Ignorance, Debt and Financial Crises

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July 20, 2020

Abstract

Debt is optimal for trading, and the optimal collateral backing that debt is also debt. Debt as collateral transfers the most value intertemporally. When that debt is used as collateral for another debt contract, the 'debt-on-debt' preserves symmetric ignorance because it minimizes the incentive to produce private information about the payoffs, so debt is least information-sensitive, i.e., liquid. But, bad public news (a shock) about the value of the collateral that backs the debt can cause information-insensitive debt to become information-sensitive. To prevent endogenous adverse selection agents reduce the amount of trade below the expected value of the debt collateral. The shock is amplified, a financial crisis.

Earlier versions of this paper circulated under different titles, including "Opacity and the Optimality of Debt for Liquidity Provision," and "Financial Crises and the Optimality of Debt for Liquidity Provision." Thanks to seminar participants at the 2009 Yale Cowles Foundation Summer Economic Theory Conference, the Financial Intermediation Research Society Florence Conference, the JME/SNB/SGC Conference, the New York Fed, Wharton, NYU, HBS, Columbia Business School, MIT, Princeton, University College London, the European Central Bank, the IMF the Milton Friedman Institute at the University of Chicago, Bonn, Mannheim, Stanford, the Philadelphia Fed, Penn State, Chinese University of Hong Kong and to Patrick Bolton, Yeonkoo Che, Peter DeMarzo, Douglas Diamond, Jon Levin, Robert Lucas, Yukitoshi Matsushita, Stewart Myers, Jean-Charles Rochet, Bernard Salanie, Ernst-Ludwig von Thadden, Robert Wilson, and Mark Wolfson for comments and suggestions.
1 Introduction

Short-term debt is not optimally designed to be accepted without information about its payoffs being produced. Such debt is opaque. No agent has an incentive to produce (costly) private information about the payoffs. To minimize the incentive to produce information the short-term debt is backed by debt collateral: debt-on-debt. Then short-term debt is maximally information-insensitive. But bad public news can cause agents to question the value of the collateral. The debt can become information-sensitive, a loss of confidence, a run. To prevent endogenous adverse selection agents reduce the amount of trade below the expected value of the debt collateral, a fire sale. Thus the shock is amplified, a financial crisis.

Financial crises are always about short-term debt. Crises are runs on short-term debt. Yet current theories of crisis assume the existence of debt, and current theories of debt do not explain the origins of crises. In this paper we provide a theory of the existence and optimality of debt-on-debt, a theory that also shows that debt - while optimal - is vulnerable to a crisis in which trade collapses. The breakdown of these markets is a manifestation of a risk that is endogenously created by agents in the economy who optimally use debt backed by debt collateral for liquidity reasons, precisely because by design it is best at maintaining symmetric ignorance between the two parties to the transaction. Since short-term debt and its collateral are designed to be information-insensitive, the model explains why there is a quiet period before a crisis. No information is produced, by design. But, then the short-term debt and the collateral can suddenly become information-sensitive, a crisis. Debt-on-debt is the defining feature of banking. Indeed, all banks sen short-term debt is backed by debt. In the U.S. free bank notes were backed by state bonds. Demand deposits are backed by portfolios of loans. Sale and repurchase agreements (repos) are backed by specific bonds. Asset-backed commercial paper is backed by asset-backed and mortgage-backed securities. Money market mutual funds are backed by short-term debt and U.S. Treasuries.

We consider a model with three dates (t= 0,1,2) and three agents {A,B,C}. Agent A owns a project that delivers some uncertain amount of consumption goods at date 2, but he has no goods to invest in the project at date 0. Agent B has goods at date 0 but wants to consume a minimum amount at date 1 (agent B has a demand for liquidity at that date). So at date 0 agent B wants to buy a security from agent A (lend to A) to store his goods and such that agent A can invest in the project. A public signal arrives at the start of date 1. Then agent B uses the security received from agent A as collateral to issue a security to agent C in exchange for (some of) agent C’s goods. Agent C can produce costly private information about the payoff of agent A’s project (i.e., the value of B’s collateral). We address two interrelated questions. First, what is the optimal collateral security for agent B to buy from agent A at date 0? Second, what is the optimal security backed by that collateral for agent B to use to trade with agent C at date 1?

In order to solve this two layer optimal security design problem with endogenous private information acquisition (as well as the exogenous arrival of public news) we introduce the concept of ‘information sensitivity’. This is a property of any security. See Dang, Gorton, and Holmström (2018). It measures the value of producing private information about the security’s payoff given an offered price for the security and thus an agent’s incentive to produce the information at a cost. When trading a security that is information-insensitive, agents have no incentive to acquire information. Other securities, quintessentially equity, are information-sensitive. We show that debt is the least information-sensitive security. And we show that debt-on-debt is optimal and the equilibrium has the following properties. At date 0, agent B lends to agent A with a debt contract that is backed by the project (the real sector). Then agent B uses that debt as collateral to issue debt to agent C obtaining consumption goods. There are two reasons why debt is the optimal collateral. The information-sensitivity of the tradable debt is (further) minimized by debt collateral.
And debt collateral is also optimal because its value is least sensitive to the arrival of public information and thus maximizes the value of the collateral when there is bad public news.

We interpret agent $B$ as a bank. The bank makes a two-period loan to agent $A$ at date 0 knowing the amount of "withdrawals" that will occur at date 1. The withdrawals (agent $B$'s desired date 1 consumption) have to be honored by trading with agent $C$ using a debt contract backed by the loan. In other words, agent $B$ will try to issue money to agent $C$. But agent $C$ may be unwilling to accept agent $B$'s money because of suspicions about the backing collateral (possibly based on information privately produced). The bank debt may then be unacceptable in which case $B$ cannot honor the withdrawals, a crisis.

Alternatively, the transaction at date 1 between agents $B$ and $C$ can be thought of as a sale and repurchase (repo) transaction. If agent $C$ requires collateral to trade, then agent $B$ can borrow from $C$ using the bond received from $A$ as collateral. Because in repo the return on the collateral accrues to the owner, in this case agent $B$, agent $B$ will receive the repayment from $A$ and can then repay $C$ with the proceeds and reclaim the collateral (unless there is a default by $A$). Agent $C$, however, may not be willing to accept the collateral offered by $B$, as occurred in the Panic of 2007-2008. See Gorton (2010).

Our theory shows how these markets can break down. A public shock about the fundamental value of the underlying project that backs the debt collateral which backs the tradable debt can create an incentive to produce private information. Bad public news about fundamentals (a shock) causes the market value of collateral debt to drop. But more severely, it may cause information-insensitive tradable debt to become information-sensitive, a financial crisis. Agents who are capable of producing information have an incentive to learn about the value of the collateral. Other agents become suspicious in the sense of fearing adverse selection. In our model there are two potential equilibrium responses. Agent $B$ who is uninformed can prevent endogenous adverse selection by reducing the amount of trade below the expected value of the debt collateral, a fire sale. Or he can give in to adverse selection in which case there is a probability that there is no trade. We show that in both cases a collapse of trade or a financial crisis is a discontinuous event.

We study two cases concerning private information. In the first, simpler case, if agent $C$ produces private information then he learns the exact realization of the project early, at date 1. This somewhat setting allows the basics of the model to be displayed and intuition developed. Our main interest is in the second case, where the information learned by agent $C$ (if private information is produced) is the distribution from which the final project realization will be drawn conditional on the public signal. The first case can explain the forty-five degree line of the debt contract but not the flat portion. For example, it does not rule out quasi-debt, debt where there is a smaller forty-five degree portion followed by a non-flat portion. The more realistic information case rules this out.

In the setting we explore there is a fixed cost of producing information. Debt minimizes the value of the private information that can be learned so that this cost is not worth bearing. In fact, if it was possible to raise the cost of producing information, say by making the security more complicated, that would be even better. A cost of infinity would be best. It is optimal for the two transacting parties (agent $B$ and agent $C$) to not know the state of the world (the realization of agent $A$'s project or the distribution from which the realization will be drawn). It is not possible to make a security contingent on the state because agent $C$ privately learns the information if he produces it. This contrasts starkly with many existing models of debt in a corporate finance setting. For example, in the seminal model of Townsend (1979) a lender must pay a cost to determine the state of the borrower’s output to see if the loan can be repaid, but only in the case where the state is that the borrower is bankrupt. In that setting, the cost of producing information would be best if it were zero. But, in the trading context it is better if no party to the transaction engages in such due
diligence and the "state" optimally remains unknown.

The recent financial crisis has been blamed in part on the complexity and opacity of financial instruments, leading to calls for more transparency. On the contrary, we show that symmetric ignorance creates liquidity in funding markets. Furthermore, we show that the public provision of information that is imperfect can trigger the production of private information and create endogenous adverse selection. Agents can most easily trade when it is mutually known that no one knows anything privately about the value of the security used to transact and no one has an incentive to conduct due diligence about the value of the security. Debt backed by debt collateral has this property.

The paper proceeds as follows. In Section 2 we very briefly review the relevant prior literature. In Section 3 we introduce and explain the model. In Section 4 we introduce "information-sensitivity" and characterize its properties. In Section 5 we analyze optimal security design and characterize the properties of equilibrium. In Section 6 concludes.

