Appendix lists the year-by-year capital gains and dividend income tax rates for investors in the 99.99th and 95th percentiles of income in each year. We focus on the 99.99th income percentile to calculate the maximum tax rate, and therefore minimum after-tax return, facing an investor (but

[5] We do not address estate taxes or taxes at time of death. These issues are beyond the scope of the paper, but we acknowledge that the level of tax costs could be different under different scenarios. However, since our focus is on the relative tax costs across equity styles, these issues are not likely to affect the relative ranking of tax efficiency across styles.

present some results for the 95th income percentile as well). Several years have a mid-year rate change, which we ignore in our analysis by using the tax rates that existed at the beginning of the year. We also ignore differential capital gains treatment for holding periods other than those less than one year and greater than one year. These changes occur rarely and are typically small. Finally, dividend income tax rates reflect that of qualified dividends, and we treat all dividend income from our portfolios accordingly, which is a reasonable assumption based on the characteristics and reconstitution frequency of all the portfolios we examine.

We consider two different tax code regimes. First, we apply the current (end of our sample period) tax code to our portfolios historically. This analysis provides an evaluation of the average after-tax returns to the portfolios under the current tax regime as a proxy for the expected after-tax return to each strategy going forward today. Second, we also employ historical tax rates as if the strategies had been run in real time by adjusting the tax rates each year with changes to the tax code (according to Table A2) and aligning them contemporaneously with returns through time.

Tax assumptions. In order to calculate the tax exposure of each portfolio and its after-tax returns, we make several assumptions. First, we must choose a procedure for determining tax lots for the cost basis for each sale in order to determine capital gains and losses. Initially, we adopt the HIFO system of identifying tax lots, which entails taking the highest priced purchases out first when applying taxes to the portfolio. Results are similar using a FIFO (first-in, first-out) or LIFO (last-in, first out) system for tax lots. In addition, an "optimal" tax lot system could be adopted that seeks to minimize taxes by selecting the optimal tax lots to relieve. In Section II, we will explore the use of an optimal tax lot system.

We compare the tax implications of every portfolio from two perspectives. First, we look at each portfolio as a stand-alone investment, where losses are netted against any gains only within that particular portfolio, and any losses exceeding gains in a calendar year that cannot be used are then carried forward according to the tax code. Second, we assume that all losses can be applied immediately (no carryforward of losses), which would be true in the context of a broader portfolio if there are always sufficient gains coming from other investments against which to net those losses.[6] The first assumption computes the maximum tax effect from capital gains, and the second assumption computes the minimum tax effect from capital gains. Since the portfolios we consider

[6] For most funds and accounts the netting of losses across investments is allowed, but for mutual funds, for instance, the IRS does not allow an investor to cross-net unused losses from one fund against gains from another. We acknowledge that other institutional structures can have an impact on the tax efficiency of our investment strategies (e.g., ETFs), but considering the optimal institutional structure for each style is beyond the scope of this paper.

are likely to be part of a broader asset allocation, but at the same time adequate gains to net against within that broader portfolio may not always exist, the true tax effects likely lie somewhere in between these bounds.

These calculations are very similar to those made by Bergstresser and Pontiff (2012) on a different set of portfolios. However, a few small differences are worth noting. Unlike Bergstresser and Pontiff (2012), we do not examine separate returns for corporate, broker-dealer, or state tax rates. We also assume taxes can be paid with idle cash and do not assume taxes must be paid by further liquidation of the portfolio (self-financing), which causes a new set of taxes that then must be iterated to a fixed point where the tax burden is exactly equal to the cash generated from the sale of securities. We also report simple raw returns and not log returns as in Bergstresser and Pontiff (2012). Finally, and most importantly, Bergstresser and Pontiff (2012) take the style portfolios as given and compute the after-tax returns on their portfolios "as is." While we initially report the after-tax returns of our real-world portfolios "as is", the main focus of the paper is on tax aware style portfolios that modify portfolios to minimize the expected tax burden, as a rational tax-sensitive investor would do. We refer to these modified style portfolios as "tax managed" portfolios, which are novel to the literature.

C. After-Tax Returns and Tax Exposure

Table 1 reports the average annualized return before and after taxes on each equity style index portfolio (before any tax management). The first column of Panel A of Table 1 shows that among large cap stocks, value outperforms the market by about 135 basis points (bps) per year before taxes. Growth underperforms the market by 173 bps per year before taxes. Momentum outperforms the market by 243 bps per year before taxes. These results are consistent with a long academic literature that finds that momentum and value outperform the market and growth underperforms the market on average.[7]

Among small cap stocks, we find that value outperforms the market before taxes by an even wider margin of 1.88% per year, growth underperforms the market by 2.38% per year, and momentum outperforms the small cap market by 3.87% per year. These results are also consistent with the literature (Fama and French (1993, 2012), Hong, Lim, and Stein (2000), Grinblatt and Moskowitz (2004), Israel and Moskowitz (20122)). Finally, an integrated combination of value and momentum in large (small) caps outperforms the market by 2.16% (3.21%) per year before taxes.

[7] While these comparisons simply take the difference in returns between portfolios without any risk-adjustment, beta-adjusted returns or alphas yield nearly identical results.

The next two columns of Panel A of Table 1 report the annualized turnover (defined as the average dollars bought and sold divided by the imputed net asset value of each index) and dividend yield of the portfolios. Two key observations stand out. First, the momentum portfolios generate substantially more turnover than the other indices.[8] The substantially higher turnover of a momentum style, however, does not necessarily mean it exposes investors to higher capital gains taxes, since exposure to capital gains is a function of short and long-term gains *and* losses, which are all caused by turnover. Second, value portfolios have much higher dividend yields than the other indices. High value stocks tend to be high dividend paying stocks (relative to their market values) and hence expose investors to high dividend income taxes. Brennan (1970) and Litzenberger and Ramaswamy (1979) discuss the tax inefficiency of being exposed to high dividend income.

C.1 Carry-forward Losses as a Stand-Alone Investment

The next four columns of Panel A of Table 1 report the annualized average after-tax returns of the portfolios under the current U.S. tax code and their effective tax rates, which are the differences between the before- and after-tax returns divided by the pre-tax returns and represent the drag on performance from taxes. We also report the effective tax rates coming from capital gains and dividends separately. Treating each index as a stand-alone investment by netting out realized losses only against realized gains generated from the portfolio itself, and carrying forward any unused capital, we find that momentum has the highest effective tax rate of 20.2%, followed by value with 13.7%, and then the market and growth indices with about 7%. These tax rates are similar to those of Bergstresser and Pontiff (2012) on a similar set of style portfolios over a different time period, where the relative ranking of effective tax rates across styles is preserved. On an after-tax basis, therefore, the outperformance of value and momentum styles diminishes, though is still significant. Value still outperforms growth by 208 bps, and momentum outperforms growth by 216 bps. A similar pattern is observed among the small cap portfolios, though the differences in effective tax rates are smaller. Momentum has the highest effective tax rate of 22.7%, followed by value at 18.5%, then growth at 14.3%. However, the source of tax exposures for the value and momentum portfolios are very different. Value's tax exposure comes more evenly from capital gains and dividend income, whereas momentum's tax exposure comes primarily from capital gains.

[8] This is partly due to quarterly rebalancing of the momentum indices as opposed to annual rebalancing for the Russell indices, but it is also driven by the nature of the momentum strategy, which uses market price data that updates more frequently than book-to-market or earnings forecasts and hence generates more frequent changes in rankings among stocks.

The remaining columns of Table 1 report results repeating the analysis using the higher historical tax rates. Since value and momentum have higher tax exposure, the higher historical tax rates mute after-tax their outperformance further, though their premia remain significant, even under the more punitive historical tax rates.

C.2 Using All Losses Immediately within a Broader Portfolio

Panel B of Table 1 repeats the analysis assuming all losses can be used immediately to offset other gains in a broader portfolio, rather than being carried forward.[9] This assumption represents the minimum capital gains tax exposure for each style. The dividend tax exposure of the style portfolios is unaffected. Not surprisingly, the effective tax rates for every portfolio decline, and therefore after-tax returns rise, when losses can be applied immediately. The impact, however, varies considerably across equity styles based on the amount of capital gains and losses each style generates. For instance, the market index and value portfolio do not generate a lot of capital losses and have more of their tax exposure coming from dividend income. Hence, the ability to use losses in a broader portfolio is more limited for these styles. As a consequence, the after-tax returns to the market and value increase negligibly (less than 5 basis points). A slightly larger improvement is found for growth, where after-tax returns go up by 23 basis points per year, but the biggest increase is for momentum, where after-tax returns rise by 116 basis points per year, where momentum's effective tax rate falls from 20.2% to 11.4%. Because of its ability to generate short-term losses, momentum is particularly valuable in the context of a broader portfolio, where those losses can be used to offset gains elsewhere in the portfolio. Among small cap stocks, there is a similar picture. While small cap momentum has the highest effective tax rate (22.7%) when viewed as a stand-alone investment in Panel A, when viewed in the context of a broader portfolio in Panel B, it has a lower effective tax rate (13.8%) than either the Russell 2000 (14.0%) or value (18.0%).

C.3 Comparison to Fama and French Portfolios (1927 to 2010)

For robustness, Table A3 in the Internet Appendix examines the returns to long-only versions of the Fama and French size, value, growth, and momentum portfolios formed from CRSP data that go back to 1927, providing an additional 47 years of performance history. Results are also reported for the sample period June 1974 to June 2010 overlapping with the style indices. Over the overlapping sample period, the before-tax average returns of the Fama and French portfolios are a little bit higher than, but pretty close to, the investable style indices. However, the effective tax rates on the Fama

[9] The tax code currently allows any losses to be used to offset any other investment capital gains, including real estate, derivatives, etc. But, these losses cannot be used against ordinary income including dividends and interest beyond the $3,000 per year allowance.

and French portfolios are also a little bit higher, such that the after-tax returns on the Fama and French portfolios are very close to and consistent with those of the style portfolios. Table A4 in the Internet Appendix reports the before and after-tax correlations of the real-world indices with the Fama and French portfolios, which range from 0.91 to 0.97. Over the full sample dating back to 1927, the effective tax rates are similar. Given the consistency of results, we focus the remainder of the paper on the investable equity style portfolios over the more recent period.[10]

C.5 Up and Down Markets

The gap between before and after-tax returns can be substantially different in rising versus falling markets. Table 2 reports the after-tax performance of the equity styles in up and down markets, defined as years in which the market index yields a positive and negative return, respectively.

In a rising market, long-only equity portfolios produce significant capital gains that expose an investor to taxes. So, naturally, the after-tax returns of all the portfolios decline. The largest declines occur for the momentum portfolios since they generate the largest capital gains during these times. The value portfolios produce the next largest declines both because of their capital gains and because of their substantial dividend income. The net effect of taxes on momentum and value reduces momentum's outperformance by only about 1%, leaving a premium relative to value of 2.47% in large cap and 3.69% per year in small cap on an after-tax basis. Since the growth portfolios produce the smallest tax consequences in up markets, the outperformance of momentum relative to growth on an after-tax basis diminishes as well, but still remains at 1.15% for large cap stocks and 2.49% per year for small caps.

In down markets, all the pre-tax average returns are negative, with growth and then momentum delivering the most negative average returns and value exhibiting the least negative returns.[11] However, when losses can be used immediately, on an after-tax basis the returns to momentum actually rise, becoming less negative after taxes. The returns to momentum increase by 4.08% per year after taxes in a down market, whereas growth returns hardly change after taxes, and value returns decrease after taxes by 1.20% per year. On a pre-tax basis, value outperforms momentum in down markets, but on a post-tax basis value is roughly equal to or, in the case of small caps, underperforms momentum in down markets, due to the additional tax benefit of short-term loss

[10] Throughout the paper we use a tax rate for an investor in the 99.99th income percentile. Considering lower income tax rates, we find an even higher after-tax outperformance for momentum and value styles. Table A5 in the Internet Appendix shows that the lower tax rates at the 95th income percentile benefit momentum style portfolios the most—adding an extra 10 to 30 bps over value and an extra 30 to 50 bps over growth.

[11] A main reason why momentum underperforms value in down markets can be attributed to the difference in their conditional betas coming into down market environments (see Daniel and Moskowitz (2016)).

realizations generated by momentum and the additional tax cost of dividend income generated by value. Likewise, momentum's superior performance over growth improves on an after-tax basis in down markets.

Momentum implicitly generates negative taxes in a down market, which can enhance returns in a broad portfolio that has gains elsewhere. Momentum produces significant short-term loss realizations and in a down market does not produce significant realized gains. If those losses can be used to offset other gains from another part of a broader asset allocation framework, they can net substantial tax savings that boost after-tax returns. Value, on the other hand, produces positive taxes in both up and down markets because most of its tax exposure comes from dividend income, which has the same tax consequences in up and down markets. Put differently, dividends are much more stable than capital gains and hence yield essentially the same tax consequences in good and bad market environments. Value loses about 1.5% per year from taxes in both up and down markets equally, whereas momentum loses about 3% in up markets but implicitly gains almost 4% in down markets from taxes (if those losses can be used to offset other gains in a broader portfolio). A taxable investor, therefore, is provided an implicit hedge in down markets from a momentum strategy.

For an extreme example, the last three columns of Table 2 examine the GFC from July 2007 to March 2009. Momentum generated more than 8% additional after-tax returns from its short term losses, while neither growth nor value offered much tax benefit. These numbers represent the maximum tax benefit from being able to use all capital losses immediately, which assumes an investor would have had sufficient gains to net these losses against. An assumption that may not have been true, especially during this period, and hence represents an upper bound on the tax benefits.

D. Decomposing Turnover

Our results highlight that turnover is not a good indicator of capital gains tax exposure. As the momentum portfolios highlight, despite having five to seven times the turnover of the other styles, the effective tax rate for momentum is similar to, and sometimes even smaller than, the other equity styles. Momentum holds on to recent past winners and sells recent past losers, thus realizing a lot of short-term losses and long-term gains, which is efficient from a tax perspective.

Table 3 decomposes the annual turnover of each equity style into short and long-term capital gains and losses, reported as a percentage of the imputed net asset value of each portfolio (i.e., per dollar), where we assume each index started with a dollar investment at the beginning of the sample period. From a tax perspective, long-term gains are more efficient than short-term gains (because they are taxed at a lower rate), and short-term losses are more efficient than long-term losses (because they can be used to offset the higher taxed short-term gains). We report the percentage of total gains from long-term realizations and total losses from short-term realizations as indicators of the portfolio's tax efficiency.

Among both large and small cap stocks, the market index is mainly exposed to long-term gains, making it quite tax efficient as a stand-alone investment. But, since the market portfolio does not generate many short-term losses, it is not very tax efficient within the context of a broader portfolio. Value also has significant long-term gains, but also has non-trivial short-term gains, and generates very little short-term losses, making value less tax efficient than the market on a stand-alone basis and even less tax efficient within a broader asset allocation strategy. Growth generates the same tax exposure as value with one key exception. Growth generates more short-term losses than value, making it more tax efficient within a broader framework, but because growth has lower returns overall than value, a significant portion of these short-term losses (about 3.3% of NAV on average) have to be carried forward, which makes growth less tax efficient as a stand-alone strategy. Finally, momentum generates a lot of long-term gains, a lot of short-term gains, and substantial short-term losses. About 6.2% of those losses are carried forward on average, so the tax efficiency of momentum is significantly improved within the context of a broader portfolio where those additional short-term losses could potentially be used immediately.

Turnover is a deceptive indicator of tax exposure. For instance, much of the high turnover to a momentum strategy has tax benefits. As a simple metric to illustrate this point, consider the ratio of effective capital gains tax rate-to-turnover for each equity style. Momentum has the lowest ratio by far among the styles—its turnover is five to seven times higher than the other styles but its tax rate is similar to the other styles. This result suggests that much of the turnover to momentum portfolios does not have negative tax consequences.

Figure 1 summarizes the results. Value portfolios expose investors to substantial dividend income and net short-term capital gains, both of which are highly tax inefficient. Growth strategies have moderate dividend yields and slightly negative short-term capital gains exposure, making growth more tax efficient. Momentum produces large positive pre-tax alphas, has reasonably low dividend exposure and small short-term capital gains, giving momentum a similar effective tax rate as value, despite having much higher turnover. But, momentum still has a high tax rate because of its large long-term gains.

Momentum and value have similar effective tax rates, but for different reasons. Momentum generates most of its tax exposure through capital gains, while value produces most of its tax exposure from dividend income. These differences are key features that affect the ability to minimize tax exposure for equity styles. The remainder of the paper focuses on tax optimized portfolios that seek to minimize taxes subject to tracking error to the style.

II. Tax Lot Optimization

We consider tax-optimized versions of the style portfolios. Portfolios analyzed in the literature and from the previous section are not designed to optimize or respond to tax considerations in any way, and hence may be quite tax inefficient. To better answer how tax efficient various investment styles are, it seems crucial to evaluate how taxes can be minimized within a style. Does growth, value, or momentum lend itself more easily to tax optimization? How tax efficient can each of these styles become if portfolios are designed to minimize taxes?

We begin by examining tax optimization without altering any of the portfolio's holdings or trades. Specifically, we attempt to minimize taxes through optimal tax lot selection to determine the cost basis. This choice does not impose any tracking error on the portfolio.

A. Tax Lots

In order to specify the tax optimization problem, we first define the capital gains tax liability for an individual stock trade and then extend this definition to the basket of stock trades. Let S_t be the number of shares of a given stock held in the portfolio at time t and $\Delta S_t = S_t - S_{t-1}$ be the change in shares from time $t-1$ to t. For the purposes of calculating the tax liability, we are concerned with trades where $\Delta S_t < 0$ (i.e., sales of shares). Once a sale occurs, it triggers a potential tax liability, depending on whether there is a gain or loss realization which is determined by comparing the dollar value of the sale at time t to the original cost basis of the position in the stock. The cost basis is determined by the traded prices and traded quantities on the acquisition date of the stock. For the purposes of computing taxes, these past acquisitions of shares are recorded in "tax lots" which are defined below.

The sale of a particular stock's shares today can involve multiple tax lots, where any or all of the past purchases of the stock's shares can be used in determining the cost basis. Therefore, to determine the cost basis for a given sale, a system of identifying tax lots must be adopted. The FIFO (first-in, first-out) system orders tax lots from oldest to most recent purchases and uses the earliest purchases first to determine tax lots. The LIFO (last-in, first-out) system uses the most recent purchases first to determine tax lots. The HIFO (highest-in, first-out) system uses the highest purchase price of past stock buys to order tax lots and applies the highest priced tax lots first. HIFO is designed to minimize taxes since it realizes smaller capital gains from larger cost bases today and pushes larger capital gains from smaller cost bases to the future. The results from the previous section used the HIFO system to determine tax lots. We can also use an optimal tax lot system which chooses tax lots to minimize taxes, which we investigate here.

Formally, we define each tax lot for a single stock at a date t, which is a day on which shares were acquired. Tax lots are represented in terms of the number of days since acquisition, the trade price on the day the shares were purchased, and the quantity of shares purchased for each acquisition date. Each tax lot has a unique trading day, where all shares purchased on a given day are aggregated at the average trade price at which those shares were acquired on that day (e.g., multiple trades on a given day are aggregated at the daily level).[12] We define the matrix of tax lots **L** pertaining to the sale of stock i on date t as,

$$\underset{(n\times 3)}{\mathbf{L}} = \left(\underset{(n\times 1)}{P} \; ; \underset{(n\times 1)}{S} \; ; \underset{(n\times 1)}{d} \right)$$

where,

d = # days since shares were acquired

P = average trade price of shares on day of acquisition, P_{t-d}

S = quantity of shares traded at time of acquisition, S_{t-d}

n = number of tax lots

Since it is often the case that the sum of shares from the tax lots exceeds the number of shares sold at time t (the exception being a liquidation of all shares held in the stock which would equal the sum of all tax lots), an investor must choose a subset of tax lots to use for the cost basis. The U.S. tax code allows an investor to choose an approach for tax lot determination that must be applied consistently throughout the portfolio. For example, a rules-based approach that orders tax lots along a dimension. One popular method is FIFO (first-in, first-out) as described above, which sorts the matrix of tax lots **L** by its third column, d, in descending order. LIFO (last-in, first-out) sorts **L** by d in ascending order. The HIFO (highest price, first-out) system sorts **L** by its first column, P, in descending order from highest to lowest price. In principle, one could sort tax lots by quantity of shares (the second column of **L**) to minimize or maximize the number of tax lots used, or sort by some function of P, S,

[12] It is standard industry practice to aggregate trades done on a particular day into one trade ticket per name and per side (if there are both buys and sells in the name on the same day) at the average trade price for that day's trades. The methodology described in this section could be applied to multiple trade tickets in a name per day without loss of generality. Given the standard tax lot identification methods described in this section, any differences in tax lot identification between whether trades are aggregated per day or not would be limited and economically small.

and *d* that minimizes taxes, often referred to as "optimal" tax lot determination. We compare the HIFO system (used for the previous results) to an optimal tax lot determination system.

