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Security Baskets and Index-linked Securities*

Security baskets and index-linked securities are securities whose values are aggregates of the cash flows or values of other assets. Creation of these "composite" securities improves the welfare of uninformed investors by reducing their trading losses. The introduction of composite securities can also affect real investment decisions. When investors are exposed to nontradeable, heterogeneous risks, different trading equilibria are possible. Markets for multiple composite securities may exist or, when the degree of heterogeneity is high, markets for a single composite security and its component securities may coexist. No results depend on the existence of exogenous "noise" traders.

I. Introduction

Financial innovation during the last two decades has produced a bewildering array of apparently redundant securities. These security baskets and index-linked securities, which we refer to as "composite securities," are securities whose values are aggregates of the cash flows or values of other assets. There are numerous examples of such composite securities that are created by intermediaries or stock or futures exchanges. Intermediaries bundle assets to create new securities whose payoffs depend on the cash flows of the underlying asset pool. Examples of such security baskets include a large variety of mortgage- and asset-backed securities, closed-end mutual funds, and real estate investment trusts. While intermediaries generally create composite securities with positive net supply, exchanges can create composite securities with zero net supply, such as stock index futures and index participations. Obviously, the creation of these composite securities gives investors a trading ve-

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hicle that is an alternative to buying or selling the underlying assets that compose the basket or index.

Today, the amount of composite securities is very large. For example, U.S. mortgage-backed securities are now a \$1 trillion market, and the amounts outstanding of publicly issued and privately placed (nonmortgage) asset-backed securities were \$44 billion and \$20 billion, respectively, during 1990. The total value of closed-end mutual funds as of December 31, 1989, was \$53.66 billion, and the outstanding (book value) amount of real estate investment trusts (REITs) in the same year was \$44.2 billion. In addition to baskets of securities, index-linked securities are also very popular. For instance, the Standard and Poor's (S & P) 500 Stock Index futures contract is one of the highest volume futures contracts with over \$10 billion traded daily.[1]

The popularity of such composite securities seems puzzling since investors, on their own, can apparently obtain the same resulting cash flow by holding a diversified portfolio of the same securities in the same proportions. Bundling securities appears to be a redundant activity. Thus, it is difficult to explain the existence of a large number of securities that appear to be simple repackagings of existing securities. In this essay we argue that the activity of creating composite securities is not redundant. When investors have immediate needs to trade, and prices are not fully revealing, the return on these composite securities cannot be replicated by holding the individual underlying assets in the same proportions.

The twin requirements of urgent needs to trade and noisy rational expectations have been analyzed separately by many others. For example, Grossman and Miller (1988) and Diamond and Dybvig (1983) study models in which some agents have an urgent, and possibly unexpected, need to trade. Moreover, Diamond and Verrecchia (1981) and Grossman (1989) provide examples of noisy rational expectations models in which prices are not fully revealing. In this essay we combine these two ingredients. We study a multiasset trading environment

1. Index participations, which are zero net supply securities whose values are directly linked to the level of a stock index, briefly traded on the Philadelphia Stock Exchange and on the American Stock Exchange (AMEX) (*Financial Times* [May 16, 1989]). Trading was halted when the U.S. Court of Appeals of the Seventh Circuit ruled that these securities fall under Commodity Futures Trading Commission (CFTC) not Securities and Exchange Commission (SEC) jurisdiction. See "Two Exchanges Cease Trading in 'Basket' Items," *Wall Street Journal* (August 11, 1989). The Chicago Board Options Exchange has also proposed trading an index participation (called "market basket securities"), and in 1989 the New York Stock Exchange began trading in baskets of stocks, called "exchange stock portfolios" (*New York Times* [June 5, 1989]). Harris (1989), Kupiec (1989), and Rubinstein (1989) provide descriptions of these security baskets and index securities. Most recently, the AMEX filed a proposal with the SEC to trade S & P 500 depository receipts (SPDRs), which are (positive net supply) claims on a trust holding shares of the stocks making up the S & P 500 (*Wall Street Journal* [March 12, 1992]).

where investors possessing superior information (insiders) can profit at the expense of fully rational, but lesser informed, agents who have unexpected consumption needs (liquidity traders). We show that the liquidity traders can optimally respond to the presence of these insiders by packaging the given primitive assets into a composite security. This composite security is a superior trading vehicle in that it minimizes the liquidity traders' losses to the insiders; that is, it reduces the information advantage of the insiders over the liquidity traders.[2]

We consider the optimal portfolio choice of traders facing uncertain timing of their consumption. These liquidity traders choose initial portfolios knowing that they may subsequently want to sell their portfolios to consume. If they sell their holdings, they face an informed trader in each security market. These informed traders behave as in the model of Kyle (1985).[3] For simplicity, Kyle (1985) took the liquidity traders to be nonoptimizing agents whose exogenous net demands for a security were represented as a normally distributed shock. He focused on characterizing the insider's optimal trading strategy. In Kyle's model, the security's price is set by a market maker who observes the total net order flow for the security but who cannot directly observe the component security demands of the liquidity traders and the insider. This allows the insider, who is assumed to know the true value of the security, to camouflage his trading activity. The market maker, who is assumed to be competitive and set the security price such that his expected profit is zero, raises (lowers) the price of the security when the demand is high (low) to help protect himself from the possibility that the demand reflects insider trading. But because this high (low)

2. Gorton and Pennacchi (1990) showed that uninformed liquidity traders could design securities (transform primitive cash flows) that could prevent trading losses to insiders. In that case, composite securities were not created, rather, primitive cash flows were split (into debt and equity) to create a portion (the debt) that was subject to fewer trading losses to insiders because it was (relatively) riskless, that is, its value was known. As will be seen below, the environment here does not admit the possibility of a leveraged intermediary as an optimal solution. Unlike Gorton and Pennacchi (1990), there are no agents without liquidity needs willing to invest in intermediary equity at the initial date. In this article, we consider whether liquidity traders can prevent or minimize losses to insiders by combining securities, rather than creating new securities by splitting the cash flows of more primitive securities. Hence, we take the menu of securities as given and ask whether combining them in various ways can improve the situation of the liquidity traders.

3. The Kyle (1985) model has been used, and extended, by many researchers. A partial list, including related models, would contain Admati and Pfleiderer (1988), Fishman and Hagerty (1989), Kumar and Seppi (1989), and Subrahmanyam (1991). Subrahmanyam (1991) has independently produced results that are qualitatively similar to some results in this article. However, his uninformed liquidity traders are assumed to be risk neutral, and their relative demands for various securities are exogenously fixed. (In most cases, liquidity transfers are assumed to have equal, normally distributed demands for individual securities.) This article places more emphasis on the portfolio choice of risk-averse liquidity traders and considers the construction of an optimal composite security.

security demand might instead be a result of high (low) demand from liquidity traders, the market maker does not raise (lower) security prices to such an extent that the strategic trades of insiders are completely neutralized. Essentially, the market maker "loses" when demand shifts are primarily the result of shifts in insider demand but "gains," at the expense of the liquidity traders, when demand shifts primarily reflect shifts in liquidity trader demands. This results in expected zero, positive, and negative profits for the market maker, insider, and liquidity traders, respectively.

Importantly, an implication of the Kyle model is that the expected profit of the insider increases when the security's variance is higher. The higher the security's variance, the greater is the value of the insider's superior information.[4] This result is intuitively clear for the limiting case in which the security's value has zero variance; that is, its value is certain. In this situation, the competitive market maker would always set the security's price equal to its known value, resulting in zero profits for all agents.

It may now be clear that one rationale for creating composite securities is simply that variance can be reduced when primitive securities are combined. By trading a composite security, rather than the primitive securities that underlie the composite, the liquidity traders can reduce their expected losses to insiders because they are trading a security with a lower rate of return variance. To illustrate this point, consider the very simple example of two risky securities that trade in different markets and that have perfectly negatively correlated returns. Liquidity traders transacting in each of the two markets will, on average, lose to insiders. But, if the two securities are combined in the right proportions, then a new, composite security can be created that has a riskless return.[5] Consequently, if the liquidity traders were to hold and trade this composite security, they would suffer no losses to insiders.