2 Previous Literature

Our paper builds on several prior literatures. With regard to "liquidity", Diamond and Dybvig (1983) and Gorton and Pennacchi (1990) study liquidity provision but assume the existence of debt. (Also see Holmström (2008)). Diamond and Dybvig (1983) associate "liquidity" with intertemporal consumption smoothing and argue that a banking system with demand deposits provides this type of liquidity. In Gorton and Pennacchi (1990) agents meet to trade. They argue that debt is an optimal trading security because it minimizes trading losses to informed traders when used by uniformed traders. Hence debt provides liquidity because it is information-insensitive. In fact, in Gorton and Pennacchi (1990) the debt is riskless, and it is not formally shown that debt is an optimal contract. Since debt is riskless there can be no crisis.

There is a large literature on the optimality of debt in firms' capital structures, based on agency issues in corporate finance. In DeMarzo and Duffie (1999) the problem is to design a security that maximizes the payoff of a seller who will exogenously become (privately) informed prior to actually selling the securities. Since there is adverse selection, the demand curve of the uninformed buyers is downward-sloping. Prior to obtaining private information but anticipating the competitive separating signaling market equilibrium at the trading stage, the seller designs a security that trades-off the price and quantity effects ex ante. The seller cannot redesign the security after obtaining the private signal. They show that under some conditions debt is the optimal security. The key driver for the optimality of debt for an informed seller is the "flat" part of the debt contract. The intuition is that the flat part excludes the smallest set of high type sellers and thus reduces the price sensitivity when the seller increases the quantity.

Our design problem is very different. Rather than analyzing how security design can mitigate exogenous adverse selection problems when the security is issued by the firm to investors, we analyze a two layer optimal security design problem with endogenous information acquisition which is relevant when two parties meet to transact. We ask which security is optimal as backing collateral in the first stage and which security is optimal to trade in the second stage such that symmetric information is preserved. We design a security that maximizes the payoff of an uninformed agent who faces a potentially informed buyer when he needs to sell.

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1 Gorton (2017) surveys safe debt in general.

2 See also Biais and Mariotti (2005) who extend DeMarzo and Duffie (1999) to a setting where buyers are strategic and derive an optimal screening mechanism at the trading stage rather than assuming a separating signaling equilibrium. DeMarzo (2005) shows that pooling reduces the adverse selection problem an uninformed agent faces when he sells to an informed intermediary while tranching increases the amount that the informed intermediary (seller) can sell to uninformed buyers subsequently.

3 We impose no restrictions on the set of eligible securities except limited liability, i.e. the security payoff cannot be larger.
In our setting efficient trade is inhibited by "transparency". There are a few papers that raise the issue of whether more information is better in the context of trading or banking. These include Andolfatto (2009), Kaplan (2006), and Pagano and Volpin (2009). Andolfatto (2009) considers an economy where agents need to trade, and shows that when there is news about the value of the money used to trade, some agents cannot achieve their desired consumption levels. Agents would prefer that the news be suppressed. Kaplan (2006) studies a Diamond and Dybvig-type model in which the bank acquires information before depositors do. He derives conditions under which the optimal deposit contract is non-contingent. Pagano and Volpin (2009) study the incentives a security issuer has to release information about a security, which may enhance primary market issuance profits, but harm secondary market trading. All these authors assume debt contracts.

There is a very large literature on financial crises. The concept of a "financial crisis" refers to a sort of "regime change" due to a bank run on some form of short-term debt. The leading example is a banking panic, which occurs when a sufficiently large number of depositors choose to withdraw their deposits, relative to the cash available to the banks, forcing a suspension of convertibility. Broadly and briefly, there are various different theories of financial crisis. First, there are self-fulfilling expectations or sun spots theories, starting with Diamond and Dybvig (1983), and refined by Goldstein and Pauzner (2005) who apply the global games method of Carlsson and van Damme (1993). In these models, agents are unsure of other agents' actions or beliefs, and the crisis is an outcome of the coordination failure. Morris and Shin (1998) also use the global games modeling technique to model a coordination game in which each player's payoff depends on his own action and the actions of others, as well as unknown economic fundamentals. This view of crises focuses on a loss of confidence, which is related to beliefs about other agents.

In the second theory there is no coordination failure, but there is asymmetric information in that market participants do not know which institutions are most at risk. A shock can occur which is big enough to cause some banks to fail, but agents do not know which banks will fail. Risk averse agents rationally respond by, for example, seeking to withdraw their money from all banks even though only a few are actually insolvent. See Gorton (1985, 1988) and Gorton and Huang (2006). Again, there is a loss of confidence in the sense that agents are no longer sure of banks’ solvency. The disruption can be large, although the overwhelming majority of banks are solvent.

The financial crisis in our economy comes from an entirely different source than the theories in the existing literature. Crises in the existing literature are not linked to the optimality of debt, while our theory follows naturally from the optimality of debt. Beliefs about the actions of other agents matter in our theory in that the fear of others producing private information when there is a shock is what can make debt information-sensitive. A 'loss of confidence'—the "regime change"—corresponds to the previously information-insensitive debt becoming information-sensitive when there is a shock, resulting in the fear of adverse selection. In our theory, the crisis is inherently linked to the underlying rationale for the existence of debt-on-debt as the optimal trading security.

3 The Model

We consider an exchange economy with three dates \( t = 0, 1, 2 \) and three agents \( \{A, B, C\} \) whose utility functions are given as follows:

\[ u_t(A, B, C) \]

than the project payoff that is used to back the security.

\(^4\)See Allen, Babus, and Carletti (2009) for a survey.
Agents $A$ and $C$ have linear utility and value consumption the same at all dates. Agent $B$ also has linear utility but he values the first $k$ units of date-1 consumption at the marginal rate $1 + \alpha$. Figure 1 illustrates the utility function of agent $B$.

We interpret $k$ as the desired amount of liquidity that agent $B$ wishes to obtain at $t = 1$. If agent $B$ is a bank, then $k$ can be interpreted as the amount of liabilities the bank has to repay. Agent $B$ is indifferent between consuming more than $k$ at date 1 and delaying that consumption to date 2, but he strictly prefers consumption at date 1 up to the amount $k$.

The agents have the following endowments of goods:

$$\omega_A = (0, 0, X)$$
$$\omega_B = (w, 0, 0)$$
$$\omega_C = (0, w_C, 0)$$
where \( w \) and \( w_C \) are constants and \( X \) is a random payoff on a project owned by agent \( A \) that is realized at date 2. The random variable \( X \) is described by a continuous distribution function \( F(x) \) and density \( f(x) \) with positive support \([x_L, x_H]\). Agent \( A \) has no endowment of goods at dates 0 and 1, but receives \( x \) units of goods at date 2 from the project, where \( x \) is a verifiable realization of the random variable \( X \). Agent \( A \) can therefore issue claims on the outcome \( x \) at date 0 (and date 1, though this will not be relevant). Agent \( B \) possesses \( w \) units of goods at date 0 and nothing at the other dates. Agent \( C \) has \( w_C \) units of goods at date 1. Goods are nonstorable and endowments are non-contractible.

These assumptions are made to create a demand for claims on \( x \) that will be traded over the two periods. The only reason for trade is that agent \( B \)'s utility function gives him an extra benefit \( \alpha \) from consuming the first \( k \) units at date 1. It is socially efficient for agent \( A \) to consume at date 0, for agent \( B \) to consume \( k \) units at date 1, and for agent \( C \) to consume at date 2. So, agents \( A \) and \( B \) will transact first and then agents \( B \) and \( C \) will transact, as we will explain below.

### 3.1 Information

There are two types of information, public information \( z \) about the distribution \( f(x) \) and private information (production) about the realization of \( X \). We assume that at date 0 agents have symmetric information.

**Public News:** At date 1, before agent \( B \) and agent \( C \) meet, a public signal \( z \) is realized. The signal \( z \) is publicly observed, but is non-contractible. We will sometimes assume that the posterior distributions are spanned by two distributions: \( F(x, z) = zF_H(x) + (1 - z)F_L(x) \), where \( z \in [z_L, z_H] \) and \( F_H(x) \) dominates \( F_L(x) \) in the first order sense. In this case \( z \) can be interpreted as better ‘news’, the higher its value. Only in the case of spanning will the prior distribution belong to a one-dimensional set of posteriors.

**Private Information Production:** We assume that agent \( C \) is a more sophisticated buyer (e.g. a hedge fund), who can privately acquire information about \( X \) at a cost of \( \gamma \). For the first information case we assume \( C \) learns \( x \) perfectly. The second information case is where agent \( C \) learns \( x \) with noise, i.e., \( C \) learns the conditional distribution from which \( x \) will be drawn. Neither \( A \) nor \( B \) can acquire private information about \( x \).

### 3.2 Securities

In order to trade, agents need to write contracts which specify a price and a security. A security \( s(x) \) maps an outcome of \( X \) to a repayment \( s(x) \). The realization \( x \) of the project at date 2 is verifiable. At date 1, having purchased \( s_0(x) \) from agent \( A \) at date 0, agent \( B \) can design a new security, \( s(m) \), using \( m \equiv s_0(x) \) as collateral and trade the new security to agent \( C \) for agent \( C \)'s \( t = 1 \) goods. The notation \( m \equiv s_0(x) \) is intended to emphasize that the security that \( B \) offers to \( C \) has \( s_0(x) \) as collateral. The collateral itself is a function of \( x \), \( m(x) \), so we will sometimes dispense with the \( m \) notation and write \( s_1(x) \).