Under the HIFO system, we reorder the matrix **L** by column one (price) from highest to lowest such that: $\mathbf{L}(1,1) \geq \mathbf{L}(2,1) \geq \mathbf{L}(3,1) \ldots \geq \mathbf{L}(n,1)$. Under the optimal system, we order the matrix **L** by tax lots that minimize the tax cost to the investor at time *t*. The number of whole tax lots, *K*, used to compute the cost basis for the stock sale is determined by:

$$K = \arg\max \ \text{s.t.} \sum_{k=1}^{K} \mathbf{L}(k,2) \leq -\Delta S_t, \ \text{where } K \leq n \tag{1}$$

The tax exposure for this single stock trade for stock *i* is then given by the following two equations, separated into short and long-term tax exposures, where $K = K_{ST} + K_{LT}$, representing the number of short and long-term tax lots separately.

Short-term tax exposure: (2)

$$STX_i = \left[\sum_{l=1}^{K_{ST}} \mathbf{L}(l,2)\left(P_t - \mathbf{L}(l,1)\right) + \left(-\Delta S_t - \sum_{l=1}^{K_{ST}} \mathbf{L}(l,2) \right)\left(P_t - \mathbf{L}(K_{ST}+1,1)\right) \right] \ \forall \ \mathbf{L}(l,3) \leq 365$$

Long-term tax exposure: (3)

$$LTX_i = \left[\sum_{l=1}^{K_{LT}} \mathbf{L}(l,2)\left(P_t - \mathbf{L}(l,1)\right) + \left(-\Delta S_t - \sum_{l=1}^{K_{LT}} \mathbf{L}(l,2) \right)\left(P_t - \mathbf{L}(K_{LT}+1,1)\right) \right] \ \forall \ \mathbf{L}(l,3) > 365$$

The first expression of the short and long-term tax exposures in equations (2) and (3) represents the *K* tax lots that are fully utilized, and the second part of each equation captures any remaining shares from the stock sale that only partially fill the last tax lot. Thus, the tax exposure for this single stock trade is the sum across all relieved tax lots of the value received upon sale minus the cost basis for each lot, categorized as short-term if there is a sale of shares within one year (365 days) of the purchase date, and long-term if there is a sale more than a year from the purchase date.[13] The short and long-term tax exposures can each be positive or negative. A positive number represents net realized gains, and a negative number represents net realized losses that we assume can either be used immediately to offset other gains or are carried forward according to the tax code for future use (without loss of generality, we assume the discount rate for carried forward losses is zero).

---

[13] The U.S. tax code uses 365 days to define a year except for leap years, when an additional day is added. Equations (2) and (3) indicate year definitions of 365 days for simplicity, but we use 366 days in leap years for our calculations.

Summing up the tax exposures across all N stock sales in the portfolio at time t, we get the following for the tax liability of the entire portfolio:

$$TL_{portfolio} = \begin{cases} \sum_{i=1}^{N}\left(STX_i + LTX_i\right)\tau_t^{LT} & \text{if } \sum_{i=1}^{N} STX_i < 0 < \sum_{i=1}^{N} LTX_i \text{ and } \left|\sum_{i=1}^{N} STX_i\right| \le \sum_{i=1}^{N} LTX_i \\ \sum_{i=1}^{N}\left(STX_i + LTX_i\right)\tau_t^{ST} & \text{if } \sum_{i=1}^{N} LTX_i < 0 < \sum_{i=1}^{N} STX_i \text{ and } \left|\sum_{i=1}^{N} LTX_i\right| \le \sum_{i=1}^{N} STX_i \\ \sum_{i=1}^{N} STX_i\,\tau_t^{ST} + \sum_{i=1}^{N} LTX_i\,\tau_t^{LT} & \text{otherwise} \end{cases} \quad (4)$$

The U.S. tax code allows for short-term losses to offset short-term gains and long-term losses to offset long-term gains such that only the net gains and losses of the portfolio are taxed. The tax code requires that short-term losses must be used first to offset short-term gains and then any remaining short-term losses can be applied to offset any remaining long-term gains. Likewise, long-term losses must first be used to offset long-term gains, and then any remaining long-term losses can be applied to any remaining short-term gains. In the event total net losses exceed total net gains the losses can be carried forward for future use, but those losses must retain their character such that carried forward short-term losses must first be applied to future short-term gains and carried forward long-term losses must first be applied to future long-term gains. Short-term losses are more valuable from a tax perspective than long-term losses since long-term losses have to be applied first, both contemporaneously and in the future, to lower taxed long-term gains.

B. HIFO vs. Optimal Tax Lot Determination

Using the notation above, the optimal tax lot system uses P, S, and d to determine the tax lots that minimize taxes in the current period. If the entire portfolio is to be liquidated today, then this system will determine the optimal tax lots necessary for tax minimization. But, if the portfolio is to be held over multiple periods, it is not necessarily the case that the optimal tax lot system will produce lower taxes than another system such as HIFO (used previously), because the optimization is myopic. That is, by lowering the tax exposure of the portfolio today, it may increase the tax exposure of the portfolio in the future. Therefore, over a multiperiod investment horizon, it is not clear whether optimal tax lot determination produces lower total taxes than HIFO.

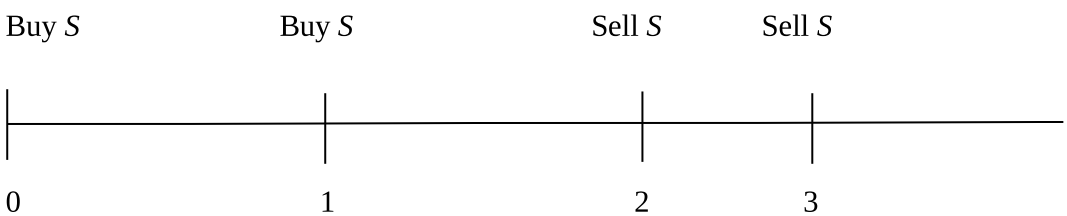
To illustrate the impact of tax lot determination on taxes, we first present a simple, stylized theoretical example highlighting when optimal tax lot determination produces more or less taxes than HIFO over a multiperiod investment horizon. We then apply an optimal tax lot system to our portfolios over our sample period and empirically quantify how much tax lot optimization matters.

Importantly, both the optimal tax lot system and the HIFO system are applied myopically, period by period, to the portfolios over time. In Section IV, we attempt to address the dynamics of the portfolios for tax optimization.

B.1 A Theoretical Example

A simple theoretical example illustrates the optimal vs. HIFO system for tax lot determination. Consider a single security whose shares are bought at two different dates and sold at two different dates. For simplicity, and without loss of generality, let the number of shares of the stock, *S*, be the same over each of the buy and sell dates and let P_t be the price of a share of the stock on date *t*. The sequence of trades in the stock is as follows:

$$\begin{array}{l} \text{at } t=0,\ \text{buy } S \text{ shares for } \mathrm{P}_0 S \\ \text{at } t=1,\ \text{buy } S \text{ shares for } \mathrm{P}_1 S \\ \text{at } t=2,\ \text{sell } S \text{ shares for } \mathrm{P}_2 S \\ \text{at } t=3,\ \text{sell } S \text{ shares for } \mathrm{P}_3 S \end{array} \tag{5}$$



The short-term tax rate is larger than the long-term tax rate, $\tau_S > \tau_L$, where the short-term rate applies to any capital gain less than two units of time in length and the long-term tax rate applies to all gains greater than two units of time.

In order to generate a tax event and a choice for tax lot, we assume P_2 is greater than both P_1 and P_0, so that the investor faces a capital gain at time 2 and a choice between two cost bases. We consider two tax lot systems: HIFO and optimal, and define the tax liability at time *t* as TL_t. At time 2, the tax liabilities under both systems are,

$$
\begin{aligned}
&\text{HIFO:}\\
&TL_2 = (P_2 - P_0)S\tau_L, \quad \text{if } P_1 < P_0\\
&TL_2 = (P_2 - P_1)S\tau_S, \quad \text{if } P_1 > P_0\\
\\
&\text{Optimal: } \min[(P_2 - P_0)S\tau_L, (P_2 - P_1)S\tau_S]\\
&TL_2 = (P_2 - P_0)S\tau_L, \quad \text{if } P_1 < P_0\\
&TL_2 = \left\{ \begin{array}{l} (P_2 - P_1)S\tau_S, \text{ if } P_2 < \dfrac{\tau_S P_1 - \tau_L P_0}{\tau_S - \tau_L} \\ (P_2 - P_0)S\tau_L, \text{ if } P_2 > \dfrac{\tau_S P_1 - \tau_L P_0}{\tau_S - \tau_L} \end{array} \right\}, \quad \text{if } P_1 > P_0.
\end{aligned}
\tag{6}
$$

Under the HIFO system, the investor selects the highest priced tax lot first, whereas under the optimal system the investor selects the tax lot that will minimize his tax liability. If prices decline from period 0 to period 1, then HIFO selects the purchases at time zero (highest priced) for the cost basis and the optimal tax lot strategy does the same because in this case not only is $P_2 - P_0$ a smaller amount, but it also gets multiplied by the smaller long-term capital gains rate τ_L. Thus, when $P_1 < P_0$, the HIFO and optimal systems choose the same tax lots and produce the same tax liability.

If, however, $P_1 > P_0$ then the two systems potentially choose different tax lots. HIFO in this case chooses P_1 as its cost basis, which gets taxed at the short-term rate since these shares have only been held for one period, but the optimal system will only similarly choose P_1 if P_2 is small enough to lower the basis in order to offset the higher short-term tax rate it faces. More formally, if P_2 is less than the short-term tax rate times P_1 minus the long-term rate times P_0 divided by the difference between the short and long-term capital gains rates, then the optimal strategy will also select the same tax lot as HIFO. But, if P_2 is greater than this ratio, then the optimal strategy will select the older tax lot of P_0, which gets applied to the lower long-term tax rate. Hence, in period 2, the optimal tax lot system generates a tax burden either equal to or lower than the HIFO system, depending on the price path. It is clear from equation (6) that for all price paths $TL_2^{optimal} \leq TL_2^{HIFO}$.

Now consider what happens in period 3. The choice of tax lot in period 2 determines the tax lot used in period 3, since there is only one tax lot remaining at that time. Since the last purchase of shares was in period 1, if period 3 is considered a long-term holding period relative to period 1, then the answer is trivial because under either method the period 3 sale is treated as a long-term gain/loss. Hence, in this case, the tax liability for both optimal and HIFO systems will be identical in period 3 and the problem becomes myopic since the only tax decision of consequence is what happens in period 2. If, however, the time between period 3 and period 2 is short enough that the gain/loss between period 1 and period 3 is considered short-term, then the two systems may produce different tax liabilities in period 3 since one tax lot will be treated as short-term, while the other long-term. In essence, this is what we have in mind in the example illustrated in (5), where the sale of shares at time 3 is recent enough so that $P_3 - P_1$ is treated as a short-term gain/loss (e.g., think of periods 1 and 2 being six months in length and period 3 being three months in length). Under this more interesting scenario, the tax liabilities at time 3 under the two systems are,

$$
\begin{aligned}
&\text{HIFO:} \\
&TL_3 = (P_3 - P_1)S\tau_S, \quad \text{if } P_1 < P_0 \\
&TL_3 = (P_3 - P_0)S\tau_L, \quad \text{if } P_1 > P_0 \\[2ex]
&\text{Optimal:} \\
&TL_3 = (P_3 - P_1)S\tau_S, \quad \text{if } P_1 < P_0 \\
&TL_3 = \left\{ \begin{array}{l} (P_3 - P_0)S\tau_L, \ \text{if } P_2 < \dfrac{\tau_S P_1 - \tau_L P_0}{\tau_S - \tau_L} \\[2ex] (P_3 - P_1)S\tau_S, \ \text{if } P_2 > \dfrac{\tau_S P_1 - \tau_L P_0}{\tau_S - \tau_L} \end{array} \right\}, \quad \text{if } P_1 > P_0.
\end{aligned} \tag{7}
$$

Once again, the tax liabilities are the same if $P_1 < P_0$, but if $P_1 > P_0$ then the time 3 tax liabilities could differ under the two systems depending on how large P_2 is relative to P_1 and P_0. In this case, the tax liability at time 3 for the optimal system could be larger than it is under HIFO. More precisely, if $P_1 > P_0$ and $P_2 > \frac{\tau_S P_1 - \tau_L P_0}{\tau_S - \tau_L}$ then the tax liability at time 3 under the optimal tax lot system will be larger than under HIFO.

In fact, the tax disadvantage of the optimal system at time 3 relative to HIFO could be large enough to wipe out the tax advantage the optimal system had at time 2. If P_3 is sufficiently larger than P_2 (and $P_2 > \frac{\tau_S P_1 - \tau_L P_0}{\tau_S - \tau_L}$), then the total tax liability, $= TL_2 + TL_3$, from the optimal system will be greater than the total tax liability from HIFO. And, for arbitrarily large P_3, the total tax burden from the optimal system can be arbitrarily larger than that from HIFO. (Table A6 in the Internet Appendix provides a simple numerical example illustrating these effects.)

Intuitively, the optimal system at time 2 selects the lower long-term gain realization to minimize taxes, but in the process generates a much larger short-term tax burden at time 3 if prices rise sufficiently. In other words, by myopically minimizing taxes in period 2, the investor may generate arbitrarily larger tax burdens in subsequent periods relative to a simple heuristic like HIFO, and those larger tax liabilities may swamp whatever tax gains were achieved previously. The issue of myopic optimization leading to potentially worse outcomes in a dynamic setting is not uncommon to any dynamic programming problem. The only point we wish to emphasize is that determining optimal tax lots in a myopic setting is not clearly more tax efficient than using a simple heuristic like HIFO to determine tax lots when a multiperiod investment horizon is considered. Ultimately, however, this is an empirical question, and could vary across styles, which we now try to answer.

B.2 An Empirical Comparison of HIFO vs. Optimal Tax Lots

Table 4 reports the annualized average after-tax returns and effective tax rates of our portfolios using an optimal tax lot determination system and compares them to those under a HIFO system. The first four columns of Table 4 report results assuming losses are carried forward according to the tax code, where each portfolio is treated as a stand-alone investment. The results are mixed. Among the large cap portfolios, three of the five portfolios have a slight increase (decrease) in after-tax returns (effective tax rates) from using an optimal tax lot system versus HIFO, but two portfolios, growth and momentum, experience a decrease (increase) in after-tax returns (effective tax rates). Hence, an optimal tax lot system, because it is myopic, can have a negative impact on after-tax returns. However, in all cases, the differences are negligible. The largest change is an underwhelming 2.9 basis points (annualized) for the value + momentum portfolio. For the small cap portfolios, the picture is very much the same—very slight improvements in four of the five portfolios, but all within a few basis points of the after-tax returns from using HIFO. The next four columns of Table 4 report results assuming all losses can be used immediately within the context of a broader portfolio. Again, the differences between optimal tax lot determination and HIFO are very small and can result in increasing tax costs.

Finally, the last four columns of Table 4 report the percentage of long-term gains and short-term losses realized as a metric of the tax efficiency of the portfolios. For the most part, the optimal tax lot system generates increased realization of long-term gains (except for small cap momentum) and short-term losses (except for growth and value + momentum), which indicates that the optimal tax lot system is moving in a more tax efficient direction, but the differences are tiny and do not result in any significant tax savings.

Overall, the impact of using optimal tax lots that minimize taxes each period versus the HIFO tax system is negligible, in part because the optimal system minimizes taxes myopically. For simplicity and brevity, we use the HIFO tax lot system for the remainder of the paper.

III. Tax Trading Optimization

In this section, we consider optimal tax trading of the securities in the portfolio. Optimal tax trading has greater scope for improving after-tax returns than tax lot optimization, but unlike tax lots, introduces tracking error to the style portfolio. We design tax optimized versions of our portfolios that attempt to minimize taxes, subject to a tracking error constraint. Comparing the after-tax returns of the original/tax unaware portfolios to those of the tax optimized portfolios provides an economic magnitude of the tax improvements across styles.

A. Minimizing Capital Gains Exposure

We start by minimizing the tax consequences from capital gains alone, ignoring dividend income. We consider altering the portfolio's dividend exposure in the next subsection. The objective is to minimize capital gains taxes subject to a tight constraint on the tracking error we allow the optimized portfolio to take. We want to optimize for capital gains tax exposure but not at the expense of producing a portfolio that is too dissimilar from the equity style itself.[14]

In order to focus exclusively on the tax consequences of trading, we assume the original portfolios are optimal in the absence of taxes with respect to their given equity styles. In other words, we assume the current portfolios maximize alpha for their given style, so that any modified trading due to tax optimization results in some loss of ex ante alpha. Essentially, we assume no scope for improving pre-tax alpha within the style portfolio, so that minimizing capital gains taxes is equivalent to maximizing expected after-tax returns. This assumption simplifies the optimization such that changing the weight on a security only becomes a tradeoff between the marginal benefit of lowering the capital gains tax versus the marginal cost of introducing more tracking error to the original portfolio. Put differently, tracking error imposes a cost on pre-tax alpha (and cannot improve pre-tax alpha). Essentially, we treat each portfolio's alpha as being completely determined by its style characteristic and therefore tracking error has a direct mapping to ex ante alpha degradation. Of course, when examining portfolios empirically ex post, realized alpha may or may not improve before taxes.[15]

We minimize the tax liability of the portfolio subject to a tracking error constraint defined relative to the original index using a risk model to measure the contribution each security makes to

[14] For example, we could buy and hold a portfolio and never trade for the entire 36-year sample period in order to minimize capital gains, but this portfolio would very quickly not look anything like its intended style.
[15] Allowing the optimization to also improve pre-tax alpha would introduce a third dimension the optimization could pursue that would not only require a model of expected returns but would also confound the impact of tax trading, making it difficult to assess tax efficiency across portfolios.

the overall tracking error of the portfolio. We use two risk models for robustness: the Fama and French (1993) three factor model augmented with a momentum factor, which we refer to as the "Fama-French four factor model", and the US Short-Term BARRA risk model (USE3S).[16]

Using these risk models, the tax optimization problem is,

$$\begin{aligned}
&\min_{\mathbf{w}_t} \; TL_{portfolio} \\
&s.t. \\
&\sqrt{\mathbf{w}^*\Omega_t\mathbf{w}^{*\prime} + \mathbf{w}^*\Sigma_t\mathbf{w}^{*\prime}} \leq c \\
&\mathbf{w}^* = \mathbf{w}_t - \mathbf{w}_B \\
&\mathbf{w}_t = \frac{\mathbf{S}_t \circ \mathbf{P}_t}{\mathbf{S}_t{}'\mathbf{P}_t}
\end{aligned} \tag{8}$$

where $\mathbf{w}_t$ is the vector of chosen portfolio weights after all trades at time *t*, defined as the vector of shares owned in each stock times their price at time *t* (where $\circ$ denotes the Hadamard or entrywise element-by-element matrix product) divided by the total dollar value of the portfolio $\mathbf{S}_t{}'\mathbf{P}_t$, and $\mathbf{w}_B$ is the vector of portfolio weights of the original index or benchmark portfolio (e.g., the optimal portfolio in the absence of taxes), where **w*** represents the change in weights between the new portfolio at time *t* and the benchmark portfolio. *Ω* is the covariance matrix of stock returns from the risk model, **Σ** is the covariance matrix of residual returns from the risk model, and *c* is a pre-specified tracking error constraint. A one month lag between the risk model estimates and portfolio weights is used to ensure the risk estimates would be available to form the portfolios in real time. Betas from the factor models are estimated using the most recent rolling five year window of monthly returns (requiring at least 24 months of valid returns), along with the covariance matrix of the factors and the residual covariance matrix over the same period.

Use of a risk model enables the optimizer to calculate the marginal contribution of each security to total tracking error and therefore allows tradeoffs between tracking error and capital gains tax exposure. These computations are based on *ex ante* measures of correlation and volatility from the

[16] The BARRA model is a factor-based model like Fama and French (1993) that contains risk factors for volatility, momentum, size, nonlinear size, trading activity, growth, earnings yield, value, earnings variation, leverage, currency exposure, and dividend yield. This model is commonly used to estimate stock variances and covariances. For details on how these factors are constructed and how betas with respect to these factors are computed see the BARRA handbook. We also ran optimizations that simply minimized the Cartesian or sum of squared distances between the new portfolio weights and the original weights, which alleviates the need for specifying a risk model. However, this method of measuring tracking error ignores the correlation structure of returns and assumes homoskedasticity across stocks. It is equivalent to assuming the identity matrix for the covariance matrix among securities. While we obtain qualitatively similar results using this method, the quantitative results were quite different, suggesting that the covariance estimates matter.

risk model. The actual tracking error ex post may be different out of sample depending on how accurately the risk model captures future return second moments. The optimization problem in (8) is solved numerically, where the tax liability of the portfolio is minimized each period in a myopic fashion. In Section IV we consider dynamic tax optimizations over various investment horizons.

