JOURNAL OF ECONOMIC THEORY 67, 327–369 (1995)

Profitable Informed Trading in a Simple General Equilibrium Model of Asset Pricing*

JAMES DOW

London Business School, London NW1 4SA, United Kingdom

AND

GARY GORTON

The Wharton School, University of Pennsylvania, Philadelphia, Pennsylvania 19104-6367

Received April 8, 1993; revised September 20, 1994

We present a simple general equilibrium model of asset pricing in which profitable informed trading can occur without any "noise" added to the model. We use an equilibrium concept similar to rational expectations equilibrium, but which explicitly allows for the possibility of adverse selection. We show that models of profitable informed trading must restrict the portfolio choices of uninformed traders: in particular, they cannot buy the market portfolio. In this model, profitable informed trading lowers the welfare of all agents when compared across steady states. *Journal of Economic Literature* Classification Numbers: G14, D50, D60, D82. © 1995 Academic Press, Inc.

I. INTRODUCTION

In this paper, we analyze a general equilibrium model of asset pricing in which profitable informed trading can occur without any "noise". We show that the existence of profitable informed trading requires a restriction on the feasible portfolios of uninformed agents, namely, that they are not able to buy the market portfolio. We use an equilibrium concept similar to rational expectations equilibrium (REE), but which explicitly allows for the

* We thank Larry Ausubel, Jonathan Berk, Tilman Börgers, Martin Hellwig, Meg Meyer, J. A. Mirrlees, Sam Orez, Ailsa Röell, and an anonymous referee for comments.

0022-0531/95 $12.00

642/67/2-3

possibility of adverse selection. Because the model is general equilibrium we are able to address issues of economic welfare.

Like REE, the equilibrium concept we use is a reduced form. It does not provide the details of how prices are formed. For this reason, we also provide a brief description of an institutional mechanism that implements the equilibrium. We stress that equilibrium with profitable trading necessarily implies adverse selection.

I.1. *Profitable Informed Trading*

We study financial markets, i.e., markets with trade in securities which are only valuable because they are claims on the consumption good ("money"). Agents who have private information about asset fundamentals have an incentive to trade and make profits if they can trade at a price which does not yet reflect the information. If private information is to be reflected in asset prices, then these informed agents must make profits to give them an incentive to trade on their information (if either information acquisition or trade is costly). But, if they make profits, it must be at the expense of other traders. This raises the paradoxical question: whom do they trade with? A large class of models in financial economics, originating with Grossman and Stiglitz [15, 16], addresses this question. Grossman and Stiglitz [15, 16] proposed that an unobservable exogenous noise term be added to the excess demand function for the asset. Informed agents can then profitably trade on their information since the uninformed, not knowing the realization of the noise term, do not know whether the informed are buying or selling. Kyle [22] interpreted the exogenous error term as the volume sold by noise or "liquidity" traders, agents whose behavior is not explicitly modelled but is motivated by liquidity needs. Glosten and Milgrom [12] also rely on the presence of exogenous liquidity traders. The liquidity trader device has now become the standard paradigm for the analysis of asset pricing models with private information.

The finance literature has developed by increasingly incorporating institutional features of price formation mechanisms while continuing to make use of exogenous noise. Our focus is in the opposite direction: we attempt to explicitly model liquidity motivated trade, and we explore the role of liquidity traders in models of price formation. On the other hand, we use an equilibrium concept similar to rational expectations equilibrium (explained below), rather than explicitly modelling the institutions of price formation. This allows us to conclude that the essential restriction necessary for profitable informed trading is that uninformed agents are not able to buy the market portfolio. The existing models (e.g., Grossman and Stiglitz [15, 16], Kyle [22], and Glosten and Milgrom [12]) all implicitly incorporate this restriction, and below we describe how it appears in these models.

If better informed agents make superior returns by trading on their private information, it must be at the expense of the others. These other agents will (collectively) make below average returns. Yet, any agent who is able to buy the market portfolio of assets can guarantee him or herself the average return. How should we model such markets? In the existing literature it is hard to understand the larger environment which would motivate these agents to participate in the market.

This paper sets out to investigate profitable informed trading in a model where all traders are explicitly modelled as rationally optimizing agents. There is no exogenous noise. We analyze an overlapping generations model. This is a natural framework for introducing non-information-based trade since the initial allocation is not Pareto efficient. Young agents invest in order to save for future consumption, while old agents liquidate their portfolios in retirement. We introduce an additional period, middle age, in which agents become privately informed about dividend realizations after their lifetime. They can speculate on the basis of this information. In our paper the uninformed earn below average returns, but choose to participate because this is preferred to the alternative of not participating in asset markets.

The existence of profitable informed trading does not depend on forcing uninformed agents to lose money *per se*. Rather, the crucial restriction concerns their inability to buy the market. In the existing models of profitable informed trading this is implicitly assumed. We discuss below how the assumption enters each of several such models (Grossman and Stiglitz [15, 16], Kyle [22], and Glosten and Milgrom [12]). In the equilibrium we consider (as in REE), the assumption enters as follows: agents are not allowed to submit demands which condition on the demands of other, informed, agents; nor can they condition on the outcome of the auctioneer's allocation; and nor are they allowed to submit "strategic" untruthful demands to the auctioneer. Our main conclusion is that the restriction preventing agents from buying the market allows for the existence of profitable informed trading.

Our model is "noiseless" in the sense that information is the only motive for preferring one asset to another. An agent who sells an asset, and invests the proceeds in another asset, will only do so if he has private information that the former asset is relatively overvalued at current market prices. This distinguishes our model from other models where prices are not fully revealing because there are other possible causes for changes in relative asset demands. For example, in Kyle [22], demand for the risky security could be high (relative to holding the riskless security) because informed agents have received good news or because there is a large liquidity demand. In Biais and Hillion [71] and Dow and Gorton [10] changes in relative asset demands can be due to private information ("news") or to hedging needs (noise). In Ausubel's [5] model of a two-good exchange economy, agents' preferences depend on two signals. He gives an example in which an increase in the demand for good x, relative to good y, could arise because informed agents know that good x is relatively more desirable to all agents (news) or because their information increases their own desire for good x while leaving the preferences of the uninformed unchanged (noise).[1]

In a financial markets model, i.e., a model of trade in securities which are only valuable because they are claims on the consumption good, it is natural to distinguish between news and noise. The REE literature models exchange of many commodities and uses noises to refer to a random error term added to the aggregate excess demand function.[2] The broader definition that we have used here is motivated by our desire to model nonrevelation of private information in financial markets without introducing other motives for preferring one asset to another.

I.2. *The Equilibrium Concept*

Our aim is to study models of profitable trading on private information in a setting which is not dependent on the details of a specific price formation process, just as Walrasian equilibrium allows us to study large markets without explicitly modelling the limit of a sequence of finite imperfectly competitive markets. REE suggests itself as the appropriate tool (see Radner [26, 27], Green [13], Grossman [14], Jordan [18], and the survey by Jordan and Radner [19]). However, fully revealing REE does not allow privately informed agents to profit, while adding noise introduces an exogenous and otherwise undesirable complication. Hence we use an alternative equilibrium concept based on REE, but modified to allow for the possibility of adverse selection.

The possibility of coexistence of adverse selection with REE was noted by Kreps [21]. He gives an example in which an agent is indifferent between elements of the demand correspondence and lets the Walrasian auctioneer make the choice. However, the only market-clearing choice systematically gives the uninformed agent an adversely selected allocation. The same issue arises in our model: uninformed agents' demands are determined by asset pricing conditions which state that they are indifferent between holding different (fully diversified) portfolios with the same value.

[1] In general, Ausubel [5] considers the case where both signals affect the utility functions of both informed and uninformed agents, but the two types of signals affect the informed agents differently from the uninformed.

[2] Ausubel [5] does not introduce "noise" in the sense of the REE literature, i.e., an error term on the excess demand function. However, as the discussion in the text indicates, he does introduce a motive for preferring one good relative to the other that plays an analogous role to the "endogenous noise" that is caused by the hedging motive in Biais and Hillion [7].

In equilibrium the portfolios which they receive are less desirable *ex post* than other portfolios which were *ex ante* perfect substitutes.

Kreps [21] argued that this was unsatisfactory because the auctioneer's choice would then contain additional information which would change the agent's demands.[3] He essentially argued that the definition of REE should be changed to exclude this possibility.

In contrast, we maintain this possibility and explore its consequences. This necessitates a more careful definition of equilibrium: a problem with the standard definition of REE is that demands are not defined appropriately in the possible presence of adverse selection. We amend the definition of demands to allow for this possibility and we define an equilibrium with adverse selection, with the standard REE as a special case (we do not take a stand on whether this equilibrium should properly be called an REE).

Adverse selection can only occur if agents' demands are multivalued. Then the auctioneer may be able to clear the market by giving uninformed agents allocations that are *ex post* less desirable than other elements of their demand correspondences. Since REE is based on the idea that agents "know the model," they should allow for this possibility when they form their demands. The amendment to the definition that we incorporate is exactly this: agents cannot avoid adverse selection, but they do anticipate it when forming their demands. To illustrate we give the following informal example.

Consider an uninformed agent buying a car. There are new cars and used cars for sale. All new cars are identical; but used cars are of two types: good used cars and lemons. Informed buyers can distinguish the lemons, while used cars appear identical to the uninformed buyer. Since he cannot distinguish between them, the uninformed agent's demand for good used cars is the same as his demand for bad used cars (if the prices do not reflect the difference in quality). However, he knows that he faces adverse selection and so his demand for used cars is "adjusted" downward relative to the demand for new cars. This allows for the fact that the uninformed agent who buys a used car is more likely to buy a bad one. This is the basic idea of the equilibrium in this model: our equilibrium with adverse selection is essentially a formalization of this idea in an REE setting. The formalization requires a description of how the auctioneer allocates goods to clear the market, thereby resulting in adverse selection. Uninformed agents understand this when they form their demands. Therefore, to define equilibrium requires a description of the auctioneer's allocation process.

Kreps [21] described a situation analogous to the above example, but with only used cars. In that case, the issue of whether consumers adjust

[3] This is a paraphrase of Kreps [21, p. 38].

their demands for new cars versus old cars does not arise. Consequently, the definition of the demands was less problematic.

I.3. *Welfare*

Profitable informed trading, in which some agents benefit at the expense of others, naturally raises questions of economic welfare.[4] Welfare analysis is not possible in models of profitable informed trading which rely on liquidity traders. In those models the informed make profits and the uninformed lose money, but no welfare statements can be made because the preferences of the uninformed are not modelled.[5] As a result, two criteria have been used to judge welfare. One is to assume that higher volatility of prices implies lower welfare. While volatile prices surely have some costs, it is clear that there is no necessary connection between volatility and welfare. For example, in Hart and Kreps [17] higher volatility raises welfare. Another criterion is based on the fact that informed agents benefit from trading on their private information, while the liquidity traders lose money. Since their preferences are not explicitly modelled, it is not possible to say whether this loss of money outweighs the liquidity traders' presumed (but unmodelled) liquidity gain from trading (for this reason, Rochet and Vila [28] call their result based on this criterion a "weak invisible hand" property).

One of the benefits of a general equilibrium approach is that it can be used to address questions of welfare economics. Our aim is to emphasize that welfare analysis of these issues may be more complicated than may initially appear. We show that agents are worse off in the equilibrium with profitable informed trading than in the equilibrium with fully revealing prices. Trading on private information in middle age earns a superior return. The welfare loss is due to the fact that this higher return distorts intertemporal consumption allocations.

The welfare result is less straightforward than it may appear at first sight. With profitable informed trading, middle-aged agents earn returns above the market, while in the equilibrium without profitable informed trading they earn the market return. It might seem to follow immediately that the

[4] In much of the finance literature this is referred to as "insider trading." At the level of abstraction in these economic models, however, there is no distinction between trading on legitimately acquired information, such as fundamental analysis, and insider trading as defined by law. The analysis in our model concerns the effects of information, however acquired, and has nothing to say about the distinction between inside information and other types of private information. Since the phrase "insider trading" has normative connotations we avoid this usage.

[5] Presumably, even if the noise trade were derived from maximization of a (simplistic) utility function, all outcomes would be Pareto efficient and welfare analysis would remain uninteresting. These models have no investment and no elasticity of consumption demands.

middle-aged save more in the equilibrium with profitable informed trading because they earn a higher return. But the rate of return on the market may differ across the two equilibria. In fact, it turns out to be lower in the equilibrium with profitable informed trading. Nevertheless, it remains true that the middle-aged earn a higher return in that equilibrium than in the equilibrium without profitable informed trading.

II. The Model

II.1. *Introduction*

The economy consists of overlapping generations of identical consumers who live for three periods, (youth, middle age, and old age) and a stock market in which claims to dividends are traded. The stock market consists of a continuum of stocks, each representing a claim to the dividend stream on a firm. A firm is a Lucas [23] fruit tree. It yields an infinite stream of dividends. The dividend realizations are independent both serially and in cross section. There is therefore no aggregate uncertainty, and indeed diversification is costless in equilibrium so agents never bear any risk.

Young consumers receive one unit of a consumption good. They do not consume, but invest their income in the stock market. Middle-aged agents rebalance their portfolio to provide some consumption, as well as some investment for old age. Old consumers liquidate their portfolios and consume the value. Middle-aged agents are privately informed about which stocks will pay dividends next period. In addition, in every period all agents receive information about the current period's dividends.