Because the creation of a composite security provides an opportunity to reduce trading losses, lesser informed agents will optimally switch to holding and trading this security rather than the individual securities that make up the composite. This "migration" to the composite security reduces liquidity trading in the individual component securities. As market makers in these component securities recognize

4. A simplifying assumption of the Kyle model, which we maintain throughout this article, is that the insider can perfectly predict the future price of the security. This implies that the variance of the security is an exact measure of the value (or variance) of the insider's superior information. If, instead, the insider makes imperfect predictions, the variance of the prediction error would also influence the insider's expected profit.

5. A riskless composite security can be created if the proportion of security 1 to security 2 equals the ratio σ_2/σ_1, where σ_i is the standard deviation of the rate of return on security i.

the reduction in liquidity trading, insiders have less camouflage with which to disguise their trades. This gives market makers more information regarding the extent of insider trading and changes their price setting behavior accordingly. Prices become more responsive to the quantities traded by insiders, thus lowering insider profits.

In this article we begin by considering a model in which liquidity traders have homogeneous preferences and endowment distributions. We find that the introduction of an optimally designed composite security eliminates all trading in the individual component securities. This is because liquidity traders' risk of losing to insiders is minimized when they place their entire wealth in the composite security. This absence of markets for individual component securities appears to be consistent with some types of composite securities, such as asset-backed securities and REITs, that lack secondary markets for their component assets. However, this result is inconsistent with other types of composite securities, such as stock index futures and certain types of closed-end stock mutual funds, where secondary markets for their component securities do exist.

Interestingly, when our model is generalized to consider heterogeneity in liquidity traders' endowment risks, we find that trading in a composite security and its individual component securities can coexist. Heterogeneous endowment risks create different portfolio hedging demands by different clienteles of liquidity traders. A trade-off between hedging this risk and minimizing losses from insider trading arises. One strategy is for liquidity traders to design multiple clientele-specific composite securities that balance these two factors. A second strategy involves liquidity traders taking a portfolio position in a single security that best hedges their endowment risk, combined with a position in a common (universal) composite security. We show that which strategy is optimal depends on the size of the clientele. If the number of liquidity traders in each clientele is large, then the volume of trading in markets for the clientele-specific composites is large and per-capita losses to insiders are small. In this case, clientele-specific composites are optimal. However, if the number of liquidity traders in each clientele is small, trading would be too thin, and the alternative strategy is optimal. In this latter case, trade in a common composite and the underlying component securities coexists.

In the remainder of the article we present a model that formalizes the preceding discussion. The model is introduced in Section II. Liquidity traders are assumed to choose an initial portfolio of securities knowing that subsequently they may have to trade in markets where insiders are present. Within this framework, we derive the expected values and covariances of asset returns and show that the liquidity traders are disadvantaged by the presence of insiders. Section III then considers the creation of composite securities and demonstrates the improvement in liquidity traders' expected utility. Section IV shows that, if liquidity traders have different portfolio demands, the effects of market thinness can result in the coexistence of markets for a composite security as well as its component securities. Section V concludes.

II. The Model Environment

There are three dates in the economy, dates 0, 1, and 2. Firms issue securities to agents in exchange for capital at date 0. At date 1, markets for the securities open where agents can trade forward contracts on firms' shares. The contracts are settled at date 2 at which time firms pay a liquidating dividend in the form of units of the consumption good. When trade occurs at date 1, some agents will be informed about the value of a firm's liquidating dividend, while other agents will not. We focus on characterizing the optimal strategies of uninformed agents, referred to as "liquidity traders," who know at date 0 that they may need to trade in markets with informed traders at date 1.

A. *Liquidity Traders*

There is a large number, μ, of liquidity traders who each receive an endowment of e units of nonstorable capital at date 0. They may choose to invest this capital in up to M different technologies in the economy, each owned by a firm. It is assumed that these technologies are in perfectly elastic supply and that they can produce a return only at date 2. For each unit of capital, a firm issues a share, which for the ith firm has a date 2 rate of return, $\tilde{v}_i$, that is distributed $N(\overline{p_i}, \Sigma_i)$.[6]

Liquidity traders' preferences are uncertain initially. At date 0, they do not know whether they will have utility from consumption at date 1 or at date 2. It is assumed that their utility function depends only on the mean and variance of consumption; that is, it takes the form $U(E[C], E[(C - E[C])^2])$, where C refers to an individual's consumption at either date 1 or date 2, and where $U_1 > 0$ and $U_2 < 0$. Liquidity traders with utility for consumption at date 1 will be referred to as "early" consumers, while those with utility for consumption at date 2 will be referred to as "late" consumers. Note that liquidity traders who turn out to be early consumers will always sell their securities at date 1 because they have immediate consumption needs.[7]

The number of liquidity traders who will be early consumers is a random variable, $\tilde{n}$. It is assumed that there is an independent probability k that a given liquidity trader will turn out to be an early consumer,

6. Negative consumption is permitted at date 2.

7. Note that, like Kyle (1985) and others, the assumption that liquidity traders have urgent consumption needs implies that they will submit market orders, as opposed to limit orders, when selling securities. Limit orders may not be executed, delaying the trades these agents want to make.

so that the random variable $\tilde{n}$ will have a binomial distribution with mean equal to μk and a variance equal to $\mu k(1 - k)$. This random number of early consumers is assumed to be uncorrelated with the return of firms' shares.

B. Markets

At date 1 a set of markets open where claims on the M firms' shares are traded. These markets allow for spot (date 1) delivery of the consumption good in exchange for forward (date 2) delivery of firms' shares. Specifically, a short seller of security i's shares receives p_i units of the consumption good at date 1 in return for delivery of one share of firm i (or $\tilde{v}_i$ units of the consumption good) at date 2.[8] An agent taking a long position in a forward contract agrees to take delivery of one share of security i at date 2 (or $\tilde{v}_i$ units of the consumption good) in exchange for paying p_i units of the consumption good at date 1. It is assumed that the consumption good can be costlessly stored from date 1 to date 2 and that there is a zero rate of interest in this interval.[9]

At date 1, early consumers will sell forward all their shares that, in total, equal $\tilde{n}e$, receiving p_i units of the consumption good for each share of firm i sold. Since the probability of each liquidity trader being an early consumer is k, the expected number of shares sold forward by early consumers is μke. Define the unanticipated (unexpected) supply of shares by liquidity traders as $\tilde{u} \equiv e(\tilde{n} - \mu k)$. The random variable $\tilde{u}$ is binomially distributed with mean zero and variance equal to $\sigma_u^2 \equiv e^2 \mu k(1 - k)$.

C. Informed Traders

There are M informed traders, each observing the realized liquidation value of one of the M securities, $\tilde{v}_i$, after the date 0 securities markets close, but prior to the opening of the forward markets at date 1.[10] A

8. Since the number of shares issued by firms at date 0 is assumed to be finite, but we wish to allow traders to take very large (possibly infinite) long or short positions in these securities at date 1, a forward market with possible cash settlement at date 2 is a simple way of accommodating these assumptions. The net supply of shares will always equal that issued by firms, but cash-settled forward markets will allow offsetting positions by insiders and market makers of any magnitude.

9. This assumption makes the price of forward delivery of security i in terms of date 2 consumption also equal p_i, its price in terms of date 1 consumption. We do this to simplify the calculation of date 2 payoffs from positions taken at date 1.