A.1 Tight Tracking Error Constraint

The tracking error constraint, c, is initially set to 25 basis points. This is a tight constraint that ensures the tax managed portfolios are highly correlated with their original style indices. Panel A of Table 5 reports results from tax optimized portfolios using the Fama and French "four factor risk model" and Panel B reports results using the BARRA risk model. The first two columns report the average annualized after-tax returns of each tax optimized portfolio and the change in after-tax returns from the original index. Across all styles there is a marked improvement in after-tax returns, with the biggest individual improvements generated for value and momentum, and an even larger improvement for the combination of value + momentum. The after tax returns to large (small) cap value increase by 37 (79) bps and to large (small) cap momentum by 29 (6) bps per year using the Fama and French risk model. An integrated 50-50 value-momentum combination among large (small) cap stocks improves by 54 (102) bps per year after optimizing for capital gains taxes, which is much larger than a simple average of the after-tax improvements to value and momentum individually. This result indicates there are significant additional tax benefits from the interaction between value and momentum. Moreover, the after-tax outperformance of a value and momentum combination over the market index is further widened through tax optimization, as the Russell 1000 and 2000 indexes only improve by 15 bps through tax optimization.

The third and fourth columns report the effective tax rates on the tax optimized portfolios and their change from the original indices. The value and momentum portfolio's tax rates decline by about three percent, but the 50-50 value-momentum combination reduces effective tax rates by about five percent. The fifth column reports the change in turnover of the tax optimized portfolios from their original versions, and the sixth and seventh columns report the change in realized long-term gains and short-term losses, respectively. Simple intuition suggests that minimizing capital gains tax exposure implies lowering turnover. However, this is not necessarily the case because of the offsetting of gains and losses and the differential tax rates between short-term and long-term gains. For example, the market portfolio optimized for capital gains taxes actually increases turnover in order to realize more short-term losses. For the other equity styles there is generally a reduction in turnover and an increase in long-term capital gain and short-term capital loss realization.[17] Momentum reduces short-term losses slightly and delays capital gains to shift them to long-term status. A momentum strategy, which holds short-term winners and sells off short-term losers, is by its nature titled toward tax efficient trading. A value strategy, on the other hand is less tax efficient since it realizes too few long-term gains and too few short-term losses according to the tax optimizer.

The same results hold, both qualitatively and quantitatively, using the BARRA risk model to measure tracking error in Panel B. The last three columns of each panel of Table 5 report the alpha, its *t*-statistic, and the ex post tracking error of the tax optimized portfolios relative to their original portfolios, by regressing the tax-optimized style returns on the original (tax unaware) portfolio's returns over the sample period. Tracking error is the standard deviation of the residual from the regression. As the table highlights, the improvement in after-tax performance is generally statistically significant and roughly the same magnitude as the raw differences, suggesting that the betas of the tax managed portfolios with respect to the original indices are very close to one. The tracking errors of the portfolios are also very low, indicating that while after-tax returns are being improved substantially, each portfolio maintains a close tie to its original style index out of sample. The ex ante tracking error constraint of less than 0.25% per year appears to hold up well out of sample, although the ex post realized tracking errors are generally larger than 0.25%. Nevertheless, all of the R^2s from these regressions are at or above 0.99. Comparing Panels A and B, the BARRA risk model provides consistently better tracking error out of sample than the Fama-French model. Since the results of the tax improvement in returns are very similar under both risk models, but the BARRA model provides a better forecast of tracking error, we use the BARRA model for the remainder of the paper.[18]

A.2 The Frontier of After-Tax Returns vs. Tracking Error

We examine how the results change across styles if we loosen the tracking error constraint. Figure 2 plots the after-tax returns for tax optimized portfolios for different levels of tracking error. We examine tracking error constraints ranging from 25 to 300 basis points in 25 bp increments. We plot the frontier of after-tax returns versus tracking error to allow us to measure the after-tax improvement

[17] The reduction in turnover would potentially lower transactions costs of the portfolios in addition to lowering their tax exposure. Although transactions costs are beyond the scope of this paper, the interaction between trading cost optimization (Frazzini, Israel, and Moskowitz (2020)) and tax optimization is an interesting dimension to explore.

[18] Rather than model tracking error, we also examine optimizations that minimize portfolio weight distances, where no ex ante risk model needs to be specified. While the portfolios produced from these "risk model-free" tax optimizations delivered qualitatively similar results, these portfolios also yielded significant tracking error, suggesting that the correlation structure among the securities is important and that the risk models provide a reasonably accurate estimate of those correlations.

in performance across styles at different levels of risk. Panel A reports the results for large cap and Panel B for small cap stocks.

Figure 2 shows that after-tax returns increase as the optimization allows more tracking error, but at a certain point the after-tax returns plateau or in some cases start to decline. The inflection point varies across styles. For the market, value, and growth portfolios there is a general increase in after-tax returns as tracking error increases, but it is not monotonic. For momentum, after-tax returns increase up to about 200 bps of tracking error, but then start to diminish. For the value + momentum combination portfolio, the after-tax improvement is flat across tracking error. In terms of magnitude, the largest improvements from tax optimization accrue to growth, as large cap growth's after-tax returns are improved from 8.50% to almost 10.0% per annum as tracking error moves from 25 to 275 bps. However, even after the additional tax improvements, the after-tax returns to growth are still significantly below the market, value, and momentum. Panel B of Figure 2 shows the same general patterns for the small cap style portfolios.

While it is tempting to conclude from Figure 2 that growth has the greatest improvement from tax optimization, followed by value and then momentum, this is misleading because tracking error also has an effect on pre-tax returns. The impact of tracking error on the before-tax returns of the portfolio may also be the reason that after-tax returns do not increase monotonically for greater tracking error. Hence, the after-tax returns plotted in Figure 2 contain two effects. The first is the distortion to pre-tax returns caused by tracking error and the second is the improvement in tax efficiency. Both increase with tracking error.

To separate out these effects, Figure 3 repeats the plots from Figure 2 separately for each equity style but also adds a plot of the before-tax returns to each style across tracking error levels. Panel A reports results for the large cap portfolios and Panel B for small cap. As Panel A shows, the Russell 1000's pre-tax returns increase at the same rate as after-tax returns improve when tracking error increases, suggesting that almost all of the improvement in after-tax returns is coming from tracking error and not tax efficiency. The equal distance between the before and after-tax lines across all tracking error levels is evidence that the main improvement to the Russell 1000 index from the optimization is embedded in pre-tax returns and not from tax trading. Likewise, the after-tax improvements in growth are predominantly coming from pre-tax returns induced by tracking error and not improved tax efficiency. For value, it seems most of the improvement in after-tax returns is also coming from tracking error inducing higher pre-tax returns, but the gap between pre and post-tax returns narrows as tracking error increases, indicating that some of the improvement is also coming from tax efficiency. Finally, for momentum, the pre-tax returns monotonically decline as tracking error increases, indicating that tracking error is particularly costly to momentum's pre-tax alpha. However, the after-tax returns to momentum do not decline as tracking error increases—increasing until about 200 bps of tracking error and then starting to decrease. Hence, there are significant after-tax gains to momentum from optimal tax trading that are offsetting the decline in pre-tax alphas due to tracking error. For tracking error less than 200 bps, the improvement from tax efficiency outweighs the decline in pre-tax alpha for momentum.

Panel B of Figure 3 plots the before and after-tax returns for the small cap style portfolios. The patterns and conclusions are the same—nearly all of the after-tax improvement in returns to the market, growth, and value styles comes from tracking error improving pre-tax returns and not from tax trading efficiency. For momentum, pre-tax returns are hurt by tracking error, which is being offset by significant after-tax return improvements from optimal tax trading.

Summarizing the evidence in Figure 3, once we account for distortions to pre-tax returns, the momentum style has the largest tax efficiency gains from tax trading optimization. However, those gains come at a cost since increased tracking error reduces pre-tax alpha.

A.3 Effective Tax Rates and Turnover vs. Tracking Error Frontier

Figure 4 plots the improvement in tax efficiency for each tax managed equity style across tracking error levels by plotting the effective tax rates, change in turnover from the original index, and changes in long-term capital gain and short-term capital loss realizations. Panel A reports results for the large cap portfolios and Panel B for the small cap portfolios.

Effective tax rates decline steeply and monotonically for momentum across tracking error levels, but are relatively flat for the other equity styles. This evidence further indicates that all of the after-tax improvements for momentum are coming from tax trading, while any improvements in after-tax returns for the other equity styles are predominantly coming from pre-tax changes to returns due to tracking error.

Momentum's improved tax efficiency is driven by lower turnover as tracking error increases, which slows the rate of capital gain realization to increase the long-term capital gain exposure of the portfolio. Momentum steadily increases its percentage of long-term gain realizations as tracking error increases, resulting in a more tax efficient portfolio. None of the other styles exhibit much change in long-term gain realization. Conversely, momentum does not realize more short-term losses through tax optimization, whereas the other styles do increase short-term losses up to about 175 bps of tracking error. Results are similar for large cap and small cap portfolios.

A.4 Optimal Tax Lot and Optimal Tax Trading

For robustness, we also compute tax managed equity style portfolios that also use an optimal tax lot system in conjunction with optimal tax trading. We repeat the tax trading optimizations above using an optimal tax lot system as described in Section II in place of the HIFO system. Although tax lot optimization did not show much, if any, improvement in after-tax returns to our portfolios previously, it is possible that combining it with optimal tax trading may yield more significant improvements if there is an interaction between trading and tax lot optimization.

Figure 5 plots the changes in long-term capital gains, short-term capital losses, effective tax rates, and after-tax returns of tax managed portfolios that use optimal tax trading at various levels of tracking error for portfolios that also use an optimal tax lot system. For comparison relative to the HIFO tax lot system, the plots in Figure 5 are for changes in these variables relative to portfolios that use the HIFO tax lot system. Hence, the numbers in Figure 5 represent the incremental contribution from using an optimal tax lot system with tax optimized trading.

Across all equity styles there is very little change in the tax efficiency of the tax managed portfolios from using an optimal tax lot system over HIFO. Long-term gain and short-term loss realization remain similar, effective tax rate differences are negligible, and after-tax return improvements are insignificantly different from zero, across all levels of tracking error. The results further support that an optimal tax lot system has a negligible impact on the tax efficiency of the portfolios relative to HIFO, even with optimal tax trading.

B. Minimizing Dividend Income Exposure

In this subsection, we optimize portfolios for taxes by minimizing dividend income exposure while ignoring capital gains exposure. We use the dividend yields on all stocks from the prior year as an expected dividend yield in the optimization.

B.1 No Dividends

We first consider eliminating all dividend paying stocks such that none of the portfolios pay any (expected) dividend income tax. However, this eliminates the majority of the market capitalization of the portfolios, particularly among the large cap portfolios. Figure 6 plots the percent of market cap remaining for the large cap value, growth, and momentum portfolios over time. For value, eliminating dividend paying stocks essentially eliminates almost all value stocks. Over the sample period less than 8% of the market cap of the value portfolio remains on average if dividend payers are eliminated, and the maximum market cap remaining at any point in time is only 14.6%. For growth, the elimination of dividend payers is less intrusive, but still only 18.8% of the growth portfolio remains on average and the maximum market cap remaining over the sample period is 54%. For momentum, excluding dividend-paying stocks is not as invasive—19.3% of the market cap remains on average and as much as 75.6% of the index remains over the sample period. Among all three styles there is also a trend, where dividend-paying stocks comprise more of the portfolios in the earlier part of the sample period, and become less significant over time. This trend is consistent with the demise of dividend payments documented by Fama and French (2001).

Panel A of Table 6 reports the after-tax returns, effective tax rates, dividend yield, and tracking error of style portfolios that eliminate all dividend paying stocks, and their differences from the original portfolios. The after-tax returns to the market, growth, and momentum portfolios are larger when dividend payers are eliminated, but the value portfolio's returns decline by 80 basis points on an after-tax basis. However, the effective tax rates to the no dividend versions of all of these style portfolios are not meaningfully better than the original portfolios. Thus, most of the difference in returns is coming from tracking error induced by eliminating dividend-paying stocks. For value, eliminating dividend-payers is especially harmful. Not only does eliminating dividend-payers remove most of the market cap of the value portfolio, but the non-dividend stocks that remain tend to underperform. For both reasons, it does not appear feasible to run a value strategy without dividend exposure (value stocks are high dividend-paying stocks, and many studies, including Fama and French (1996, 2012) and Israel and Moskowitz (2012), use dividend yield as a value measure). For growth and momentum, the non-dividend paying stocks outperform their original portfolios by 1.16% and 1.63%, respectively, and hence reduce income taxes while also increasing pre-tax returns.

As the last column of Panel A of Table 6 reports, the ex post tracking error of the non-dividend portfolios is high, which is consistent with the significant reduction in market cap from removing dividend-paying stocks shown in Figure 6. Eliminating dividends altogether results in portfolios that are simply too different from their original style indices. We now explore more moderate changes to the portfolios in an attempt to limit dividend exposure.

B.2 Minimize Dividend Exposure Subject to a Tight Tracking Error

Another way of gauging how easy it is to reduce dividend income for the various equity styles is to impose a tracking error constraint on the portfolios. Specifically, we solve the same optimization problem in equation (8), but replace the objective function $\min_{\mathbf{w}_t} \; TL_{portfolio}$ with $\min_{\mathbf{w}_t} \; \mathrm{D/P}_{portfolio}$. Panel B of Table 6 reports results for portfolios that minimize dividend income exposure subject to a tracking error constraint of 25 basis points. We focus exclusively on the large cap portfolios here since they have significant dividend exposure.

As the last three columns of Panel B of Table 6 show, the tracking error constraint quickly binds. Dividend yields on the portfolios do not decrease very much (on the order of 0.23% to 0.40%) because the tracking error constraint does not allow it. Consequently, the after-tax returns of the equity styles and effective tax rates do not change much either. In every case, the after-tax returns to the equity styles decline or stay the same. These results are much different than those for minimizing capital gains exposure subject to the same tight tracking error constraint of 25 bps. For capital gains, we were able to improve tax efficiency and generally increase after-tax returns under the tight tracking error constraint. This evidence suggests that managing dividend income tax exposure is more difficult than managing capital gains exposure in terms of tracking error consequences. Put differently, managing dividends imposes more significant tracking error costs on the portfolios than managing capital gains. This cost seems to be greatest for value strategies since dividend yields and value stocks are synonymous. Imposing a tight tracking error constraint severely limits the ability to reduce dividend exposure. We now explore dividend tax optimization under looser constraints.

B.3 The Dividend vs. Tracking Error Frontier

Figure 7 plots the effective dividend tax rates, effective capital gains tax rates, effective total tax rates, and after-tax returns of tax managed equity style portfolios that seek to minimize dividend income exposure subject to tracking error constraints that range from 25 to 300 basis points per year.

As the upper left graph of Figure 7 shows, effective dividend tax rates decline monotonically as tracking error increases, indicating that the optimization is doing what it is supposed to. This decrease appears to be steepest for value, which makes sense since value has the largest dividend exposure. However, as the upper right graph shows, effective capital gains taxation tends to rise at the same time. Hence, as dividend taxes decline from tax optimization, capital gains taxes increase, primarily because reducing dividend income entails selling the security before the ex-date, which incurs more capital gains realizations. The net effect is a fairly flat total effective tax rate frontier across tracking error, as evidenced by the lower left graph. Tax managed versions of value, growth, and the market portfolio that minimize dividends actually exhibit a slight increase in effective total tax rates as tracking error increases, indicating that the increase in capital gains is imposing heavier taxes than the reduction in dividend income from the optimization.

As a result, the last graph in Figure 7, which plots the after-tax returns to the dividend-tax-managed equity style portfolios, shows no significant improvement and, for value and the market portfolio, a decrease in after-tax performance. Tax managing dividend exposure imposes unfavorable capital gains exposure across equity styles that results in no overall tax improvement and possibly some degradation in after-tax returns. These effects reiterate the difficulty in managing dividend tax exposure, which is most pronounced for value strategies.

C. Minimizing Total Taxes: Capital Gains and Dividend Exposure

Table 7 examines tax optimized versions of our equity style portfolios that try to simultaneously minimize dividend *and* capital gains tax exposure. Specifically, we seek to minimize the following expression, representing the total tax burden of the portfolio that includes capital gains and dividends:

$$\min_{\mathbf{w}_t} \; TL_{portfolio} + Div_{portfolio} \times \tau_{income} \tag{9}$$

where the latter term represents the dividend income of the portfolio times the income tax rate, subject to the same constraints from equation (8).

Table 7 reports the after-tax returns, effective total tax rates, effective capital gains tax rates, effective dividend tax rates, and turnover of tax optimized versions of the large cap equity style portfolios. Panel A reports results under a tight tracking error constraint of 25 basis points per annum and Panel B reports results for a much wider 2% tracking error.

Under the tight tracking error constraint in Panel A, we see improvements in after-tax returns across all equity styles, ranging from 12 basis points for growth to 35 bps for momentum and 40 bps for value + momentum. In all cases, effective tax rates decline, with the most significant reduction occurring for momentum with a 3.87% decline in its tax rate. Consistent with our previous results, almost all of the gain in taxes comes from managing capital gains and not dividends. For example, of the 3.87% reduction in effective tax rate for momentum, 3.80% (or more than 98 percent) is from lowering the effective capital gains rate and only 0.07% comes from reducing dividends. Even for value, whose effective total tax rate reduction is 2.82%, 2.70% (96 percent) comes from capital gains reduction and only 0.12% from dividend reduction. Since capital gains impose much lower tracking error costs than dividends, the optimizer chooses to minimize taxes through capital gains.

Panel B reports results under a much looser tracking error constraint of 300 basis points. Here, the reduction in effective tax rates is, of course, much more substantial and ranges from a minimum reduction of 2.99% for the market to a maximum of 12.15% for momentum. Once again, even at higher tracking error, most of the improvement in tax efficiency comes from capital gains and not dividend management, though the split varies across equity styles. For momentum, nearly all of the improvement in tax efficiency is from capital gains (11.31% of the 12.15% reduction in tax rates), with only 0.84% coming from reduction in dividend taxes. For value, of the 7.00% decline in effective tax rate, 4.98% comes from capital gains and 2.03% from dividends. Thus, even at higher tracking error and for the equity style most exposed to dividend income, the optimization still has an easier time reducing tax burden through capital gains rather than dividends. This theme is echoed for the other equity styles, too.

Finally, both panels show that the optimization reduces the turnover of each equity style portfolio in order to minimize taxes, with the exception of the market portfolio, where the optimization raises turnover. The reduction in turnover for the other equity styles is driven by the optimization pushing short-term gains to the long-term in order to reduce capital gains taxes, but in the case of the market portfolio, which is a buy and hold strategy that predominantly only has long-term gains, the optimization wants to increase the realization of losses and hence increases turnover. These effects are stronger at higher tracking error.

One interesting result is the change in turnover for the value + momentum combination portfolio. At the higher 200 bp tracking error, the tax managed portfolio for momentum has 159.7% less turnover than the original index and tax managed value has an 10.5% reduction in turnover. However, the value + momentum combination has only a 16.1% reduction in turnover, suggesting that the combination of value and momentum nets out many trades that a tax optimizer would want to do independently. This evidence suggests another benefit of combining these two styles of investment (in addition to those of Asness, Moskowitz, and Pedersen (2012) and Frazzini, Israel, and Moskowitz (2020)) that would benefit a tax sensitive investor.

For completeness, Figure 8 plots the frontier of effective dividend, capital gains, and total tax rates as well as after-tax returns across various levels of tracking error. Effective dividend tax rates for the tax optimized equity styles decline steadily as tracking error is allowed to increase, with the steepest decline for value. Capital gains tax rates decline monotonically and steeply with tracking error for momentum, and much more modestly for value, but are virtually flat for the other styles. Therefore, total tax rates decline the most for momentum and value through tax optimization as tracking error increases, but for different reasons. For momentum, the optimizer is able to manage capital gains effectively to significantly reduce the tax burden. For value, it is a combination of managing both dividends and capital gains that reduces its tax exposure. As the last graph in Figure 8 shows, the improvement in after-tax returns is small across the equity styles because increased tracking error from tax management results in a general loss in pre-tax returns.

D. The Cost of Tracking Error: Style Drift

We show that tax managed versions of the equity style portfolios introduce tracking error, but how much of that tracking error comes from changes in style versus idiosyncratic movements? If tracking error imposes significant style drift to our portfolios, then that is a much bigger cost than if it simply induces idiosyncratic noise.