We will consider stationary equilibria of the economy. The economy has a fully revealing REE, as well as an equilibrium in which informed traders earn superior returns. We investigate the former equilibrium as a benchmark, but our main interest is in the latter equilibrium.

II.2. *Timing within Each Period of the Model*

The sequence of events within each period is as follows. First, the young receive their endowment income. The middle-aged receive private information about *next* period's dividends. Everybody receives information about *this* period's dividends. Second, the stock market and the market for the consumption good open and trading takes place. Stocks trade *cum* dividend (i.e., ownership of this period's upcoming dividend is transferred to the buyer). Third, consumption takes place: middle-aged and old consumers consume, and they consume this period's income of the young, together with the dividends from the end of last period. Finally, this period's dividends arrive and are held over until next period. See Fig. 1.

FIG. 1. Time line.

II.3. *Definitions and Notation*

II.3.1. *Time.* There is an infinite number of discrete periods indexed by $t = -\infty, ..., \infty$.

II.3.2. *The Consumption Good.* There is a single, perishable, consumption good.

II.3.3. *Agents and Endowments.* In period t, an agent is born and lives for three periods: youth (time t), middle age (time $t+1$), old age (time $t+2$). This agent should be thought of as representing a large number of identical agents. Young agents are each endowed with 1 unit of the consumption good.

II.3.4. *Preferences.* Agents' preferences are additively separable over consumption in middle age, c^{m}, and in old age, c^{o}. Agents discount the future and preferences are represented by $U(c^{\mathrm{m}}) + \lambda U(c^{\mathrm{o}})$, where λ is the discount factor. We assume that U is continuously differentiable and strictly concave with $U'(0) = \infty$ (which implies that consumption in each period is strictly positive).

II.3.5. *Stocks.* There is a continuum of stocks indexed by $s \in [0, 1]$. A stock yields an infinite stream of dividends. In each period the dividend on a share of any stock is equally likely to be either 0 or a positive amount d (as explained in Section II.3.8, we assume $d < \frac{1}{5}$). Dividends are paid in the consumption good (because the dividends are paid at the end of the period, they are consumed next period). Stocks trade *cum* dividend. The supply of stocks is given by the uniform density on $[0, 1]$. The total dividend payments equal $\frac{1}{2}d$.

II.3.6. *Uncertainty.* Let Ω be the set of states of nature for a given time period (i.e., the full sample space of the model is $\cdots \times \Omega \times \Omega \times \Omega \cdots$), then ω_t: $[0, 1] \to \{\mathrm{H}, \mathrm{L}\}$ is a realization of the state of nature. The interpretation of ω_t is that it describes all the time t dividends. L stands for "low" (actually, zero) and H for "high." Dividends are given by d_t: $[0, 1] \times \Omega \to \{0, d\}$ with $d_t(s, \omega_t) = 0$ if $\omega_t(s) = \mathrm{L}$ and $d_t(s, \omega_t) = d$ if $w_t(s) = \mathrm{H}$. For notational simplicity we will suppress the dependence on ω_t and simply write $d_t(s)$. The dividend realizations are independent both serially and in cross section. There is therefore no aggregate uncertainty: in any set of stocks of positive measure which is chosen without information on the realization of d_t, half will pay dividends and half will not (almost surely).[6]

We introduce a number of functions below (prices, portfolios, demands, etc.) that also depend on the state of nature, and again this dependence will be suppressed in the notation. The reason for this is that the integrals of these functions over s are not random. We assume that all such functions are appropriately measurable.

We will often refer to "the H stocks" or the "HL stocks," etc. The H stocks at time t, also referred to as the "dividend paying stocks," are $\{s: \omega_t(s) = \mathrm{H}\}$; the complement of this set is the L stocks, or non-dividend

[6] Statements of this type are common in economics, although the construction of a probability measure with the required properties is not straightforward. Judd [20] shows that there is a probability measure, μ, on Ω such that: (i) $\mu(\{\omega_t : \omega_t(s) = D\}) = \frac{1}{2}$ for $D = \mathrm{H}, \mathrm{L}$; (ii) $\mu(\{\omega_t : \omega_t(s_1) = D_1\} \cap \cdots \cap \{\omega_t(s_k) = D_k\}) = \mu(\{\omega_t : w_t(s_1) = D_1\}) \times \cdots \times \mu(\{w_t : w_t(s_k) = D_k\})$ for $D_j = \mathrm{H}$ or L, $j = 1, \ldots, k$; (iii) $\{s : \omega_t(s) = \mathrm{H}\}$ is Lebesgue measurable a.s.; (iv) $\mu(\{\omega_t : \ell(\{s : \omega_t(s) = \mathrm{H}\}) = \frac{1}{2}\})$ (where ℓ is the Lebesgue measure).

Property (i) states that the dividend on each stock follows the specified distribution (in our case, 0 or d with equal probability). Property (ii) is an independence condition. Property (iii) states that for almost all realizations of ω_t the set of stocks with high dividends is Lebesgue measurable, so that we can define the fraction of stocks paying dividends. Property (iv) is the Strong Law of Large Numbers: almost surely this fraction will be one-half. Judd's construction is somewhat arbitrary. There are a number of suggestions to solve this problem; see Section 5 of Anderson [3] for a description and exposition.

We will also have to define the measure of events describing dividend realizations in two successive periods $(t, t+1)$. Define $\mu^*((A, B)) = \mu(A)\,\mu(B)$, for $(A, B) \in \Omega \times \Omega$. Note that $\{s: \omega_t(s) = D_t,\ w_{t+1}(s) = D_{t+1}\}$, for D_t, $D_{t+1} = \mathrm{H}$ or L, is Lebesgue measurable a.s.

paying stocks. The HL stocks at time t are $\{s: \omega_t(s) = \mathrm{H}\ \&\ \omega_{t+1}(s) = \mathrm{L}\}$; the HH, LH, and LL stocks are defined similarly.

II.3.7. *Information Sets.* In period t, ω_t is public information, while ω_{t+1} is private information at time t and is known only by the middle-aged.

II.3.8. *Limited Smart Money.* The equilibrium with profitable informed trading will exist only if the amount of wealth invested by the informed (middle-aged) agents is not too large (we call this the "limited smart money" condition). As will be seen below, their demands will otherwise exceed the available supply of undervalued assets. There are two requirements needed to bound the value of the middle-aged portfolio. First, we assume $d < \frac{1}{5}$. Second, we assume that the utility function, U, and the discount factor, λ, are such that the optimal choice of c^{o} is less than $\frac{1}{20}$ on every budget set in the set

$$\{(c^{\mathrm{m}}, c^{\mathrm{o}}) \mid c^{\mathrm{m}} + c^{\mathrm{o}}/R = k \text{ for some } k, R \in [1, (1+d)/(1-d)]\}.$$

The first assumption ensures that the value of the dividends the middle-aged receive is not too large compared to the endowment of the young agents. Otherwise, the value of portfolios held by the young will be a small fraction of the total, so the middle aged will hold the large remaining fraction of the stock market. (Since we have assumed endowments are 1, d should be interpreted as dividend income relative to endowment income.) The second assumption implies that the middle-aged consume enough that their portfolio held to finance consumption in old age is not too large, or, equivalently, their consumption in old age is not too large. This is a joint restriction on the utility function and the rate of time preference, λ. The way that these bounds arise will be seen in Step 5 of the Proof of Proposition 2.

II.3.9. *Prices.* The prices of stocks at date t are denoted by a (Lebesgue measurable) function p_t: $[0, 1] \to \mathbf{R}_+$, with $p_t(s)$ giving the price of stock s at time t.

II.3.10. *Price Function.* The prices of stock s is determined by a function ϕ: $\{\mathrm{H}, \mathrm{L}\} \times \{\mathrm{H}, \mathrm{L}\} \to \mathbf{R}_+$ giving the price for each stock in a given period depending on both the public and the private information for that period about that stock's dividends: $p_t(s) = \phi(\omega_t(s), \omega_{t+1}(s))$. Note that this definition imposes stationarity, independance from (irrelevant) past dividends, independence from the (irrelevant) dividends on other stocks, independence from information which is not yet known, and independence from extrinsic uncertainty.[7]

II.3.11. *Inferences from Prices.* In the equilibrium agents may learn from prices. A young agent (without private information) knows today's dividend on a given stock, but not tomorrow's dividend. If the stock price, given today's dividend, is the same for both values of tomorrow's dividend, then the agent learns nothing. On the other hand, if today's price is different for different values of tomorrow's dividend, then the price reveals tomorrow's dividend. Since ω_t is public information, knowing $p_t(s)$ will induce the following beliefs about ω_{t+1} in a young agent:[8]

$$\mathrm{Prob}(\omega_{t+1}(s)=\mathrm{H} \mid \phi(\omega_t(s), \omega_{t+1}(s))=p_t(s) \ \& \ w_t(s)=D)$$

$$=\begin{cases} 0 & \text{if} \quad \phi(D, \mathrm{H}) \neq p_t(s) \\ \frac{1}{2} & \text{if} \quad \phi(D, \mathrm{H})=p_t(s) \ \& \ \phi(D, \mathrm{L})=p_t(s) \\ 1 & \text{if} \quad \phi(D, \mathrm{H})=p_t(s) \ \& \ \phi(D, \mathrm{L}) \neq p_t(s) \end{cases}$$

for $D=\mathrm{H}$ or L.

II.3.12. *Fully Revealing Price Function.* A fully revealing price function satisfies $\phi(\mathrm{H}, \mathrm{H}) \neq \phi(\mathrm{H}, \mathrm{L})$ and $\phi(\mathrm{L}, \mathrm{H}) \neq \phi(\mathrm{L}, \mathrm{L})$.

II.3.13. *Nonrevealing Price Function.* A nonrevealing price function satisfies $\phi(\mathrm{H}, \mathrm{H})=\phi(\mathrm{H}, \mathrm{L})$ and $\phi(\mathrm{L}, \mathrm{H})=\phi(\mathrm{L}, \mathrm{L})$. We do not consider price functions which are revealing for low-dividend stocks and nonrevealing for high-dividend stocks or vice versa.

II.3.14. *Portfolios.* A portfolio is denoted by a (Lebesgue measurable) function $x: [0, 1] \rightarrow \mathbf{R}_+$. Intuitively, a portfolio is a list of the desired amounts of each stock. The definition assumes that there are no atoms in the portfolio; i.e., we only consider fully diversified portfolios (because agents are risk averse and diversification is costless). For example, the equally weighted portfolio is represented by the uniform measure ($x(s)$ equals a constant). The support of a portfolio is defined in the usual way: the closure of the set of points s at which $x(s)>0$. The definition of a portfolio assumes that agents may not sell stocks short. The value of portfolio x at prices p_t is given by the (Lebesgue) integral $\int_{[0,1]} x(s)\, p_t(s)\, ds$, and the portfolio earns dividend income:[9] $\int_{[0,1]} x(s)\, d_t(s)\, ds$.

Let $x_t^y: [0, 1] \rightarrow \mathbf{R}_+$ be the portfolio agents born in period t acquire when young. Thus, they will enter period $t+1$ as middle-aged agents with

[7] Note that if w_t and w_{t+1} are Lebesgue measurable, setting prices according to ϕ will result in $p_t(s)$ being measurable.

[8] Viewed as a function of s, this conditional probability is measurable a.s.

[9] The assumptions on the uncertainty imply that this is defined a.s.

these portfolios. Similarly, $x_t^{\mathrm{m}}: [0, 1] \rightarrow \mathbf{R}_+$ is the portfolio acquired by middle-aged agents in period t (i.e., agents born in period $t-1$). Old agents liquidate their existing portfolios and do not reinvest.

II.3.15. *Demand.* Given an equilibrium pricing rule, agents submit demands to the auctioneer. The demand has two components: a portfolio demand and a consumption demand. The portfolio demand of the young at time t, ξ_t^{y}, is a set of portfolios. The portfolio demand of the middle-aged at time t, ξ_t^{m}, is also a set of portfolios. (The old simply liquidate their portfolios; i.e., their demand is a singleton containing the portfolio which is identically zero.) In middle age the consumption demand is denoted $c_t^{\mathrm{m}} \in \mathbf{R}_+$ and in old age, $c_t^{\mathrm{o}} \in \mathbf{R}_+$.

The portfolio demands will be multi-valued since agents will be indifferent between many different portfolios because any portfolio whose support is of positive measure is fully diversified. Although demands for stocks are multi-valued, demands for consumption levels are not because the utility function for consumption is strictly concave.[10]

Demands must satisfy agents' budget constraints. (We will describe the utility maximization problem below in Sections II.3.17 and II.3.18.) The budget constraint for the young is

$$\int_{[0,1]} x_t^{\mathrm{y}}(s)\, p_t(s)\, ds = 1,$$

for the middle-aged

$$\int_{[0,1]} x_{t-1}^{\mathrm{y}}(s)\, p_t(s)\, ds + \int_{[0,1]} x_{t-1}^{\mathrm{y}}(s)\, d_{t-1}(s)\, ds = c_t^{\mathrm{m}} + \int_{[0,1]} x_t^{\mathrm{m}}(s)\, p_t(s)\, ds,$$

and for the old

$$c_t^{\mathrm{o}} = \int_{[0,1]} x_{t-1}^{\mathrm{m}}(s)\, p_t(s)\, ds + \int_{[0,1]} x_{t-1}^{\mathrm{m}}(s)\, d_{t-1}(s)\, ds.$$

Demands, as defined here, are sets of portfolios and consumption quantities and we will later define equilibrium by the condition that these demands are optimal given the prices, and clear the market. It is perhaps more common, in general equilibrium theory, to define demands as correspondences that given the optimal quantities as a function of the price and then require zero excess demand in equilibrium. These two approaches are clearly equivalent. In this model we have chosen the former approach

[10] Note that the demands are *not* net of the agents' existing portfolios. Clearly, it is equivalent to work with net demands, i.e., trades, but that will turn out to be less convenient.

because it would be difficult, and counterintuitive, to define demands at out-of-equilibrium prices, which the second approach requires us to do.[11]

We will assume that agents cannot guarantee themselves the market rate of return. In other words, they cannot protect themselves against adverse selection. This aspect of the equilibrium should be regarded as a reduced form: at this stage we do not describe the specific features of the market institution which prevent agents from achieving the market rate of return (see Section VI). It should be noted that this condition does not imply that adverse selection will necessarily prevail in equilibrium. As will be seen below, the model has two equilibria, only one of which has adverse selection.