10. Because the model rules out information asymmetries at date 0 when securities are issued, we are abstracting from corporate financing problems that have been the subject of numerous papers; see, e.g., Campbell and Kracaw (1980), Diamond (1984), and Myers and Majluf (1984). Our focus in this article is on information asymmetries in a trading context. However, the creation of composite securities may serve not only to reduce trading losses but also to reduce adverse selection problems in primary security markets. For instance, if new composite security issues were known to represent claims on a broad spectrum of assets with random or above average qualities, security buyers might be less prone to be purchasing only the poorer quality assets, i.e., "lemons."

trader who has information about the liquidation value of security i is assumed to optimally choose a quantity of firm i's shares to purchase or sell forward. Let $x_i(\tilde{v}_i)$ be the net forward purchase of firm i's shares by the ith informed trader that maximizes expected profits. In choosing x_i, the informed trader does not know the level of liquidity traders' sales of security i. However, it is known that the level of liquidity trader sales is distributed as above and is independent of the distribution of $\tilde{v}_i$.

D. *Trading Equilibrium*

Each security market described above is similar to the single market analyzed by Kyle (1985). Like Kyle, we assume there is a competitive market maker in each market who observes the order flow $\tilde{x}_i - \tilde{u}_i$, then determines a price in terms of consumption goods, $p_i = p_i(\tilde{x}_i - \tilde{u}_i)$, and a market position that clears the market.[11] Note that market makers are assumed to observe only the order flow for their own market.[12] In addition, the market makers and the informed traders are assumed to be endowed with sufficient quantities of the consumption good at dates 1 and 2 such that their budget constraints are not binding. However, one difference between our model and Kyle's model is that our liquidity trader sales are derived to be binomially distributed, whereas Kyle assumed noise trader sales were normally distributed.

Under the assumption of normality, Kyle shows there is an equilibrium in which the insider optimality submits an order that is a linear

Hence, initial information asymmetries may help explain the *method* used to create composite securities. For example, in Bank of America's 1986 sale of $500 million of car loans, only car loans that were rated highest by the bank's credit scoring system qualified to be placed in the pool. Of these top-rated loans, those actually placed in the pool were randomly selected.

11. In Kyle's (1985) model, the net sale of securities (by both liquidity and informed traders) has an unconditional expectation of zero, while in our model, the net sale of securities has an unconditional expectation of μk. This difference is of no importance. The competitive market maker in Kyle's model makes expected security purchases of zero to clear the market, while the competitive market maker in our model makes expected purchases of μk to clear the market.

12. This assumption, shared by other multiasset trading models such as Subrahmanyam (1991), is important to our result that liquidity traders benefit from trading a composite security relative to trading its individual component securities. By observing order flows in other markets, market makers could better discriminate between liquidity trading and insider trading in their own market, reducing the advantage of the insider. Under the assumption that market makers can perfectly observe the contemporaneous order flows in every other market, the advantages to insider trading might be reduced to the extent that no further gains could be derived from introducing and trading composite securities. One could argue, however, that this is an unreasonable amount of information for a market maker to instantaneously acquire and analyze. A lag in information transmission between markets of one period or more, as is assumed in Kumar and Seppi (1989), would preserve our results. In any case, composite securities provide a simple mechanism for benefiting liquidity traders without requiring the assumption that market makers possess a high degree of intermarket information.

function of $\tilde{v}_i$ and the market maker optimally responds by setting a price that is a linear function of the total order flow, $\tilde{x}_i - \tilde{u}_i$. His agents' linear least squares decision rules are optimal because these rules are equivalent to maximum-likelihood inference when order flow is normally distributed. When the liquidity traders' component of the order flow is binomially distributed, as in the present article, linear least squares decision rules are the best linear unbiased decision rules of the insider and market maker in the sense of minimizing their mean-squared errors, but these rules are not necessarily optimal within the class of all (linear and nonlinear) estimators.[13] However, for analytical tractability, we confine our analysis to the class of linear decision rules by defining an equilibrium at date 1 as[14] (i) a linear trading strategy for the informed agent in each market, i, that maximizes his expected profits knowing how the market maker sets the price and (ii) a linear price setting rule, chosen by the market maker, that is constrained to earn zero expected profits knowing how the informed agent behaves.

Given these linear trading and pricing strategies, an equilibrium similar to that analyzed by Kyle (1985) will result. In this equilibrium, informed traders in each security market make expected profits at the expense of the liquidity traders. Define w_i as the proportion of liquidity traders' aggregate initial endowment invested in the ith security. Then the informed agent's trading strategy, $x_i(\tilde{v}_i)$, and the market maker's price setting rule, $p_i(\tilde{x}_i - \tilde{u}_i)$, are given by

$$x_i = \beta_i(\tilde{v}_i - \overline{p}_i) \tag{1}$$

and

$$p_i = \overline{p}_i + \lambda_i(\tilde{x}_i - \tilde{u}_i), \tag{2}$$

where $\beta_i = w_i\sigma_u/\Sigma_i^{1/2}$ and $\lambda_i = \Sigma_i^{1/2}/(2w_i\sigma_u)$.

13. This is just a restatement of the Gauss-Markov theorem. Because the liquidity traders' order flow has a discrete binominal distribution, rather than a continuous normal distribution, the (continuous) linear rules given by eqq. (1) and (2) will only approximate optimal rules. To see this, note that an insider would optimally avoid trading an amount that would result in a total order flow that did not coincide with one of the discrete points of the liquidity traders' binomially distributed order flow, as the difference between the total order flow and these points would be revealed as the result of insider trading. The optimal price set by the market maker would then not necessarily be linear in the total order flow. Numerical solutions, rather than closed-form solutions, would generally be required for this discrete case. See Bagnoli and Holden (1991) for an example of this solution technique. A partial defense of our use of linear least squares decision rules is that, as the number of liquidity traders increases, the binomial distribution will converge to the normal distribution. Since the unanticipated aggregate security sales of liquidity traders, $\tilde{u}_\mu = e(\tilde{n} - \mu k)$, has a zero mean and a standard deviation equal to $\sigma_u = e[\mu k(1 - k)]^{1/2}$, the DeMoivre-Laplace limit theorem states that $P(a \leq u_\mu/\sigma_u \leq b) \rightarrow \Phi(b) - \Phi(a)$ as $\mu \rightarrow \infty$, where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function.

14. In spite of the fact that linear least squares decision rules are not fully optimal (i.e., rational), they have been of interest to other researchers. For example, Marcet and Sargent (1988) review research that studies the effect of linear least squares decision rules on convergence to rational expectations equilibria.

From equations (1) and (2), we can now calculate a liquidity trader's date 0 expected return on asset i and the covariance of asset i and asset j. These unconditional moments account for the possibility that the asset will need to be sold at time 1, with a possible loss to insiders, if the liquidity trader is an early consumer, whereas if the liquidity trader is a late consumer, the asset will be liquidated at date 2.

PROPOSITION 1. A liquidity trader's expected rate of return on asset i and the covariance between a liquidity trader's rates of return on asset i and asset j, for $0 < k < 1$, are given by

$$E[r_i] = \bar{p}_i - \varphi\Sigma_i^{1/2} \tag{3}$$

and

$$\mathrm{cov}(r_i, r_j) = \Sigma_i^{1/2}\Sigma_j^{1/2}\left\{\left(1 - \frac{3k}{4}\right)\rho_{ij} + \frac{1}{4}\left[k + \frac{(k-1)^2 - k}{\mu}\right]\right\}, \tag{4}$$

where

$$\varphi = \frac{1}{2}\left[\frac{k(1-k)}{\mu}\right]^{1/2}.$$

Proof. See the Appendix. Q.E.D.

As a benchmark, compare these moments to the moments that would prevail if no insiders were present. In that case, we would have

$$E[r_i] = \bar{p}_i \tag{5}$$

and

$$\mathrm{cov}(r_i, r_j) = \Sigma_i^{1/2}\Sigma_j^{1/2}\rho_{ij}(1 - k). \tag{6}$$

The factor $(1 - k)$ appears in the formula for the covariance because if a liquidity trader turns out to be an early consumer, the market maker's price for security i will always equal $\bar{p}_i$ when there are no insiders present; that is, the price of selling a share is certain.