To address the imposition tax optimization has on style drift, we examine the betas of the equity style portfolios with respect to the Fama-French four factors consisting of the excess return on the CRSP value-weighted market portfolio, *RMRF*, the size factor, *SMB*, the value factor, *HML,* and the momentum factor, *UMD*, obtained from Ken French's website. We compute betas using the entire sample period of returns from July 1974 to June 2010 for the original (tax unaware) style portfolios, tax managed portfolios that minimize capital gains (ignoring dividends), tax managed portfolios that minimize dividends (ignoring capital gains), and tax managed portfolios that minimize total taxes subject to a 25 (tight) and 300 (loose) basis point tracking error constraint.

Figure 9 plots the betas of the various versions of the style portfolios. Minimizing capital gains does not seem to impose much, if any, style drift for value, growth, or momentum. However, minimizing dividends generates significant style drift. In fact, for value, the most dividend-heavy style, minimizing dividends all but eliminates the portfolio's exposure to value. These results are consistent with our previous findings that managing dividends imposes significant tracking error on the portfolio, and that tracking error is not idiosyncratic but dramatically changes the style of the portfolio itself. This style drift fundamentally changes the nature of the portfolio.

The last two sets of bars for each style report the betas of tax managed portfolios that try to minimize total taxes (capital gains plus dividends) at tight and loose tracking error constraints. Value experiences the largest style drift as tracking error increases from 25 to 300 bps. Growth and momentum are able to maintain their styles fairly closely even at the higher tracking error threshold. This evidence is consistent with value having much higher dividend exposure than the other styles, and tax managing dividends has much larger style drift consequences. Tax managed versions of growth and momentum are less prone to style drift because their tax exposure is predominantly through capital gains.

IV. Dynamic Tax Trading Optimization

The tax optimizations in the previous section minimize the tax liability of the portfolio each period myopically. As we show in Section II with tax lots, myopic and dynamic optimizations can give different results. In this section, we ask whether a dynamic tax trading optimization could yield even greater tax benefits to our style portfolios?

Dynamic tax optimization requires an additional set of assumptions in order to make it tractable. For example, we need to specify an investment horizon and apply a discount rate to future tax burdens. More importantly, though, we need to have information or a forecast of what the future portfolio will look like in order to have some sense of what the tax consequences of today's trades could have on future tax liabilities.

Rather than attempt to forecast future portfolio weights for each style, which entails developing a model for the evolution of the portfolio, we conduct a simpler exercise designed to elicit the maximum tax benefit we could hope to gain from taking into account the dynamic implications of tax trades. Specifically, we specify an investment horizon (e.g., five years) and allow our tax optimization to have perfect foresight of the future portfolio over that horizon. That is, we provide the optimizer with the future portfolio weights of the equity style portfolio in the absence of taxes. We assume here, just as we did for the previous static optimizations, that the optimal portfolio in the absence of taxes maximizes pre-tax returns. We then minimize taxes over the entire investment horizon by taking into account what we know the portfolio will look like (in the absence of taxes) at the end of that horizon. Perfect foresight of future portfolio weights provides an upper bound on the maximum tax benefits from dynamic optimization.

We choose three investment horizons over which we consider dynamic optimization: one year, two year, and five year holding periods. Two sets of optimizations are run under a tight (25 bp) tracking error constraint and a loose (200 bp) tracking error constraint. The optimization is conducted every month, but the future portfolio weights at the end of the investment horizon are included in the information set. Because we allow perfect foresight of the future portfolio over the given investment horizon, statistics such as after-tax returns and effective tax rates are biased due to look-ahead bias. Hence, we only report the percentage of long-term gain and short-term loss realizations as measures of tax efficiency improvements, which should not be biased. To measure the marginal impact on tax efficiency from dynamic trading, we report the difference between dynamica and the myopic optimized portfolios at the same tracking error constraint.

Table 8 reports the results. The first seven columns report the results for long-term gain realization and the second set of seven columns reports results for short-term loss realization. The first column in each set reports the results from the myopic optimization, with the remaining columns reporting results for one, two, and five-year perfect foresight optimizations and their differences from the myopic portfolios. As Table 8 indicates, in most cases the dynamically optimized portfolios realize fewer long-term gains and more short-term losses than the myopically optimized portfolios, with the differences increasing when allowing for higher tracking error. This suggests that the dynamically optimized portfolios are moving in a more tax efficient direction and improving tax efficiency relative to the myopic optimization. However, the differences are economically small and in several cases of the opposite sign (suggesting that the myopic optimization is more tax efficient, which can occur because our chosen investment horizon is not the same as the full sample period). Moreover, there is little evidence that the magnitudes are larger for longer investment horizons, where the five year foresight results are similar to the one and two year results.

Overall, the results in Table 8 suggest that dynamic tax trading does not add significantly to the tax efficiency of the equity style portfolios over and above myopic tax optimization. In some cases, dynamic optimization even lowers the tax efficiency of the portfolio. Since perfect foresight provides the maximum benefit dynamic tax trading could provide, these results suggest that for practical purposes the tax benefits of dynamic trading for the equity style portfolios are are small and second order relative to myopic optimization that captures the majority of the tax benefits.

V. Conclusion

We examine the after-tax performance of equity style portfolios and the ability to improve performance through tax optimization. Value and momentum face the largest tax rates, but for different reasons. Value's tax exposure is dominated by dividend income, while momentum's comes predominantly from capital gains. Attempting to improve tax efficiency through tax optimization, we examine both optimal tax lot selection, which incurs no tracking error, and optimal tax trading, which imposes a tracking error cost. Optimal tax lot selection has only a small effect on the portfolios' tax efficiency relative to a simple rule like HIFO. Optimal tax trading to manage capital gains, however, provides substantial tax savings with only modest tracking error costs, but managing dividends imposes significant tracking error consequences. Hence, tax optimization benefits capital gain-heavy styles such as momentum more than income-heavy styles such as value. Consequently, tax optimization improves the tax efficiency of momentum more than other styles, widening the relative after-tax return premium of momentum.

Tax optimization exacerbates the performance differences across equity styles on an after-tax basis, where momentum, and to a lesser extent value, receive the largest improvements from optimization. Further exploration of tax implications we do not consider in this paper are tax location decisions, tax harvesting within and across portfolios, and the potential interaction between tax optimization and trading cost optimization, which may be interesting avenues for further research.

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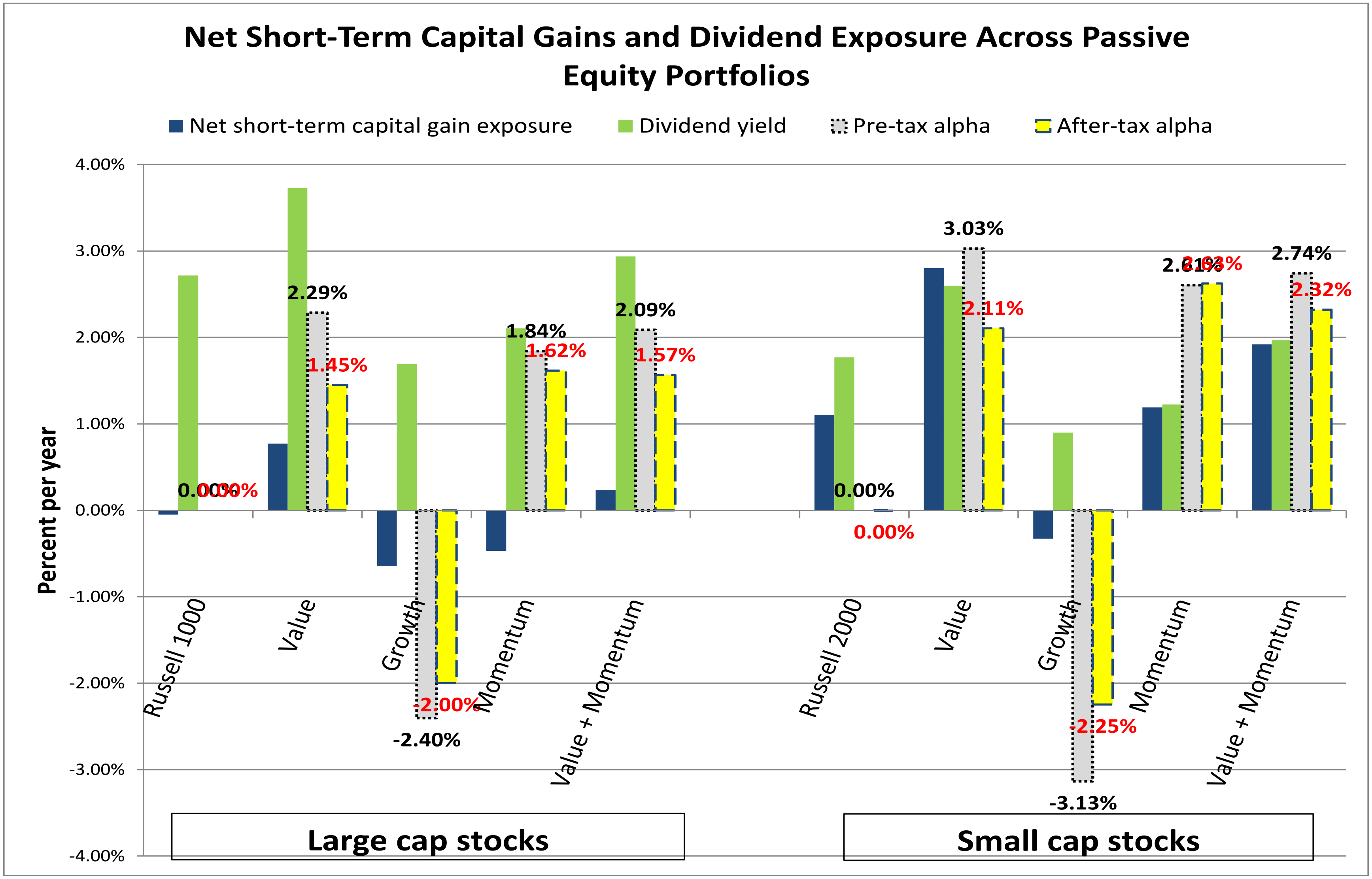
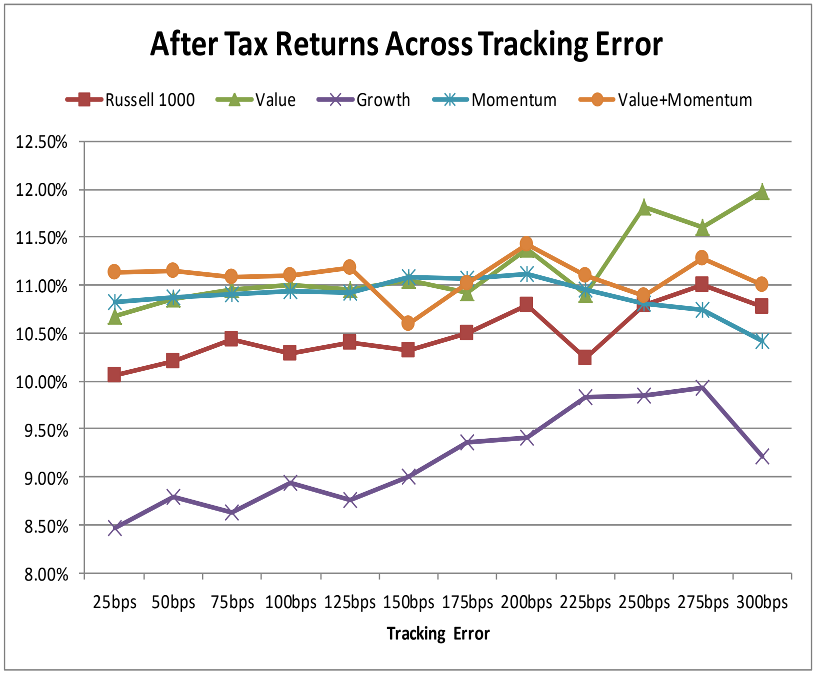


Figure 1: Net Short-Term Capital Gains, Dividend Exposure and Pre- and Post-Tax Alphas Across Equity Styles
Plot of the average annualized net short-term capital gains exposure, dividend yield and pre- and post-tax alpha of the equity style indices from July 1974 to June 2010.

PANEL A: LARGE CAP STOCKS



PANEL B: SMALL CAP STOCKS

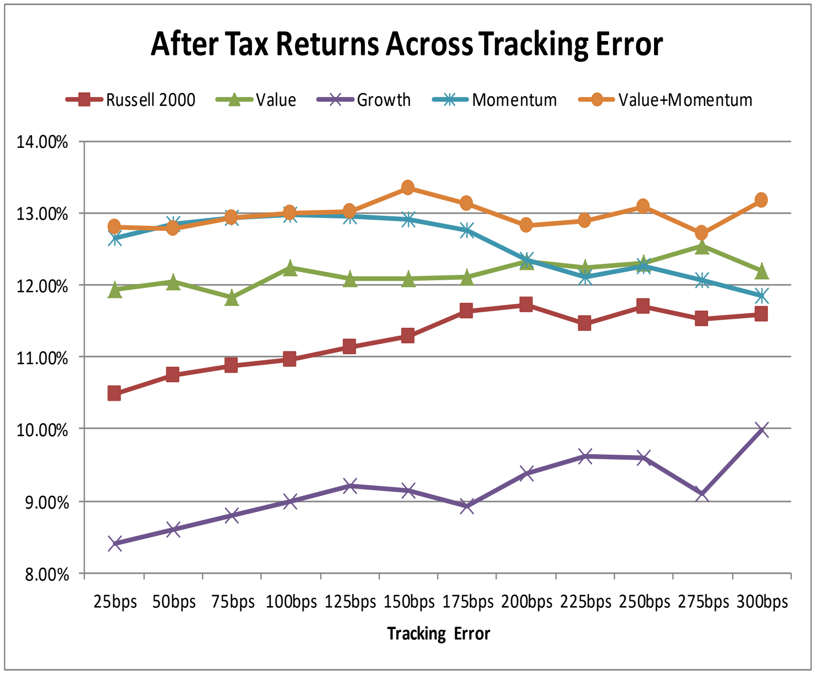
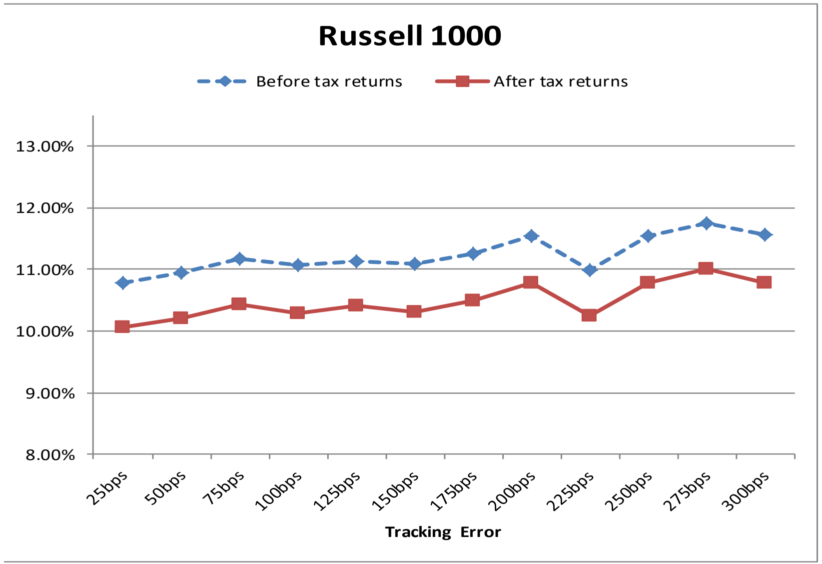
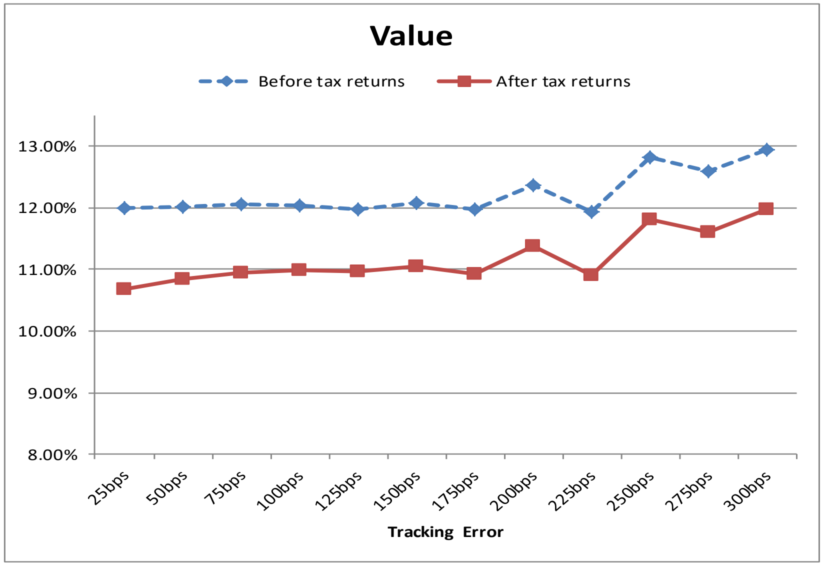


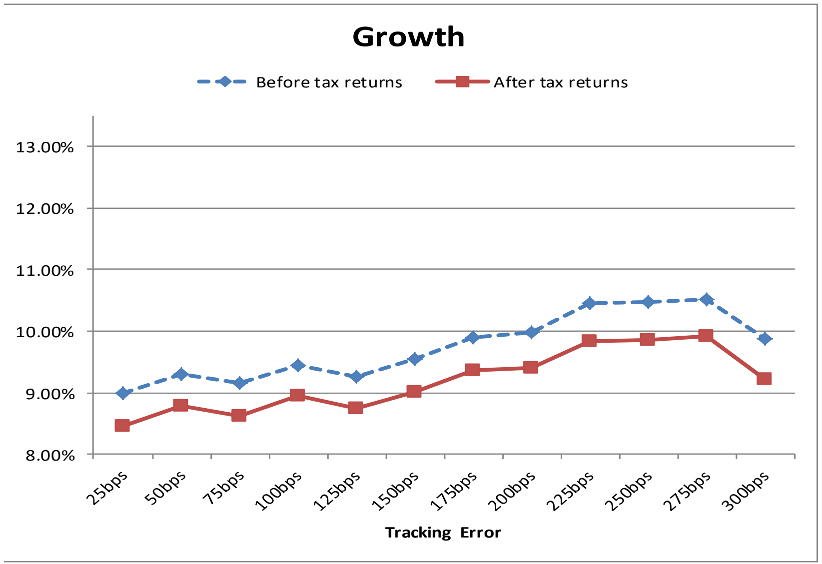
Figure 2: After-Tax Returns of Tax Optimized Portfolios Across Tracking Error—Capital Gains Only

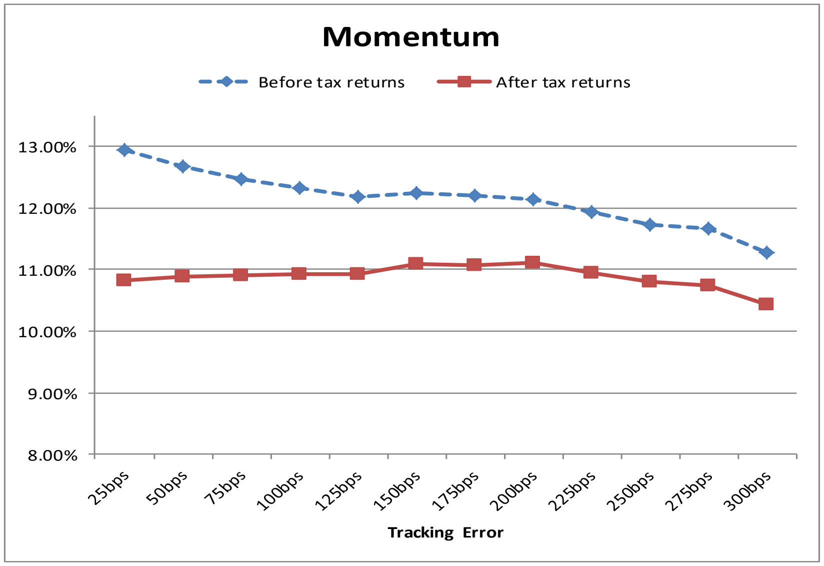
Plot of the average annualized after-tax returns of tax managed portfolios that minimize capital gains only subject to a tracking error constraint that ranges from 25, 50, . . . , 300 basis points across equity style indices for the market, value, growth, momentum, and value + momentum. Panel A plots results for large cap portfolios and Panel B for small cap portfolios. Returns are computed over the period July 1974 to June 2010 and assume the current U.S. tax code and rates apply with all excess capital losses carried forward according to the tax code.