**Date 0 Securities:** Let \( S_0 \) denote the set of feasible date 0 securities, i.e., functions, \( s_0(x) \), which satisfy the resource (or limited liability) constraint and are non-decreasing. Two examples are:

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5 The key assumption is that \( w_C \) is non-contractible. For example, agent \( C \) is not present at date 0 or has the utility function \( U_C = \beta C_{C_0} + C_{C_1} + C_{C_2} \) where \( \beta < 1 \). In these cases agents \( B \) and \( C \) will not trade at \( t = 0 \).

6 This common assumption is designed to keep a borrower from destroying output in order to repay less.
Figure 2: The Sequence of Moves

(i) Equity: \( s_0(x) = \beta x \) where \( \alpha \in (0, 1] \) is the share of \( x \);

(ii) Debt: \( s_0(x) = \min[x, D] \) where \( D \) is the face value of the debt.

Date 1 Securities: At date 1 agent B owns \( s_0(x) \), which he can use as collateral for a new security \( s_1(x) \). The set of feasible securities at \( t = 1 \) that agent B can use to trade with agent C is given by \( S_1 = \{ s_1 : s_1(x) \leq s_0(x), \text{ for all } x \} \).

3.3 The Sequence of Moves

The sequence of moves is shown in Figure 2. At \( t = 0 \), agent B wants to buy a security from agent A to allow him to store some of his endowment until date 1. He makes a take-it-or-leave-it offer \( (s_0(x), p_0) \) to agent A, the owner of the project X. The offer consists of a price \( p_0 \), the amount of goods that agent B intends to pay A for a security \( s_0(x) \), that promises the payment \( s_0(x) \) at date 2 to the holder of the security. If A declines the offer, the game ends and parties just consume their endowments. At \( t = 1 \), agent B makes a take-it-or-leave-it offer \( (s_1(m), p_1) \) to agent C, where \( m \equiv s_0(x) \) is the collateral that backs B’s promise to pay \( C s_1(m) \) at date 2. If C accepts, he pays B the price \( p_1 \) at date 1. The consumptions of the agents are described in Figure 2. Note that because agent B makes take-it-or-leave-it offers to agents A and C, they receive no rents. This simplifies the analysis since we can then focus on maximizing agent B’s utility subject to participation by agents A and C.
4 The Information-Sensitivity of a Security

At $t = 1$ agent $B$ owns an asset $m$ with induced distribution $F(m)$. Agent $B$ can use $m$ as collateral for a contract $s_1(m)$ which will be sold to agent $C$. Agent $B$ can choose any security from the set $\{s_1 : s_1(m) \leq m\}$ and a price, $p_1$, to maximize his utility subject to the constraint that agent $C$ is willing to buy and can produce information. To save on notation, in this section we will use $p$ and $s(m)$.

Suppose agent $B$ proposes the contract $(p, s(m))$ to agent $C$, i.e., agent $C$ can buy the security $s(m)$ at price $p$. The value of information for agent $C$ is defined as $\pi \equiv E(U_C(I)) - E(U_C(NI))$ where $E(U_C(I))$ is the expected utility based on the optimal transaction decision in each state under perfect information about $x$ (I), and $E(U_C(NI))$ denotes the expected utility of an optimal transaction decision based on the initial information only, i.e., no information about the true state (NI).

Define $\pi_L(p) \equiv \int_{x_L}^{x_H} \max[p - s(x), 0] f(x) dx$ and $\pi_H(p) \equiv \int_{x_L}^{x_H} \max[s(x) - p, 0] f(x) dx$.

**Proposition 1.** (Value of Information) Suppose an agent is offered a security $s(m)$ at price $p$. The value of information to the agent, $\pi$, of $s(m)$, is given as follows: (i) If $p \leq E(s(m))$, then $\pi(p) = \pi_L(p)$; (ii) If $p \geq E(s(m))$, then $\pi(p) = \pi_H(p)$; (iii) At $p = E(s(m))$, $\pi_L(p) = \pi_H(p)$.

**Proof:** (i) For $p < E(s(m))$, without information, the agent buys the security (because it is undervalued). If agent $C$ becomes informed he will buy the security in states where $s(m) < p$. The value of information is the amount he avoids overpaying for the security in low states. Integrating over all $m$ with $p - s(m) > 0$ gives $\pi(p) = \pi_L(p)$. See Figure 3.

(ii) For $p > E(s(m))$, without information, the agent does not buy the security (because it is overvalued). If the agent is informed, he will only buy the security in states where $s(m) > p$. The value of information is the amount of profit he makes in high states. Integrating over all $m$ with $s(m) - p > 0$ gives $\pi(p) = \pi_H(p)$.

(iii) At $p = \pi(p)$, the expected loss in low states equals the expected gains in high payoff states since $p = E(s(m))$. So, $\pi_L(p) = \pi_H(p)$. QED

Now we consider which security, $s(m)$, minimizes both $\pi_L(p)$ and $\pi_H(p)$.

**Proposition 2.** Consider the set of all securities, $\{s : s(m) \leq m\}$, with the same expected payoff. For any $f(m)$ and any price $p$, debt minimizes the value of information.

**Proof:** We compare debt, $s^D(m) = \min[m, D]$, where $D$ is the face value of the debt, with a generic contract, $s^g(m)$, where both contracts have the same expected value $V$, i.e., $E[s^D(m)] = E[s^g(m)] = V$. From Proposition 2 (Value of Information), for $p \leq E[s(m)]$, the value of information of debt is $\pi_L^D = A$, where $A \equiv \int_{Q_D} (p - m) f(m) dm$ and $Q_D = \{m : m \leq p\}$. See Figure 4. The value of information of $s^g(m)$ is $\pi = \pi_A + B$ where $A + B \equiv \int_{Q^g} (p - m) f(m) dm$ and $Q^g = \{s(m) : m \leq p\}$. It is obvious that $\pi_L^D \leq \pi_L^g$ for any $f(m)$. The inequality is strict if $s^g(m)$ is such that $s^g(m) < m$ for some $m < p$. For $p \geq E[s^g(m)]$, the value of information of debt is $\pi_R^D = D + E$ where

$$D + E = \int_{x_L}^{x_H} \max[s^D(m) - p] f(m) dm.$$

The value of information of $s^g(m)$ is $\pi_R^g = E + F$ where
Figure 3: The Value of Information

\[ E[s^D(m)] = E + F = D + E + B + C = \int_{x_L}^{x_H} \max[s^D(m) - p] f(m) dm + \int_{x_L}^{x_H} \max[s^D(m), p] f(m) dm \]

and

\[ E[s(m)] = E + F + C = \int_{x_L}^{x_H} \max[s(m) - p] f(m) dm + \min[s(m), p] f(m) dm. \]

Finally, note that \( E[s^g(y)] = Es^D(y) = V \) implies that \( B + C + D + E = C + E + F \) and thus \( D + B = F \). If \( s(y) < y \) for some \( y < p \), then \( B > 0 \) and therefore, \( \pi^D_R = D < F = \pi^g_R \). QED

**Corollary 1.** If a debt contract triggers information acquisition by agent \( C \), then so does any other contract with the same price and same expected payoff.

**Proof:** Suppose agent \( C \) acquires information under a debt contract, i.e., \( \gamma < \pi^D \). Then by Proposition 2, \( \gamma < \pi^D \leq \pi^g \), so agent \( C \) acquires information under the generic contract. QED
5 Security Design

In section 5.1 we analyze the $B - C$ game and show that given arbitrary collateral, debt is an optimal security for agent $B$ to sell to agent $C$ at date 1. Section 5.2 shows that debt-on-debt is optimal in the $A - B - C$ game. Section 5.3 derives the equilibrium prices and amount of trade and characterizes the properties of the deb-on-debt equilibrium. Section 5.4 discusses the assumptions and results.

5.1 The Optimality of Debt in the B-C Game

At date 1, agent $B$ owns a security $s_0(x) \equiv m$ that he bought from agent $A$. Agent $B$ can use $s_0(x) \equiv m$ as collateral for a security $s_1(m)$ that he can sell to agent $C$. To save on notation we omit the subscript and use $s(m)$. In this section we analyze an optimization problem in the $(s(m), p)$-space where $p \in R$ and $s(m) \in \{s | s(m) \leq y\}$ where $s(m)$ is non-decreasing.

Proposition 3. Debt is optimal in the $B - C$ game.

Proof: Let $\{s^g(x), p^g\}$ be a generic contract that $C$ finds acceptable. We need to show that there exists a debt contract that $C$ also accepts and that gives $B$ as high an expected utility as the generic contract $\{s^g(x), p\}$. There are two cases to consider.

Case A: Information acquisition is not triggered under the generic contract. Since $C$ does not acquire information, $p^g \leq E(s^g(x)) \equiv V^g$, else $C$ would not accept the generic contract. Consider debt $D$ such that $V^D \equiv E(s^D(x)) = p^D = p^g$. Since debt is minimally information-sensitive for any given price, $C$ does not acquire information when offered this debt contract and will accept it since its price is fair. The debt contract
gives $B$ the same consumption at date 1 and no less consumption at date 2 as the generic. So it is weakly better than the generic.

Case B: Information acquisition is triggered by the generic contract. In this case the value of information is $\pi^g_R(p^g) \geq \gamma$. Consider the debt contract $\{s^D(x), p^D\}$ with $p^D = p^g$ and $D$ such that $\pi^D_R = \gamma$. This is possible since $\pi^D_R \geq \pi^g_R$ for $D = x_H$, the highest possible value of $x$.