From the point of view of the middle-aged, we can divide the stocks at time t into four *categories* according to their current and future dividends: high dividends today and tomorrow (H, H); high dividends today and low dividends tomorrow (H, L); and similarly the remaining two, (L, H) and (L, L). By the assumptions on the price function ϕ, all stocks in the same category have the same price. We assume that middle-aged agents will not distinguish in their demands between different stocks in the same category:

$$x_t^{\mathrm{m}} \in \xi_t^{\mathrm{m}} \Rightarrow \hat{x} \in \xi_t^{\mathrm{m}} \text{ for all } \hat{x} \text{ satisfying}$$
$$\int_{\{s:\, \omega_t(s) = \mathrm{H}\ \&\ \omega_{t+1}(s) = \mathrm{H}\}} \hat{x}(s)\, ds = \int_{\{s:\, \omega_t(s) = \mathrm{H}\ \&\ \omega_{t+1}(s) = \mathrm{H}\}} x_t^{\mathrm{m}}(s)\, ds \tag{**}$$

and similarly for the integrals over the sets with information (H, L), (L, H), and (L, L).

The young cannot distinguish between different stocks with the same dividend today but different dividends tomorrow, except if the price function, ϕ, reveals the information. If it does reveal the information, then the above condition holds for the young also. If it does not reveal the information, we require that young agents do not distinguish in their demands between different stocks paying the same dividends today:

$$x_t^{\mathrm{y}} \in \xi_t^{\mathrm{y}} = \hat{x} \in \xi_t^{\mathrm{y}} \text{ for all } \hat{x} \text{ satisfying}$$
$$\int_{\{s:\, \omega_t(s) = \mathrm{H}\}} \hat{x}(s)\, ds = \int_{\{s:\, \omega_t(s) = \mathrm{H}\}} x_t^{\mathrm{y}}(s)\, ds \tag{**}$$

and similarly for the integrals over the set of stocks paying low dividends today (L).

[11] This is because, in our model, demands depend on the allocation rule used by the auctioneer. In order to define out-of-equilibrium demands, one would have to construct allocation rules for nonmarket-clearing allocations! The dependence of demands on the allocation rule is explained in Sections II.3.16, 17, and 18.

The two conditions (**) require that agents cannot protect themselves from adverse selection. For example, buying the market portfolio corresponds to a uniform distribution over stocks. The conditions require that if that demand is submitted then the agent must also submit all other distributions with the same support. On the other hand, if dividend-paying stocks are underpriced compared to non-dividend-paying stocks, the agent can demand only non-dividend-paying stocks. For example, the degree of adverse selection may be different, in equilibrium, between stocks currently paying and not paying dividends. Agents will take this into account in forming their demands and therefore the relative prices of H and L stocks must reflect this. Thus, although agents cannot protect themselves against adverse selection, they understand that it occurs and they take it into account when forming their demands. We can describe this as a "rational expectations" property.

We will define the support of the demand in the obvious way, namely, it is the union of the supports of the portfolios in the demand. The support of demand ξ_t^i ($i =$ y or m) is denoted $\operatorname{supp}(\xi_t^i)$.

II.3.16. *The Auctioneer's Allocation Rule.* In order to define equilibrium in a model where there may be equilibria with adverse selection, we cannot consider demands independently of this possible adverse selection. Therefore, we now explain how the auctioneer will allocate stock portfolios to clear the market. This will enable us to define the demands and hence equilibrium.[12]

An *allocation rule* determines the portfolio holdings of the young agents and those of the middle-aged as a function of the demands submitted by each. If there is only one way to clear the market, the allocation rule selects this pair of portfolios. If there are several ways, the rule specifies one of them. The auctioneer selects an allocation $(x_t^{\mathrm{y}}, x_t^{\mathrm{m}})$ such that $x_t^{\mathrm{y}} \in \xi_t^{\mathrm{y}}$, $x_t^{\mathrm{m}} \in \xi_t^{\mathrm{m}}$, and demand equals supply, i.e., $x_t^{\mathrm{y}} + x_t^{\mathrm{m}} = \mathbf{1}$.[13]

We will require that if agents do not distinguish between a set of stocks in their demands, the auctioneer will not distinguish between the stocks in the allocation. This condition on the allocation rule is somewhat weaker than the condition on the price function (that a stock's price depends only on its dividends today and tomorrow). We can therefore assume that the auctioneer's allocated portfolios are constant on each of the four sets of stocks: (H, H), (H, L), (L, H), (L, L). The allocation problem is therefore reduced to determining how much of each category of stock to allocate

[12] Note that, although Kreps [21] gave an example of equilibrium with adverse selection, he did not formally define this type of equilibrium in the presence of adverse selection. We discuss this further in Sections II.5.5 and VII.

[13] The symbol $\mathbf{1}$ represents the function which takes the value 1 everywhere on [0, 1]. The equality holds a.e.

between the young and middle-aged. There is only one way for the auctioneer to do this (if it is possible to clear the market at all), as follows. First, consider the support of the middle-aged agents' demand and the support of the young agents' demand. Any stock which is in the support of only one of these generations' demands is allocated to that generation. For each generation we can now define the value of the remaining part of their portfolio as the value of their desired portfolios less the value of the stocks thus allocated. Any stock in the supports of both generations' demands is allocated in proportion to these values.

Let the support of the young agents' demands be S^y and the support of the middle-aged agents' demands be S^m. The market clearing allocation is

$$x_t^y(s) = 1 \qquad \text{for} \quad s \in S^y \backslash S^m$$

$$x_t^m(s) = 1 \qquad \text{for} \quad s \in S^m \backslash S^y$$

since the allocation must equal the total supply. The expenditure of the young agents on stocks $S^y \backslash S^m$ is therefore $\int_{[S^y \backslash S^m]} p_t(s)\, ds$. By their budget constraint, the remaining amount is

$$y \equiv 1 - \int_{[S^y \backslash S^m]} p_t(s)\, ds.$$

Similarly, the amount the middle-aged have remaining is

$$m \equiv \int_{[0,1]} x_{t-1}^y(s)\, p_t(s)\, ds + \int_{[0,1]} x_{t-1}^y(s)\, d_{t-1}(s)\, ds - \int_{[S^y \backslash S^m]} p_t(s)\, ds.$$

Therefore, the remaining market clearing allocations are

$$x_t^y(s) = y/(y+m) \qquad \text{for} \quad s \in S^y \cap S^m$$

$$x_t^m(s) = m/(y+m) \qquad \text{for} \quad s \in S^y \cap S^m.$$

II.3.17. *Stock Returns.* The return on stock s from time t to time $t+1$ is given by

$$r(s, p_{t+1}, p_t, d_t) = [p_{t+1}(s) + d_t(s) - p_t(s)]/p_t(s).$$

Since all stocks in the same category have the same price, we will give the expected returns by category of stock. The middle-aged agents' expected returns are given by

$$\begin{aligned} r_t^m(s) &= E(r(s) \mid \omega_t, \omega_{t+1}) \\ &= [\tfrac{1}{2}\phi(\omega_{t+1}(s), \mathrm{H}) + \tfrac{1}{2}\phi(\omega_{t+1}(s), \mathrm{L}) + d_t(s) - p_t(s)]/p_t(s). \end{aligned}$$

For the young agents there are two cases: revealing prices and nonrevealing prices. If ϕ reveals the private information, knowing the prices will allow them to earn the same expected returns as the middle-aged agents. They will not face adverse selection and the returns will be given by the formula above.

If ϕ does not reveal the private information, then for the young there are two categories of stocks, those with high and low dividends today. We can work out the returns on these two categories from the auctioneer's allocation rule and from the middle-aged agent's demands. These returns can only be defined in equilibrium since they make use of the allocation rule, which only applies when demands are compatible with market clearing. However, we can define returns in a "candidate" equilibrium in which markets can be cleared, and then use the returns to verify whether it is an equilibrium.

Since we restrict attention to candidate equilibria we consider demands from young and middle-aged agents which have the property that the union of the supports of these demands is the entire set of stocks, $[0, 1]$. Note that the support of the young agents' demands can be either all stocks, all H stocks (paying high dividends today), or all L stocks (paying low dividends today). Since the middle-aged agents can distinguish between all four categories of stocks, their support can, in principle, be any nonempty combination of these four categories (of which there are $2^4 - 1 = 15$).

Consider first the case where the support of the young agents' demands is the set of all stocks. We will compute the expected return for the H stocks. There are four possibilities. If the middle-aged support contains no H stocks or if it contains all H stocks, then there is no adverse selection among the H stocks and their expected return is

$$r_t^y(s) = E(r(s) \mid \omega_t) = [\tfrac{1}{4}\phi(\mathrm{H}, \mathrm{L}) + \tfrac{1}{4}\phi(\mathrm{H}, \mathrm{H}) + \tfrac{1}{4}\phi(\mathrm{L}, \mathrm{L}) + \tfrac{1}{4}\phi(\mathrm{L}, \mathrm{H}) + d_t(s) - p_t(s)]/p_t(s).$$

Since a nonrevealing ϕ is, by definition, constant in its second argument, this is simply

$$r_t^y(s) = [\tfrac{1}{2}\phi(\mathrm{H}, \mathrm{L}) + \tfrac{1}{2}\phi(\mathrm{L}, \mathrm{L}) + d_t(s) - p_t(s)]/p_t(s).$$

Next, suppose that the middle-aged support includes only a subset of the H stocks. This could occur if the middle-aged support includes either HL stocks or HH stocks. For the sake of argument suppose the HL (and not the HH stocks) are included. Then the allocation rule will give all the HH stocks to the young and a fraction, $y/(y+m)$, of the HL stocks. Thus, among the H stocks allocated to the young, the fraction of HH stocks is

$$1/(1 + y/(y+m)) = (y+m)/2y + m).$$

Recall that of today's HH stocks, half will be HL tomorrow and half will be HH. Similarly, today's HL stocks become either LL or LH tomorrow. Consequently, the return to the young on H stocks is

$$r_t^y(s) = \{[\tfrac{1}{2}\phi(\mathrm{H}, \mathrm{L}) + \tfrac{1}{2}\phi(\mathrm{H}, \mathrm{H})](y+m)/(2y+m) + [\tfrac{1}{2}\phi(\mathrm{L}, \mathrm{L}) + \tfrac{1}{2}\phi(\mathrm{L}, \mathrm{H})]\, y/(2y+m) + d_t(s) - p_t(s)\}/p_t(s).$$

The other cases are similar.

II.3.18. *Definition of Equilibrium.* An equilibrium is a pricing rule ϕ such that, for all t, there exist demands ξ_t^y, (ξ_t^m, c_t^m), c_t^o and allocations $x_t^y \in \xi_t^y$ and $x_t^m \in \xi_t^m$ with:

(i) Market Clearing: $1 + \frac{1}{2}d = c_t^m + c_t^o$ and $x_t^y + x_t^m = 1$.

(ii) Budget Balance: $\int_{[0,1]} x_t^y p_t = 1$ (young);

$$\int_{[0,1]} x_{t-1}^y (p_t + d_{t-1}) = c_t^m + \int_{[0,1]} x_t^m p_t \text{ (middle-aged)};$$

$$c_t^o = \int_{[0,1]} x_{t-1}^m (p_t + d_{t-1}) \text{ (old)}.$$

(iii) Utility Maximization:

(iiia) Maximization of Portfolio Returns:

$$s \in \mathrm{supp}(\xi_t^m) \Rightarrow \forall s' \in [0,1]: r_t^m(s) \geq r_t^m(s') \text{ (middle-aged)};$$

$$s \in \mathrm{supp}(\xi_t^y) \Rightarrow \forall s' \in [0,1]: r_t^y(s) \geq r_t^y(s') \text{ (young)}.$$

(iiib) Optimal Consumption Choice:

$$c_t^m = \arg\max U(c_t^m) + \lambda U(c_{t+1}^o)$$

$$\text{s.t.} \int_{[0,1]} x_{t-1}^y (p_t + d_{t-1}) = c_1^m + \int_{[0,1]} x_t^m p_t$$

$$\text{and } c_{t+1}^o = \int_{[0,1]} x_t^m (p_{t+1} + d_{t+1}).$$

II.4. *The Asset Pricing Property*

Condition (iiia) implies that all stocks in the support of a young agent's demand must earn the same return; in case this support includes all stocks, it follows that all stocks must earn the same return. Note that these returns are from the young agents' point of view; i.e., they are not conditioned on the private information but they are corrected for possible adverse selection. We will refer to this as the *asset pricing property* because all stocks are perfect substitutes at these prices.