PANEL A: LARGE CAP STOCKS









PANEL B: SMALL CAP STOCKS

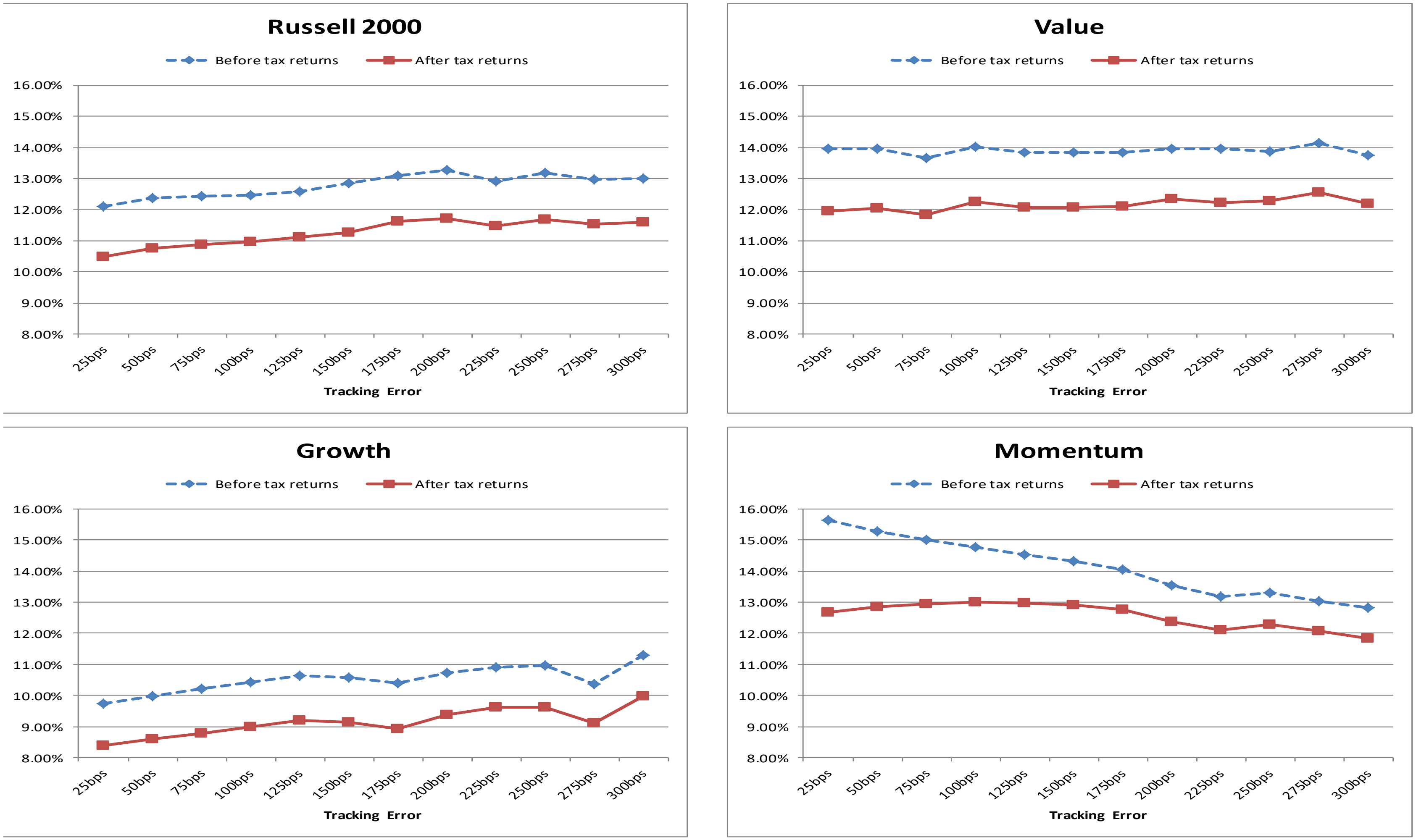
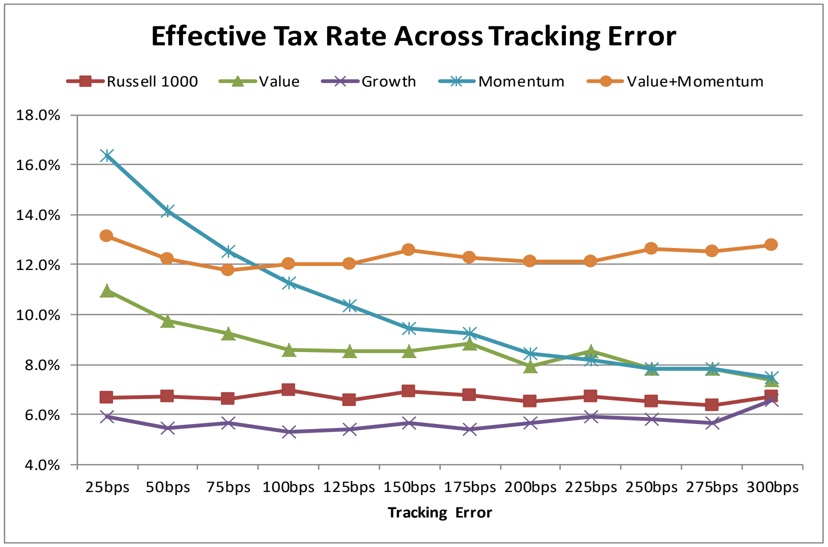
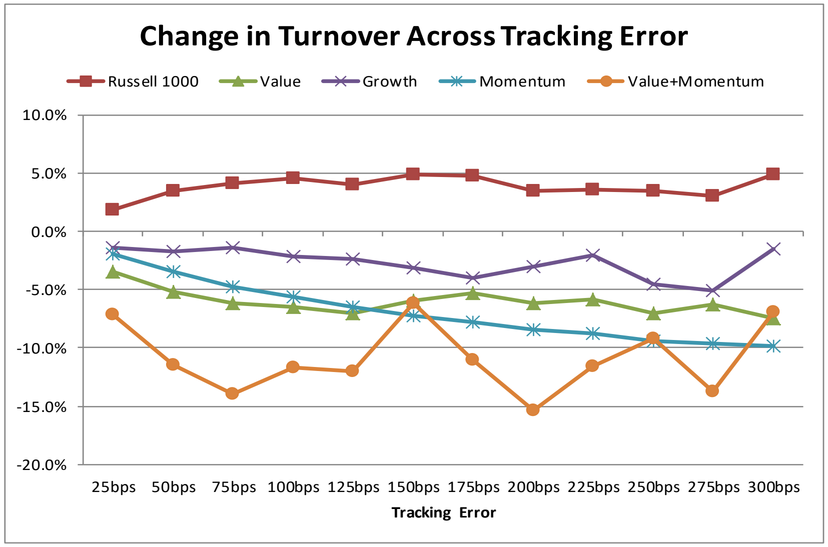


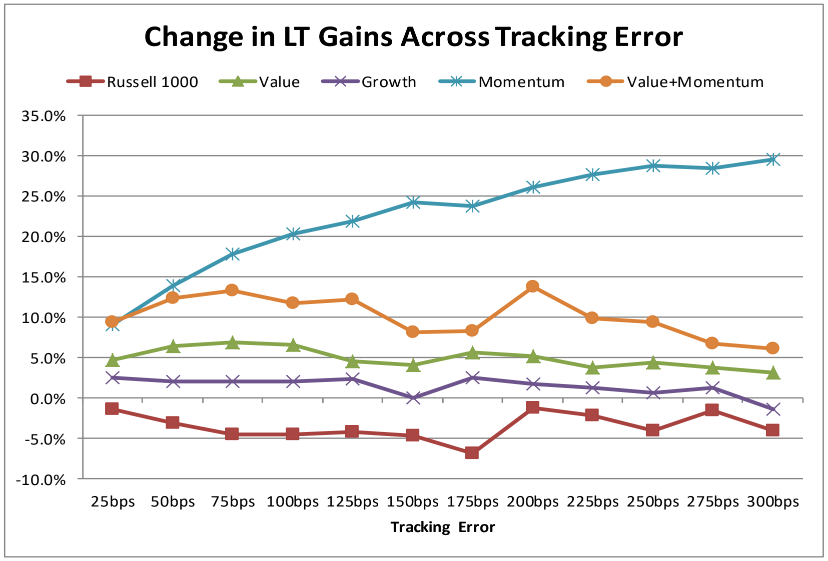
Figure 3: Before vs. After-Tax Returns of Tax Optimized Portfolios Across Tracking Error—Capital Gains Only

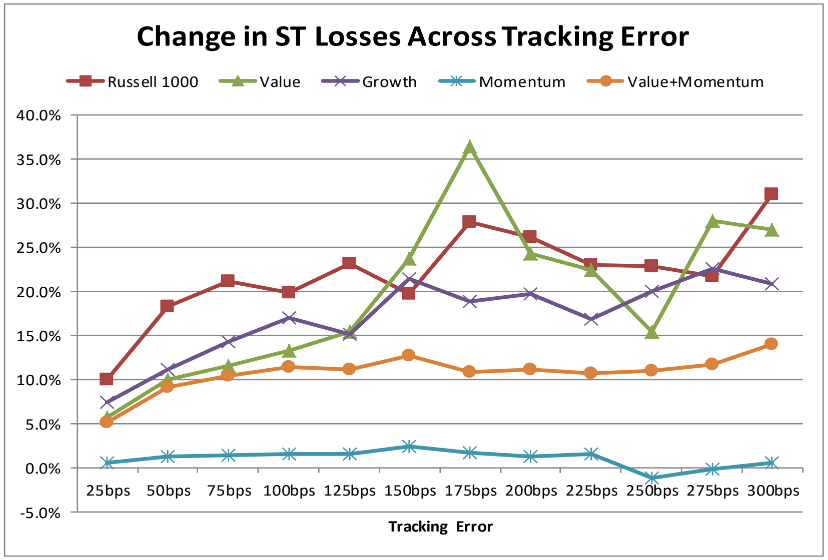
Plot of the average annualized before and after-tax returns of the tax managed portfolios from Figure 3 that minimize capital gains only subject to a tracking error constraint that ranges from 25, 50, . . . , 300 basis points across equity style indices for the market, value, growth, and momentum. Panel A plots results for large cap portfolios and Panel B for small cap portfolios. Returns are computed over the period July 1974 to June 2010 and assume the current U.S. tax code and rates apply with all excess capital losses carried forward according to the tax code.

PANEL A: LARGE CAP STOCKS









PANEL B: SMALL CAP STOCKS

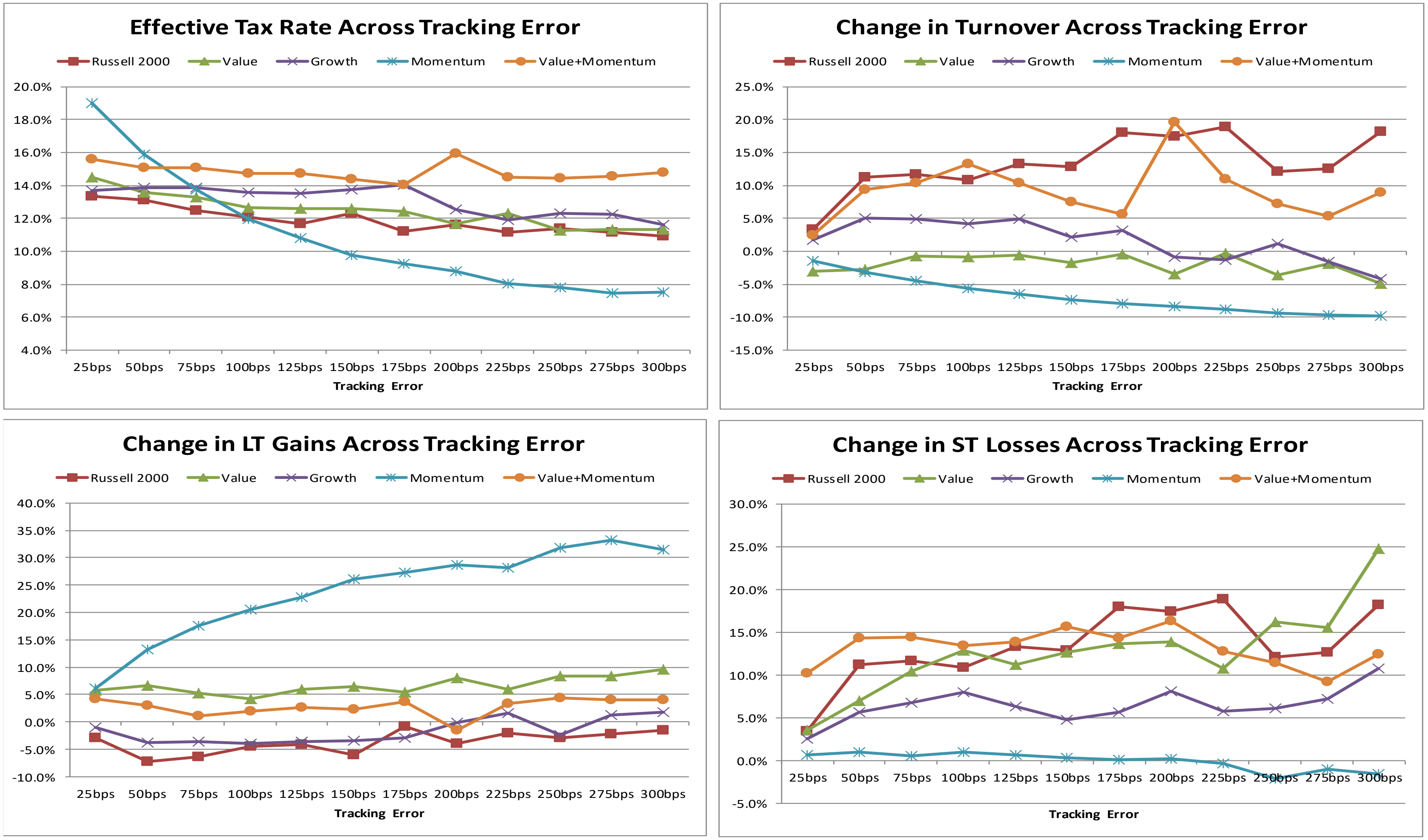
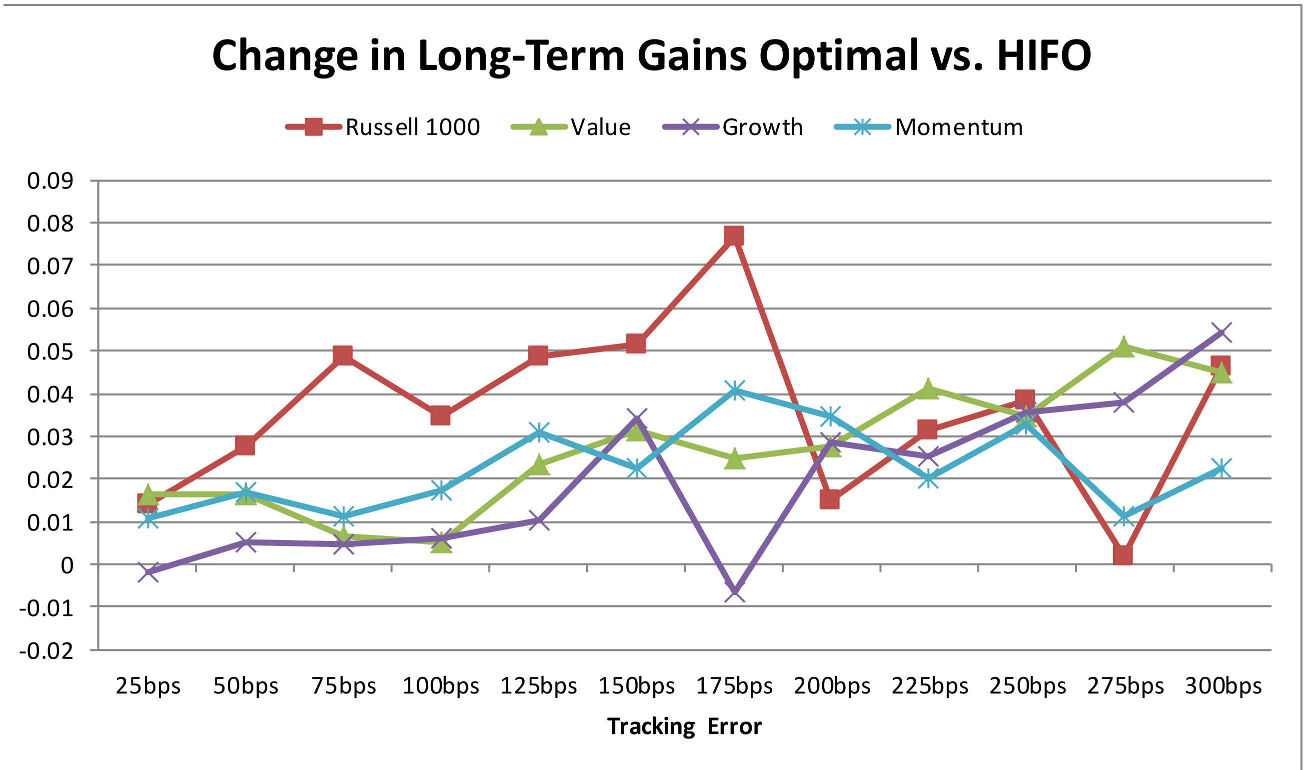
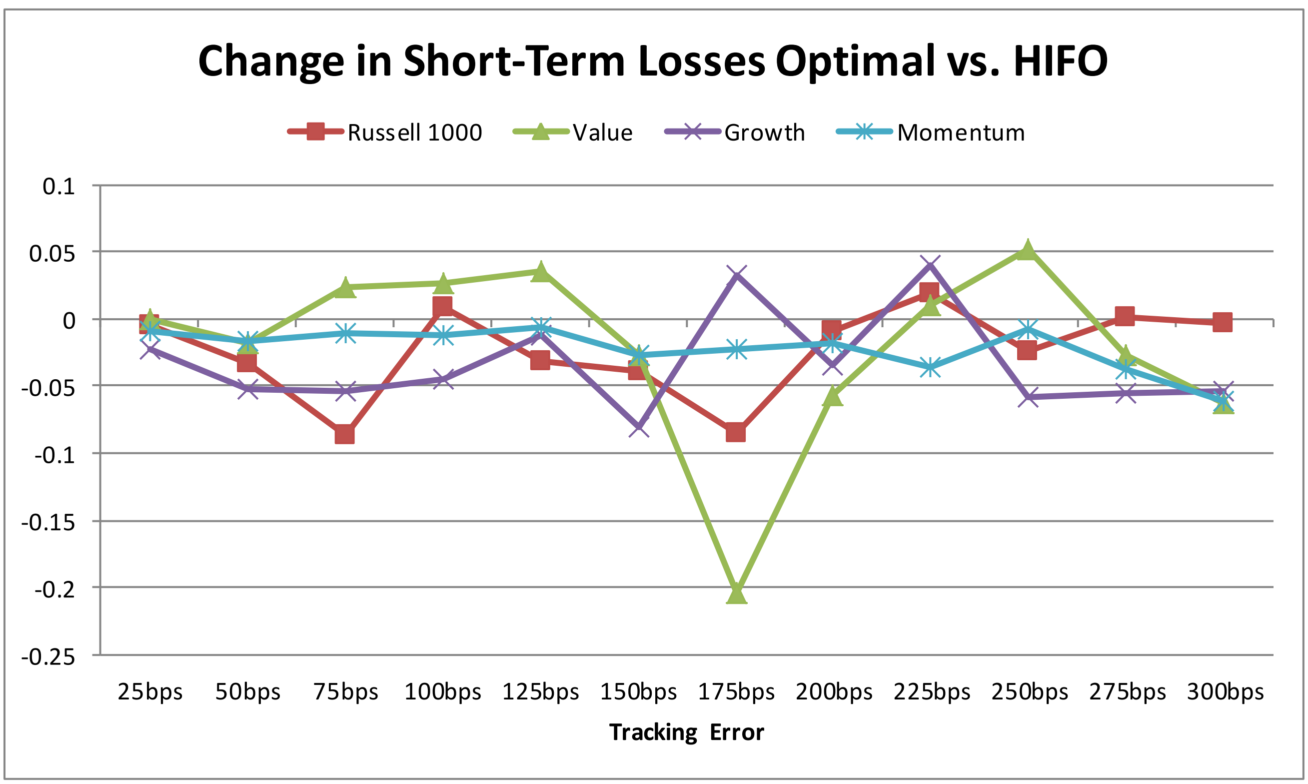


Figure 4: Effective Tax Rate and Turnover of Tax Optimized Portfolios Across Tracking Error—Capital Gains Only
Plot of the average annualized effective tax rate, change in turnover from the original index, and changes in the percentage of long-term capital gains and short-term capital losses realized of the tax managed portfolios from Figure 3 that minimize capital gains only subject to a tracking error constraint that ranges from 25, 50, . . . , 300 basis points across equity style indices for the market, value, growth, momentum, and value + momentum. Panel A reports results for large cap portfolios and Panel B for small cap portfolios. Statistics are computed over the period July 1974 to June 2010 and assume the current U.S. tax code and rates apply with all excess capital losses carried forward according to the tax code.