If $V^D \geq p^D$, then raise $p^D$ to $\tilde{p}^D = V^D$. By construction $\pi^D_R(\tilde{p}^D) \leq \gamma$, so this debt contract will not trigger information acquisition. The contract is fair so $C$ will accept. Consequently, there will be trade with certainty at date 1, which makes debt strictly better than the generic, which fails to trade if $C$ finds that $x < p^g$.

Suppose $V^D < p^D = p^g$. By construction of $\{s^D(x), p^D\}$, $C$ will acquire information and therefore accepts the contract. Because $p^D = p^g$, trade occurs with debt whenever it occurs with the generic (and generally more frequently). Both debt and the generic are priced the same and the debt line cuts the horizontal price line to the (weakly) left of where the generic cuts that same line. If the generic has detached itself from the 45 degree line at the shared price, then debt will trade strictly more often. Therefore, total surplus is higher. Since $\pi^D_R(p^D = \gamma$, debt gives weakly less surplus to $C$ than the generic. So, $B$ must be at least as well off (and generally better off)) with debt than with the generic. QED

5.2 The Optimality of Debt-on-Debt in the A-B-C Game

In this section we solve the full game, i.e. an optimization problem in the space: $\{s_0(x), p_0, s_1(m), p_1\}$, where $p_0, p_1 \in R, s_0(x) \in \{s_0|s_0(x) \leq x\}$ and $s_1(m) \in \{s_1|s_0(x) \leq s_0(x)\}$ (recall $m \equiv s_0(x)$). We impose no assumptions on the set of posteriors $f(x|z)$ induced by the public signal $z$. We denote the value function of $s_0(x)$ at date 1 as: $V \equiv E[s_0(x)|z] = \int s_0(x)f(x|z)dx$

In the main text we analyze the case where $V^D(z) \geq k$ for all $z$.\footnote{The other case is in Appendix 1.} In other words, agent $B$ has enough resources to buy debt collateral at date 0 that maintains value above $k$ for all realizations of the public signal $z$. Note, if agent $B$ is a bank, $k$ can be interpreted as the amount that depositors will withdraw at date 1. This assumption implies that the bank is always solvent at date 1. But we will show that it might still become illiquid such that depositors cannot withdraw $k$. We will elaborate more on the economics of this assumption in the context of bank capital regulation and its effectiveness for avoiding a financial crisis in subsection 6.1. We will analyze the case where $V^D(z) < k$ for some $z$ and provide a sufficient condition for debt-on-debt to remain optimal in Appendix 1. We will also show that it is least costly for agent $B$ to buy debt collateral in order to have the expected collateral value stay above $k$ and always remain solvent at date 1.

Consider a generic contract $\{(s^g_0(x), w_0), (s^g_1(x, z), p_1^g(z))\}$. We will construct a pure debt contract $\{(s^D_0(x), w_0), (s^D_1(x, z), p_1^D(z))\}$ which has the same initial price $w_0$, and $E_x V^D(z) = E_x(s^D_0(x)) = w_0 = E_x(s^g_0(x)) = E_z V^g(z)$, as the generic contracts and for which the continuation contract, also debt, dominates the generic contracts for each realization of $z$. In particular, we will show that for each $z$:

- $C$’s expected rent, defined as $C$’s expected consumption over the two periods $t = 1, 2$ in excess of $w_C$, is no larger with debt-on-debt contract than with the generic contracts, and
- $B$’s expected consumption premium, defined as $E[\alpha \cdot \min\{C_{B1}, k\}]$, is no smaller with the debt-on-debt contract than with the generic contracts. Here the expectation is both over $z$ and over consumption
The optimality of debt follows from the two conditions above, because given a contract \( \{(s_0(x), w_0), (s_1(x, z), p_1(z))\} \), B’s expected utility over the two periods \( t = 1, 2 \) is

\[
w_0 + E[\alpha \cdot \min\{C_{B1}, k\}] - E[s_1(x|z) - p_1(z)].
\]

The first term is the total market value of the contract B purchases from A. The second term is the expected consumption premium at date 1. The third term is the expected rent paid to C.

Our strategy will be to show that for each \( z \), we can do as well with a modified debt contract as with the generic contract. Sometimes the modification entails lowering the face value \( D \), while adjusting the price \( p_D^D(z) \) so that it is fair (equals the market value of the modified debt); we refer to this as "writing down debt". In other cases we will just lower the face value \( D \) without changing the initial price; we refer to this as 'tightening debt'.

**Proposition 4.** Let \( D \) be a debt contract with value \( w_0 \) and \( g \) a generic contract with value equal to \( w_0 \) and \( E_x V^D(z) \equiv E_x (s_0|x)(x) = w_0 = E_x (s_0^g(x)) \equiv E_x V^g(z) \). Then debt-on-debt is optimal in the A-B-C- game if \( V^D(z) \geq k \) for every \( z \).

**Proof:** Let \( z \) be an arbitrary realization of the public signal at date 1. Define \( p^D_1(z) = \min\{k, p^D_1(z)\} \).

**Case A:** The generic continuation contract does not trigger information acquisition. Define a continuation debt contract with face value \( D_1(z) \) so that \( V^{D_1(z)} = p^D_1(z) \), as defined above. This is possible since \( V^D(z) \geq k \geq p^D_1(z) \). Write-down the face value of the debt contract, \( D \), to the minimum of \( k \) and \( p^D_0 \). If to \( p^g \), then by Corollary 1, it does not trigger information acquisition and the rent is zero. If the write-down is to \( k \), then it is even less information-sensitive, so information is not triggered and the rent is zero. Figure 5 shows this case. Note that the consumption premium is greater than or equal to \( op^g \).

**Case B:** The generic continuation contract triggers information acquisition. Define \( D_1(z) \) as above. If the debt contract \( (s_1^{D_1(z)}(z), p^{D_1(z)}_1(z)) \) does not trigger information acquisition, then debt is trading with probability 1 at a price that is either as high as the generic price \( p^g_1(z) \) or at a price equal to \( k \). In either case, the consumption premium is strictly higher than the consumption premium of the generic, which trades with a probability strictly less than 1.

If the debt contract \( (s_1^{D_1(z)}(z), p^{D_1(z)}_1(z)) \) triggers information acquisition at fair value, then tighten this debt contract, that is, lower its face value to \( D \) (without altering the price) so that the rent going to C is just sufficient to make C acquire information: \( \pi^D_0(z, p^{D_1(z)}_1(z)) = \gamma \). If \( p^{D_1(z)}_1(z) = p^g_1(z) \) the (tightened) debt contract trades with at least as high, and possibly higher, probability as the generic. If \( p^{D_1(z)}_1(z) < p^g_1(z) \), then the debt contract trades with a strictly higher probability than the generic. See Figure 6. In either case, the rent is minimal (\( \gamma \)) for information acquisition, hence no larger than with the generic. QED

Note that agent B uses one of two strategies implicit in the above proof. When there is bad news, agent B might reduce the face value of the debt such that the debt trades at a price below the conditional expected value in order to avoid triggering private information production. Call this Strategy I. This case is a "fire sale" since the equilibrium price is below the conditional expected value of the debt. There is no adverse selection but there is an inefficiently low volume of trade. Alternatively, there is bad news which causes the debt to become information-sensitive, such that agent B's best response is to acquire information in order to adverse
Figure 5: Generic Does Not Trigger Information Acquisition

Figure 6: More Trade Under Debt
selection. Call this Strategy II. Agent $B$ optimally offers a debt contract, which agent $C$ may or may not accept. Consequently, public information about fundamentals can cause a partial or total collapse of trade.

6 Numerical Example (Debt-on-Debt)

Suppose $F_1 \sim u[0, 0.8], F_2 \sim u[0.2, 1.2], F_3 \sim u[1.2, 2]$ and the probabilities of each distribution occurring are $\lambda_1 = \lambda_2 = \varepsilon$, and $\lambda_3 = 1 - 2\varepsilon$. Suppose $\varepsilon = 0.00001$, $w = 1$, $k = 0.3$, $\gamma = 0.001$, and $\alpha = 1.003$. Note, $E[x] = 1.6$. In this example, at date 0 agent $B$ buys debt with face value $D_0 = 1$ and price $p_0 = 1$. Equilibrium outcomes at $t=1$ are as follows.

(i) If $F_3$ is the true distribution, then $V^D(z_3) = 1$ and $\pi^D_1(z_3) = 0$. Agent $B$ sells debt with $D_1(z_3) = 0.3$ for $p_1(z_3) = E[s^D_1(\cdot)] = 0.3$. Note, he can also sell the original debt (i.e. $D_1 = D_0 = 1$) for price $p_1 = 1$ but his utility is the same.

(ii) If $F_2$ is the true distribution, then $V^D(z_2) = 0.68$ and $\pi^D_1(z_2) = 0.1152 > \gamma$. Agent $B$ sells debt with face value $D_1(z_2) = 0.245$ and $p_1(z_2) = E[s^D_1(\cdot)] = 0.244$.

(iii) If $F_1$ is the true distribution, then $V^D(z_1) = 0.4$ and $\pi^D_1(z_1) = 0.1 > \gamma$. Agent $B$ sells debt with face value $D(z_1) = 0.041$ and $p_1(z_1) = E[s^D_1(\cdot)] = 0.04$.

Agent $C$ does not acquire information in all three cases.