II.5. *Comments on Assumptions*

II.5.1. *Number of Agents.* The analysis could be conducted either with a finite number of agents or with a continuum of agents. We have chosen a finite number to simplify the exposition. The main advantage of a continuum would be in making the price-taking assumption underlying REE seem more natural. Formally, we can define REE (or Walrasian equilibrium) for only one or a finite number of agents, but the price-taking assumptions are more reasonable when there are a large number of agents. As in many papers in the Walrasian and REE literature we use a single consumer to represent this "large" number of consumers (see, for example, Ausubel [5, Footnote 8, p. 99]).

Also, with a continuum of agents it would be possible for all agents to hold nondiversified portfolios. For example, if agents are indexed by $i \in [0, 1]$, agent i could hold stock $s = i$. With a finite number of agents, each agent has a positive amount of wealth. But the total supply of any given stock or any nondiversified portfolio has zero value. In principle, we can allow our agents to submit nondiversified demands but these cannot arise in equilibrium. It should be clear in the sequel that diversification is costless in equilibrium, and therefore agents will not submit such demands even though we have not explicitly considered this possibility.

II.5.2. *Perishability of the Consumption Good.* We have simplified the analysis by assuming that the consumption good is perishable. It will be verified below that, in equilibrium, the restriction that the consumption good cannot be stored is not binding, because the uninformed can earn a positive return on holding a stock portfolio. The equilibrium would be unchanged if they were able to hold the consumption good until next period rather than investing in the market.

II.5.3. *Timing of Dividends.* It might seem more natural that what we call "this" period's dividends, which arrive at the end of this period and are consumed next period, should be called "next" period's dividends and arrive at the beginning of next period, for consumption later in the same period. We make this convention to simplify terminology later in the paper: instead of referring to "today's dividend" and "tomorrow's dividend," we would have to refer to "tomorrow's dividend" and "the day after tomorrow's dividend."

II.5.4. *Limited Smart Money Condition.* We have assumed that dividends are small (relative to endowments) and that agents discount the future enough that consumption in old age is also small. Our assumptions are chosen to ensure that middle-aged consumers will sell most of their portfolio to generate consumption. They will be left with a small amount of wealth invested in the stock market. It is this wealth which is used for speculative trading, in other words, invested on the basis of private information. The equilibrium with profitable informed trading requires that the quantity of undervalued stocks bought by this "smart money" not exceed the available supply. For the same reason, we assume that agents are not allowed to borrow the consumption good or to sell the stock short, although it would be straightforward to allow positive limits on short sales and borrowing. This limit on the amount of smart money is analogous to that in Rock's [29] model of initial public offerings.

The importance of the assumptions is that the amount of private information should be small, as measured by the fraction of total wealth invested by privately informed agents. For simplicity, we have assumed that all agents become informed and so consume mostly in middle age. An alternative approach would be for a small fraction of the population to become informed, with preferences unrestricted. We chose the approach of restricting old-age consumption, with *ex post* identical agents, because it makes the exposition and the welfare analysis simpler.

II.5.5. *The Allocation Rule and Returns Conditional on Allocations.* We define this allocation rule explicitly because in the nonrevealing equilibrium which is the focus of this paper, young agents will face adverse selection. A formal definition of demand requires this allocation rule to be defined because agents take this adverse selection into account when forming their demands.

Note that Kreps [21] does not explicitly define the allocation rule in his example. He assumes that agents do not take this into account in forming their demands, but because both the demanded assets are subject to the same degree of adverse selection it would make no difference if the agents did understand the adverse selection and condition accordingly. In our model, as will be seen below, there may be different degrees of adverse selection for different stocks. Consequently, the allocation rule will result in returns to the uninformed being scaled down by different amounts for different stocks, and relative prices will have to adjust to reflect this. In Section VII we discuss the relationship with Kreps [21] in more detail.

II.5.6. *Comments on Equilibrium.* We have restricted attention to stationary equilibria, in which stocks are treated symmetrically by investors. There are other equilibria: for example, the price of each asset, and the demand of each cohort for the asset, could depend on a number of irrelevant variables. It could be that young people favor stocks with certain index numbers and middle-aged people favor the remaining stocks.

This could also change as a function of time or as a function of an extrinsic random variable. Such issues are not the subject of this paper.

Both of the equilibria we analyze satisfy the additional requirement suggested by Anderson and Sonnenschein [4] and Diamond and Verrechia [8] that the auctioneer's price function can be written as a function of the demands and hence only indirectly a function of the true state of nature. rather than as an arbitrary function of the state.

III. Profitable Informed Trading and Adverse Selection

Milgrom and Stokey [24] prove what has come to be known as the "Groucho Marx" or No Trade Theorem; namely, then when allocations are *ex ante* Pareto efficient (and this is common knowledge) and then agents receive private information, nobody will want to accept a trade offered by another agent.[14] In the model we study, allocations are not efficient so agents may be willing to trade. We now show that if they do trade, then the middle-aged can profit at the expense of the young only if the portfolios allocated to the young are adversely selected. Like the No Trade Theorem, this is virtually a tautology, but we set it out here to emphasize what is required for profitable informed trading to occur. It should be clear that this result is not specific to our model, and in Section VII below we discuss how adverse selection arises in Grossman and Stiglitz [15], Kyle [22], Glosten and Milgrom [12], and related models.

III.1. *Definition of Adverse Selection*

Above we have only defined returns on portfolios consisting entirely of stocks whose expected returns (conditioned appropriately) are all identical. When agents choose their portfolios to maximize return they will always choose portfolios with this property. In this section we will need to define returns on portfolios where the returns are not necessarily equal. Define the portfolio weights (the value-weighted distribution of stocks in the portfolio) for portfolio x at prices p_t by

$$w(s) \equiv x(s)\, p_t(s) \Big/ \left[\int_{[0,1]} x(s)\, p_t(s)\, ds \right].$$

[14] "Please accept my resignation. I don't want to belong to any club that will accept me as a member," *Groucho and Me* (1959, Chap. 26). If allocations are already efficient, an agent who offers to sell an asset must have received bad news about it. See Rubinstein and Wolinsky [30] and Dow *et al.* [11] for further discussion of the assumptions on common knowledge and preferences under which Milgrom and Stokey's [24] result holds.

The return on a portfolio x from period t to period $t+1$ is given by

$$\int_{[0,1]} w(s)\, r(s, p_{t+1}, p_t, d_t)\, ds.$$

Uninformed young agents may face adverse selection. They submit a demand to the auctioneer which consists of a set of portfolios which from their point of view all contain assets with the same expected return. The auctioneer's allocation rule and the demands of the informed middle-aged agents may result in this return being less than the expected return on a portfolio which is uniform on the supports of their demands. If this happens, the portfolio received by the young is adversely selected. *An adversely selected portfolio* is a portfolio x with weights w, at prices p_t, such that

$$\int_{[0,1]} w(s)\, r(s, p_{t+1}, p_t, d_t)\, ds < \int_{\operatorname{supp}(x)} (1/\alpha)\, r(s, p_{t+1}, p_t, d_t)\, ds,$$

where α is the Lebesgue measure of $\operatorname{supp}(x)$.

III.2. *Definition of Profitable Informed Trading*

Profitable informed trading is said to occur when the return that the informed middle-aged agents earn exceeds the return on the stock market as a whole

$$\int_{[0,1]} w^{\mathrm{m}}(s)\, r(s, p_{t+1}, p_t, d_t)\, ds > \int_{[0,1]} r(s, p_{t+1}, p_t, d_t)\, ds,$$

where w^{m} gives the weights for the middle-aged portfolio x_t^{m} at prices p_t.

III.3. *A Theorem on Profitable Informed Trade*

THEOREM. *Profitable informed trading by the middle-aged occurs if and only if the portfolios of the young are adversely selected.*

Proof. We first show that the return on a uniform portfolio over the support of the young agents' demands cannot exceed the return on the stock market. Suppose it did. Then the return on the portfolio with support S^{y} exceeds the return on the stock market. Hence the return on a portfolio with support S^{y} exceeds the return on a uniform portfolio on the complement $([0,1]\backslash S^{\mathrm{y}})$, but $([0,1]\backslash S^{\mathrm{y}})$ is contained in S^{m} by market clearing. This implies that the middle-aged are not maximizing the return on their portfolios, since maximization means that $s \in S^{\mathrm{m}} \Rightarrow r(s) > r(s')$ for all s'. Thus, the return on a uniform portfolio over S^{y} cannot exceed the return on the market.

Now suppose that there is adverse selection; i.e., that r^y is less than the return on a uniform portfolio over S^y. Together with the fact that the return on a uniform portfolio over S^y cannot exceed the return on the market, this implies that r^y is less than the return on the market (a uniform portfolio over $[0, 1]$). Profitable informed trading means that r^m exceeds the return on the market. But, by market clearing, the market return is a weighted average of r^y and r^m. Thus, we must have profitable informed trading. The converse is immediate. ∎

IV. Fully Revealing Prices

To provide a benchmark for welfare analysis, we start by describing the REE in which prices fully reveal the private signals concerning next period's dividends.

IV.1. *Existence of Equilibrium*

PROPOSITION 1. *There exists an equilibrium in which prices fully reveal the private signals concerning next period's dividends.*

Proof. In this equilibrium, all agents buy and sell the market. In other words, their portfolios are constant functions. To maximize utility, it must be that the portfolio they hold (the market) is as good as any other portfolio with the same value. This implies that they are indifferent between stocks with different dividend streams. In other words, the asset pricing property holds: the return, r, to holding each type of stock is the same.

The equilibrium is characterized by four numbers: $p_{HH} = \phi(H, H)$, $p_{HL} = \phi(H, L)$, $p_{LH} = \phi(L, H)$ and $P_{LL} = \phi(L, L)$. So p_{HH} is the price of a stock if both this period's and next period's dividends are high; p_{HL} is the price if this period's dividend is high but next period's dividend is low. p_{LH} and p_{LL} are interpreted similarly. We will refer to these as the four "types" of stocks.

The definition of fully revealing equilibrium states that $p_{HH} \neq p_{HL}$ and $p_{LH} \neq p_{LL}$; that is, stocks with different dividends tomorrow, but the same dividend today, have different prices.[15] Define the average price:

$$\bar{p} = \int_{[0,1]} p_t(s)\, ds = \tfrac{1}{4}(p_{HH} + p_{HL} + p_{LL} + p_{LH}). \tag{1}$$

[15] In principle, stocks with different dividends this period but the same dividends tomorrow could have the same price since today's dividends are public information, although it will turn out that all four numbers will be different.

The asset pricing condition may be stated as

$$p_{\mathrm{LL}} = \bar{p} - \tfrac{1}{2}d/(1+r) - \tfrac{1}{2}d/(1+r)^2 \tag{2}$$

$$p_{\mathrm{HL}} = \bar{p} + \tfrac{1}{2}d/(1+r) - \tfrac{1}{2}d/(1+r)^2 \tag{3}$$

$$p_{\mathrm{LH}} = \bar{p} - \tfrac{1}{2}d/(1+r) + \tfrac{1}{2}d/(1+r)^2 \tag{4}$$

$$p_{\mathrm{HH}} = \bar{p} + \tfrac{1}{2}d/(1+r) + \tfrac{1}{2}d/(1+r)^2. \tag{5}$$

This simply states that deviations from the average price $\bar{p}$ are given by the discounted values of the deviations of the dividends from the average level, $\frac{1}{2}d$. Note that for any $\bar{p}$ and $r > 0$ these prices are indeed four distinct numbers.

It remains to show that there is an equilibrium value for the amount invested and the quantities consumed. These will be functions only of the average stock price, $\bar{p}$. We now show that there is such an equilibrium average price. Let γ be the fraction of the stock market bought each period by the young generation (i.e., $x_t^{\mathrm{y}}(s) = \gamma$, for all s). Let δ be the fraction of the stock market sold every period by the middle-aged (i.e., $x_t^{\mathrm{m}}(s) = \gamma - \delta$, for all s). The budget constraint for the young is

$$1 = \int_{[0,1]} x_t^{\mathrm{y}}(s)\, p_t(s)\, ds = \int_{[0,1]} \gamma p_t(s)\, ds = \gamma\bar{p}. \tag{6}$$

For the middle-aged the budget constraint is

$$\begin{aligned} c^{\mathrm{m}} &= \int_{[0,1]} x_{t-1}^{\mathrm{y}}(s)\, p_t(s)\, ds - \int_{[0,1]} x_t^{\mathrm{m}}(s)\, p_t(s)\, ds + \int_{[0,1]} x_{t-1}^{\mathrm{y}}(s)\, d_{t-1}(s)\, ds \\ &= \int_{[0,1]} \delta p_t(s)\, ds + \int_{[0,1]} \gamma d_{t-1}(s)\, ds \\ &= \delta\bar{p} + \tfrac{1}{2}\gamma d. \end{aligned} \tag{7}$$

In old age the budget constraint is

$$\begin{aligned} c^{\mathrm{o}} &= \int_{[0,1]} x_{t-1}^{\mathrm{m}}(s)(p_t(s) + d_{t-1}(s))\, ds = \int_{[0,1]} (\gamma-\delta)(p_t(s) + d_{t-1}(s))\, ds \\ &= (\gamma-\delta)(\bar{p} + \tfrac{1}{2}d). \end{aligned} \tag{8}$$

Goods market clearing is

$$c^{\mathrm{m}} + c^{\mathrm{o}} = 1 + \tfrac{1}{2}d. \tag{9}$$

Equations (6) through (9) can be reduced to

$$c^{\mathrm{m}} = \tfrac{1}{2}\gamma d + 2 - 1/\gamma \tag{10}$$

$$c^{\mathrm{o}} = (1-\gamma)(1/\gamma + \tfrac{1}{2}d). \tag{11}$$

The return on the market, r, is given by $r = (\bar{p} + \frac{1}{2}d)/\bar{p} - 1 = \frac{1}{2}d\gamma$, so that all the unknown variables may be written as functions of γ (hence of $\bar{p}$, by (6)).