Change in Long-Term Gains Optimal vs. HIFO
Russell 1000
Value
Growth
Momentum
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
25bps
50bps
75bps
100bps
125bps
150bps
175bps
200bps
225bps
250bps
275bps
300bps
Tracking Error


Change in Short-Term Losses Optimal vs. HIFO
Russell 1000
Value
Growth
Momentum
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
25bps
50bps
75bps
100bps
125bps
150bps
175bps
200bps
225bps
250bps
275bps
300bps
Tracking Error

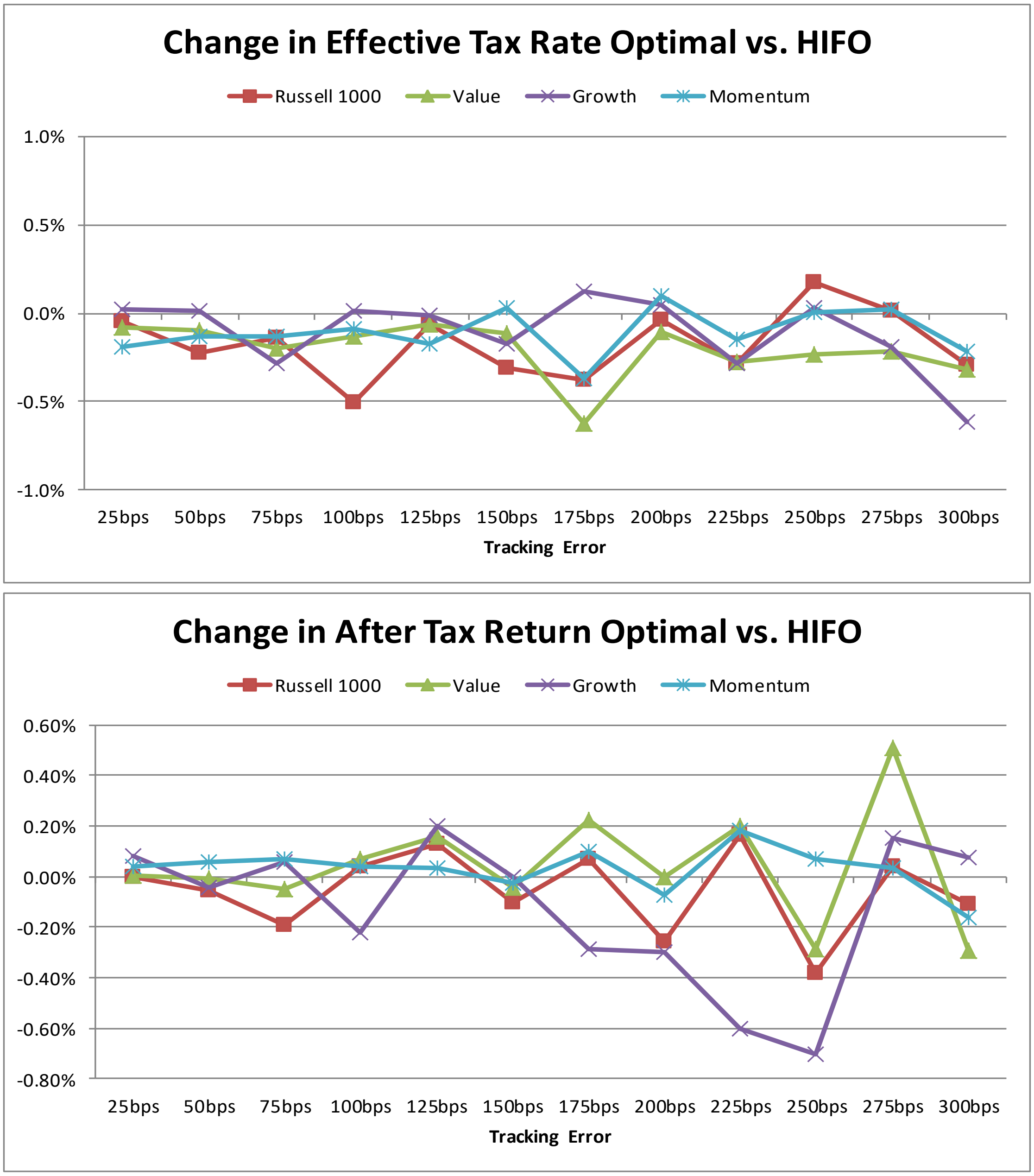


Figure 5: Optimal vs. HIFO Tax Lot Comparison for Tax Optimized Portfolios
Plot of the difference between Optimal tax lot and HIFO tax lot systems for the tax managed portfolios from Figure 3. The difference between Optimal and HIFO average annualized change in the percentage of long-term realized gains, percentage of short-term realized losses, effective tax rates, and after tax returns are plotted in each graph for each of the tax managed portfolios from Figure 3 that minimize capital gains only subject to a tracking error constraint that ranges from 25, 50, . . . , 300 basis points across equity style indices for the market, value, growth, and momentum. Statistics are computed over the period July 1974 to June 2010 and assume the current U.S. tax code and rates apply with all excess capital losses carried forward according to the tax code.

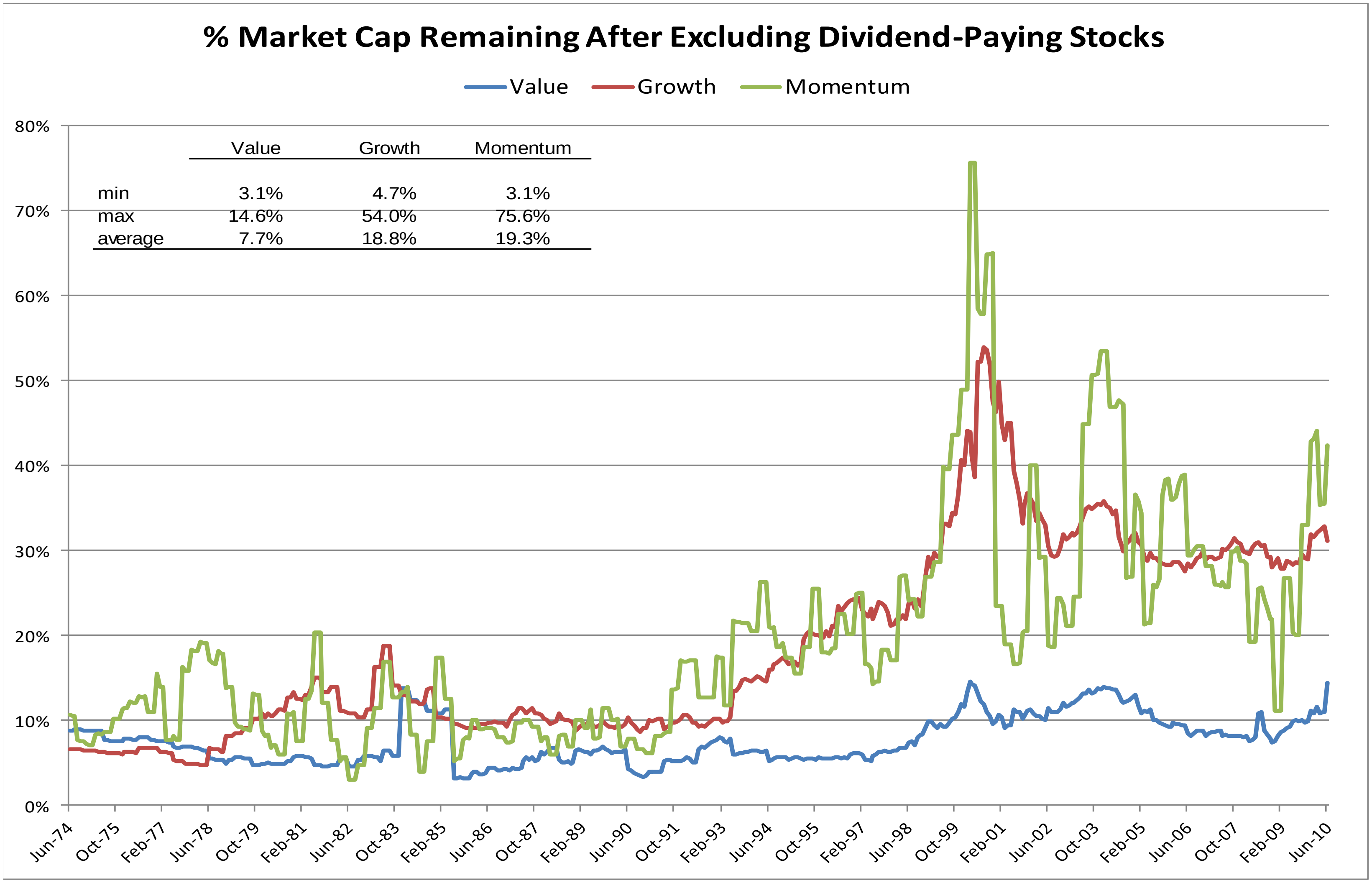


Figure 6: Percent of Market Cap Remaining After Excluding Dividend-Paying Stocks
Time-series plot of the monthly percent of market capitalization of the original index remaining after excluding dividend-paying stocks from the large cap value, large cap growth, and large cap momentum indices.

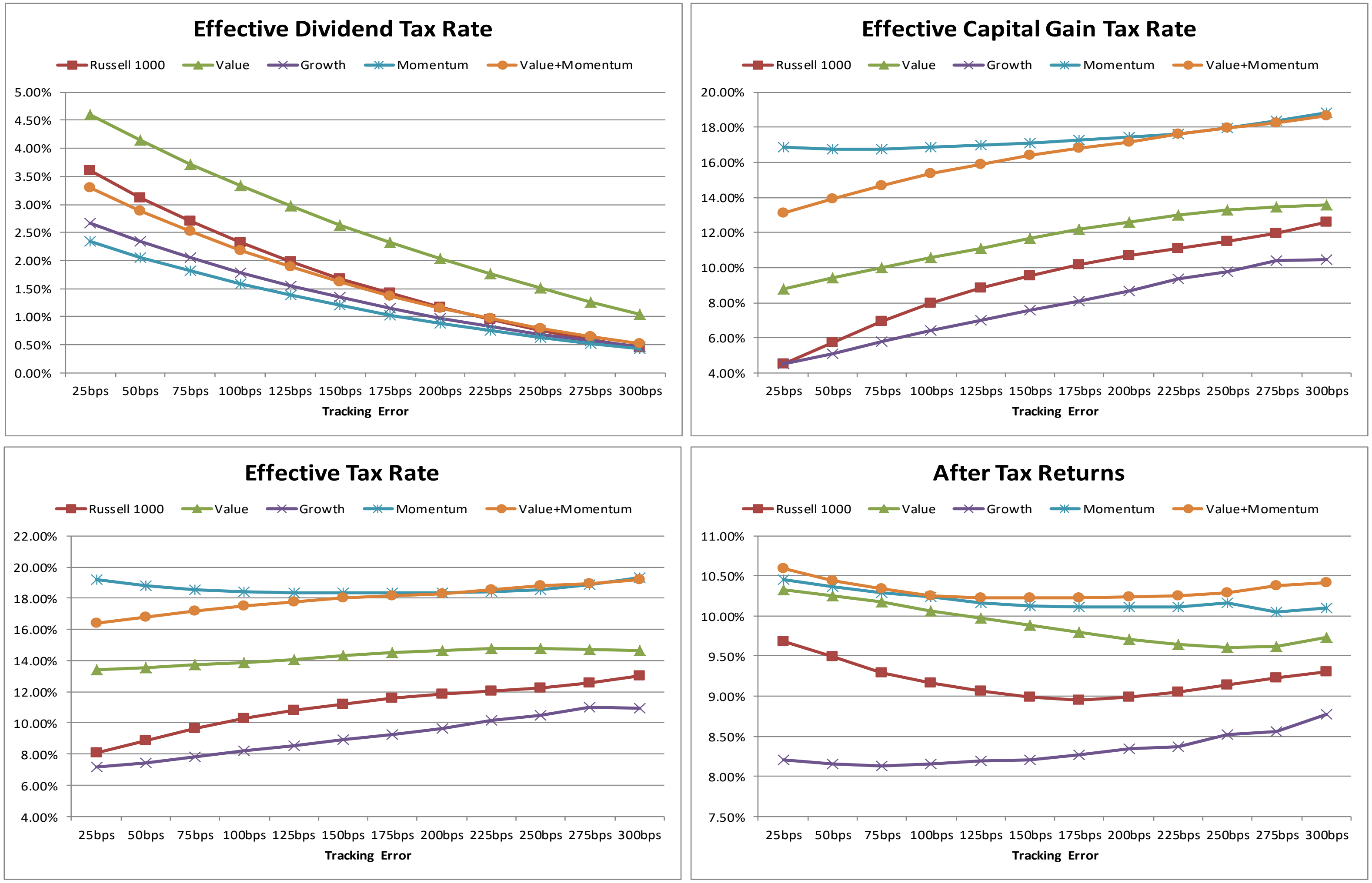


Figure 7: Tax Optimized Portfolios Across Tracking Error—Dividend Income Only

Plot of the average annualized effective dividend tax rate, effective capital gain tax rate, total effective tax rate, and after-tax returns of tax managed portfolios that minimize dividend income only subject to a tracking error constraint that ranges from 25, 50, . . . , 300 basis points across equity style indices for the market, value, growth, momentum, and value + momentum. Statistics are computed over the period July 1974 to June 2010 and assume the current U.S. tax code and rates apply with all excess capital losses carried forward according to the tax code.

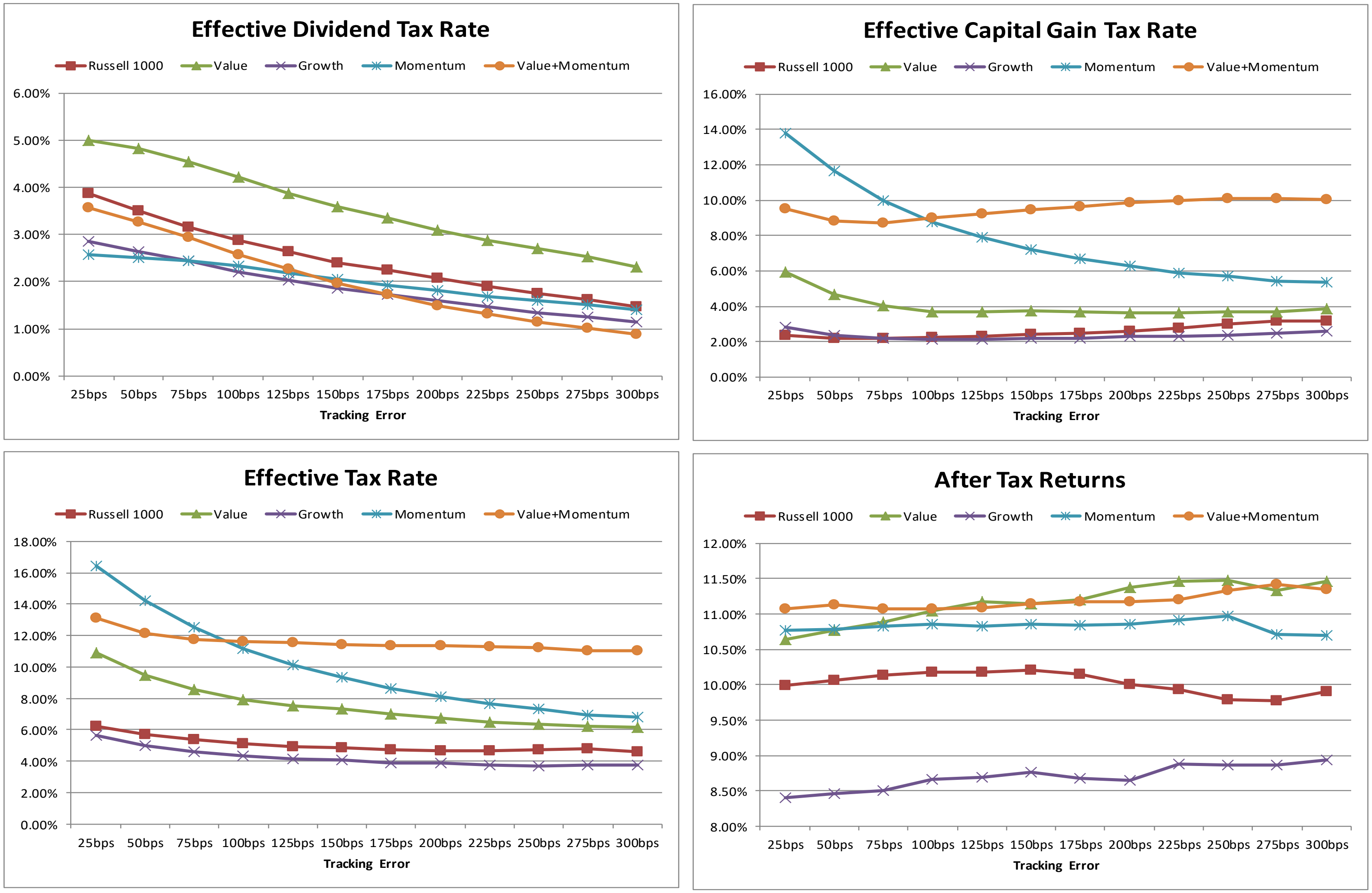


Figure 8: Tax Optimized Portfolios Across Tracking Error—Total Taxes (Capital Gains + Dividends)
Plot of the average annualized effective dividend tax rate, effective capital gain tax rate, total effective tax rate, and after-tax returns of tax managed portfolios that minimize total taxes = capital gains + dividend income, subject to a tracking error constraint that ranges from 25, 50, . . . , 300 basis points across equity style indices for the market, value, growth, momentum, and value + momentum. Statistics are computed over the period July 1974 to June 2010 and assume the current U.S. tax code and rates apply with all excess capital losses carried forward according to the tax code.

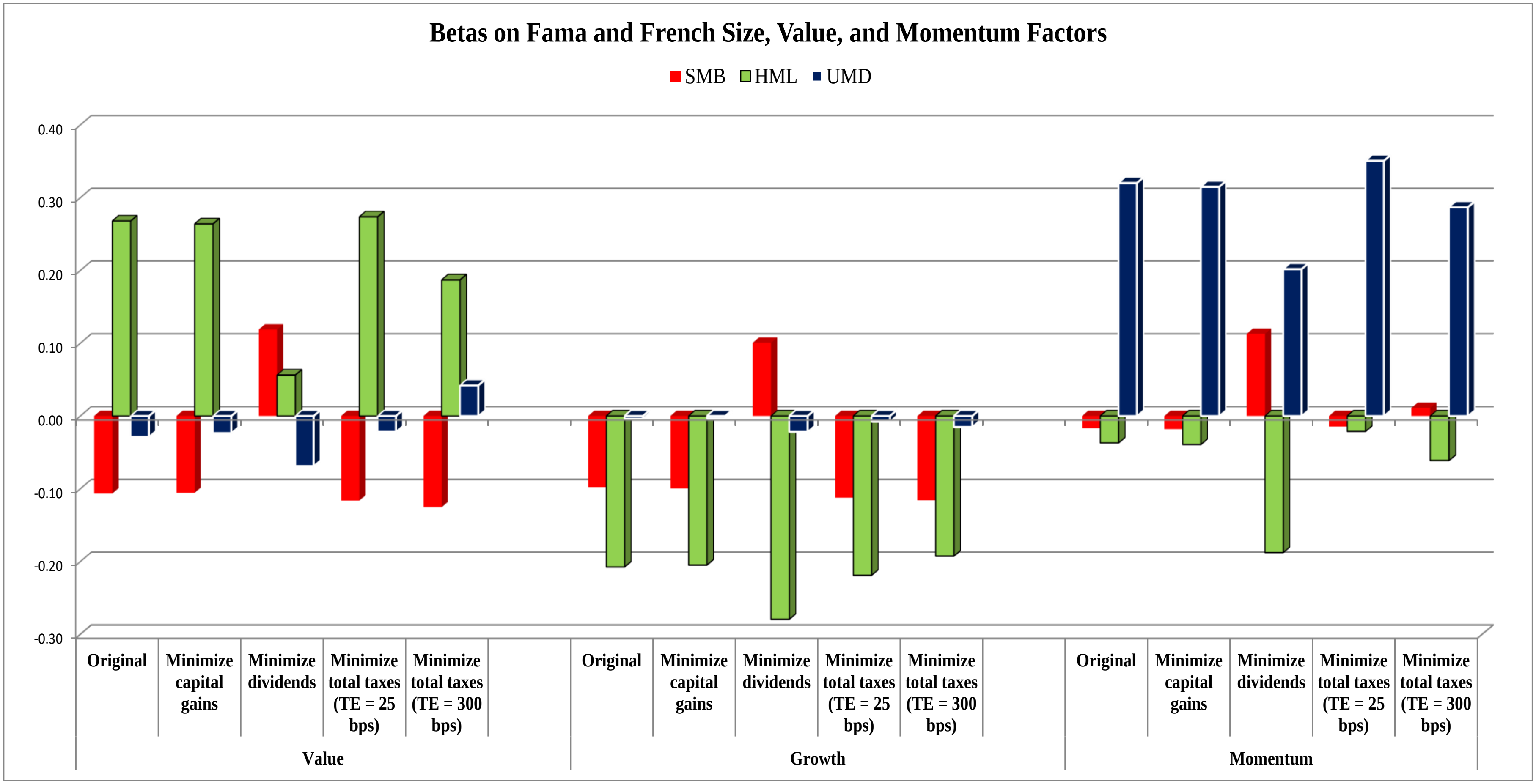


Figure 9: Fama-French Factor Exposure of Original and Tax Optimized Equity Style Portfolios

Plot of the factor exposures or betas on the Fama-French factors *SMB* (size), *HML* (value), and *UMD* (momentum) for the original equity style indices for value, growth, and momentum and four tax managed versions of these styles: minimizing capital gains only, minimizing dividend income only, minimizing total taxes (capital gains + dividends) subject to a tracking error constraint of 25 bps, and minimizing total taxes subject to a tracking error of 300 bps. Betas are estimated over the period July 1974 to June 2010.