For $0 < k < 1$, insiders unambiguously decrease the expected return on any asset, i, by a factor that is linear in the asset's standard deviation. Insiders profit at the expense of liquidity traders even though each security price is "fair" in the sense that the expected profit of each market maker is zero. An insider has positive expected profits because the market maker does not adjust the security price enough, on average, to fully offset the probability of insider trading. This is because the market maker cannot perfectly disentangle the cause of supply shifts as being due to the insider or liquidity traders. The liquidity traders end up being penalized when their supply shifts are misinterpreted as being partially the result of insider trading.[15]

15. The model restricts liquidity traders' actions to the choice of a quantity to submit, but the result that prices are not fully revealing and liquidity traders experience losses

As in Kyle (1985), the insider's expected profit (and, therefore, the total expected loss of the liquidity traders) is proportional to the standard deviation of liquidity traders' orders. This standard deviation is $e[\mu k(1 - k)]^{1/2}$. Since the total amount invested by liquidity traders in each security is proportional to $e\mu$, the expected loss per unit of investment is proportional to the ratio of these two quantities, $[k(1 - k)/\mu]^{1/2}$. This explains the form of the φ term in equation (3). Thus, while the insider's expected profit (and the total expected loss of the liquidity traders) grows at the rate $\mu^{1/2}$, the expected loss per unit of investment of the liquidity traders declines at the rate $\mu^{1/2}$. In other words, ceteris paribus, in thinly traded markets (markets where the number of liquidity traders, μ, is low), the expected total profit of the insider will be relatively low. However, the liquidity traders' expected loss per unit of investment will be higher the thinner the trading in this market. This effect of thin trading will be an important consideration when we allow for liquidity trader heterogeneity in Section IV.

If equations (4) and (6) are compared for the case in which $i = j$, and hence $\rho_{ij} = 1$, we see that the rate of return variance of any asset is unambiguously larger when insiders are present. This increase in variance is due to the market maker changing the security price at date 1 in response to the perceived presence of insider trading.

Also, as shown in the Appendix, the covariance between asset rates of return depends on both second and third moments of the binomial distribution. This results in equation (4) containing the term $[(k - 1)^2 - k]/\mu$, implying that for $(k - 1)^2 > k$, or $k < (3 - 5^{1/2})/2 \approx .382$, rate of return covariances decline with increases in the number of liquidity traders, μ. For $k > (3 - 5^{1/2})/2$, covariances increase with increases in the number of liquidity traders.

Proposition 1, which gives each asset's return and covariance with any other asset, provides sufficient information to calculate the liquidity traders' efficient portfolio frontier in the presence of insiders. See Merton (1972) for the analytic derivation of the solution to this problem. We could then compare this efficient portfolio frontier to that for the case in which insiders are not present, which can be constructed using equations (5) and (6). However, without explicitly deriving these frontiers, the following proposition implies that the efficient frontier when insiders are present is dominated by the efficient frontier when insiders are absent.

PROPOSITION 2. The presence of insiders diminishes the expected utility of liquidity traders.

Proof. Since we have assumed that liquidity traders' utility de-

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is not sensitive to this assumption. While this restriction on strategies is a convenient specification, prices need not be fully revealing even when agents are allowed to submit supply schedules. See, e.g., Dubey, Geanakopolos, and Shubik (1987) and Kyle (1989).

pends on only the mean and variance of their consumption, the expected utilities of holding two portfolios can be compared by simply comparing the first two moments of the portfolios' return distributions. The proof consists of showing that, for *any* set of portfolio weights (i.e., both efficient and inefficient), liquidity traders receive a lower expected portfolio return and face a higher portfolio variance when insiders are present. First, note that the expected return on any portfolio is lowered by the presence of insiders, by proposition 1.[16] Next, denote Σ^* as the $M \times M$ covariance matrix of asset returns when insiders are present. Its elements are given by equation (4). Also let Σ denote the $M \times M$ covariance matrix of date 2 security payoffs, that is, the covariance matrix whose i, jth element is $\Sigma_i^{1/2}\Sigma_j^{1/2}\rho_{ij}$. Then for any set of portfolio weights, w, we must show that $w'\Sigma^* w > w'\Sigma(1 - k)w$; that is, the variance of any portfolio when insiders are present exceeds the variance of the same portfolio when insiders are not present.

Define $K \equiv \Sigma^* - \Sigma(1 - k)$ and note that

$$\begin{aligned} w'Kw = {} & \frac{k}{4}\sum_{i=1}^{M}\sum_{j=1}^{M} w_i w_j \Sigma_i^{1/2}\Sigma_j^{1/2}\rho_{ij} \\ & + \frac{1}{4}\left[k + \frac{(k-1)^2 - k}{\mu}\right]\sum_{i=1}^{M}\sum_{j=1}^{M} w_i w_j \Sigma_i^{1/2}\Sigma_j^{1/2}. \end{aligned}$$

The first term is just a multiple of the quadratic form for the case of no insiders and is, therefore, positive definite. The second term equals a positive constant times

$$(w_1\Sigma_1^{1/2} + w_2\Sigma_2^{1/2} + \dots + w_M\Sigma_M^{1/2})^2 \geq 0.$$

Therefore, the covariance matrix when insiders are present exceeds the covariance matrix without insiders by a positive definite matrix. Q.E.D.

Proposition 2 says that the welfare of the liquidity traders is reduced in the presence of insiders. The proof involved showing that the presence of insiders resulted in a lower expected return and higher variance for *any* portfolio held by the liquidity traders. Hence, it must be the case that the presence of insiders results in a lower expected return and higher variance for all *efficient* portfolios held by the liquidity traders. Let us now consider how the liquidity traders might react after being confronted with this problem.

16. This result depends on the restriction that liquidity traders cannot engage in short sales at the initial date. Recall that, since liquidity traders are identical and assets must be in positive supply, the feasibility of short sales at date 0 is ruled out.

III. Designing an Optimal Composite Security

In this section we will consider an intermediary that seeks to design a composite security that maximizes the utility of a representative liquidity trader. At date 0, the balance sheet of the intermediary is composed of assets consisting of the newly issued shares of firms. The intermediary finances these assets by issuing shares to liquidity traders in return for the liquidity traders' endowments. The intermediary agrees to liquidate its shares at date 2 when it receives the proceeds from the stock of the firms it has purchased. Hence, this intermediary can be thought of as a closed-end mutual fund or a grantor trust issuing an asset-backed security. Liquidity traders purchase "closed-end mutual fund shares" or "asset-backed securities" that can be traded in a secondary market at date 1.[17] To start with, we show that the creation of the composite security improves the welfare of the liquidity traders. Then we will describe how such a composite security is optimally designed.

As in the case of primitive security markets, it is assumed that at date 1 there is a market maker and an insider present in the market for the composite security. At date 1, the insider is assumed to know the date 2 liquidation value of the composite security, which equals the liquidation values of the primitive securities underlying the composite security.[18] Thus, we have the following proposition.

PROPOSITION 3. A liquidity trader's expected return and variance of a composite security having the $M \times 1$ vector of portfolio weights c is given by

$$E[r_c] = c'\overline{p} - \varphi(c'\Sigma c)^{1/2} \tag{7}$$

17. The closed-end mutual funds are assumed to buy a portfolio at date 0 and liquidate at date 2. At date 1, claims on the fund can be traded, but the fund itself does not alter its portfolio. Unit trusts are examples of closed-end funds that do not alter their portfolio.