It remains only to verify that there exists a value of γ at which the above quantities also satisfy utility maximization. In other words, given a value of γ, we can find the return on agents' portfolios and, hence, the optimal consumption levels. These must equal the quantities given in Eqs. (7) and (8). A simple continuity argument can be used to show that there exists a value of γ satisfying these conditions (for the details of a similar argument, see the Proof of Proposition 2, Step 6). ▮

IV.2. *Remark*

Assets are priced as the discounted value of all the future dividends. The market portfolio has price $\bar{p}$ and yields a dividend stream of $\frac{1}{2}d$ in perpetuity. The discounted value of the future dividends is $\frac{1}{2}d/r$, where $r = (\bar{p} + \frac{1}{2}d)/\bar{p} - 1$ is the rate of return. The discounted value of future dividends is then $\frac{1}{2}d/r = \frac{1}{2}d/[\bar{p} + \frac{1}{2}d)/\bar{p} - 1] = \bar{p}$. This shows that the market portfolio is priced as the value of all future dividends (see Dow and Gorton [9]). Combined with the expressions for relative prices given in Proposition 1, this implies that assets are priced as the discounted value of the expected future dividends.

Compare this result to the standard overlapping generations model of fiat money. Here, stock is valued by discounting the fundamental in the usual way. Money in overlapping generations models is viewed as a "bubble" since it yields a zero return, but it can be viewed as a limiting case of the present model as $d \to 0$.[16]

IV.3. *Example and Relation to the Standard Overlapping Generations Model*

The fully revealing equilibrium may be illustrated by considering the case of logarithmic utility as an example. Then the optimal choice of consumption satisfies $c^{\mathrm{m}} = c^{\mathrm{o}}/[\lambda(1+r)] = (1+r)/(1+\lambda)$. Using (9) and $r = \frac{1}{2}d\gamma$, we obtain $\gamma^2(\frac{1}{4}\lambda d) + \gamma(\frac{1}{2} + \gamma) - \frac{1}{2}(1+\lambda) = 0$. Given values for the exogenous parameters, d and λ, this equation can be solved for the equilibrium value of γ, and the equilibrium average price is given by $\bar{p} = 1/\gamma$. For example, when d and λ satisfy the equation $d = 6(1-\lambda)/[\lambda(1+\lambda)]$, then the solution is $\gamma = \frac{2}{3}$; this corresponds to the case where $\lambda = 1/(1+r)$ so $c^{\mathrm{m}} = c^{\mathrm{o}}$. The

[16] In case $d = 0$, we cannot price the asset by using the formula $\frac{1}{2}d/r$ since both numerator and denominator are 0.

value of r is $\frac{1}{3}d$ and $\bar{p}=\frac{3}{2}$. The individual prices are easily computed. For example, $p_{\mathrm{HH}}=\frac{3}{2}+\frac{1}{2}d/[1+\frac{1}{3}d]+\frac{1}{2}d/[1+\frac{1}{3}d]^2$.

To further explore the relationship between asset pricing in this model and in the standard overlapping generations model of fiat money, suppose $d=0$ (stocks do not pay dividends so they correspond to fiat money). Stock will have a positive value because it enables agents of different generations to trade with each other (Allais [2] and Samuelson [31]). The solution in this case is $p=(1+2\lambda)/(1+\lambda)$. Note that when $\lambda=0$, we have the standard two-generation overlapping generations model since no consumption occurs in old age. In that case $p=1$ as usual. For $\lambda>0$, the price will be higher than one because in each period only part of the total amount of securities will be exchanged for goods. The stock price is then determined by the quantity theory of money. For example, with $\lambda=1$, $c^{\mathrm{m}}=c^{\mathrm{o}}$, so one-third of the shares will be held by middle-aged consumers saving for old age, one-third will be exchanged for goods by middle-aged consumers, and the remaining third will be exchanged for goods by old consumers. The price level is $p=\frac{3}{2}$. Similarly when $\lambda\to\infty$, $p\to 2$.

IV.4. *Discussion of Portfolio Demands*

The fact that agents' demands are multi-valued is important for the equilibrium with adverse selection, to be analyzed subsequently. To understand this, we briefly discuss agents' demands in this fully revealing REE. Given the asset pricing condition agents are indifferent between all diversified portfolios of the same value,

$$\xi_t^{\mathrm{y}}=\left\{x_t^{\mathrm{y}}:\int_{[0,1]}x_t^{\mathrm{y}}(s)\,p_t(s)\,ds=1\right\}$$

$$\xi_t^{\mathrm{m}}=\left\{x_t^{\mathrm{m}}:\int_{[0,1]}x_t^{\mathrm{m}}(s)\,p_t(s)\,ds=(\gamma-\delta)\,\bar{p}\right\}$$

at the equilibrium prices, $p_t(s)=\phi(\omega_t(s),\omega_{t+1}(s))$. In equilibrium the auctioneer selects portfolios $x_t^{\mathrm{y}}=\gamma\mathbf{1}$, $x_t^{\mathrm{m}}=(\gamma-\delta)\,\mathbf{1}$, which belong to these demands; i.e., $\gamma\mathbf{1}\in\xi_t^{\mathrm{y}}$ and $(\gamma-\delta)\,\mathbf{1}\in\xi_t^{\mathrm{m}}$ (where the symbol $\mathbf{1}$ represents the function that takes the value 1 everywhere on $[0,1]$).

We now illustrate the application of the auctioneer's allocation rule, specified above. $S^{\mathrm{m}}=S^{\mathrm{y}}=[0,1]$ so in the notation of Section II.3.16, $y=1$, $m=(\gamma-\delta)\,\bar{p}$, so the allocations on $S^{\mathrm{m}}\cap S^{\mathrm{y}}=[0,1]$ are given by

$$x_t^{\mathrm{y}}(s)=y/(y+m)=1/(1+(\gamma-\delta)\,\bar{p})=\gamma$$

(since $\bar{p}=1/\gamma$ by Eq. (6) and $\delta=2\gamma-1$ by Eqs. (7)–(9) and similarly,

$$x_t^{\mathrm{m}}(s)=m/(y+m)=(\gamma-\delta)\,\bar{p}/(1+(\gamma-\delta)\,\bar{p})=\gamma-\delta.$$

V. Profitable Informed Trading: The Nonrevealing Equilibrium

In this section we show that there is an equilibrium in which the informed middle-aged can profitably speculate. In this equilibrium, prices will reflect the dividends to be paid this period. but not private signals about next period's dividends. Prices do not reflect private information.

PROPOSITION 2. *There exists an equilibrium in which asset prices do not reflect the private information and there is profitable informed trading.*

Proof. According to the definition of the nonrevealing price function, this equilibrium will be characterized by two numbers, $p_L = \phi(L, L) = \phi(L, H)$ and $p_H = \phi(H, L) = \phi(H, H)$. In each period, p_H is the price of shares about to pay a dividend. It is public knowledge that they will pay dividends this period and they are priced accordingly. Only half of these currently dividend-paying stocks will pay dividends next period, but only the informed agents know which ones and this is not reflected in the current price. We will refer to these as "undervalued" dividend-paying stocks. The other half of today's dividend-paying stocks will not pay dividends next period and these are referred to as "overvalued." Similarly, p_L is the price of shares which pay no dividend this period, half of which are undervalued and half overvalued.

The proof is in six steps. In Step 1 we derive the asset pricing property. In Step 2 we set out the market clearing conditions for goods and for stocks. In Step 3 we express all of the unknown variables as functions of a single one of them, γ_L. Then we turn to the agent's maximization problem. In Step 4, we verify that the (informed) middle-aged agent chooses to hold only undervalued non-dividend-paying stocks (a condition that was assumed in Steps 2 and 3). In Step 5, we verify that the limited smart money condition holds. In Step 6, we express the optimum consumption level in old age as a function of γ_L. A simple continuity argument shows existence of equilibrium.

Step 1. Since there is adverse selection, portfolios allocated to young agents will contain a disproportionately low fraction of undervalued stocks. In principle, this fraction could differ between the dividend-paying and non-dividend-paying stocks: indeed, this will turn out to be the case. We define π_H to be the fraction of currently dividend-paying stocks, among those allocated to the (uninformed) young, that are undervalued. Similarly, π_L is the fraction of currently non-dividend-paying stocks, of those allocated to the young, that are undervalued.

Adverse selection means that (at least one of) π_L and π_H are less than $\frac{1}{2}$, which is the fraction in the population of stocks. It turns out that $\pi_L \neq \pi_H$ in equilibrium, for reasons which will become apparent. In fact,

$\pi_H = \frac{1}{2}$ while $\pi_L < \frac{1}{2}$. Intuitively, the reason for this is that among undervalued shares, informed agents are not indifferent between dividend-paying and non-dividend-paying shares because they have higher required rates of return than the uninformed. Hence, the middle-aged value dividends less than the uninformed (this is verified in Step 3 below). The limited smart money condition ensures that when the informed demand portfolios with support (L, H), they will not exceed the total supply (verified in Step 4 below). For ease of exposition we will set out the next part of the proof without imposing $\pi_H = \frac{1}{2}$. We make this substitution in Step 3.

In this equilibrium the support of the young agents' demands is the entire interval; i.e., they are indifferent between buying a non-dividend-paying stock (p_L) and a dividend-paying stock (p_H). The difference ($p_H - p_L$) reflects not only the different dividends which will be paid this period, but also the different degrees of adverse selection, π_H and π_L. This gives the asset pricing property:

$$(p_H\pi_L + p_L(1-\pi_L))/p_L = (d + p_H\pi_H + p_L(1-\pi_H))/p_H. \tag{12}$$

Step 2. The young have one unit of the good to invest. The support of their demand is the whole set of stocks. Let γ_H be proportion of dividend-paying stocks bought by the young, and define γ_L similarly. The fraction of dividend-paying stocks is $\frac{1}{2}$ so

$$\tfrac{1}{2}\gamma_H = \int_{\{s\,:\,\omega_t(s)=H\}} x_t^y(s)\,ds,$$

is the mass of dividend-paying stocks bought by the young. Similarly, the mass of non-dividend-paying stocks bought by the young is

$$\tfrac{1}{2}\gamma_L = \int_{\{s\,:\,\omega_t(s)=L\}} x_t^y(s)\,ds.$$

Their budget constraint is therefore

$$\tfrac{1}{2}p_H\gamma_H + \tfrac{1}{2}p_L\gamma_L = 1. \tag{13}$$

In middle age consumers receive private information. They own the shares they bought when young and they have received some dividend income. On entering middle age, a consumer's asset holdings consist of

$\frac{1}{2}\gamma_H d$	Dividends
$\frac{1}{2}\gamma_H\pi_H + \frac{1}{2}\gamma_L\pi_L$	Dividend-paying shares
$\frac{1}{2}\gamma_H(1-\pi_H) + \frac{1}{2}\gamma_L(1-\pi_L)$	Non-dividend-paying shares.

Let δ be the mass of shares of undervalued non-dividend-paying stock sold by the middle-aged. The remainder of the portfolio is held until old age and then liquidated. These transactions may be summarized:

$\frac{1}{4}[\gamma_H \pi_H + \gamma_L \pi_L]$	Undervalued dividend-paying shares, all sold
$\frac{1}{4}[\gamma_H \pi_H + \gamma_L \pi_L]$	Overvalued dividend-paying shares, all sold
$\frac{1}{4}[\gamma_H(1-\pi_H) + \gamma_L(1-\pi_L)]$	Undervalued non-dividend-paying shares, of which δ sold
$\frac{1}{4}[\gamma_H(1-\pi_H) + \gamma_L(1-\pi_L)]$	Overvalued non-dividend-paying shares, all sold.

The middle-aged consume their dividends and the proceeds from liquidating part of their portfolio. They face the budget constraint

$$\begin{aligned} c^m = \tfrac{1}{2}\gamma_H d + \tfrac{1}{2} p_H[\gamma_H \pi_H + \gamma_L \pi_L] \\ + \tfrac{1}{4} p_L[\gamma_H(1-\pi_H) + \gamma_L(1-\pi_L)] + \delta p_L. \end{aligned} \tag{14}$$

The old consume the value of their remaining shares. An old agent owns a mass

$$\int_{[0,1]} x^m_{t-1}(s)\, ds = \tfrac{1}{4}[\gamma_H(1-\pi_H) + \gamma_L(1-\pi_L)] - \delta$$

of shares, all of which are about to pay dividends. Thus, his budget constraint is

$$c^o = p_H\{\tfrac{1}{4}[\gamma_H(1-\pi_H) + \gamma_L(1-\pi_L)] - \delta\}. \tag{15}$$

We now turn to the market clearing. conditions. As before, the goods market must clear:

$$c^m + c^o = 1 + \tfrac{1}{2}d. \tag{16}$$

Also, the stock market must clear. Since allocated portfolios are constant on the sets (H, H), (H, L), (L, H), and (L, L), it is sufficient to impose market clearing in aggregate on each of these sets. Equivalently, we can impose market clearing in aggregate over each of the four sets (H), (L), (H, H), and (L, H). In other words, we will check that demand equals supply for dividend-paying and non-dividend-paying stocks and for undervalued dividend-paying and non-dividend-paying stocks.