Table 1:
After-Tax Returns and Tax Exposures of (Long Only) Equity Styles

Reported are the annualized average returns before taxes, turnover, and dividend yields of the equity style portfolios from July 1974 to June 2010 as well as the annualized average after-tax returns, effective tax rates [(before-tax returns - after-tax returns)/before-tax returns] and effective tax rates coming from capital gains and dividend income exposure separately. After-tax returns and tax exposures are computed under two tax regimes: the current tax code and historical tax code lined up contemporaneously with returns in real time. Panel A reports results treating each strategy as a stand-alone investment, where capital losses are netted only against capital gains generated from the portfolio itself and any unused losses are carried forward according to the tax code. Panel B reports results treating each portfolio in the context of a broader portfolio where we assume all losses can be used immediately to offset other gains in the portfolio. All results are presented for an investor at the 99.99th income percentile.

	PANEL A: CARRYFORWARD LOSSES AS IF A STAND-ALONE INVESTMENT										
				Using current (2011) tax code				Using historical tax code at the time			
	Annualized before-tax return	Annualized turnover	Dividend yield	Annualized after-tax return	Effective Tax rate	Effective capital gain tax rate	Effective dividend tax rate	Annualized after-tax return	Effective Tax rate	Effective capital gain tax rate	Effective dividend tax rate
Large cap stocks											
Russell 1000	10.66%	7%	2.7%	9.84%	7.7%	3.5%	4.2%	8.74%	18.0%	5.3%	12.7%
Value	12.01%	17%	3.7%	10.36%	13.7%	8.6%	5.1%	8.59%	28.5%	13.1%	15.4%
Growth	8.93%	17%	1.7%	8.28%	7.3%	4.2%	3.1%	7.52%	15.8%	6.5%	9.3%
Momentum	13.09%	161%	2.1%	10.44%	20.2%	17.6%	2.7%	8.71%	33.5%	25.4%	8.1%
Value + Momentum	12.82%	90%	2.9%	10.67%	16.8%	13.0%	3.8%	8.93%	30.3%	18.9%	11.5%
Momentum - Value	1.08%			0.08%				0.12%			
Momentum - Growth	4.17%			2.17%				1.19%			
Val+Mom - Val+Gro	2.16%			0.83%				0.19%			
Small cap stocks											
Russell 2000	12.00%	27%	1.8%	10.21%	14.9%	12.5%	2.4%	8.81%	26.5%	19.1%	7.5%
Value	13.88%	35%	2.6%	11.32%	18.5%	15.4%	3.1%	9.41%	32.2%	22.8%	9.4%
Growth	9.62%	40%	0.9%	8.25%	14.3%	12.8%	1.5%	7.13%	25.9%	21.0%	4.9%
Momentum	15.87%	168%	1.2%	12.27%	22.7%	21.4%	1.3%	10.06%	36.6%	32.7%	3.9%
Value + Momentum	15.20%	103%	2.0%	12.12%	20.3%	18.1%	2.2%	10.03%	34.0%	27.5%	6.6%
Momentum - Value	1.99%			0.95%				0.65%			
Momentum - Growth	6.25%			4.02%				2.94%			
Val+Mom - Val+Gro	3.21%			1.91%				1.22%			

	Annualized before-tax return	Annualized turnover	Dividend yield	Annualized after-tax return (current 2011 tax code)	Effective Tax rate (current 2011 tax code)	Effective capital gain tax rate (current 2011 tax code)	Effective dividend tax rate (current 2011 tax code)	Annualized after-tax return (historical tax code at the time)	Effective Tax rate (historical tax code at the time)	Effective capital gain tax rate (historical tax code at the time)	Effective dividend tax rate (historical tax code at the time)
	PANEL B: USE ALL LOSSES IMMEDIATELY AS PART OF A PORTFOLIO										
Large cap stocks											
Russell 1000	10.66%	7%	2.7%	9.88%	7.3%	3.1%	4.2%	8.79%	17.5%	4.8%	12.7%
Value	12.01%	17%	3.7%	10.40%	13.4%	8.3%	5.1%	8.63%	28.1%	12.7%	15.4%
Growth	8.93%	17%	1.7%	8.51%	4.6%	1.6%	3.1%	7.83%	12.2%	2.9%	9.4%
Momentum	13.09%	161%	2.1%	11.60%	11.4%	8.7%	2.7%	10.03%	23.4%	15.2%	8.2%
Value + Momentum	12.82%	90%	2.9%	11.29%	12.0%	8.2%	3.8%	9.63%	24.9%	13.4%	11.5%
Momentum - Value	1.08%			1.20%				1.40%			
Momentum - Growth	4.17%			3.09%				2.20%			
Val+Mom - Val+Gro	2.16%			1.40%				0.84%			
Small cap stocks											
Russell 2000	12.00%	27%	1.8%	10.31%	14.0%	11.6%	2.4%	8.91%	25.7%	18.2%	7.5%
Value	13.88%	35%	2.6%	11.39%	18.0%	14.9%	3.1%	9.50%	31.5%	22.2%	9.4%
Growth	9.62%	40%	0.9%	8.73%	9.3%	7.8%	1.5%	7.65%	20.5%	15.5%	5.0%
Momentum	15.87%	168%	1.2%	13.68%	13.8%	12.5%	1.3%	11.61%	26.8%	22.9%	4.0%
Value + Momentum	15.20%	103%	2.0%	12.88%	15.3%	13.1%	2.2%	10.85%	28.6%	22.0%	6.6%
Momentum - Value	1.99%			2.29%				2.11%			
Momentum - Growth	6.25%			4.95%				3.96%			
Val+Mom - Val+Gro	3.21%			2.57%				1.94%			

Table 2:
After-Tax Returns in Up and Down Markets When Losses Can Be Used Immediately

Reported are the pre-tax and after-tax annualized average returns, as well as the difference between them, of each of the equity styles under the current U.S. tax code. After-tax returns assume all losses can be used immediately in a portfolio context. The first three columns report mean returns in up market environments only, defined as years for which the Russell 1000 delivered a positive return, the second three columns reports results in down markets only, defined as years for which the Russell 1000 delivered a negative return and the last three columns reports results for the global financial crisis from July 2007 to March 2009.

	Up Markets			Down Markets			July 2007 to March 2009		
	Pre-tax mean return	After-tax mean return	Difference	Pre-tax mean return	After-tax mean return	Difference	Pre-tax mean return	After-tax mean return	Difference
Large cap stocks									
Russell 1000	18.83%	18.05%	-0.77%	-14.49%	-14.99%	-0.49%	-30.48%	-30.87%	-0.38%
Value	18.63%	17.10%	-1.53%	-8.91%	-10.12%	-1.20%	-35.99%	-36.64%	-0.65%
Growth	18.94%	18.42%	-0.53%	-20.29%	-20.21%	0.07%	-25.17%	-25.40%	-0.23%
Momentum	22.56%	19.57%	-3.00%	-15.34%	-11.26%	4.08%	-27.77%	-19.49%	8.27%
Value + Momentum	20.77%	18.53%	-2.24%	-12.24%	-10.83%	1.41%	-32.11%	-28.47%	3.64%
Momentum - Value	3.93%	2.47%		-6.43%	-1.15%		8.23%	17.14%	
Momentum - Growth	3.62%	1.15%		4.95%	8.95%		-2.60%	5.90%	
Val+Mom - Val+Gro	1.94%	0.47%		2.25%	4.16%		-1.63%	2.39%	
Small cap stocks									
Russell 2000	19.58%	17.89%	-1.69%	-8.87%	-9.85%	-0.99%	-32.11%	-32.89%	-0.78%
Value	19.44%	17.08%	-2.36%	-2.28%	-4.08%	-1.81%	-34.19%	-34.65%	-0.46%
Growth	19.43%	18.28%	-1.15%	-15.41%	-15.12%	0.29%	-29.97%	-30.27%	-0.30%
Momentum	24.21%	20.77%	-3.44%	-6.23%	-2.97%	3.26%	-34.30%	-22.30%	12.00%
Value + Momentum	22.13%	19.26%	-2.87%	-4.37%	-3.71%	0.66%	-34.88%	-28.96%	5.92%
Momentum - Value	4.76%	3.69%		-3.95%	1.11%		-0.11%	12.35%	
Momentum - Growth	4.77%	2.49%		9.19%	12.15%		-4.33%	7.98%	
Val+Mom - Val+Gro	2.55%	1.37%		4.49%	6.14%		-2.77%	3.93%	

Table 3:
Short and Long-Term Capital Gain and Loss Exposure of Equity Styles

Reported are the annualized turnover, long and short-term capital gains and losses as a percent of the net asset value of each style (per $), percentage of long-term capital gains and short-term losses realized and the average loss carry-forward (unused losses) per year as a percent of net asset value for each index under the current U.S. tax code.

		Gains as a % of NAV			Losses as a % of NAV			
	Annualized turnover	Long-term	Short-term	% Long-term gains	Long-term	Short-term	% Short-term losses	Average loss carryforward (%NAV)
Large cap stocks								
Russell 1000	7%	2.77%	0.19%	96.1%	0.68%	0.24%	21.4%	0.3%
Value	17%	5.13%	1.07%	85.4%	1.01%	0.30%	19.6%	0.4%
Growth	17%	3.93%	0.51%	90.9%	1.59%	1.16%	44.0%	3.3%
Momentum	161%	7.74%	9.89%	45.9%	0.24%	10.35%	96.8%	6.2%
Value + Momentum	90%	6.41%	5.39%	58.2%	0.83%	5.16%	81.1%	2.3%
Small cap stocks								
Russell 2000	27%	8.60%	2.45%	79.1%	2.88%	1.34%	30.4%	0.3%
Value	35%	8.16%	4.16%	67.6%	2.62%	1.35%	31.1%	0.5%
Growth	40%	9.27%	3.81%	69.6%	4.07%	4.14%	48.2%	5.8%
Momentum	168%	8.51%	17.72%	29.1%	0.32%	16.53%	97.5%	6.0%
Value + Momentum	103%	8.93%	10.63%	45.6%	1.95%	8.71%	79.2%	2.3%

Table 4:
The Impact of Optimal Tax Lot Determination vs. HIFO

Reported are the annualized average after-tax returns and effective tax rates of the equity style portfolios using an optimal tax lot determination that chooses tax lots to minimize the current period's capital gain exposure. Differences in after-tax returns and effective tax rates from the HIFO system of tax lot determination are also reported. After-tax returns and effective tax rates are computed using the current tax code, where the first four columns report results assuming any excess losses are carried forward according to the tax code as if each portfolio is a stand-alone investment and the next four columns report results assuming all losses can be used immediately as part of a broader portfolio. The last four columns report the percentage of long-term gains and short-term losses as well as their differences from the HIFO tax lot system. Results are reported for the sample period June 1974 to June 2010.

	Optimal Tax Lot vs. HIFO Tax Lot Accounting											
	Carryforward losses as if a stand-alone investment				**Use all losses immediately as part of a Portfolio**							
	Annualized after-tax return	Δ from HIFO	Effective Tax rate	Δ from HIFO	Annualized after-tax return	Δ from HIFO	Effective Tax rate	Δ from HIFO	% Long-term gains	Δ from HIFO	% Short-term losses	Δ from HIFO
Large cap stocks												
Russell 1000	9.84%	0.004%	7.62%	-0.035%	9.89%	0.005%	7.21%	-0.047%	96.86%	0.79%	21.61%	0.17%
Value	10.38%	0.017%	13.58%	-0.146%	10.42%	0.012%	13.29%	-0.102%	86.89%	1.50%	20.19%	0.62%
Growth	8.28%	-0.001%	7.26%	0.008%	8.50%	-0.014%	4.80%	0.157%	92.06%	1.14%	39.84%	-4.14%
Momentum	10.44%	-0.001%	20.24%	0.010%	11.61%	0.006%	11.34%	-0.044%	45.87%	0.00%	96.90%	0.12%
Value + Momentum	10.70%	0.029%	16.53%	-0.227%	11.28%	-0.003%	12.00%	0.025%	62.43%	4.20%	79.04%	-2.09%
Small cap stocks												
Russell 2000	10.22%	0.005%	14.84%	-0.042%	10.31%	0.001%	14.03%	-0.009%	79.44%	0.31%	30.64%	0.21%
Value	11.35%	0.035%	18.23%	-0.250%	11.42%	0.034%	17.72%	-0.243%	68.59%	1.00%	31.64%	0.56%
Growth	8.24%	-0.005%	14.33%	0.053%	8.70%	-0.031%	9.60%	0.320%	69.91%	0.32%	47.68%	-0.53%
Momentum	12.26%	0.029%	22.75%	-0.067%	13.66%	0.015%	13.95%	0.115%	28.62%	-0.39%	97.58%	0.07%
Value + Momentum	12.19%	0.075%	19.79%	-0.509%	12.91%	0.033%	15.05%	-0.238%	47.24%	1.62%	79.22%	-0.03%

Table 5:
Tax Managing Capital Gains Exposure

Reported are results based on the current tax code from tax optimized portfolios of the equity indices by minimizing capital gains tax exposure subject to a tracking error constraint that requires use of an ex ante risk model. Panel A reports results that use the one-month lagged USE3S BARRA risk model (US Short-Term model) to estimate ex ante tracking error, and Panel B reports results that use the Fama and French four factor model to estimate ex ante tracking error (using rolling five year beta estimates and covariance matrices). Ex ante tracking error is constrained to be less than 25 basis points. In both panels, the annualized average after-tax returns and effective tax rates of the tax managed portfolios are reported along with their differences from the original indices (that are not tax managed). Also reported is the change in turnover, long-term gains, and short-term losses between the tax managed versions and original indices. The last three columns report the alpha, *t*-stat of alpha and *ex post* tracking error from a time-series regression of the tax managed portfolios on the original indices.

								Regression on original portfolio		
	After-tax return	Δ from original	Effective tax rate	Δ from original	Δ Turnover	Δ Long-term gains	Δ Short-term losses	Alpha	T-stat of alpha	Tracking error
	PANEL A: TRACKING ERROR ESTIMATED FROM BARRA USE3S RISK MODEL									
Large cap stocks										
Russell 1000	10.05%	0.21%	6.7%	-1.0%	2%	-2.0%	15.0%	0.19%	(2.69)	0.38%
Value	10.68%	0.32%	11.0%	-2.7%	-4%	7.1%	5.1%	0.33%	(2.95)	0.59%
Growth	8.52%	0.25%	5.9%	-1.4%	-1%	3.1%	7.2%	0.25%	(2.26)	0.58%
Momentum	10.77%	0.32%	16.9%	-3.4%	-20%	8.7%	-0.3%	0.22%	(1.99)	0.59%
Value + Momentum	11.09%	0.42%	13.3%	-3.5%	-10%	10.7%	3.8%	0.28%	(3.30)	0.45%
Small cap stocks										
Russell 2000	10.40%	0.19%	14.1%	-0.8%	7%	-5.5%	8.2%	0.14%	(0.92)	0.81%
Value	11.92%	0.60%	14.9%	-3.6%	-3%	7.0%	7.7%	0.54%	(3.58)	0.81%
Growth	8.29%	0.04%	14.2%	-0.1%	3%	-1.8%	4.0%	-0.03%	(-0.16)	1.02%
Momentum	12.46%	0.18%	20.3%	-2.4%	-11%	3.9%	-0.7%	0.06%	(0.45)	0.68%
Value + Momentum	12.73%	0.62%	16.0%	-4.3%	-4%	5.6%	6.5%	0.39%	(3.29)	0.64%
	PANEL B: TRACKING ERROR ESTIMATED FROM FAMA-FRENCH 4-FACTOR MODEL									
Large cap stocks										
Russell 1000	9.93%	0.15%	7.3%	-0.9%	2%	-1.0%	21.5%	0.06%	(0.55)	0.55%
Value	10.65%	0.37%	11.3%	-2.8%	-4%	5.3%	7.0%	0.27%	(1.71)	0.85%
Growth	8.38%	0.14%	6.5%	-1.2%	-2%	2.6%	5.4%	0.17%	(1.25)	0.72%
Momentum	10.74%	0.29%	17.0%	-3.3%	-22%	8.5%	-0.3%	0.20%	(1.39)	0.78%
Value + Momentum	11.18%	0.54%	12.6%	-4.4%	-15%	7.8%	6.3%	0.33%	(2.77)	0.65%
Small cap stocks										
Russell 2000	10.72%	0.15%	13.8%	-1.0%	8%	-4.6%	7.0%	0.43%	(1.96)	1.19%
Value	12.33%	0.79%	14.4%	-3.8%	-2%	5.8%	9.1%	0.84%	(4.00)	1.13%
Growth	8.74%	0.08%	14.4%	-0.1%	2%	-1.7%	2.5%	0.39%	(1.47)	1.43%
Momentum	12.55%	0.06%	20.4%	-2.5%	-13%	3.7%	-1.6%	0.03%	(0.18)	1.00%
Value + Momentum	13.33%	1.02%	15.1%	-5.0%	-5%	5.2%	9.8%	0.93%	(5.45)	0.91%

Table 6:
Tax Managing Dividend Exposure

Reported are results based on the current tax code from tax optimized portfolios of the equity indices by minimizing dividend exposure. Panel A excludes all dividend paying stocks so that expected dividend income is zero. Panel B minimizes the dividend yield of the portfolio subject to a tracking error constraint of 25 basis points (using the one-month lagged USE3S BARRA risk model to estimate ex ante tracking error). Both panels report the annualized average after-tax returns, effective tax rates and dividend yields of the tax managed portfolios, along with their differences from the original indices (that are not tax aware). Also reported is the *ex post* tracking error from a time-series regression of the tax optimized portfolio on the original index.

	After-tax return	Δ from original	Effective tax rate	Δ from original	Dividend yield	Δ from original	Tracking error
	PANEL A: MINIMIZE DIVIDENDS (EXCLUDE ALL DIVIDEND-PAYING STOCKS)						
Russell 1000	10.08%	0.24%	9.0%	1.4%	0.0%	-2.7%	10.10%
Value	9.56%	-0.80%	18.5%	4.8%	0.1%	-3.7%	10.22%
Growth	9.44%	1.16%	8.0%	0.8%	0.0%	-1.7%	8.87%
Momentum	12.08%	1.63%	18.5%	-1.8%	0.0%	-2.1%	10.10%
Value + Momentum	11.90%	1.22%	17.1%	0.4%	0.0%	-2.9%	10.42%
	PANEL B: MINIMIZE DIVIDEND YIELD SUBJECT TO (25 bp) TRACKING ERROR						
Russell 1000	9.69%	-0.15%	8.11%	0.45%	2.32%	-0.40%	0.41%
Value	10.33%	-0.03%	13.38%	-0.34%	3.33%	-0.39%	0.37%
Growth	8.21%	-0.07%	7.19%	-0.06%	1.46%	-0.23%	0.34%
Momentum	10.45%	0.00%	19.19%	-1.04%	1.83%	-0.27%	0.33%
Value + Momentum	10.59%	-0.08%	16.41%	-0.35%	2.53%	-0.40%	0.35%

Table 7:
Tax Managed Equity Styles—Total Taxes (Capital Gains + Dividends)

Reported are results based on the current tax code from tax optimized portfolios of the equity indices by minimizing total taxes = capital gains + dividend exposure, subject to a tracking error constraint. Panel A minimizes total taxes subject to a tracking error constraint of 25 basis points and Panel B uses a tracking error constraint of 200 basis points (using the one-month lagged USE3S BARRA risk model to estimate ex ante tracking error). Both panels report the annualized average after-tax returns, effective tax rates, effective capital gains tax rate, effective dividend tax rate, and annual turnover of the tax managed portfolios, along with their differences from the original indices (that are not tax aware). Also reported is the *ex post* tracking error from a time-series regression of the tax optimized portfolio on the original index.