18. We assume that the information set of the insider operating in the composite security market is the union of the information sets of all individual insiders operating in each individual security market. This assumption regarding the amount of inside information in the composite market is the worst possible case from the point of view of the liquidity traders. It implicitly assumes that insiders can act as a monopoly to optimally utilize their individual pieces of information. If this collusion were not possible, the level of composite market inside information would be only a noisy signal on the value of the composite security. This would strengthen our result regarding the advantage of a composite security. The idea that insiders in a composite security market have less accurate information relative to insiders in individual security markets is what Gammill and Perold (1989) argue constitutes the (only) advantage to uninformed agents in trading composite securities. Thus, the (reduction in variance) advantage of composite securities modeled in our article is wholly distinct from theirs. However, at the cost of further complexity, our model could be generalized to assume that the composite market insider possesses only a noisy signal of the security's liquidation value. In this case, liquidity traders' losses would depend on the accuracy of the insider's signal.

and

$$\mathrm{var}(r_c) = c'\Sigma c\left[1 - \frac{k}{2} + \frac{(k-1)^2 - k}{4\mu}\right], \tag{8}$$

where $\overline{p} = (\overline{p}_1 \ldots, p_M)$.

Proof. The proof is an application of proposition 1 for the single, composite security. Q.E.D.

The following proposition gives the rationale for creation of a composite security.

PROPOSITION 4. A composite security can always be created that increases the expected utility of the liquidity traders.

Proof. The proof consists of showing that for *any* set of individual security portfolio weights that would be chosen by a liquidity trader, if a composite security was constructed with these same portfolio weights, then the liquidity trader would receive a higher expected return, and face a lower variance, by holding this composite security rather than the individual securities that make up the portfolio. The first step is to compare a liquidity trader's expected return on the composite security and the portfolio of individual securities. The second step is to compare variances.

Step 1: When securities are unbundled, the expected return is given by

$$w'\overline{p} - \varphi(w_1\Sigma_1^{1/2} + \ldots + w_M\Sigma_M^{1/2}). \tag{9}$$

From proposition 3, we can compute the expected return on a composite security with these same weights. The expected return on the composite security will then exceed that on the portfolio of individual securities if

$$\sum_{i=1}^{M} w_i\Sigma_i^{1/2} > \left(\sum_{i=1}^{M}\sum_{j=1}^{M} w_i w_j \Sigma_{ij}\right)^{1/2}. \tag{10}$$

Since both sides of the above inequality are positive, we can square both sides to obtain

$$\left(\sum_{i=1}^{M} w_i\Sigma_i^{1/2}\right)^2 > \sum_{i=1}^{M}\sum_{j=1}^{M} w_i w_j \Sigma_{ij}. \tag{11}$$

Expanding the squared term on the left-hand side, and subtracting the right-hand side from the left, we obtain

$$2\sum_{i=1}^{M}\sum_{j=1}^{M} w_i w_j(1 - \rho_{ij})\Sigma_i^{1/2}\Sigma_j^{1/2}, \tag{12}$$

which is greater than zero if at least one of the asset correlation coefficients, ρ_{ij}, is less than one. (Recall that, in equilibrium, the portfolio of individual securities chosen by the representative liquidity trader must have positive security weights, w_i.)

Step 2: Using proposition 1, we know that when securities are unbundled the variance of a liquidity trader's return is

$$\left(1 - \frac{3k}{4}\right) w'\Sigma w + \frac{1}{4}\left[k + \frac{(k-1)^2 - k}{\mu}\right]\left(\sum_{i=1}^{M} w_i \Sigma_i^{1/2}\right)^2, \tag{13}$$

while from proposition 3 the variance of the return on the composite security is

$$w'\Sigma w \left\{1 - \frac{3k}{4} + \frac{1}{4}\left[k + \frac{(k-1)^2 - k}{\mu}\right]\right\}. \tag{14}$$

Note that the variance of the composite security will be less than that on the portfolio of individual securities if

$$w'\Sigma w < \left(\sum_{i=1}^{M} w_i \Sigma_i^{1/2}\right)^2. \tag{15}$$

In step 1, we proved that this inequality must hold if at least one ρ_{ij} is less than unity. Q.E.D.

Using the results of proposition 3, we can also compute a set of asset weights to create an optimal composite security, that is, that composite security that would maximize the utility of a liquidity trader. Let ϵ be an M dimensional vector of ones, and let $\overline{R}$ be a given expected return. Then we can state the following proposition.

Proposition 5. The optimal composite security is a set of portfolio weights, c, that solves

$$\min_{\{c\}} c'\Sigma c, \tag{16}$$

subject to

$$(i)\ c'\overline{p} - \varphi(c'\Sigma c)^{1/2} = \overline{R};\quad (ii)\ c'\epsilon = 1;\quad (iii)\ c_i \geq 0,\quad i = 1, \ldots, M.$$

Proof. This follows from the results of proposition 3 giving the expected return and variance of a composite security for a given set of portfolio weights. The optimal set of weights for a liquidity trader whose utility depends on only the mean and variance of the return distribution are those minimizing the variance of the composite security return subject to a given expected return. Q.E.D.

A closed-form solution does not exist to problem (16), the constrained minimization problem to determine the optimal portfolio weights. However, a numerical solution can be computed. Of course these portfolio weights will, in general, be different from those of the representative liquidity trader for the case in which a composite security was not available. Hence, if the M primitive production technologies are in perfectly elastic supply, the equilibrium supplies of these primitive securities will differ when a composite security is available versus when it is not. In other words, the introduction of an optimal composite security will affect firms' real investment decisions.

IV. Optimal Design of Composites in the Presence of Clienteles

The previous sections provide an argument for why basket securities and index-linked securities exist. There are two important implications of the argument. First, trading in the individual securities would cease once the composite security was introduced. This implication seems realistic for certain examples of assets for which only a market for a basket of these assets exists—for example, mortgage-backed and asset-backed securities. However, it is often the case that both the composite security and its component securities trade in separate markets, such as the case of stocks and stock index futures. A second implication of our previous analysis is that a single composite security would satisfy the optimal portfolio choice of all liquidity traders. Again, this might be a reasonable result if one considers the large volume of trade in stock index futures, such as the S & P 500 contract. However, the large number of other types of composite securities, such as mortgage- and asset-backed securities and closed-end mutual funds, appears to contradict this result. In this section we consider a simple extension of the previous model that admits the possibility that liquidity traders may, on the one hand, choose to hold and trade a composite security as well as some of the individual securities that make up this composite. On the other hand, their needs may best be satisfied by designing a number of different composite securities whose component securities are not traded.

We show that heterogeneity among liquidity traders can result in cases where individuals may be better off holding a portion of their wealth in a single security rather than holding their entire wealth in a basket of securities. Heterogeneity is introduced by assuming that liquidity traders can be categorized at date 0 into N different publicly observable types, or clienteles, where $N \leq M$, the total number of individual primitive assets in the economy. In addition, for simplicity, we assume that the number of liquidity traders in each clientele or type is the same, equal to μ/N. Each liquidity trader of clientele i, $i = 1, \ldots, N$, expects to receive a nontradeable endowment at date 2 equal to $-\delta\tilde{v}_i$ if and only if he is a late consumer. In other words, if an agent in clientele i is a late consumer, he receives an endowment shock that is perfectly negatively correlated with primitive asset i. This creates a hedging demand for asset i on the part of clientele i liquidity traders. As before, all liquidity traders are assumed to have identical utility functions that depend on only the mean and variance of their consumption.

To highlight the effect of different types of endowment shocks and their influence on asset demands, we specialize the payoff distributions of the M primitive assets by assuming that they are independently and identically distributed as $\tilde{v}_i \sim N(\bar{p}, \sigma^2)$, $i = 1, \ldots, M$. This assumption creates a symmetry that simplifies the portfolio choice problem of the N different clienteles of liquidity traders.