First, consider dividend-paying stock. The old sell $\frac{1}{4}[\gamma_H(1-\pi_H)+\gamma_L(1-\pi_L)]-\delta$, and the middle-aged sell $\frac{1}{2}\gamma_H\pi_H+\frac{1}{2}\gamma_L\pi_L$. The young buy $\frac{1}{2}\gamma_H$. Thus, market clearing requires that

$$\tfrac{1}{4}[\gamma_H(1-\pi_H)+\gamma_L(1-\pi_L)]-\delta+\tfrac{1}{2}\gamma_H\pi_H+\tfrac{1}{2}\gamma_L\pi_L=\tfrac{1}{2}\gamma_H. \tag{17}$$

Similarly, the market clearing condition for the non-dividend-paying stock is

$$\delta+\tfrac{1}{4}\gamma_H(1-\pi_H)+\tfrac{1}{4}\gamma_L(1-\pi_L)=\tfrac{1}{2}\gamma_L. \tag{18}$$

Next, market clearing requires that the mass of undervalued dividend-paying stock bought equals the mass sold:

$$\tfrac{1}{4}\gamma_H\pi_H+\tfrac{1}{4}\gamma_L\pi_L+\tfrac{1}{2}\{\tfrac{1}{4}[\gamma_H(1-\pi_H)+\gamma_L(1-\pi_L)]-\delta\}=\tfrac{1}{2}\gamma_H\pi_H. \tag{19}$$

The left-hand-side consists of all the undervalued dividend-paying stocks sold by the middle-aged, plus half of all the stocks sold by the old. The right-hand-side is the mass allocated to the young, by definition of π_H. Finally, the market clearing condition for undervalued non-dividend-paying stock is

$$\delta=\tfrac{1}{2}\gamma_L\pi_L. \tag{20}$$

The left-hand side is the mass sold by the middle-aged; the right-hand side is the mass bought by the young by definition of γ_L and π_L.

Step 3. We now impose the substitution $\pi_H=\frac{1}{2}$. Since middle-aged agents are not allocated any dividend-paying stock, the definition of γ_H implies that $\gamma_H=1$. In other words, all of the dividend-paying stocks are allocated to the young. Making the substitutions $\gamma_H=1$ and $\pi_H=\frac{1}{2}$ in the market clearing condition for dividend-paying stocks (17),

$$\gamma_L\pi_L+\gamma_L-4\delta=\tfrac{1}{2}, \tag{21}$$

the asset pricing property (12) becomes

$$(p_H\pi_L+p_L(1-\pi_L))/p_L=(d+\tfrac{1}{2}p_H+\tfrac{1}{2}p_L)/p_H, \tag{22}$$

and Eq. (13) gives

$$p_H=2-p_L\gamma_L. \tag{23}$$

Now use the market clearing condition for undervalued non-dividend paying stocks (20) to eliminate δ from (14), (15), and (21), and substitute $\gamma_H=1$ and $\pi_H=\frac{1}{2}$:

$$c^{\mathrm{m}} = \tfrac{1}{2}d + \tfrac{1}{2}p_{\mathrm{H}}[\tfrac{1}{2} + \gamma_{\mathrm{L}}\pi_{\mathrm{L}}] + \tfrac{1}{4}p_{\mathrm{L}}[\tfrac{1}{2} + \gamma_{\mathrm{L}}\pi_{\mathrm{L}} + \gamma_{\mathrm{L}}] \tag{24}$$

$$c^{\mathrm{o}} = \tfrac{1}{4}p_{\mathrm{H}}[\tfrac{1}{2} + \gamma_{\mathrm{L}} - 3\gamma_{\mathrm{L}}\pi_{\mathrm{L}}] \tag{25}$$

$$\gamma_{\mathrm{L}} - \gamma_{\mathrm{L}}\pi_{\mathrm{L}} = \tfrac{1}{2}. \tag{26}$$

Equation (26) may be rewritten as

$$\pi_{\mathrm{L}} = 1 - 1/(2\gamma_{\mathrm{L}}) \tag{27}$$

Eliminating p_{H} and π_{L} from the asset pricing property, using (23) and (27), gives

$$p_{\mathrm{L}}^2[\gamma_{\mathrm{L}}^3 - \gamma_{\mathrm{L}}] + p_{\mathrm{L}}[-4\gamma_{\mathrm{L}}^2 + \gamma_{\mathrm{L}} - \gamma_{\mathrm{L}}d + 1] + [4\gamma_{\mathrm{L}} - 2] = 0. \tag{28}$$

We have now derived equations which define values of consumption, and the other endogenous variables, which are consistent with market clearing, budget balance, and the asset pricing property, but not necessarily all conditions for utility maximization (there are three equations which we did not use: goods market clearing (16), market clearing for non-dividend paying stocks (18), and market clearing for undervalued dividend paying stocks (19): it may be verified that these three equations are redundant so the system is not overdetermined). These values are all functions of γ_{L}:

$$\gamma_{\mathrm{H}} = 1 \tag{29}$$

$$\pi_{\mathrm{H}} = \tfrac{1}{2} \tag{30}$$

$$\pi_{\mathrm{L}} = 1 - (\tfrac{1}{2}\gamma_{\mathrm{L}}) \tag{31}$$

$$\delta = \tfrac{1}{2}\gamma_{\mathrm{L}} - \tfrac{1}{4} \tag{32}$$

$$p_{\mathrm{L}}^2[\gamma_{\mathrm{L}}^3 - \gamma_{\mathrm{L}}] + p_{\mathrm{L}}[-4\gamma_{\mathrm{L}}^2 + \gamma_{\mathrm{L}} - \gamma_{\mathrm{L}}d + 1] + [4\gamma_{\mathrm{L}} - 2] = 0 \tag{33}$$

$$p_{\mathrm{H}} = 2 - p_{\mathrm{L}}\gamma_{\mathrm{L}} \tag{34}$$

$$c^{\mathrm{m}} = 1 + \tfrac{1}{2}d - \tfrac{1}{2}p_{\mathrm{H}}(1 - \gamma_{\mathrm{L}}) \tag{35}$$

$$c^{\mathrm{o}} = \tfrac{1}{2}p_{\mathrm{H}}(1 - \gamma_{\mathrm{L}}). \tag{36}$$

Step 4. Above we asserted that the informed middle-aged agents prefer to hold undervalued non-dividend-paying stock rather than undervalued dividend-paying stock. The middle-aged have a fixed amount of money to invest in the market. Intuitively, of each dollar invested in dividend-paying stocks, part buys the current dividend payment, and the rest buys capital appreciation/loss (the claim to all future dividends, which can be resold next period). Each dollar invested in non-dividend-paying stocks goes entirely on the capital appreciation/loss. Since the middle-aged have no comparative advantage in buying current dividends, but do have a comparative advantage in correctly predicting which stocks will pay dividends next period and accordingly will be priced higher, they prefer non-dividend-paying stocks. They will if $(p_H + d)/p_H < p_H/p_L$, i.e.,

$$1 + d/p_H < p_H/p_L. \tag{37}$$

We now provide an algebraic verification of this inequality. Solving (31) and (34) simultaneously to eliminate γ_L and substituting into the asset pricing property (22) to eliminate π_L gives

$$[2p_H - p_H^2 - p_H p_L + p_L^2 + p_L - (p_L^2/p_H)]/(2 - p_H)\, p_L = d/p_H + 1,$$

which is the left-hand side of inequality (37). Making this substitution gives

$$(p_L/p_H)(1 - p_H)(p_H - p_L) < 0.$$

This inequality holds since $p_H > p_L$ and $p_H > 1$. To verify that $p_H > 1$, recall the young's budget equation (13): $\frac{1}{2} p_H \gamma_H + \frac{1}{2} p_L \gamma_L = 1$. Since $\gamma_H = 1$ and $\gamma_L < 1$, $\frac{1}{2} p_H + \frac{1}{2} p_L > 1$. Now $p_H > p_L$ implies that $p_H > 1$. This completes the demonstration that middle-aged agents choose to hold undervalued non-dividend-paying stock instead of undervalued dividend-paying stock.

Step 5. In this step we will verify that the limited smart money condition is satisfied; i.e., that the market value of middle-aged portfolios is less than the market value of the undervalued non-dividend-paying shares. We have assumed that $c^o < \frac{1}{20}$ on all budgets in a specified set, and that $d < \frac{1}{5}$. We now show, first, that the budgets that can arise in the model do lie within this set, and second, that if $c^o < \frac{1}{20}$ and $d < \frac{1}{5}$, then the limited smart money condition holds.

The middle-aged choose consumption to solve the maximization problem

$$\text{Max}_{(c^m, c^o)}\; U(c^m) + \gamma U(c^o) \qquad \text{s.t. } 1 + r^y = c^m + c^o/(1 + r^m),$$

where r^m is the return on a middle-aged portfolio, and r^y is the return on a young portfolio:

$$1 + r^y = (d + \tfrac{1}{2} p_H + \tfrac{1}{2} p_L)/p_H = [p_H \pi_L + p_L(1 - \pi_L)]/p_L$$
$$1 + r^m = p_H/p_L.$$

We start by considering the asset pricing property (22). Since $\pi_L > 0$, this implies that

$$p_H < p_L + 2d; \tag{38}$$

the price of dividend-paying stocks must be less than it would be if there were complete adverse selection among non-dividend-paying stocks ($\pi_L = 0$). Since $1 + r^m = p_H/p_L$, we have

$$1 + r^m < 1 + 2d/p_L. \tag{39}$$

The budget constraint of the young is $\frac{1}{2}p_H + \frac{1}{2}p_L\gamma_L = 1$ (substituting $\gamma_H = 1$ into (13)). Again using $1 + r^m = p_H/p_L$, we have

$$1 + r^m + \gamma_L = 2/p_L. \tag{40}$$

Eliminating $2/p_L$ from (39) using (40) gives $1 + r^m < (1 + d\gamma_L)/(1 - d) < (1 + d)/(1 - d)$, where the last inequality follows from $\gamma_L < 1$. Now observe that $1 + r^y = [p_H\pi_L + p_L(1 - \pi_L)]/p_L > 1$, since $p_H > p_L$. Thus, $1 < 1 + r^y < 1 + r^m < (1 + d)/(1 - d)$. Setting $k = 1 + r^y$ and $R = 1 + r^m$ shows that the budget lies within the set specified in the limited smart money condition of Section II.3.8.

The limited smart money condition states that $\frac{1}{4}p_L > 1 + r^y - c^m$. The left-hand side is the market value of the total amount of non-dividend-paying undervalued stocks (half the stocks are non-dividend paying, of which half are also undervalued). The right-hand side is the value of the portfolio of the middle-aged: they enter middle age with a portfolio (including dividend income) worth $1 + r^y$, of which they consume c^m. Substituting, $1 + r^y = (d + \frac{1}{2}p_H + \frac{1}{2}p_L)/p_H$, and eliminating c^m using goods market clearing (16), we write the limited smart money condition as

$$\begin{aligned} c^o &< 1 + \tfrac{1}{2}d + \tfrac{1}{4}p_L - (d + \tfrac{1}{2}p_H + \tfrac{1}{2}p_L)/p_H \\ &= [1 - (\tfrac{1}{2}p_H + \tfrac{1}{2}p_L)/p_H] + [d(1 - 1/p_H)] + [\tfrac{1}{4}p_L - \tfrac{1}{2}d]. \end{aligned}$$

The first term in square brackets is positive since $p_H > p_L$. The second term is positive since $p_H > 1$. Thus, it suffices to show that

$$c^o < \tfrac{1}{4}p_L - \tfrac{1}{2}d. \tag{41}$$

By (38) above, $p_L > p_H - 2d > 1 - 2d$, where the second inequality follows since $p_H > 1$. Since by assumption $d < \frac{1}{5}$, we have $\frac{1}{4}p_L > \frac{1}{4}(1 - 2d) > \frac{1}{4}(1 - \frac{2}{5}) = \frac{3}{20} = \frac{1}{10} + \frac{1}{20} > \frac{1}{2}d + \frac{1}{20}$. Thus, the right hand side of inequality (41) exceeds $\frac{1}{20}$ so the assumption on c^o ensures that the limited smart money condition is satisfied. This completes Step 5.

Step 6. Next we turn to the agent's consumption-savings decision. Given rates of return on the young and middle-aged portfolios, the agent will choose utility-maximizing consumption levels in middle age and old age. As we will show, given the market clearing and asset pricing conditions, these rates of return are also functions of γ_L. Equilibrium requires that these utility-maximizing consumption choices be equal to the market-clearing consumption levels derived in Step 3, (35) and (36).

Let $c^o(\gamma_L)$ denote the consumption level in old age consistent with market clearing and the asset pricing property. Substituting (34) into (36) gives

$$c^o(\gamma_L) = \tfrac{1}{2}(2 - p_L\gamma_L)(1 - \gamma_L) \tag{42}$$

where p_L solves Eq. (33).

The agent chooses consumption in middle age and old age. Let $\psi^o(\gamma_L)$ be the agent's optimal choice of consumption in old age; i.e., (ψ^m, ψ^o) is the solution to

$$\mathrm{Max}_{(c^m, c^o)}\ U(c^m) + \gamma U(c^o) \qquad \text{s.t.} \quad 1 + r^y = c^m + c^o/(1 + r^m),$$

where r^m is the return on a middle-aged portfolio, and r^y is the return on a young portfolio

$$1 + r^y = (d + \tfrac{1}{2}p_H + \tfrac{1}{2}p_L)/p_H = [p_H\pi_L + p_L(1 - \pi_L)]/p_L$$

$$1 + r^m = p_H/p_L,$$

and π_L, p_L, and p_H are determined as functions of γ_L according to Eqs. (31), (33), and (34).