To summarize this example, at date 0 agent $B$ buys debt from agent $A$. At date 1 agent $B$ uses the date 0 debt as collateral for a (new) debt contract that he sells to agent $C$. In normal times (i.e., $F = F_3$), there is efficient trade between agents $B$ and $C$ at date 1. If fundamentals deteriorate (i.e., $F = F_2$), then the market value of debt collateral drops to 0.4. Agent $B$ sells a (new) debt with face value 0.245 for price of 0.244. Thus, he consumes less than $k$. If there is crisis news (i.e., $F = F_1$), then the market value of the debt collateral drops to 0.4. Agent $B$ offers to sell debt with face value 0.0411 for price 0.04. There is inefficient low consumption in equilibrium but this is best that agent $B$ can achieve.

In the above example, $\alpha = 1.003$ and is small so the best response is Strategy I if there is adverse selection concerns. If $\alpha$ is larger, say $\alpha = 1.4$, then when the true distribution is $F_1$, the best response of agent $B$ is to induce agent $C$ to acquire information. Agent $B$ offers to sell debt with face value $D_1(z_1) = 0.34$ and $p_1(z_1) = 0.3$. In equilibrium agent $C$ acquires information and only buys if $x \geq 0.3$. With probability $0.3875$ there is no trade.

6.1 Discussion of the Debt-on-Debt Equilibrium

The numerical example illustrates a number of points. Suppose the posterior distributions induced by the public signal $z$ are ordered by first order stochastic dominance (FOSD) such that $f(x|z_j) \succ^{\text{FOSD}} f(x|z_k)$ for $z_j > z_k$. There exists a signal $z'$ such that $\pi^D_L(z') = \int_{x_L}^{x_H} \max[K - x, 0] \cdot f(x|z') dx = \gamma$ or just larger than $\gamma$ (where $K$ is the efficient amount of trade described in Proposition 4). Then $\pi^D_L(z) > \gamma$, for all $z < z'$. In other words, if agent $B$ proposes to trade the efficient amount $M$, agent $C$ acquires information under signal $z$. In such states, there is a drop of trading volume below the conditional expected value of the security. Although agent $B$ owns a collateral with expected value $V^D(z)$ larger than $k$, he can only consume strictly less than $k$. In other words, there is a "fire sale" problem.

*Note, if agent $B$ buys equity with $E[s^E(x)] = w$ at date 0, then at date 1 agent $B$ sells debt backed by that equity. As a consequence there is adverse selection even when there is good news. For all three public signals agent $B$ consumes strictly less at date 1 than with the debt collateral.*
If market participants observe $f(x|z)$ but the econometrician or the regulator does not, then it is not possible for them to "predict" the (equilibrium) outcome. Even though the market value of the collateral is higher than $k$ and publicly observable, the amount of trade or the drop in trading volume is not predictable. Furthermore, suppose the amount of trade under Strategy I is monotonic in $z$ and the expected amount of trade under Strategy II is also monotonic in $z$ (given some restrictions on the signal structure). Still, in equilibrium the realized amount of trade might not be monotonic in the public signal since it might be optimal for agent $B$ to switch back and forth between the two strategies. The (realized) amount of trade under Strategy II is strictly larger than the amount of trade under Strategy I. In addition, if agent $B$ chooses Strategy II, there is a positive probability that no trade occurs.

Therefore, our model captures an interesting feature of a financial crisis. The reduction of trade or even the collapse of trade is not necessarily a monotonic function of the drop in collateral value triggered by public news. This implication is consistent with the historical experiences of financial crises. A slight decrease in collateral value can sometimes cause a large drop in trading volume while a large drop in collateral value is not necessarily followed by large reduction of trade.

Our model also has an interesting implication for capital regulation. Proponents of capital requirements argue that by imposing a high enough capital buffer, a financial crisis can be avoided. Our model is able to speak to that issue. In the context of our model, when $E[s(x)|z] \geq k$ for all public signals $z$, the bank is always solvent. We show that even if the bank is solvent, it can be illiquid because the amount the bank can sell is much lower than the market value of its assets. The "liquidity" or "illiquidity" of a security is an equilibrium object and depends on the outcome of strategic interactions and the best responses of agents rather than being a purely statistical property. Suppose debt becomes information-sensitive. If the gains from trade (i.e. $\alpha$) are large relative to information costs (i.e. $\gamma$) then Strategy II is more likely to be played. Otherwise Strategy I is more likely to be played. Since $\{\alpha, \gamma, z\}$ are typically private information of market participants the dynamics of a financial crisis are not easy to predict by the regulator.

The arguments above do not establish that debt is uniquely optimal. This depends on parameter values. For instance, while it is possible to assure $B$ a date 1 consumption level $k$ for sure with debt, there is also a quasi-debt contract that can achieve the same unless the debt contract that provides $k$ cannot be increased without triggering information acquisition. In that case all other contracts (including quasi-debt) will trigger information acquisition or the consumption they can provide with certainty at date 1 will be smaller. Debt has the lowest information-sensitivity of all contracts with same value, including quasi-debt. In that respect it is unique.

7 The Game with Noise

We now turn to the second information case, a more complicated, but more realistic, version of the model. In this version agent $C$ does not learn the exact date 2 realization of the project at date 1, but instead (privately) learns the distribution of $x$ conditional on the public signal. In this case we will show that the pure debt contract is uniquely optimal.

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9Quasi-debt with the same expected value as the straight debt has a 45 degree line, as with straight debt, but does not have the flat part of the straight debt contract. With quasi-debt, the 45 degree line, however, stops before the kink in straight debt and then it increases, perhaps non-linearly. It stops before the kink due to the non-decreasing assumption.
7.1 Preliminaries

To begin, we provide some preliminary results which will be needed subsequently.

First, we provide a characterization of the value function of the collateral security $s_0(x)$, i.e. how $V(z) \equiv E[s_0(s)|z] = \int_{x_L}^{x_H} s_0(x)f(x|z)dx$ changes with the public signal $z$. We make two additional assumptions.

Assumption 1: $S_0 = \{s_0 : s_0 \leq x \text{ and } s_0 \text{ monotonic}\}$

Assumption 2: $\{f(x|z)\}$ satisfies the Strict Monotone Likelihood Ratio Property (SMLRP)

In particular, A2 means that we are assuming spanning, i.e. that there are two possible distributions $F_L$ and $F_H$ from which the final outcome $x$ is drawn. The probability that $x$ is drawn from $F_H$ is denoted $z = \Pr(F_H)$. Without loss of generality, we can assume that the public signal that the agents observe is the posterior $z$. The posterior is distributed $z \sim G$. The prior is denoted $z_0$, and satisfies $z_0 = E[z]$. Given any posterior $z$ the distribution of $x$ is given by: $x \sim F(x|z) \equiv zF_H(x) + (1 - z)F_L(x)$.

Since we assume that $F_L$ and $F_H$ satisfy SMLRP (the Strict Monotone Likelihood Ratio Property), the likelihood ratio:

$$l(x) \equiv \frac{f_H(x)}{f_L(x)}$$

is strictly increasing in $x$.

We say that the parameterized family of $x$ distributions $F(x|z)$ satisfies the SMLRP if for every $z_2 > z_1; z_1, z_2 \in [0,1]$, the likelihood ratio

$$l(x) \equiv \frac{z_2f_H(x) + (1 - z_2)f_L(x)}{z_1f_H(x) + (1 - z_1)f_L(x)}$$

is strictly increasing in $x$.

**Lemma 1.** Suppose $F_L$ and $F_H$ satisfy SMLRP, then the family parametrized by $F(x|z)$ satisfies SMLRP.

**Proof:** Let $z_2 > z_1; z_1, z_2 \in [0,1]$. We need to show that $l(x)$ is strictly increasing in $x$. Differentiating we get:

$$\frac{dl(x)}{dx} = \frac{z_2(z_1l(x) + (1 - z_1)) - z_1(z_2l(x) + (1 - z_2))l'(x)}{(z_1l(x) + (1 - z_1))^2}$$

where the strict inequality follows because the numerator equals $z_2 - z_1 > 0$ and $l(x)$ is strictly increasing by assumption. QED

Definition: A function $s_1(x)$ is said to cross the function $s_2(x)$ from below if there exists an $x^*$ such that $s_1(x) \leq s_2(x)$ for $x < x^*$ and $s_1(x) > s_2(x)$ for $x \geq x^*$ and $s_1(x) \neq s_2(x)$. If in addition the two functions are equal only at a single point, then $s_1(x)$ is said to cross the $s_2(x)$ strictly from below.
Lemma 2. Let $h(x|z)$ be the conditional density function of $x$ given a continuous signal $z$. Assume $h(x|z)$ is differentiable in $z$ for every $x$ and the family $\{h(x|z)\}$ satisfies SMLRP. Let $s_1(x)$ and $s_2(x)$ be two nondecreasing functions such that $s_1(x)$ crosses $s_2(x)$ from below. Assume there is a $z_0$ such that $E[Δs(x)|z_0] = 0$, i.e. $V_1(z_0) = V_2(z_0)$. If the functions $V_1 ≡ E[s_1(x)|z]$ and $V_2(z) ≡ E[s_2(x)|z]$ cross, then $V_1$ crosses $V_2$ strictly from below.