Given that asset payoffs are independent and identically distributed, our previous results make it clear that if $\delta = 0$ (no endowment shocks), all liquidity traders would maximize their expected utility by placing their date 0 endowment in a composite security consisting of equal proportions of the M primitive assets. Thus, in the absence of heterogeneity, a single equally weighted market basket would meet the investment needs of all liquidity traders. However, when $\delta > 0$, a single equally weighted market basket may not be sufficient to maximize utility, as liquidity traders of clientele i will now have an additional hedging demand for security i. This naturally leads to consideration of two alternative symmetric equilibria consisting of different sets of securities held by different clienteles of liquidity traders.

One equilibrium that we consider is that of N composite security markets, where composite security i is an unequally weighted portfolio that is "customized" to meet the portfolio choice of liquidity traders of clientele i. Liquidity traders of clientele i place their entire date 0 endowment in composite security i, and they are the only clientele of liquidity traders who trade in this security. Similar to our previous analysis, this equilibrium implies that trade occurs only in composite securities.

The second equilibrium that we analyze is one in which all liquidity traders place one portion of their date 0 endowment in a market basket consisting of equal proportions of the M primitive assets. In addition, liquidity traders of clientele i satisfy their hedging demand for asset i by placing the other portion of their date 0 endowment in the individual asset i. Hence, this equilibrium implies that trading in $N + 1$ markets will occur at date 1: markets for N different individual assets and a market for the single equally weighted composite security. Coexistence of markets for individual and composite securities results.

This section shows that, if the number of different clienteles of liquidity traders, N, is sufficiently small, the first equilibria described by the N different composite securities will be preferred by liquidity traders. However, for N sufficiently large, liquidity traders will prefer to hold a common equally weighted market basket security along with the individual primitive asset that is correlated with their endowment shock. The intuition for this finding comes from the results of proposition 1. Equation (3) states that liquidity traders' expected loss per unit of investment is proportional to the inverse of the square root of the number of traders in that security, $1/\mu^{1/2}$. In other words, liquidity traders suffer greater expected rate of return losses in more thinly traded security markets. Therefore, if each clientele of liquidity traders holds a separate composite security, their expected rate of return losses will be proportional to $(N/\mu)^{1/2}$. Hence, the greater the number of different liquidity trader clienteles, the higher will be each of their expected losses when trading these different composite securities.

Instead of trading N different composite securities, all liquidity traders may be better off holding an equally weighted composite security along with the individual primitive asset that is correlated with their endowment shock. Since they all hold the equally weighted composite, their expected rate of return losses on this single composite will be proportional to $1/\mu^{1/2}$ rather than $(N/\mu)^{1/2}$. The disadvantage of this trading strategy is that their hedging demand will cause them to hold a part of their wealth in an individual security that suffers from thin trading as well as a lack of diversification that increases their losses to insiders. However, since it is only a portion of their wealth that they hold in this individual security, with the rest in the equally weighted market basket, they may be better off relative to the case of holding all their wealth in a thinly traded clientele-specific composite security.[19]

We now formally prove the previous assertions. The assumptions made in this section allow for a symmetric treatment of the N different clienteles of liquidity traders. Define ω as the proportion of a liquidity trader's date 0 endowment that is invested, either directly or through a composite security, in the asset that is correlated with his date 2 endowment shock. Thus a proportion $1 - \omega$ is invested in the other $M - 1$ assets. The following lemma gives the optimal composite security portfolio weights for the equilibrium in which each clientele of

19. Throughout this section we do not consider the possibility of a liquidity trader holding only the composite market basket at date 0 and then, if he turns out to be a late consumer, adjusting his portfolio at date 1 to hedge endowment risk at date 2. (Though this action would also admit date 1 trade in both composite and single securities.) This strategy would require the liquidity trader, at date 1, to trade against insiders when selling part of the composite security and also when buying some of the individual security best suited to hedge his endowment risk. Our presumption is that this strategy is dominated by the strategy of avoiding trade at the interim date unless the trader's preferences turn out to be for early consumption. There is also a problem modeling the strategy of altering the portfolio at date 1 under our assumption that traders submit market orders since the proceeds from a date 1 market-order sale of a portion of the composite security would be uncertain. Further, the date 1 funds needed to pay for the market-order purchase of the single security would also be uncertain. The trader may not be able to pay for his order in the single security market. Thus, the situation is more complicated than can be tractably handled in the context of Kyle-type price formation.

liquidity traders invests in a separate unequally weighted composite security.

LEMMA 1. If liquidity traders of different clienteles invest in separate unequally weighted composite securities, their expected value and variance of consumption are given by

$$E[C] = e\left\{\bar{p} - \frac{\sigma}{2}\left[\frac{Nk(1-k)}{\mu}\right]^{1/2}\left[\omega^2 + \frac{(1-\omega)^2}{M-1}\right]^{1/2}\right\} - \delta\bar{p}(1-k) \tag{17}$$

and

$$E[(C - E[C])^2] = e^2\left\{\left[\omega^2 + \frac{(1-\omega)^2}{M-1}\right]\sigma^2\left[1 - \frac{k}{2} + \frac{(k-1)^2 - k}{4\mu/N}\right]\right\} + (1-k)\delta(\delta\sigma^2 + \delta\bar{p}^2k - 2e\omega\sigma^2), \tag{18}$$

where the optimal proportion of the composite security invested in the asset that is correlated with their future endowment, ω^*, satisfies

$$\frac{1}{M} < \omega^* < \frac{1}{M} + \frac{(M-1)\delta(1-k)}{Me\left[1 - \frac{k}{2} + \frac{(k-1)^2 - k}{4\mu/N}\right]}. \tag{19}$$

The term on the left-hand side of inequality (19) is the value of ω that maximizes liquidity traders' expected consumption, and the value on the right-hand side of inequality (19) is the value of ω that minimizes liquidity traders' variance of consumption.

Proof. The term in braces in equation (17) is the expected rate of return on the composite security. It is a direct application of proposition 3, where c is an $M \times 1$ vector with all elements equal to $(1 - \omega)/(M - 1)$ except for the ith element (referring to the security whose return is correlated with the liquidity trader's future endowment), which equals ω, and the total population of liquidity traders holding this security is μ/N rather than μ. The second term in equation (17) is simply the expected value of the liquidity trader's date 2 endowment times the probability of being a late consumer. The term in braces in equation (18) is the variance of the rate of return on the composite security, which is also a direct application of proposition 3. It is straightforward to show that the remaining term in equation (18) is the variance of the liquidity trader's date 2 endowment plus two times the covariance of this endowment with the return on the composite security. Inequality (19) is found by taking the derivatives of (17) and (18) with respect to ω. Q.E.D.

Note that the optimal value of the proportion of wealth held in the security correlated with the liquidity trader's future endowment, ω^*, will depend on the liquidity traders' utility weights placed on the mean and variance of consumption; that is, it will depend on the point on the efficient portfolio frontier selected by liquidity traders. The term in addition to $1/M$ on the right-hand side of (19) represents a security demand for the purpose of hedging against future endowment uncertainty and can be made arbitrarily large by increasing the value of δ.

Similar to lemma 1, the following lemma gives liquidity traders' expected value and variance of consumption from holding part of their initial endowment in an equally weighted composite security and the rest in an individual asset correlated with their future endowment. As before, we define ω as the portion of their wealth invested in the asset that is correlated with their future endowment, including that portion contained in the composite security. Thus the portion of their wealth held in the individual correlated asset is given by $\omega - (1 - \omega)/(M - 1)$, which must be greater than or equal to zero since short sales cannot occur at date 0.