	After-tax return	Δ from original	Effective tax rate	Δ from original	Effective capital gains tax rate	Δ from original	Effective Dividend tax rate	Δ from original	Annual Turnover	Δ from original	Tracking error
	PANEL A: MINIMIZE TOTAL TAXES SUBJECT TO (25 bp) TRACKING ERROR CONSTRAINT										
Russell 1000	10.00%	0.16%	6.22%	-1.44%	2.35%	-1.13%	3.87%	-0.31%	15.21%	0.93%	0.34%
Value	10.63%	0.27%	10.91%	-2.82%	5.91%	-2.70%	5.00%	-0.12%	27.92%	-6.92%	0.51%
Growth	8.40%	0.12%	5.67%	-1.58%	2.81%	-1.37%	2.86%	-0.21%	30.53%	-3.76%	0.36%
Momentum	10.79%	0.35%	16.36%	-3.87%	13.78%	-3.80%	2.58%	-0.07%	283.10%	-39.56%	0.50%
Value + Momentum	11.08%	0.40%	13.13%	-3.63%	9.60%	-3.37%	3.53%	-0.26%	162.28%	-16.98%	0.49%
	PANEL B: MINIMIZE TOTAL TAXES SUBJECT TO (200 bp) TRACKING ERROR CONSTRAINT										
Russell 1000	10.01%	0.17%	4.66%	-2.99%	2.59%	-0.89%	2.07%	-2.11%	24.40%	10.13%	2.33%
Value	11.38%	1.01%	6.72%	-7.00%	3.63%	-4.98%	3.09%	-2.03%	24.39%	-10.45%	2.56%
Growth	8.64%	0.37%	3.88%	-3.37%	2.29%	-1.89%	1.59%	-1.48%	30.60%	-3.69%	2.46%
Momentum	10.72%	0.28%	8.08%	-12.15%	6.27%	-11.31%	1.81%	-0.84%	162.92%	-159.74%	2.31%
Value + Momentum	11.17%	0.50%	11.36%	-5.40%	9.87%	-3.10%	1.49%	-2.30%	163.14%	-16.11%	2.33%

Table 8:
Dynamic vs. Myopic Tax Optimization (with Perfect Foresight)

Reported are the percentage of long-term gains and short-term losses realized for tax optimized portfolios that minimize total taxes = capital gains + dividend exposure, subject to a tracking error constraint of 25 and 200 basis points. Four separate optimizations are used with four different objective functions. The first objective is myopic and seeks to minimize taxes in the current month only. The other three objectives seek to minimize taxes dynamically by taking into account the impact of today's trades on current and future taxes. To implement the dynamic optimizations, we allow the objective function to know what the future portfolio will look like one year, two years, and five years ahead (e.g., perfect foresight of the portfolio's future holdings), where the optimization seeks to minimize both today's taxes and taxes at a future date one, two, and five years ahead. Since with perfect foresight returns are biased, we only report the net percentage change in long-term capital gains and short-term capital losses realized. Differences between the dynamic and myopic optimizations are also reported.

DYNAMIC VS. MYOPIC TAX OPTIMIZATION WITH PERFECT FORESIGHT

	% Long-term gains							% Short-term losses						
	Myopic	**Dynamic with 1-year foresight**	Difference	**Dynamic with 2-year foresight**	Difference	**Dynamic with 5-year foresight**	Difference	**Myopic**	**Dynamic with 1-year foresight**	Difference	**Dynamic with 2-year foresight**	Difference	**Dynamic with 5-year foresight**	Difference
Tracking error = 25 bps														
Russell 1000	93.74%	93.44%	-0.3%	94.7%	0.9%	96.7%	3.0%	37.0%	42.0%	5.0%	42.1%	5.1%	39.4%	2.4%
Value	91.67%	88.49%	-3.2%	88.5%	-3.2%	90.6%	-1.1%	27.4%	29.0%	1.7%	30.1%	2.7%	38.0%	10.6%
Growth	93.97%	91.48%	-2.5%	92.7%	-1.3%	95.1%	1.2%	48.8%	53.8%	5.0%	51.1%	2.3%	51.7%	2.9%
Momentum	54.88%	52.85%	-2.0%	52.7%	-2.2%	54.1%	-0.8%	96.4%	96.7%	0.3%	96.6%	0.2%	99.0%	2.7%
Value + Momentum	68.03%	65.08%	-3.0%	56.4%	-11.6%	59.0%	-9.0%	86.0%	84.3%	-1.7%	80.9%	-5.1%	89.1%	3.1%
Tracking error = 200 bps														
Russell 1000	92.56%	87.71%	-4.8%	81.3%	-11.3%	87.5%	-5.1%	45.5%	66.9%	21.4%	57.3%	11.7%	58.0%	12.4%
Value	91.35%	80.60%	-10.8%	76.1%	-15.2%	72.9%	-18.5%	91.3%	80.6%	-10.8%	76.1%	-15.2%	72.9%	-18.5%
Growth	93.77%	90.01%	-3.8%	88.2%	-5.5%	87.6%	-6.2%	60.2%	59.6%	-0.6%	59.8%	-0.4%	60.7%	0.5%
Momentum	74.51%	71.47%	-3.0%	73.1%	-1.4%	75.1%	0.6%	95.4%	96.1%	0.6%	95.9%	0.4%	97.7%	2.3%
Value + Momentum	68.56%	61.56%	-7.0%	56.4%	-12.1%	59.0%	-9.5%	89.9%	91.8%	1.9%	80.9%	-9.0%	89.1%	-0.8%

Appendix

Table A1: Correlations between Replicated and Actual Indices

Reported are the return correlations between actual indices versus replicated indices over the overlapping live period of the actual index returns. Reported is the return correlation of each index with its replicated version.

Large cap indices	Correlation	Small cap indices	Correlation
Russell 1000	0.989	Russell 2000	0.956
Russell 1000 Value	0.982	Russell 2000 Value	0.967
Russell 1000 Growth	0.989	Russell 2000 Growth	0.923
AQR Momentum (large cap)	0.997	AQR Momentum (small cap)	0.995

Table A2: Historical Tax Rates

Tax rates by year for an investor in the 95th and 99.99th income percentile of the U.S. tax code over the sample period from 1974 to 2012. Tax rates are obtained from the Federal Individual Income Tax Rates History 1913 - 2012 from the Tax Foundation in Washington, D.C. and from the Department of the Treasury, Office of Tax Analysis (November 3, 2008).

	Tax Rates by Year (99.99th income percentile)			**Tax Rates by Year (95th income percentile)**		
Year	Short-term rate (%)	Long-term rate (%)	Dividend income (%)	Short-term rate (%)	Long-term rate (%)	Dividend income (%)
1974	70.0	36.5	70.0	45.0	36.5	45.0
1975	70.0	36.5	70.0	45.0	36.5	45.0
1976	70.0	39.9	70.0	45.0	39.9	45.0
1977	70.0	39.9	70.0	45.0	39.9	45.0
1978	70.0	39.9	70.0	50.0	39.9	50.0
1979	70.0	28.0	70.0	49.0	28.0	49.0
1980	70.0	28.0	70.0	55.0	28.0	55.0
1981*	70.0	20.0	70.0	55.0	20.0	55.0
1982	50.0	20.0	50.0	50.0	20.0	50.0
1983	50.0	20.0	50.0	45.0	20.0	45.0
1984	50.0	20.0	50.0	42.0	20.0	42.0
1985	50.0	20.0	50.0	48.0	20.0	48.0
1986	50.0	20.0	50.0	48.0	20.0	48.0
1987	38.5	28.0	38.5	38.5	28.0	38.5
1988	28.0	28.0	28.0	33.0	28.0	33.0
1989	28.0	28.0	28.0	33.0	28.0	33.0
1990	28.0	28.0	28.0	33.0	28.0	33.0
1991	31.0	28.0	31.0	31.0	28.0	31.0
1992	31.0	28.0	31.0	31.0	28.0	31.0
1993	39.6	28.0	39.6	31.0	28.0	31.0
1994	39.6	28.0	39.6	31.0	28.0	31.0
1995	39.6	28.0	39.6	31.0	28.0	31.0
1996	39.6	28.0	39.6	31.0	28.0	31.0
1997*	39.6	20.0	39.6	31.0	20.0	31.0
1998	39.6	20.0	39.6	31.0	20.0	31.0
1999	39.6	20.0	39.6	31.0	20.0	31.0
2000	39.6	20.0	39.6	31.0	20.0	31.0
2001	39.1	20.0	39.1	31.0	20.0	31.0
2002	38.6	20.0	38.6	30.0	20.0	30.0
2003*	35.0	15.0	15.0	28.0	15.0	15.0
2004	35.0	15.0	15.0	28.0	15.0	15.0
2005	35.0	15.0	15.0	28.0	15.0	15.0
2006	35.0	15.0	15.0	28.0	15.0	15.0
2007	35.0	15.0	15.0	28.0	15.0	15.0
2008	35.0	15.0	15.0	28.0	15.0	15.0
2009	35.0	15.0	15.0	28.0	15.0	15.0
2010	35.0	15.0	15.0	28.0	15.0	15.0
2011	35.0	15.0	15.0	28.0	15.0	15.0

**Mid-year rate changes*

Table A3:
After-Tax Returns and Tax Exposures of Fama-French Portfolios from 1927 to 2012

Reported are the annualized average before-tax and after-tax returns and effective tax rates of the long-only counterparts of the Fama and French small, value, growth, and momentum portfolios. After-tax returns and effective tax rates are computed using the current tax code and the historical tax code lined up contemporaneously with returns in real time. Results are reported for the sample period June 1974 to June 2010 overlapping with our indices as well as over the maximal sample period of data availability from June 1927 to June 2012. Unused losses in any period are assumed to be carried forward according to the tax code.

FAMA-FRENCH PORTFOLIOS, LONG-ONLY WITH CARRYFORWARD LOSSES

	June 1974 - June 2010					June 1927 - June 2012				
		Using current (2011) tax code		Using historical tax code			Using current (2011) tax code		Using historical tax code	
	Annualized before-tax return	Annualized after-tax return	Effective tax rate	Annualized after-tax return	Effective tax rate	Annualized before-tax return	Annualized after-tax return	Effective tax rate	Annualized after-tax return	Effective tax rate
Large cap stocks										
Market	13.28%	11.49%	13.50%	9.95%	25.03%	11.27%	9.70%	13.93%	7.39%	34.45%
Value	12.73%	10.33%	18.87%	7.97%	37.39%	11.32%	9.33%	17.58%	6.37%	43.74%
Growth	9.86%	9.00%	8.74%	7.99%	18.93%	8.93%	8.01%	10.27%	6.23%	30.18%
Momentum	13.76%	10.60%	23.00%	8.48%	38.40%	12.89%	10.01%	22.32%	6.80%	47.25%
Value + Momentum	13.49%	10.76%	20.23%	8.54%	36.71%	12.40%	10.00%	19.32%	6.93%	44.14%
Small cap stocks										
Small	14.52%	12.29%	15.33%	10.84%	25.33%	12.25%	10.33%	15.71%	7.94%	35.15%
Value	17.36%	13.87%	20.11%	11.81%	31.97%	14.24%	11.53%	19.08%	8.70%	38.95%
Growth	9.75%	8.11%	16.83%	7.01%	28.08%	9.04%	7.40%	18.17%	5.20%	42.47%
Momentum	19.13%	14.12%	26.22%	11.64%	39.16%	17.31%	12.79%	26.12%	8.54%	50.67%
Value + Momentum	18.53%	14.40%	22.28%	12.15%	34.44%	15.95%	12.56%	21.27%	9.13%	42.72%

Table A4: Correlations between Indices and Fama-French Portfolios

Reported are the return correlations between the equity style indices and the Fama and French (long-only) portfolios for both pre-tax and after-tax returns.

Correlation between indices and Fama-French portfolios		
	Pre-tax returns	After-tax returns
Large cap stocks		
Market	0.938	0.938
Value	0.911	0.910
Growth	0.921	0.920
Momentum	0.946	0.945
Small cap stocks		
Market	0.969	0.969
Value	0.965	0.964
Growth	0.966	0.966
Momentum	0.962	0.952

Table A5:
After-Tax Returns for Investors at Different Income Percentiles

Reported are the annualized average returns after taxes of the equity portfolios from July 1974 to June 2010 for investors at the 99.99th and 95th income percentiles. After-tax returns and tax exposures are computed under two tax regimes: the current tax code and historical tax code lined up contemporaneously with returns in real time. Each strategy is treated as a stand-alone investment where unused losses are carried forward according to the tax code.

	Annualized after-tax return			
	Using current (2011) tax code		Using historical tax code at the time	
	99.99th Percentile	95th Percentile	99.99th Percentile	95th Percentile
Russell 1000	9.84%	9.85%	8.74%	8.99%
Value	10.36%	10.43%	8.59%	9.00%
Growth	8.28%	8.29%	7.52%	7.67%
Momentum	10.44%	10.73%	8.71%	9.22%
Value + Momentum	10.67%	10.86%	8.93%	9.38%
Momentum - Value	0.08%	0.31%	0.12%	0.23%
Momentum - Growth	2.17%	2.44%	1.19%	1.55%
Val+Mom - Val+Gro	0.83%	1.01%	0.19%	0.40%
Russell 2000	10.21%	10.32%	8.81%	9.15%
Value	11.32%	11.54%	9.41%	9.99%
Growth	8.25%	8.34%	7.13%	7.39%
Momentum	12.27%	12.77%	10.06%	10.90%
Value + Momentum	12.12%	12.47%	10.03%	10.72%
Momentum - Value	0.95%	1.22%	0.65%	0.92%
Momentum - Growth	4.02%	4.42%	2.94%	3.51%
Val+Mom - Val+Gro	1.91%	2.15%	1.22%	1.57%

Table A6:
A Numerical Example of Tax Lot Determination Under HIFO vs. Optimal

Using the formulas from Section III, we compute a numerical example of total tax liability over a multiyear period under the HIFO tax lot system and compare it to the tax liability under an optimal tax lot system.

Assume that the short term tax rate is 15%, and the long term tax rate is 35% (as is currently the case under the 2012 U.S. tax code). Consider a security that is purchased on two separate dates less than one year apart and then subsequently sold on two separate dates in the future. Without loss of generality, suppose 100 units of the security are bought and sold each time and suppose the discount rate is zero over the periods. For concreteness, we specify the dates below.

At the time of the first sale of shares on 12/31/2011, there are two tax lots to choose from: the 100 shares purchased on 12/31/2010 and the 100 shares purchased on 6/30/2011. The first tax lot would be treated as a long-term gain or loss because the transactions are a year apart, while the second tax lot would be treated as a short-term gain or loss.

In this first scenario, the first purchase of shares on 12/31/2010 is for $5.00/share, the second purchase of shares on 6/30/2011 is for $10.00/share, the price of shares at the sale on 12/31/2011 is $15.00/share. The optimal tax lot choice to minimize taxes on 12/31/2011 is to take the long-term gain, which generates a tax burden of $150. The HIFO system, on the other hand, takes the highest priced tax lot out first, so it uses the short-term and higher priced tax lot and therefore incurs a $175 tax burden.

At the second sale date, 3/31/2012, there is only one tax lot left. Under the optimal tax lot system, the 12/31/2010 lot has already been used and so the 6/30/2011 tax lot must be used. Under HIFO, the 12/31/2010 lot must now be used at the second sale date. In this case, if the price on 3/31/2012 is lower than it was on 12/31/2011, then the optimal tax lot system will generate a total tax liability over the entire time period that is lower than that generated under HIFO. For example, if the price per share on 3/31/2012 falls back to $10.00 per share, the optimal system generates a total tax liability of $150 while the HIFO system generates a total tax liability of $250.

				Tax lot system			
				Optimal		**HIFO**	
Period	Price	Units bought	Units sold	**Long-term**	**Short-term**	**Long-term**	**Short-term**
12/31/2010	$ 5.00	100	0				
6/30/2011	$ 10.00	100	0				
12/31/2011	$ 15.00	0	100	150			175
3/31/2012	$ 10.00	0	100		0	75	
Total tax liability				**150**		**250**	

If, however, the price on 3/31/2012 rose to $20 per share, then the optimal system would actually generate a higher total tax liability than HIFO. This is because minimizing the tax burden at the first sale under the optimal tax lot system, which used the only long-term lot, leaves only one remaining tax lot for the second sale, which in this case is a short-term lot. Hence, if the rise in price between the two sales is large enough, the total tax liability under the optimal system can be larger than under HIFO. This is because the optimal tax lot system optimizes or minimizes taxes myopically, which in a dynamic multi-period setting can be suboptimal relative to a simple rule of thumb like HIFO.

				Tax lot system			
				Optimal		**HIFO**	
Period	Price	Units bought	Units sold	**Long-term**	**Short-term**	**Long-term**	**Short-term**
12/31/2010	$ 5.00	100	0				
6/30/2011	$ 10.00	100	0				
12/31/2011	$ 15.00	0	100	150			175
3/31/2012	$ 20.00	0	100		350	225	
Total tax liability				**500**		**400**	

Using the formulas from Section III, we can compute the break-even price for the first sale where the price for the second sale is irrelevant. If the price of shares is $13.75 on 12/31/2011, then it doesn't matter what the price is on 3/31/2012, the two tax lot systems will generate the same total tax liability.

				Tax lot system			
				Optimal		**HIFO**	
Period	Price	Units bought	Units sold	**Long-term**	**Short-term**	**Long-term**	**Short-term**
12/31/2010	$ 5.00	100	0				
6/30/2011	$ 10.00	100	0				
12/31/2011	$ 13.75	0	100	131.25			131.25
3/31/2012	$ 20.00	0	100		225	225	
Total tax liability				**356.25**		**356.25**	

Also, if the price at 12/31/2011 < 13.75, then the optimal and HIFO systems produce the same total tax burden.

				Tax lot system			
				Optimal		**HIFO**	
Period	Price	Units bought	Units sold	**Long-term**	**Short-term**	**Long-term**	**Short-term**
12/31/2010	$ 5.00	100	0				
6/30/2011	$ 10.00	100	0				
12/31/2011	$ 13.74	0	100	130.9			130.9
3/31/2012	$ 20.00	0	100		225	225	
Total tax liability				**355.9**		**355.9**	

But, if the price at 12/31/2011 > 13.75 (and the price at 3/31/2012 is higher than 12/31/2011) then the optimal system will produce greater total tax liability than HIFO. For example, even $0.01 more than $13.75 produces $124.80 (= $481.40 - $356.60) more taxes under the optimal tax lot system than under the HIFO tax lot system.

				Tax lot system			
				Optimal		**HIFO**	
Period	Price	Units bought	Units sold	**Long-term**	**Short-term**	**Long-term**	**Short-term**
12/31/2010	$ 5.00	100	0				
6/30/2011	$ 10.00	100	0				
12/31/2011	$ 13.76	0	100	131.4			131.6
3/31/2012	$ 20.00	0	100		350	225	
Total tax liability				**481.4**		**356.6**	

Moreover, the difference in tax liability for optimal versus HIFO could be arbitrarily large, depending on the price at 3/31/2012. If prices soar to $200/share, then the tax liability under optimal is more than twice that under HIFO and amounts to a $3,724.80 (= $6,781.40 - $3,056.60) increase in tax liability.

				Tax lot system			
				Optimal		**HIFO**	
Period	Price	Units bought	Units sold	**Long-term**	**Short-term**	**Long-term**	**Short-term**
12/31/2010	$ 5.00	100	0				
6/30/2011	$ 10.00	100	0				
12/31/2011	$ 13.76	0	100	131.4			131.6
3/31/2012	$ 200.00	0	100		6650	2925	
Total tax liability				**6781.4**		**3056.6**	

The bottom line is that because the optimal tax lot system optimizes or minimizes taxes myopically, period by period, this may be suboptimal in a dynamic multi-period setting, where tax lots chosen one period have an effect on available tax lots for future periods. As a result, the myopically optimal tax lot strategy may be inferior relative to a simple rule of thumb like HIFO in a dynamic setting.