Proof: Let $Δs(x) = s_1(x) - s_2(x)$. Then:

$$\frac{d}{dz}E[Δs(s)|z_0] = \int Δs(x)h_z(s|z_0)dz$$

$$= \int Δs(x)\frac{h_z(x|z_0)}{h(x|z_0)}h_z(x|z_0)dz$$

$$= \int Δs(x)[\frac{h_z(x|z_0)}{h(x|z_0)} - \frac{h_z(x^*|z_0)}{h(x^*|z_0)}]h_z(x|z_0)dz$$

where $h_z(x|z_0) = \frac{d}{dz}h(x|z_0)$, i.e. $E[Δs(x)|z_0] = 0$. From SMLRP, $\frac{h_z(x|z_0)}{h(x|z_0)}$ is strictly increasing in $x$. Therefore,

$$\int Δs(x)[\frac{h_z(x|z_0)}{h(x|z_0)} - \frac{h_z(x^*|z_0)}{h(x^*|z_0)}]h_z(x|z_0)dz \geq 0,$$

with strict inequality on the set where $s_1(x) ≠ s_2(x)$. Therefore,

$$\frac{d}{dz}E[Δs(s)|z_0] = \frac{d}{dz}[V_1(z_0) - V_2(z_0)] > 0.$$ 

In other words, $V_1$ crosses $V_2$ strictly from below. QED

Lemma 3. Let $\{s(x), p\}$ be a contract that $B$ finds acceptable if $C$ were to acquire information. Then $B$ is at least as well off if $C$ accepts the contract without acquiring information.

Proof: Let $y ≡ \text{prob}(H|z)$ be agent $C$’s signal. Suppose $C$ acquires information. Since $V(y)$ is strictly increasing, $C$ will accept the contract $\{s(x), p\}$ if and only if he observes a $y ≥ \bar{y}$, where $\bar{y}$ is defined implicitly by

$$p = V(\bar{y}).$$

If $p > V(1)$ we set $\bar{y} = 1$ and if $p < V(0)$ we set $\bar{y} = 0$.

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10Lemma 2 is a variant of a lemma in DeMarzo et al (2010). They use that lemma to show that debt is the worst security as a mean of payments in a private value auction context. We will use this lemma to show that debt is optimal in our trading context.
The expected utility of $B$ is in this case:

$$
\int_0^y V(y)f(y)dy + E_x[x - s(x)] + \int_y^1 u(p)f(y)dy, \tag{3}
$$

where, recall, $u(p) = p + \alpha \min\{p, k\}$. The first and the second terms in expression (??) constitute B’s expected consumption at date 2. The third term is B’s utility from consuming $p$ at date 1.

If $C$ does not acquire information, B’s expected utility is

$$
u(p) + E_x[x - s(x)]. \tag{4}$$

Since $u(p) \geq V(y)$ for all $y \leq \bar{y}$, expression (??) is larger than expression (??). QED

**Proposition 5.** Consider a debt contract $s^D(x) = \min[x, D]$, where $D$ is face value of the debt and a generic contract $s^g(x)$ that intersects the debt from below and where $E[s^g(x)|z_0] = E[s^D(x)|z_0] = w$. At date 1 debt maintains the maximum value for all $z < z_0$.

**Proof:** $s^g(x)$ intersects $s^D(x)$ strictly from below. Lemmas 2 implies $V^g(z) < V^D(z)$ for all $z < z_0$. QED

**Corollary 2.** Define $z_L$ as the signal such that $f(x|z) \sim FOSD f(x|z_L)$ for all $z$. Consider a debt collateral that has $V^D(z_L) = k$. Suppose the (fair) price of this debt is $p_D$ at date 0. (i) Then any other collateral security with price $p^D$ has $V(z_L) < k$. (ii) Any other collateral security with $V(z_L) = k$ has (fair) price $p > p^D$ at date 0.

**Proof:** This follows from the fact that $V(z)$ intersects $V^D(z)$ strictly from below. QED

### 7.2 The B-C Game with Noise

**Proposition 6.** Debt is optimal in the B-C game with noisy information acquisition.

**Proof:** Let $(s(x), p)$ be a generic contract that agent $C$ finds acceptable. We need to show that there exists a debt contract that $C$ also accepts and that gives agent $B$ as high an expected utility as the generic contract $(s^g(x), p^g)$. There are two cases to consider.

**Case A:** Information acquisition is not triggered under the generic contract. Consider a debt contract with the same expected payoff and the same price as the generic contract: $E[s^D(x)] = E[s^g(x)] = w_0$ and $p^D = p^g$. Let $V^D(y)$ and $V^g(y)$ be the corresponding value functions conditional on the signal $y$. Both value functions are strictly increasing by SMLRP and linear because of spanning. By Lemma 2 $V^D(y)$ cuts $V^g(y)$ strictly from above at $y_0$, the prior probability that $F_H$ is the distribution from which $x$ is drawn; also, $y_0 = E(y)$. See Figure 7.

Because $C$ accepts the generic contract without information acquisition, we must have $p \leq w_0$. Therefore, the relevant value of information measures are $\pi^g_p(p)$ and $\pi^D_p(p)$, respectively. Note that with noisy information acquisition, $p < w_0$ is possible, unlike in the case without noise, where one can reduce $C'$s payment for values $s(x) > p$ until the contract is priced fairly.
Because $V^D(y)$ has smaller slope than $V^g(y)$, we have $\pi^D_L(p) < \pi^g_L(p) \leq \gamma$, so the debt contract does not trigger information acquisition either. The contracts offer identical expected consumption streams, so debt is as good as the generic.

**Case B): Information acquisition is triggered by the generic contract.** There are two subcases.

Case (B1): $p = p_L < w_0$. Since $p$ is below $w_0$, $V^g(y)$ intersects the line $p = p_L$ to the left of $y_0$, at $y = \bar{y}$ (see Figure 7). Consider the debt contract in Case A above, with value function $V^D(y)$ but with $p^D = p_L$. Because $V^D(\bar{y}) > V^g(\bar{y})$, we can reduce the face value of the debt contract to a level $\tilde{D} < D$ such that $V^D(y)$ intersects the line $p = p_L$ at the same point $\bar{y}$ as the value function $V^g(y)$ (point A in Figure 7). Note that the amended value function $V^D(y) < V^D(y)$ for every $y$. Applying Lemma 2 again, $V^g(y)$ crosses $V^D(y)$ strictly from below. Suppose the amended debt contract triggers information acquisition. Evaluating expression (1), we find that the amended debt contract provides agent B with higher expected utility because (i) when $y < \bar{y}$, B consumes in expectation more at date 1 with the amended debt than with the generic (by virtue of $V^D(y) > V(y)$); (ii) when $y < \bar{y}$, agent B gives away less rent to C with the amended debt ($\pi^D_R(p_L) < \pi^g_R(p_L)$ as $V^D(y) \leq V^g(y)$). Finally, if the amended debt contract does not trigger information acquisition, B is even better off according to Proposition 2.

Case (B2): $p = p_H \geq w_0$. Now $\pi_R$ is the relevant measure of value of information. We construct a debt contract that is as good as the generic as follows. Construct a debt contract with face value $\tilde{D}$ and $p = p_H$ such that the value function $V^D(y)$ goes through point B in Figure 7. Because $p_H > w_0$, B is to the right of the vertical line $y_0$, and hence $\tilde{D} > D$. Note that this implies that the value function $V^D(y) > V^D(y)$ for every $y$. Also $p_H > V^D(y_0)$. That is, the expected value of the debt contract $\tilde{D}$ is strictly higher than the expected value of the original debt contract $D$.

![Figure 7: The B-C Game with Noise](image-url)
Suppose the constructed debt contract \( \tilde{D} \), priced at \( p_H \) triggers information. Let \( B \) will offer this contract to \( C \). Agent \( C \) will accept either contract, acquire information and accept the contract if and only if \( y > \hat{y} \) (that is, the observed signal is to the right of \( B \)). By Lemma 2, the \( \tilde{D} \) contract has smaller slope than the \( D \) contract (recall that Lemma 2 makes no reference to expected values for this conclusion to hold). It follows that the value of information for the \( \tilde{D} \) contract is no larger than the value of information for the generic contract (in the figure it is strictly smaller). Since the cut-off for \( C \) accepting is the same for both contracts and the cost of information acquisition is lower for the debt contract \( \tilde{D} \), we conclude that the \( \tilde{D} \) contract dominates (weakly) the generic contract in this situation.

Suppose now that the constructed \( \tilde{D} \) contract does not trigger information acquisition. Since \( p_H > V^{\tilde{D}}(y_0) \), \( C \) will not buy the \( \tilde{D} \) contract at price \( p_H \); it is not worth it for \( C \) to acquire information and it is not worthwhile for \( C \) to buy the contract without acquiring information. Lowering the price to \( V^{\tilde{D}}(y_0) \) would make the debt contract fairly priced, but does not guarantee a higher value (the contract would trade with certainty and offer \( C \) no rent, but at a price that is lower than \( p_H \) making the result inconclusive). Instead, raise the debt level \( \tilde{D} \) further, keeping the price at \( p_0 \). This will increase the value of information of the \( \tilde{D} \) contract and move the point of intersection between \( V^{\tilde{D}}(y) \) and the horizontal line \( p_H \) to the left of \( B \). We have denoted this intersection point \( F(\tilde{D}) \) in Figure 7. Keep raising \( \tilde{D} \), while keeping the price at \( p_H \), until either the \( \tilde{D} \) contract becomes information sensitive (the value of information becomes equal to \( \gamma \)) or the intersection point \( F(\tilde{D}) \) coincides with the intersection between \( p_H \) and \( y_0 \) (point \( \chi \) in Figure 7). In the first case, we have arrived at a debt contract \( \tilde{D} \) with the same price as the generic, offering minimal information rent \( \gamma \) to \( C \) (so information is acquired), and will trade for a super-set of values \( y \) compared to the generic. Consequently, the constructed debt contract is no worse than the generic for \( B \) and in general strictly better. In the second case, the debt contract does not trigger information, but it is now fairly priced (the intersection between the \( y_0 \) line and the value function \( V^{\tilde{D}}(y) \) occurs at the value \( p_H \)). \( C \) will accept the contract without information acquisition, which therefore strictly dominates the generic. We have covered the case where the generic does not trigger information acquisition and where it does and in both cases shown that there exists a debt contract that dominates the generic. QED

7.3 The A-B-C Game

Proposition 7. Debt-on-debt is optimal in the A-B-C game with noisy information acquisition under the following assumptions:

A1. \( V^{D}(z) \geq k \) for every \( z \).