Lemma 2. If liquidity traders of different clienteles invest in the same equally weighted composite security, as well as the individual asset that is correlated with their future endowment, their expected value and variance of consumption are given by

$$E[C] = e\left\{\bar{p} - \frac{\sigma}{2}\left[\frac{k(1-k)}{\mu}\right]^{1/2} \times N^{1/2}\left[\omega + \frac{1-\omega}{M-1}\left(\left(\frac{M}{N}\right)^{1/2} - 1\right)\right]\right\} - \delta\bar{p}(1-k), \tag{20}$$

and

$$\begin{aligned} E[(C - E[C])^2] = e^2\Bigg\{&\left(\omega - \frac{1-\omega}{M-1}\right)^2 \sigma^2\left[1 - \frac{k}{2} + \frac{(k-1)^2 - k}{4\mu/N}\right] \\ &+ \left[\frac{(1-\omega)M}{M-1}\right]^2 \frac{\sigma^2}{M}\left[1 - \frac{k}{2} + \frac{(k-1)^2 - k}{4\mu}\right] \\ &+ 2\left(\omega - \frac{1-\omega}{M-1}\right)\frac{(1-\omega)}{M-1}\sigma^2 \\ &\times\left[1 - \frac{3}{4}k + \frac{1}{4}\left(\frac{M}{N}\right)^{1/2}\left(k + \frac{(1-k)^2 - k}{\mu/N}\right)\right]\Bigg\} \\ &+ (1-k)\delta(\delta\sigma^2 + \delta\bar{p}^2 k - 2e\omega\sigma^2), \end{aligned} \tag{21}$$

where the optimal proportion of liquidity traders' wealth invested in the asset correlated with their future endowment, ω^*, satisfies

$$\frac{1}{M} < \omega^* < \frac{1}{M} + \frac{M-1}{M}$$
$$\times \frac{\dfrac{\delta(1-k)}{e} - \dfrac{1}{4(M-1)}\left\{k\left[\left(\dfrac{M}{N}\right)^{1/2} - 1\right] + \dfrac{(k-1)^2 - k}{\mu}(N^{1/2}M^{1/2} - 1)\right\}}{1 - \dfrac{k}{2} - \dfrac{1}{4(M-1)}\left\{2k\left[\left(\dfrac{M}{N}\right)^{1/2} - 1\right] - \dfrac{(k-1)^2 - k}{\mu}(N^{1/2}M^{1/2} - 1)^2\right\}}. \tag{22}$$

The term on the left-hand side of inequality (22) is the value of ω that maximizes liquidity traders' expected consumption, and the value on the right-hand side of inequality (22) is the value of ω that minimizes liquidity traders' variance of consumption, given the short sales restriction that $\omega \geq 1/M$.

Proof. The calculations are similar to those in lemma 1. The term in braces in equation (20) is the expected rate of return on holding the proportion $\omega - (1 - \omega)/(M - 1)$ of the individual asset plus the proportion $(1 - \omega)M/(M - 1)$ of the equally weighted composite security. The expected rate of return on the individual security is given by equation (3), with the population of liquidity traders given by μ/N. The expected rate of return on the equally weighted composite is a simple application of proposition 3. The second term in equation (20) is simply the expected value of the liquidity trader's date 2 endowment times the probability of being a late consumer.

The term in braces in equation (21) is the squared proportion invested in the individual security times its rate of return variance plus the squared proportion invested in the equally weighted composite security times its variance plus two times the security proportions times the covariance between the individual security and the equally weighted composite. This covariance can be calculated in a similar manner to that done in step 2 of the Appendix. Note that this covariance is positive. As with lemma 1, the remaining term in equation (21) is the variance of the liquidity trader's date 2 endowment plus two times the covariance of this endowment with the return on the individual and composite securities. Inequality (22) is found by taking the derivatives of (20) and (21) with respect to ω. Q.E.D.

Using the results of lemmas 1 and 2, we can now determine the conditions under which the utility of liquidity traders can be greater in an equilibrium in which they all hold an equally weighted composite security and the individual security that is correlated with their future endowment than in an equilibrium in which different clientele liquidity traders hold unequally weighted composite securities.

Proposition 6. If liquidity traders' probability of early consumption is sufficiently small ($k \leq (1 - k)^2$), then there always exists a sufficiently large number of liquidity trader clienteles, N, such that their utility from trading individual securities and an equally weighted composite security exceeds their utility from trading unequally weighted composite securities.

Proof. The proof involves showing that, when N is sufficiently large, liquidity traders will have greater expected consumption and smaller or equal variance of consumption for the equilibrium where the equally weighted composite security and individual securities are held and traded relative to the equilibrium where the unequally weighted composite securities are traded.

Comparing the values of expected consumption in equations (17) and (20), it is straightforward to show that expected consumption for the equally weighted composite and individual securities equilibrium, (20), exceeds that for the unequally weighted composite equilibrium, (17), when

$$N > \frac{M}{1 + \frac{M-1}{1-\omega}\left\{\left[\omega^2 + \frac{(1-\omega)^2}{M-1}\right]^{1/2} - \omega\right\}}. \tag{23}$$

Note that the right-hand side of equation (23) is strictly less than M, so that there exists an finite interval for $N < M$ for which liquidity traders have higher expected consumption from holding the equally weighted composite and individual securities.

Similarly, the values of the variance of consumption in equations (18) and (21) can be compared. The variance of consumption for the equally weighted composite and individual security equilibrium, equation (21), is less than the variance of consumption for the unequal composite equilibrium, equation (18), when the following inequality holds:

$$\begin{aligned} &(N-1)(1-\omega)\frac{(k-1)^2 - k}{2\mu} \\ &> \left(\omega - \frac{1}{M}\right)\left(\frac{M^{1/2}}{N^{1/2}} - 1\right)\left[k + \frac{(k-1)^2 - k}{\mu/N}\right]. \end{aligned} \tag{24}$$

Notice that, from equation (19) and the fact that short sales are not possible at date 0, the optimal value of ω satisfies $1/M < \omega < 1$. Therefore, the left-hand side of equation (24) is nonnegative as long as $(k - 1)^2 \geq k$. The right-hand side of equation (24) is also nonnegative since $\omega > 1/M$ and $N \leq M$. However, as the number of different liquidity trader clienteles approaches the number of primitive assets in the economy, that is, $N \rightarrow M$, the right-hand side of (24) can be made arbitrarily close to zero. Hence, for N sufficiently close to M, the inequality holds. Q.E.D.

We end this section with an example that illustrates the effect of introducing a composite security when liquidity traders have heterogeneous endowment risks. Consider the case in which $N = M$, that is, the number of clienteles equals the number of primitive securities. If trading in a composite security is not permitted, a symmetric equilibrium exists in which primitive security i is held in proportion ω by clientele i (that clientele whose endowment risk is correlated with security i) while all other $M - 1$ clienteles hold security i in the proportion $(1 - \omega)/(M - 1)$. With this level of liquidity trading in each security, it is straightforward to show that the expected consumption of each liquidity trader is given by[20]

$$E[C] = e\left\{\overline{p} - \frac{\sigma}{2}\left[\frac{k(1-k)}{\mu}\right]^{1/2} M^{1/2}\left[\omega^2 + \frac{(1-\omega)^2}{M-1}\right]^{1/2}\right\} - \delta\overline{p}(1-k). \tag{25}$$

Now consider the changes that occur when trading in composite securities is introduced. Assuming $k \leq (1 - k)^2$, and since N is assumed equal to its maximum value of M, proposition 6 ensures that liquidity traders prefer to trade a single composite security, as well as its component securities, rather than multiple clientele-specific composite securities. In this equilibrium, only liquidity traders of clientele i remain trading individual security i. All other liquidity traders migrate to the composite security market. As the market maker in security i recognizes this reduction in liquidity trading, he infers a greater level of insider trading from a given order flow, increasing the order flow sensitivity of prices. As shown in Section II*D*, this reduction in liquidity trading, while increasing the *per-unit* expected losses of clientele i in security market i, decreases the *total* expected profits of insiders.