It remains to find an equilibrium value of γ_L, i.e., a value at which $\psi^o(\gamma_L) = c^o(\gamma_L)$. Since γ_L is the fraction of non-dividend-paying stocks which are traded every period, and since all of the overvalued non-dividend-paying stocks are traded, it follows that $\gamma_L \in [\tfrac{1}{2}, 1]$. We now show that for large values of γ_L, $\psi^o(\gamma_L > c^o(\gamma_L)$, while for small γ_L the reverse is true. The existence of an equilibrium value of γ_L follows by continuity. First consider $\gamma_L = \tfrac{1}{2}$. Then (33) becomes $-\tfrac{3}{8}p_L^2 + \tfrac{1}{2}p_L(1 - d) = 0$. The solution for p_L is[17] $p_L = \tfrac{4}{3}(1 - d)$. Thus, by Eq. (42), $c^o(\tfrac{1}{2}) = \tfrac{1}{3}(1 + \tfrac{1}{2}d)$. Since our (limited smart money) condition on preferences (see Section II.3.8) is $\psi^o(\gamma_L) \leq \tfrac{1}{20}$, it follows that $\psi^o(\tfrac{1}{2}) < \tfrac{1}{20} < \tfrac{1}{3} < c^o(\tfrac{1}{2})$. Now consider $\gamma_L = 1$. Then (42) gives $c^o(1) = 0$. Since, by assumption (see Section II.3.3), $U'(0) = \infty$, $\psi^o(\gamma_L) > 0$, $\psi^o(1) > 0 = c^o(1)$. Since Eqs. (29)–(36) are all continuous, $c^o(\gamma_L)$ is continuous in γ_L. Since the utility function is strictly concave, $\psi^o(\gamma_L)$ is continuous in γ_L. By the Intermediate Value Theorem, there exists γ_L such that $\psi^o(\gamma_L) = c^o(\gamma_L)$. ∎

[17] The solution $p_L = 0$ is not economically meaningful. Also, note that $p_L = \tfrac{4}{3}(1 - d)$ is the limit of the positive root as γ_L approaches $\tfrac{1}{2}$.

VI. Implementation

Fully revealing REE is a familiar concept, but the reader might question the relevance of our equilibrium with profitable informed trading and adverse selection, since the definition of equilibrium used above does not specify explicitly the set of strategies available to each agent and then determine the Nash equilibria. In other words, it does not describe an institutional arrangement to support the allocations and prices of the stocks. In this section, we briefly describe an example of an institution that does implement the equilibrium with profitable informed trading.

Consider the following institution. Stocks are traded via a competitive central broker who acts in a pure agency capacity (he matches buyers and sellers rather than meeting orders out of his own inventory). Agents who wish to sell turn their stocks over to the broker (they cannot both buy and sell in the same period). The broker posts a price for each stock, but does not reveal the quantity of each stock available for sale. Buyers observe the prices and are then allowed to submit a list of stocks they are willing to buy and a total amount of money to be invested. The broker may then choose any fully diversified portfolio composed only of stocks on the list, and with a total value that exhausts their budget.

In setting the share prices, the broker has the following objective: he must set prices such that all shares offered for sale are in fact sold, and he maximizes sales proceeds to the sellers. Note that this objective could be enforced as the result of competition among potential brokers who advertise their price-setting rules. If the central broker announced a price-setting rule which would result in some stocks remaining unsold, or the buyers' budget not being exhausted, then a competitor could offer a contract preferred by the sellers in exchange for a small brokerage fee (of course competition would then eliminate this fee). It may be verified that the above non-revealing equilibrium is an equilibrium of this game. Indeed, our definition of equilibrium with adverse selection corresponds quite closely to the Nash equilibria of this mechanism.

The above description specified that middle-aged agents could not sell all of their stocks and then buy some back. But this makes no difference: with the institution as described above, the broker will have different quantities of different stocks for sale, but the buy orders will all ask for the entire set of stocks. We can consider an alternative institution where, instead, the middle-aged sell all their stocks and then repurchase some. Then the broker will have the entire stock market for sale, but will receive some orders for all stocks, and some orders only for the undervalued non-dividend-paying stock. The same equilibrium prices will clear the markets as before.

We can also give a description of a sequential allocation process in which each stock is sold by a broker who posts a price. Buyers (as a group) visit brokers for each stock in succession and, observing the price, each decides how much to demand. If the demands exceed the available supply, the broker rations the stock in proportion to the demands submitted. Buyers continue until their budget is exhausted. This mechanism is reminiscent of Rock [29], with the main difference being that our model is general equilibrium which requires that investors' budgets be exhausted in the stock buying process.

As before, the allocation process could be designed in two alternative ways. First, the middle-aged could simply withhold the stocks they want to keep, and the only buyers would be the uninformed young. In this case, rationing would occur for the undervalued non-dividend-paying stocks because the supply of these is smaller. Alternatively, if the middle-aged were to sell their entire portfolios and then buy back the shares they want, then the same stocks would be rationed because the supply would be the same as other stocks but the demand would be greater. The middle-aged would have to inflate their demands (knowing the scaling factor that would result from the rationing rule) so that the scaled-down amount they actually received would be their desired amount. With this sequential process, the prices and allocations from our equilibrium with profitable informed trading would clear the market, with agents' budgets exactly exhausted at the last stock.[18]

VII. Profitable Informed Trading: Discussion

How can the middle-aged profit at the expense of the young in equilibrium? The asset-pricing property of the model means that the young are indifferent between all portfolios whose value equals their wealth. In equilibrium, however, the portfolio they receive is adversely selected and so their return is less than the return on the market portfolio. The equilibrium depends upon the inability of the uninformed to buy the market portfolio, or to mimic the demands of the informed agents. We discuss below how this inability arises in the equilibrium.

The inability to buy the market portfolio is an essential restriction imposed in a number of important papers on profitable informed trading: Grossman and Stiglitz [15], Kyle [22], Glosten and Milgrom [12], and related models. In any model of profitable informed trading, the uninformed earn lower returns than the informed. Clearly, they would ideally like to be able to mimic the informed and earn a superior return, but are

[18] Note that as they visit successive stocks, agents do not receive information about the remaining stocks because at any point in the process, exactly half the non-dividend-paying stocks visited would be revealed as undervalued through rationing.

unable to do so. Since there are two categories of agents, informed and uninformed, and the uninformed earn a lower return than the informed, it follows that the uninformed earn a below-average return. However, if they could at least buy the market portfolio, they would earn the average return. Thus, any such model must not only prevent the uninformed from mimicking the informed—which seems an innocuous restriction—but must also prevent them from buying the market (see the Theorem of Section III.3 above). In the following discussion, the reader should bear in mind that each of the above three models, Grossman and Stiglitz [15], Kyle [22], and Glosten and Milgrom [12], has two assets that the agent can hold: "money" and "the stock." Therefore, the market portfolio of assets consists of a quantity of money and a quantity of stock. (In these models, the stock of the riskless asset is endogenous since it is viewed as a riskless lending technology in infinitely elastic supply.) In these models the return on the market is zero. Uninformed agents would like to be able to hold the two assets in proportion to their net supply, but are unable to do so. In the equilibrium, they end up holding a disproportionately large amount of money when the stock is undervalued, and a disproportionately large amount of stock when the stock is overvalued.

Grossman and Stiglitz [15] is an REE model in which there is a random supply of the risky asset about which uninformed agents have no information. They are unable to buy the market because they do not know what the market portfolio is.

In Kyle [22], agents submit market orders to a specialist who then executes trades at a price which is set conditional on the orders submitted. There are "liquidity traders" who must trade a random amount. They can be viewed as exogenous perturbations to supply, as in Grossman and Stiglitz [15]. Alternatively, they can perhaps be viewed as uninformed agents trading for exogenous reasons.[19] In the latter case, the agents submit an order for a fixed quantity of the stock before they find out the price. This determines the amount of cash in their portfolio, and therefore they cannot choose to hold any predetermined portfolio, in particular, the market portfolio. The specialist adversely selects the portfolio of the uninformed by choosing a price for the stock which (on average) gives the uninformed a disadvantageous amount of cash. The specialist earns zero expected profit, which is the market return. Informed agents earn positive expected profit

[19] The liquidity traders are usually interpreted as trading for exogenous "liquidity" needs, which are interpreted symmetrically with respect to buying and selling. If we confine attention to sales, we can motivate them as reflecting immediate needs for cash. Since the liquidity traders do not know how much cash their stock trades will net and cannot even guarantee a lower bound on any stock sale (because the expected price is normally distributed, hence has unbounded support), it is hard to provide a detailed rationalization of their behavior.

(a return higher than the market return), at the expense of the uninformed (whose return is below the market return).

Glosten and Milgrom [12] model the behavior of a specialist who posts bid and ask prices, knowing that the population of traders consists of some informed traders and some uninformed traders. Bid and ask prices are set to be "regret free," that is, to be the correct valuation of the stock conditional on a sell or a buy, respectively. There is a positive spread between the ask and the bid, reflecting the adverse selection. The specialist makes zero profits on average (the market return). The informed traders earn positive profits despite the bid–ask spread (above the market return). Hence, the uninformed make less than the market return, as a result of the bid–ask spread.

In our model, the restriction that agents cannot buy the market portfolio may seem surprising. Since all agents are optimizing, have the same strategies available, and understand the model, why can the uninformed not buy the market? We now discuss how the restriction results from the presence of adverse selection in our equilibrium with profitable informed trading.

In the equilibrium with profitable informed trading, uninformed traders earn lower returns because the middle-aged can distinguish good stocks from bad stocks. Consider the agents' demands at the equilibrium asset prices. The asset pricing property of the model states that, at the equilibrium prices, all assets are perfect substitutes from the uninformed agents' point of view. In other words, since the young are uninformed they submit multi-valued demands (containing all portfolios satisfying their budget conditions) to the auctioneer in equilibrium.[20] On the other hand, the demands of the informed middle-aged only contain portfolios with positive quantities of good non-dividend-paying stocks, and zero quantities of all other stocks. To clear the market the auctioneer must assign portfolios to the young which contain a correspondingly low proportion of good non-dividend-paying stocks. Young agents receive an adversely selected portfolio.

Although young agents are *ex ante* indifferent between all portfolios satisfying their budget constraint (by the asset pricing condition, Eq. (12)), they would prefer, *ex post*, to receive a different allocation to that they

[20] Kreps [21, Section 5] gives an example of a two-person speculative insurance market, which depends on the uninformed agent being risk-neutral over a specific range of wealth. This leads to a multivalued demand. Since the utility function is not strictly concave, the example is nongeneric. The counterpart in our model is the asset pricing property: agents' demands are completely elastic at the equilibrium prices, or, in other words, their demands are multivalued. This suggests that our model may also be nonrobust. For example, in a world with a large but finite number of assets there will always be a small amount of non-diversifiable market risk, and risk aversion will lead to highly elastic, but single-valued, demands (the asset pricing property would reappear if agents were risk-neutral). However, the asset pricing framework used here seems of economic interest because it is the standard paradigm in the theory of finance.

actually received. This is the point noted by Kreps [21], who viewed it as an undesirable feature of the equilibrium.

In what ways are agents not indifferent *ex post*, and what is it they cannot do, *ex ante*, to prevent this? Their first preference would be to receive a portfolio consisting only of good stocks. Failing that, they would prefer to have received the market portfolio.[21] Selecting only the good stocks would require them to submit demands which condition not only on the price, but also on other agents' demands. REE does not allow this, and it seems quite reasonable to rule out such strategies. On the other hand, obtaining the market portfolio could be achieved simply by selecting one element from their (multi-valued) demand and submitting this "untruthful" demand to the auctioneer. Our equilibrium concept (and REE) does not admit this possibility.

The reader may take one of two points of view concerning the role of the restriction that agents cannot buy the market in models of informed trading. First, the reader may feel that the restriction is unjustifiable. In that case he or she will find the above models, and any model of profitable informed trading, unpersuasive (see the Theorem of Section III.3). Second, he may feel that profitable informed trading is possible in reality. In that case the restriction arises in the context of whatever trading institution is used. Our equilibrium concept is a reduced form, and therefore does not give the details of how the restriction arises (see Section VI on implementation for an example).

Note that the uninformed could avoid adverse selection by only buying dividend-paying stocks ($\pi_H = \frac{1}{2}$; see Step 1 of the Proof of Proposition 2), but this would not benefit them.[22] By the asset pricing condition, these stocks are priced high enough that uninformed agents are indifferent between buying dividend-paying stocks and non-dividend-paying stocks.

VIII. Profitable Informed Trading: Welfare

The model can be used to address questions of welfare economics. Our model is a pure exchange economy in which the only decisions concern the

[21] Note that there is no adverse selection among the dividend-paying stocks. Therefore, among portfolios which contain 50% bad stocks and 50% good stocks, they would prefer one consisting only of non-dividend-paying stocks. One consisting only of dividend-paying stocks would yield a return r_y, the same return as the adversely selected portfolio assigned by the auctioneer. The market portfolio, containing equal fractions of dividend-paying and non-dividend-paying stocks, yields an intermediate return. This return still exceeds the return on their equilibrium portfolio.