A2. Agent \( C \)'s signal \( y \) is conditionally independent of \( z \) and satisfies the strict MLRP condition\(^{11}\).

A3. In state \( H \) debt is information-insensitive\(^{12}\).

The proof is in Appendix 2.

\(^{11}\)Note that conditional independence here means that the private signal \( y – C \)'s probability of \( H – \) is conditionally independent of \( z – \) the date-1 public probability of \( H \).

\(^{12}\)Though not strictly necessary, this is the interesting case because of the contrast with state \( L \).
7.4 Discussion

In the previous case where agent $C$ learns the realization $x$ at date 1 if private information is produced, because of the seniority of repayment, private information production may be valuable to a buyer (agent $C$) if it helps him to avoid a loss in low payoff states by not buying the security. With seniority, where the holder is repaid first and gets all that is available in low payoff states, this expected loss is the smallest and therefore the value of information and the incentive to acquire information is minimized. In that situation quasi-debt is not ruled out. That is, the contract can have a forty-five degree line portion but not a flat part.

Quasi-debt, like the example shown in Figure 8, is in the set of generic contracts considered in the above propositions. Since quasi-debt has parts of the contract above the flat portion, it must have a shorter forty-five degree line in order to have the same expected value as pure debt. But, then as shown it cuts debt from below. Intuitively, quasi-debt *wastes* area above the flat portion of pure debt and that makes it suboptimal relative to debt.

8 Conclusion

Financial crises have been difficult to explain. Systemic financial crises concern short-term debt. Since short-term debt and its collateral are designed to be information-insensitive, there is a quiet period before a crisis. No information is produced, by design. But, then the short-term debt and the collateral can suddenly become information-sensitive, a crisis. In such a crisis, agents holding debt somehow "lose confidence", usually modeled as a coordination failure. We show that crises and the optimality of debt for liquidity provision are inextricably intertwined. The crisis that can occur with debt is due to the fact that the debt is not riskless.

Figure 8: Quasi-Debt Example
But, it is not the risk per se that is the problem. Debt is designed so that no agent has an incentive to produce information about the states of the world where the risk will cause a low pay-out. But, sometimes information is produced, a crisis.

In our model an agent wants to buy a security to store wealth from date 0 to date 1. Then the agent uses this security as collateral to back a new security that he wants to sell to raise cash at date 1. We address two interrelated questions. What is the optimal collateral and what is the optimal trading security? In order to solve this two layer optimal security design problem with private information acquisition and the arrival of public news we introduce the concept of "information-sensitivity". This measure captures an agent’s incentive to produce private information. We show that debt-on-debt is optimal because with this structure the trading debt is least information-sensitive. While debt-on-debt is optimal, it can lead to a collapse of trade in debt funding markets, a financial crisis. The financial crisis is a discontinuous event and occurs when public news about fundamentals makes information-insensitive debt become information-sensitive.

The crisis is not just the bad shock about the fundamentals of the debt collateral that backs the tradable debt. Instead, the crisis is a bad enough shock to cause information-insensitive debt to become information-sensitive. This is precisely the 'loss of confidence'. Agents who are capable of producing private information have an incentive to do so about the backing collateral. Other agents become 'suspicious' in the sense of fearing adverse selection. There are two potential best responses. There is information acquisition and adverse selection. In this case, there is a positive probability that no trade occurs. Another potential equilibrium outcome is that agents avoid private information production by trading at a price that is less than the fundamental value of the debt conditional on the public news. Such a write-down of debt, to "fire sale" prices, can be preferred because it recovers information-insensitivity where no agent has an incentive to produce information. But in this case an inefficient amount is traded and consumed.

If maintaining symmetric ignorance is central for liquidity provision, then this has implications for the regulation of the financial system. For example, should money market funds be required to reveal their net asset value in a timely fashion? Should banks that create short term liabilities for trade, provide more information about the value of their assets on the balance sheet? Should the regulator announce the outcomes of stress tests of banks so that investors have better information about individual banks and can run their own valuation models? Public provision of imperfect information can reduce liquidity because it can make information-insensitive debt become information-sensitive and trigger endogenous adverse selection concerns. When agents have an incentive and need to conduct due diligence about the value of money-like instruments, these financial instruments will lose their money-like property.
9 Appendix 1: The Case Where $E[x(x)|z] < k$ for some $z$

In this appendix we provide sufficient conditions for debt-on-debt to remain optimal in the case where $E[x(x)|z] < k$ for some $z$.

**Proposition 8.** Debt-on-debt is optimal if $V^D(z) < k$, provided that the following additional conditions hold:

A1. $\pi_R^D(z) \geq k$, that is, the original debt contract is always information-sensitive when its value drops below $k$.

A2. Spanning: $f(x|z) = (1 - z)f_L(x) = zf(x)$ and strict MLRP.

**Proof.** We break the proof into several cases depending on the market value of the generic continuation contract.

*Case 1.* The generic contract does not trigger information acquisition. We can assume w.l.o.g. that the generic contract in this case is priced fairly. Write down the face value of debt to $\tilde{D} < D$, defined so that the amended debt contract equals the value of the generic: $V^{\tilde{D}}(z) = V^g(z)$ This is possible by Lemma 3 and spanning. The amended debt contract does not trigger information acquisition because debt is minimally information-sensitive. It performs as well as the generic contract: both result in the same consumption spanning. The amended debt contract does not trigger information acquisition because debt is minimally

When the generic contract triggers information acquisition, there are three subcases to consider. Note that the original debt contract always triggers information acquisition by A1. Also, we can assume, w.l.o.g. that the generic has a price $p^g_1(z) > V^g(z)$, such that the rent is minimal: $\pi_R^D(z, p^g_1(z)) = \gamma$.

*Case 2a:* The generic contract triggers information acquisition and $p^g_1(z) \geq k > V^D(z) > V^g(z)$.

We amend the debt contract to $(s^D_1(x, z), p^D_1(z))$ defined by setting $p^D_1(z) = k$ and by 'tightening', i.e., lowering the face value to $\hat{D}$ so that $\pi^{D}_R(z, k) = \gamma$. This can be done by assumption A1. The amended debt contract triggers information acquisition with a probability of trade that is (weakly) higher than the generic’s. Since the amended debt contract is priced at $k$, it strictly dominates the generic unless $s^D_1(k, z) = k$, i.e. the generic runs along the 45 degree line at least up to the level $k$ (this is the case for quasi-debt, for instance). The consumption premium is higher for debt than the generic and the rent is minimal.

*Case 2b:* The generic contract triggers information acquisition and $k > p^g_1(z) > V^D(z) > V^g(z)$.

We amend the debt contract to $(s^D_1(x, z), p^D_1(z))$ by setting $p^D_1(z) = p^g(z)$ and by 'tightening' to a face value $\hat{D} < D$ such that $\pi^{D}_R(z, p^g_1(z)) = \gamma$. The amended debt contract triggers information acquisition by A1 and provides $B$ the same level of date 1 consumption with a (weakly) higher probability than the generic. Since the rent is minimal in both cases the amended debt contract (weakly) dominates the generic contract.

*Case 2c:* The generic contract triggers information acquisition and $k > V^D(z) > p^g_1(z) > V^g(z)$.

Write down the face value of debt and set price so that $\tilde{V}^{\tilde{D}}(z) = p^g_1(z) = p^D_1(z)$. If the amended debt contract does not trigger information, it strictly dominates the generic, since it trades with probability 1 for the same price as the generic without giving away any rent. If the amended debt contract triggers information acquisition, then "tighten" by lowering the face value of debt further to $\hat{D}$ so that $\pi^{D}_R(z, p^g_1(z)) = \gamma$. The twice amended debt contract triggers information acquisition, but B consumes $p^g_1(z)$ with a (weakly) higher probability than the generic contract with both giving away the minimal rent $\gamma$. So the amended contract (weakly) dominates the generic. QED
10 Appendix 2: The A-B-C Game with Noise

Proposition 9. Debt-on-debt is optimal in the A-B-C game with noisy information acquisition under the following assumptions:

A1. \( V^D(z) \geq k \) for every \( z \).