After the opening of the composite security market, liquidity traders face increased losses from trading in the individual security that hedges their endowment risk. However, this increase is more than offset by the reduction in losses achieved by moving the remaining portion of their portfolios to the composite security market. Given the same proportion, ω, of their portfolio invested in the hedge asset, their gain in expected

20. The computation is similar to previous ones in this section, but in this case liquidity traders' date 1 supply of shares of security i consists of $e\omega(\tilde{n}_i - \mu k/M)$ shares sold by liquidity traders of clientele i and $e[(1 - \omega)/(M - 1)][\tilde{n}_o - (M - 1)\mu k/M]$ shares sold by all other liquidity traders, where $\tilde{n}_i$ and $\tilde{n}_o$ are the numbers of early consumers in clientele i and all other clienteles, respectively. Note that, if $\delta = 0$, so that $\omega = 1/M$, the term in brackets in eq. (25) equals its minimum value of $1/M$, and liquidity traders' expected losses will be independent of M since their loss per unit of investment in each security will be the same. If $\delta > 0$, then $\omega > 1/M$, and liquidity trader losses increase because their trading is more concentrated in their single hedge security.

consumption is the difference between equations (25) and (20):[21]

$$e\frac{\sigma}{2}\left[\frac{k(1-k)}{\mu/M}\right]^{1/2}\left\{\left[\omega^2+\frac{(1-\omega)^2}{M-1}\right]^{1/2}-\omega\right\}>0. \quad (26)$$

V. Concluding Remarks

Financial innovation has led to a large number of securities that are simple aggregates of existing securities and thus appear to be redundant. In this article we present a positive theory of composite securities and explore the optimal design of composite securities for different types of investor clienteles. We determine when trade will occur simultaneously in both a composite and the component securities and when a variety of composites will be preferred.

When security prices are not fully revealing, informed agents can take advantage of lesser informed agents who have urgent needs to trade, and thus cannot wait for information to be revealed. These lesser informed agents will be motivated to reduce their trading losses by creating and trading in composite securities rather than individual primitive securities. By aggregating assets in the form of a composite security, a trading vehicle is created that has a lower rate of return variance and, hence, is characterized by a lower level of information asymmetries. The introduction of composite securities not only increases the expected utility of lesser informed agents but also affects equilibrium investment in the individual primitive securities.

In our initial model, the motivation for creating composite securities is so strong that the market for its component assets closes upon introduction of the composite. We showed, however, that this need not be the case if traders are not exposed to uniform risks. If, in addition to the risk of having to trade against insiders, traders face heterogeneous endowment shocks, then more interesting results can occur. If there is a small number of liquidity trader clienteles, with a large number of traders in each clientele, then liquidity traders will prefer to hold a clientele-specific composite security. The reason is that the large volume of trading in each clientele-specific composite security will result in small per-capita losses from insider trading. However, if there are many different clienteles with the number of liquidity traders in each one being small, then liquidity traders will prefer to trade in a uniform composite security, as well as individual securities, so as to avoid losses from thin trading in a clientele-specific composite. In this latter case, markets for the composite security and the component securities coexist.

21. While not presented here, it can also be shown that, when μ is sufficiently large, liquidity traders' variance of consumption decreases upon introduction of the composite security, so that overall utility will unambiguously increase. This rather lengthy derivation is available from the authors on request.

Appendix

Proof of Proposition 1

Step 1: Calculation of the liquidity traders' expected return on asset i when insiders are present. The date 0 unconditional expected return on asset i, $E(r_i)$, equals

$$E[r_i] = E\left[\tilde{p}_i\left(\frac{\tilde{n}}{\mu}\right) + \tilde{v}_i\left(1 - \frac{\tilde{n}}{\mu}\right)\right]. \tag{A1}$$

Substituting for $\tilde{p}_i$ from equation (2) in the text, we have

$$\begin{aligned} E[r_i] &= E\left[[\bar{p}_i + \lambda_i \beta_i(\tilde{v}_i - \bar{p}_i) - \lambda_i e w_i(\tilde{n} - \mu k)] \left(\frac{\tilde{n}}{\mu}\right) + \tilde{v}_i\left(1 - \frac{\tilde{n}}{\mu}\right)\right] \\ &= \bar{p}_i k - \frac{\lambda_i e w_i}{\mu} E[\tilde{n}\tilde{n} - \mu k \tilde{n}] + \bar{p}_i(1 - k) \qquad \text{(A2)} \\ &= \bar{p}_i - \frac{\lambda_i e w_i}{\mu}\{[(k\mu)^2 + \mu k(1 - k)] - (\mu k)^2\}. \end{aligned}$$

Substituting in for λ_i, we have

$$E[r_i] = \bar{p}_i - \frac{k(1 - k)e}{2\sigma_u}\Sigma_i^{1/2}. \tag{A3}$$

Finally, substituting in for σ_u, we arrive at

$$E[r_i] = \bar{p}_i - \frac{1}{2}\left[\frac{k(1 - k)}{\mu}\Sigma_i\right]^{1/2} \equiv \bar{p}_i - \varphi\Sigma_i^{1/2}. \tag{A4}$$

Step 2: Calculation of the covariance beween the returns to asset i and asset j. For $0 < k < 1$, this covariance is given by

$$\begin{aligned} E[(r_i - E[r_i])(r_j - E[r_j])] = E\Bigg[&\{\lambda_i\lambda_j\beta_i\beta_j(\tilde{v}_i - \bar{p}_i)(\tilde{v}_j - \bar{p}_j) + \lambda_i\lambda_j\tilde{u}_i\tilde{u}_j \\ &- \lambda_i\tilde{u}_i\varphi\Sigma_j^{1/2} - \lambda_j\tilde{u}_j\varphi\Sigma_i^{1/2} + \varphi^2\Sigma_i^{1/2}\Sigma_j^{1/2}\}\left(\frac{\tilde{n}}{\mu}\right) \\ &+ (\Sigma_{ij} + \varphi^2\Sigma_i^{1/2}\Sigma_j^{1/2})\left(1 - \frac{\tilde{n}}{\mu}\right)\Bigg] \\ = \frac{k\Sigma_{ij}}{4} &+ \frac{\lambda_i\lambda_j w_i w_j e^2}{\mu}E[(\tilde{n} - \mu k)^2\tilde{n}] \qquad \text{(A5)} \\ &- \frac{\lambda_i\varphi\Sigma_j^{1/2}w_i e}{\mu}E[(\tilde{n} - \mu k)\tilde{n}] \\ &- \frac{\lambda_j\varphi\Sigma_i^{1/2}w_j e}{\mu}E[(\tilde{n} - \mu k)\tilde{n}] + \varphi^2 k\Sigma_i^{1/2}\Sigma_j^{1/2} \\ &+ (\Sigma_{ij} + \varphi^2\Sigma_i^{1/2}\Sigma_j^{1/2})(1 - k). \end{aligned}$$

Using properties derived from the binomial moment generating function, it is straightforward to show that $E[(\tilde{n} - \mu k)\tilde{n}] = \mu k(1 - k)$ and $E[(\tilde{n} - \mu k)^2 \tilde{n}] = \mu k(1 - k)(1 - 2k + \mu k)$. Substituting this along with the definitions of λ_i and λ_j into (A5), we have

$$E[(r_i - E[r_i])(r_j - E[r_j])]$$
$$= \Sigma_{ij}\left(1 - \frac{3k}{4}\right) + \Sigma_i^{1/2}\Sigma_j^{1/2}\left[\frac{e^2 k(1 - k)(1 - 2k + \mu k)}{4\sigma_u^2} - \frac{\varphi e k(1 - k)}{\sigma_u} + \varphi^2\right]. \tag{A6}$$

Recalling that $\sigma_u^2 = e^2 \mu k(1 - k)$, the above expression equals

$$\Sigma_{ij}\left(1 - \frac{3k}{4}\right) + \frac{1}{4}\Sigma_i^{1/2}\Sigma_j^{1/2}\left[k + \frac{(1 - k)^2 - k}{\mu}\right]. \tag{A7}$$

Q.E.D.

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