[22] It would be equivalent to buy stock only from old people, who sell a nonadversely selected portfolio of dividend-paying stocks.

intertemporal allocation of consumption. Agents are symmetric, that is, all will be uninformed when selecting initial portfolios, all will become informed upon reaching middle age, and all will sell their portfolios in old age. We can show that the consumption allocation is distorted in the equilibrium with profitable informed trading, causing a welfare loss.[23] The welfare criterion used is based on utility in stationary equilibrium, as in the literature on the "golden rule" (Allais [2], Phelps [25]).

PROPOSITION 3. *Agents' utility is higher in the fully revealing equilibrium than in the equilibrium with profitable informed trading.*

The proof, which follows below, depends on showing that middle-aged consumers save less, and so consume more, in the fully revealing equilibrium than in the equilibrium with profitable informed trading. A revealed preference argument then shows that in the fully revealing equilibrium the agents could have afforded the consumption levels from the equilibrium with profitable informed trading with income to spare, but made other choices. Thus, agents' welfare is higher in the fully revealing equilibrium.

The demonstration that the middle-aged save less in the fully revealing equilibrium is not straightforward. If the rate of return on the market were the same in the two equilibria, then this would follow immediately from the first-order conditions for utility maximization. Middle-aged agents would save more the higher the rate of return they earn. However, because middle-aged agents save more in the equilibrium with profitable informed trading, a smaller fraction of shares change hands every period. This lower "velocity of circulation" causes the average share price to be higher in the equilibrium with profitable informed trading. Hence, the return on the market is lower. In other words, the middle-aged agents earn a higher return than the market, but the market return is, itself, lower than in the fully revealing equilibrium. Thus, it does not follow immediately that the middle-aged agents earn a higher return than they would in fully revealing equilibrium.

To show this requires a more complex argument. If consumption in middle age is lower in the fully revealing equilibrium, then the return on the market in the fully revealing equilibrium must exceed the return to the middle-aged with profitable informed trading by the first-order conditions for utility maximization. But if consumption in middle age is higher in the profitable informed trading equilibrium, the velocity of circulation of the shares will also be higher. This implies that share prices, on average, are lower in the profitable informed trading equilibrium. Hence, there is higher return on the market (since the market return is the dividend yield divided

[23] In fact, even without profitable informed trading the return on the market would exceed the socially optimal discount rate in this type of model. See Dow and Gorton [9].

by the average share price). This higher return on the market is inconsistent with the conclusions from the first-order conditions for utility maximization which imply a lower return for middle-aged agents with profitable informed trading than in fully revealing equilibrium. In other words, the assumption that consumption in middle age is higher with profitable informed trading leads to a contradiction.

The proof is more involved than the intuitive argument given in the previous paragraph because the heterogeneity of the shares must be accounted for; i.e., different stocks have different velocities of circulation.

Proof. Let $(c^{\mathrm{m}}, c^{\mathrm{o}})$ be the consumption choices in the fully revealing equilibrium, and $(c^{\mathrm{m}*}, c^{\mathrm{o}*})$ be the consumption choices in the equilibrium with profitable informed trading. By goods market clearing, $c^{\mathrm{m}} + c^{\mathrm{o}} = c^{\mathrm{m}*} + c^{\mathrm{o}*} = 1 + \frac{1}{2}d$. Let r be the rate of return on the market in the fully revealing equilibrium, r^* the rate of return on the market in the equilibrium with profitable informed trading, and $r^{\mathrm{m}*}$ the return to the middle-aged in the profitable informed trading equilibrium. As before, $\bar{p}$ is the average price in the fully revealing equilibrium, while p_{L} and p_{H} are the prices in the equilibrium with profitable informed trading.

Step 1. We first show that $c^{\mathrm{m}} > c^{\mathrm{m}*}$. Suppose that, on the contrary, $c^{\mathrm{m}} \leqslant c^{\mathrm{m}*}$. Then $c^{\mathrm{o}} \geqslant c^{\mathrm{o}*}$ by the goods market clearing condition. We will derive a contradiction from the following three conditions:

(A) By the first-order conditions for utility maximization, $c^{\mathrm{o}} \geqslant c^{\mathrm{o}*}$ implies that $r \geqslant r^{\mathrm{m}*}$.

(B) $c^{\mathrm{o}} \geqslant c^{\mathrm{o}*}$ implies that $(1 - 1/\bar{p})(\bar{p} + \frac{1}{2}d) \geqslant \frac{1}{2}(p_{\mathrm{L}} + p_{\mathrm{H}} - 2)\, p_{\mathrm{H}}/p_{\mathrm{L}}$.

(C) $r^{\mathrm{m}*} > r^*$.

We will now derive conditions A and B. Condition C is obvious.

Derivation of A. Suppose $c^{\mathrm{o}} \geqslant c^{\mathrm{o}*}$. Since $(c^{\mathrm{m}}, c^{\mathrm{o}})$ and $(c^{\mathrm{m}*}, c^{\mathrm{o}*})$ are chosen by the agents,

$$U'(c^{\mathrm{m}})/[\lambda U'(c^{\mathrm{o}})] = (1 + r)$$

$$U'(c^{\mathrm{m}*})/[\lambda U'(c^{\mathrm{o}*})] = (1 + r^{\mathrm{m}*})$$

by the first-order conditions for the utility maximization problems. Since the utility function is concave, $U'(c^{\mathrm{m}}) \geqslant U'(c^{\mathrm{m}*})$ and $U'(c^{\mathrm{o}}) \leqslant U'(c^{\mathrm{o}*})$, so $U'(c^{\mathrm{m}*})/\lambda U'(c^{\mathrm{o}*}) \leqslant U'(c^{\mathrm{m}})/\lambda U'(c^{\mathrm{o}})$. By the first-order conditions, $1 + r^{\mathrm{m}*} \leqslant 1 + r$.

Derivation of B. Suppose $c^{\mathrm{o}} \geqslant c^{\mathrm{o}*}$, i.e., $(1 - \gamma)(1/\gamma + \frac{1}{2}d) \geqslant \frac{1}{2}(1 - \gamma_{\mathrm{L}})\, p_{\mathrm{H}}$. The right-hand side is given by Eq. (36) and the left-hand side is given by

Eq. (11). Condition B is obtained by substituting for γ_L and γ using Eqs. (6) and (34), i.e., $\gamma = 1/\bar{p}$ and $\gamma_L = (2 - p_H)/p_L$.

Completion of Step 1. We will now use conditions A, B and C to derive a contradiction. Condition B can be rewritten $\frac{1}{2}d/\bar{p} \leqslant \bar{p} - 1 + \frac{1}{2}d - \frac{1}{2}(p_L + p_H - 2)\, p_H/p_L$. Combining this with Condition A, we obtain $(p_H - p_L)/p_L \leqslant \bar{p} - 1 + \frac{1}{2}d - \frac{1}{2}(p_L + p_H - 2)\, p_H/p_L$. Rewriting this,

$$\tfrac{1}{2}(p_H/p_L)(p_L + p_H) \leqslant \bar{p} + \tfrac{1}{2}d. \tag{43}$$

Condition C states that $p_H/p_L > \frac{1}{2}d[\frac{1}{2}(p_H + p_L) + 1]$. We can replace p_H/p_L in (43) with this smaller number,

$$\tfrac{1}{2}(p_L + p_H)[\tfrac{1}{2}d + \tfrac{1}{2}p_H + \tfrac{1}{2}p_L]/(\tfrac{1}{2}p_H + \tfrac{1}{2}p_L) < \bar{p} + \tfrac{1}{2}d,$$

or $\frac{1}{2}(p_H + p_L) < \bar{p}$. Combining condition A $(r \geqslant r^{m*})$ with condition C $(r^{m*} > r^*)$, it follows that $r > r^*$ which means that average prices are lower: $\bar{p} < \frac{1}{2}(p_L + p_H)$, a contradiction. This shows that $c^o < c^{o*}$.

Step 2. Note that

$$\begin{aligned} c^m + c^o/(1+r) &= [(1+r)\, c^m + c^o]/(1+r) = [c^m + c^o + rc^m]/(1+r) \\ &= [c^{m*} + c^{o*} + rc^m]/(1+r) > [(1+r)\, c^{m*} + c^{o*}]/(1+r) \\ &= c^{m*} + c^{o*}/(1+r), \end{aligned}$$

where the inequality follows from Step 1 and we have made use of goods market clearing:

$$c^m + c^o = 1 + \tfrac{1}{2}d = c^{m*} + c^{o*}.$$

In the fully revealing equilibrium the agents could have afforded the consumption levels (c^{m*}, c^{o*}) with income to spare, but chose (c^m, c^o) instead. By revealed preference, agents' welfare is higher in the fully revealing equilibrium. ∎

In the equilibrium with profitable informed trading, agents save too much in middle age because they earn an excessive rate of return. Naturally, it is not our contention that this intertemporal consumption distortion would be the predominant welfare effect in alternative models of profitable informed trading. Our model has no production and no investment.[24] Our purpose is to illustrate how a general equilibrium approach can be used to address these questions.

[24] Ausubel [6] discusses welfare in a model with investment.

References

1. G. A. Akerlof, The market for "lemons": Quality uncertainty and the market mechanism, *Quart. J. of Econ.* **84** (1970), 488–500.
2. M. Allais, "Economie et Interêt," Imprimerie Nationale, Paris, 1947.
3. R. Anderson, Non-standard analysis with applications to economics, *in* "Handbook of Mathematical Economics" (W. Hildenbrand and H. F. Sonnenschein, Eds.), Vol. IV, Chap. 39, North-Holland, Amsterdam, 1991.
4. R. Anderson and H. F. Sonnenschein, On the existence of rational expectations equilibrium, *J. Econ. Theory* **26** (1982), 261–278.
5. L. Ausubel, Partially-revealing rational expectations equilibrium in a competitive economy, *J. Econ. Theory* **50** (1990), 93–126.
6. L. Ausubel, Insider trading in rational expectations economy, *Amer. Econ. Rev.* **80** (1990), 1022–1041.
7. B. Biais and P. Hillion, Insider and liquidity trading in stock and options markets, *Rev. Finan. Stud.* **7** (1994), 743–780.
8. D. Diamond and R. Verrecchia, Information aggregation in a noisy rational expectations economy, *J. Finan. Econ.* **9** (1981), 221–235.
9. J. Dow and G. Gorton, The return on the market and the social discount rate: A note on welfare in the overlapping generations model, *Econ. Lett.* **43** (1993), 23–26.
10. J. Dow and G. Gorton, Arbitrage chains, *J. Finance* **49**(3) (1994), 819–850.
11. J. Dow, V. Madrigal, and S. R. Werlang, Preferences, common knowledge and speculative trade, London Business School, working paper, 1988.
12. L. Glosten and P. R. Milgrom, Bid, ask, and transaction prices in a specialist market with heterogeneously informed traders, *J. Finan. Econ.* **14** (1985), 71–100.
13. J. R. Green, Information, Efficiency, and equilibrium, Discussion Paper 284, Harvard Institute of Economic Research, 1973.
14. S. J. Grossman, The existence of futures markets, noisy rational expectations and informational externalities, *Rev. Econ. Stud.* **64** (1977), 431–449.
15. S. J. Grossman and J. E. Stiglitz, Information and competitive price systems, *Amer. Econ. Rev.* **66** (1976), 246–253.
16. S. J. Grossman and J. E. Stiglitz, On the impossibility of informationally efficient markets, *Amer. Econ. Rev.* **70** (1980), 393–408.
17. O. D. Hart and D. M. Kreps, Price destabilizing speculation, *J. Polit. Econ.* **94** (1986), 927–952.
18. J. S. Jordan, On the efficient markets hypothesis, *Econometrica* **51**(5) (1983), 1325–1343.
19. J. S. Jordan and R. Radner, Rational expectations in microeconomic models: An overview, *J. Econ. Theory* **26** (1982), 201–223.
20. K. L. Judd, The law of large numbers with a continuum of IID random variables, *J. Econ. Theory* **35** (1985), 19–25.
21. D. M. Kreps, A note on "fulfilled expectations" equilibria, *J. Econ. Theory* **14** (1977), 32–43.
22. A. S. Kyle, Continuous auctions and insider trading, *Econometrica* **53** (1985), 1315–1336.
23. R. E. Lucas, Asset prices in an exchange economy, *Econometrica* **46** (1978), 1429–1445.
24. P. R. Milgrom and N. Stokey, Information, trade and common knowledge, *J. Econ. Theory* **26** (1982), 17–27.
25. E. S. Phelps, The golden rule of accumulation: A fable for growthmen, *Amer. Econ. Rev.* **51** (1961), 638–643.
26. R. Radner, Équilibre des marchés à terme et au comptant en cas d'incertitude, *Cah. d'Écon.* **4** (1967), 35–52.
27. R. Radner, Rational expectations equilibrium: Generic existence and the information revealed by prices, *Econometrica* **47** (1979), 655–678.

28. J.-C. ROCHET AND J.-L. VILA, Insider trading without normality, *Rev. Econ. Stud.* **61** (1994), 131–152.

29. K. ROCK, Why new issues are underpriced, *J. Finan. Econ.* **15** (1986), 187–212.

30. A. RUBINSTEIN AND A. WOLINSKY, On the logic of "agreeing to disagree" type results, *J. Econ. Theory* **51** (1990), 184–193.

31. P. A. SAMUELSON, An exact consumption-loan model of interest with or without the social contrivance of money, *J. Polit. Econ.* **66** (1958), 467–482.