A2. Agent C’s signal \( y \) is conditionally independent of \( z \) and satisfies the strict MLRP condition\(^{13}\)

A3. In state \( H \) debt is information-insensitive\(^{14}\)

Proof. Because of spanning the value functions (as functions of the probability of \( H \)) are linear, regardless of the contract. Strict MLRP implies that the value function for a generic contract cuts the value function of a debt contract strictly from below. In other words, the slope of \( V^g(z) \) is strictly steeper than the slope of \( V^D(z) \). Note also that this property does not depend on the expected values of the debt and the generic contract (see Lemma 2). In particular, when we lower the face value of the debt contract or alter the generic contract, the changed generic contract will still cut the debt contract from below and hence have a steeper slope. Also note that when we consider agent C’s acquisition of information at date 1, agent C’s value function \( V^D(y) \) coincides with \( V^D(z) \) – the values are identical given the probability of \( H \). Finally, note that neither agent B nor agent C care about randomness in consumption at date 2. Therefore, the value functions conditional on \( y \) describe the assessments of agent B as well as agent C. The difference between B and C only concerns how they value the price at which they trade at date 1. B enjoys a premium from consuming at date 1, so the price at which they trade has extra value.

We consider all the relevant cases.

Case 1: \( z > z_0 \). Because of spanning, this is ‘good news’ regardless of the initial contract; in particular, \( V^D(z) > w_0 \). Write down the face value of debt to \( \tilde{D} < D \) so that \( V^\tilde{D}(z) = w_0 \). By assumption A3, agent C will not acquire information under the amended debt contract.

Case 2. \( z < z_0 \). Because of spanning, the news is bad regardless of the contract. In particular, \( w_0 > V^D(z) > V^g(z) \). There are three subcases to consider.

Case 2.1. \( p^g_1(z) \geq V^D(z) > V^g(z) \). This case is illustrated in Figure 9. The prior probability of \( H \) is \( z_0 \) and is to the right of the realization \( z \). We have drawn the value functions when C acquires information and observes \( y \) under the generic contract, \( V^g(y) \), and under the original debt contract, \( V^D(y) \). As mentioned above, these two functions coincide with the corresponding value functions conditional on observing \( z \). The functions intersect at the prior \( z_0 \): \( V^D(z_0) = V^g(z_0) \), a point that lies to the right of \( z \) (point A in Figure 9). Agent C’s prior belief about \( H \), before observing \( y \), is \( y_0(z) = z \).

Since \( p^g_1(z) > V^g(z) \), the generic contract must be triggering information acquisition and therefore the value of information under the generic is \( \pi^g_1(z, y_1(z)) \geq \gamma \). Consider the original debt contract priced fairly, \( p^D_1(z) = V^D(z) > k \), where the inequality follows from assumption A1. Suppose the debt contract does not trigger information acquisition. Then C will accept it and B can consume \( p^D(z) > k \) at date 1, without

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\(^{13}\) Note that conditional independence here means that the private signal \( y – C’s probability of \( H \) – is conditionally independent of \( z \) – the date-1 public probability of \( H \).

\(^{14}\) Though not strictly necessary, this is the interesting case because of the contrast with state \( L \).
Figure 9: A-B-C Case 2.1
giving away any rents to $C$. In that case, the original debt contract strictly dominates the generic contract, because the latter trades with probability less than one at the same price as debt and also gives away rent $\gamma$.

Suppose now that the original debt contract triggers information acquisition. In that case, amend the debt contract by keeping its face value the same at $D$, while raising its price to $p^C$. Here $p^C = V^D(\bar{z})$ is the price level corresponding to point $C$ in Figure 9, defined by the equality $p^C_1(z) = V^g(\bar{z})$. In other words, at $\bar{z}$ the generic value function equals the generic price $p^C_1(z)$ (point E in Figure 8), the debt value function equals the price $p^C$. This move will reduce the value of information of debt to $\pi^D_R(z, p^C) < \pi^D_R(z, p^D)$.

Suppose $\pi^D_R(z, p^C) \geq \gamma$. Then the original debt contract priced at $p^C$ will be acceptable to $C$, who will acquire information. Since $V^g(z)$ is steeper than $V^D(z)$, we have $\pi^D_R(z, p^C_1(z)) > \pi^D_R(z, p^C) \rightarrow$ the former wedge (triangle $CHD$) is a subset of the latter (triangle $EGF$) when one translates it by the difference in prices (i.e., moves the base of the triangle $CHD$ up to have the base $EF$), so the rent from the original debt contract priced at $p^C$ is smaller than the rent of the generic contract priced at $p^C_1(z)$. Both of these contracts are constructed so that they trade with equal probability. Finally, since $p^C > k$, the debt contract will provide the same expected consumption premium as the generic contract. We conclude that the debt contract is strictly better than the generic contract if $\pi^D_R(z, p^C) > \gamma$.

What if $\pi^D_R(z, p^C) < \gamma$? In that case, we raise the price of the debt contract only to the level $p^C < p^C$ defined by $\pi^D_R(z, p^C) = \gamma$. Debt will in that case trade with a strictly higher probability than the generic and at a price $\bar{p}^C > k$. The rent is strictly smaller and the expected consumption premium is strictly larger for the debt contract than the generic contract.

Now consider the case where $p^g$ falls below $w_0$ and hence $\bar{z}$ is to the left of $z_0$. Still $p^g > V^D(z)$ even in this case, by assumption. Also, we assume still that at fair value this debt contract triggers information
production. To construct a debt contract that dominates the generic, start by raising the price \( p^D \), keeping the original debt level fixed, until one of two things happens.

(i) The debt price is so high that the value of information acquisition, \( \pi_R \), equals \( \gamma \). We can’t go higher, since that would not induce information acquisition by agent C.

(ii) \( p^D \) is so high that it reaches the intersection between \( \bar{\pi} \) and \( V^D(\bar{\pi}) \), that is, the price is set so that it equals the value of debt evaluated at \( \bar{\pi} \). Note that in this case \( p^D > p^g \), since we are to the left of \( z_0 \).

Importantly, in both cases we end up with a price \( p^D > V^D(\bar{\pi}) > k \). So, if we can show that the rent to agent C is no larger and the probability of trade no smaller than for the generic, we are done.

In case (i) the rent is at \( \gamma \) (i.e. the minimum) so it is no larger than the generic, and the probability of trade is larger since we are to the left of \( \bar{\pi} \).

In case (ii) the generic and the debt contract priced at \( p^D \) trade identically, but the debt contract will give away less rent because as in the case in Figure 9, we can compare triangles now, since the contracts trade over identical \( y \)-regions (as a function of \( y \) which agent C observes). This is evident from the fact that the debt line has a lower slope than the generic line.

**Case 2.2** \( V^D(z) > p^L_1(z) > V^g(z) \). Figure 10, with \( p^L_1(z) = p_H \), illustrates this case. Start by writing down \( D \) to \( \bar{D} \) defined by \( V^D(z) = p_H \) and set the price of the amended debt fairly: \( p^D_1(z) = V^\bar{D}(z) \). The amended debt intersects the vertical line \( z \) at point \( B \) in the figure. If the fairly priced amended contract does not trigger information acquisition, it strictly dominates the generic contract, since it transfers \( \pi_H \) to agent B with probability 1. If the debt contract triggers information acquisition, then we lower the debt level further, keeping the price fixed at \( p_H \). The rent of the amended debt, \( \pi^{\bar{D}}_H(z, p_H) \), is being reduced in the process. We keep going until either the intersection between the value function of debt and the horizontal line \( p_H \) reaches point \( A \) or \( \pi^{\bar{D}}_H(z, p_H) = \gamma \), whichever happens first. If \( A \) is reached, the debt contract will trade at price \( p_H \) with the same probability as the generic contract, but the rent will be strictly smaller, since the wedge \( \pi^{\bar{D}}_H(z, p_H) \) is a strict subset of the wedge \( \pi^g_H(z, p_H) \). If \( A \) is not reached, debt will trade with a higher probability and same price as the generic. [Note: if we assume, wolg, that the generic only pays rent \( \gamma \), then we never reach point \( A \); the process stops before]. In either case, debt strictly dominates the generic.

**Case 2.3.** \( V^D(z) > V^g(z) \geq p^L_1(z) \). Figure 10, with \( p^L_1(z) = p_L \) illustrates this case. We reduce the face value of the original debt to the point where the value function of debt intersects the value function of the generic at the level \( p_L \) (point \( C \) in the figure). Point \( C \) is to the left of the vertical line \( z \), because \( p_L < V^g(z) \). Therefore, \( p_L < V^\bar{D}(z) \), implying that the relevant value of information for debt as well as for the generic is the left regions \( \pi_L \). Since the slope of the value function of debt is strictly smaller than the slope of the value function of the generic (by Lemma 2), the wedge \( \pi^{\bar{D}}_L(z, p_L) \) is a strict subset of the wedge \( \pi^g_L(z, p_L) \). If \( \pi^L_L(z, p_L) \geq \gamma \), both contracts will trigger information. Trade will occur with equal probabilities at price \( p_L \), but when trade occurs, the rent that agent B pays to C will be strictly smaller with debt than with the generic \( \pi_L^D(z, p_L) < \pi_L^g(z, p_L) \). So, the debt contract strictly dominates the generic. Finally, if \( \pi^L_L(z, p_L) < \gamma \), the debt contract will not trigger information. But this case dominates the case where information acquisition is triggered (see Lemma 3).

We have covered all possible cases and conclude that in each one, we can, using debt as collateral, do strictly better than using a non-debt contract as collateral. Note that this includes the case where the generic is quasi-debt. QED
Figure 10: A-B-C Cases 2.2, 2.3
References


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