# Noise Trading, Delegated Portfolio Management, and Economic Welfare

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We consider a model of the stock market with delegated portfolio management. Managers try, but sometimes fail, to discover profitable trading opportunities. Although it is best not to trade in this case, their clients cannot distinguish "actively doing nothing," in this sense, from "simply doing nothing." Because of this problem, (i) some portfolio managers trade even though they have no reason to prefer one asset to another (noise trade). We also show that (ii) the amount of such noise trade can be large compared to the amount of hedging volume. Perhaps surprisingly, (iii) noise trade may be Pareto-improving. Noise trade may be viewed as a public good. Results i and ii are compatible with observed high levels of turnover in securities markets. Result iii illustrates some of the possible subtleties of the welfare economics of financial markets. In our model, all agents are rational: some trade for hedging reasons, investors optimally contract with portfolio managers who may have stock-picking abilities, and portfolio managers trade optimally given the incentives provided by this contract.

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## I. Introduction

We show the possibility of rational trade in securities markets that is not motivated by an informational advantage or by "genuine" motives such as hedging, portfolio rebalancing, or liquidity needs. It is pure noise trade: it does not maximize the utility of the owner of the portfolio. We also provide an example in which this noise trade makes all agents better off. We explain this apparent paradox in what follows.

We view these results in the context of two sets of issues. First, there is a long-standing debate concerning whether prices and trading volumes in securities markets reflect fundamentals or "animal spirits." To date there is no conclusive empirical test that has resolved this debate. For this reason the two points of view are essentially dogmas. On the one hand, there is the belief that participants in markets are "rational," and the few who are not are quickly eliminated by the natural selection effects of arbitrage. On the other hand, there is the belief that many individuals may not be rational and that there is no necessary tendency for irrationality to disappear in the aggregate. In other words, a market may display aggregate "irrationality," "fads," or "herding."

In the finance literature, this debate appears in the form of differing interpretations of "noise trade." Noise trade is a random error term introduced into net asset demand to allow agents with private information to profit by trading on their information (see Grossman and Stiglitz 1976, 1980; Kyle 1985). Noise traders lose money (on average) to privately informed traders. The rationale for noise trade is theoretical: noise is necessary for profitable informed trading, and profitable informed trading is necessary for market efficiency. Since the origin of noise trade was not explicitly modeled, different interpretations have arisen. It may be viewed as rational agents trading for liquidity or hedging motives, consistent with the rational or efficient-markets perspective (e.g., Diamond and Verrecchia 1981; Ausubel 1990*a*, 1990*b*; Biais and Hillion 1994; Dow and Gorton 1994, 1996; Dow 1995). Alternatively, noise trade may represent the actions of irrational agents (e.g., Black 1986; De Long et al. 1990). In this paper, we suggest a third possibility.

The second set of issues concerns not the existence, but the level, of trading volume: trading volume seems high. There appears to be a consensus that trading volume or turnover (trading volume as a fraction of total market value) is inexplicably high. For example, "it is difficult to imagine that the volume of trade in securities markets has very much at all to do with the modest amount of trading required to accomplish the continual and gradual portfolio rebalancing inherent in our current intertemporal models" (Ross 1989, p. 94). Hard evidence for this is difficult to find because economic models have not been developed to predict trading volume.[1] Dynamic hedging strategies without transactions costs would result in infinitely high volume. In the presence of transactions costs, optimal dynamic hedging would presumably lead to much reduced volume, but the literature has not developed to the point of providing a benchmark prediction (this seems like an interesting area for research). Below we briefly examine the evidence of "high" turnover in two markets: foreign exchange and the New York Stock Exchange. The evidence is not conclusive but is suggestive. There also appears to be an empirical link between turnover and agency problems, as proxied by the fraction of the market controlled by institutions and intermediaries.[2]

The daily trading volume of foreign exchange transactions in all currencies (including forwards, swaps, and spot transactions) in 1992 was U.S. $880 billion, according to the comprehensive survey for April 1992 carried out by the Bank for International Settlements (1993). To put this number in perspective, compare this *daily* volume to the total value of *annual* world trade in 1992. The total value of world trade in 1992 was $3,646 billion and foreign direct investment was $220 billion. Thus roughly one-quarter of the annual trade and investment flow is traded each day in the foreign exchange market.

Furthermore, of the $880 billion traded daily in the foreign exchange market, $648 billion was traded between financial intermediaries and dealers (see Bank for International Settlements 1993). In other words, most of the trading volume is made up of interbank transactions. Note that in the Bank for International Settlements survey, interbank transactions specifically exclude all transactions made on behalf of bank customers. Figure 1 shows the total and interbank trading volume in major currencies for spot transactions, forwards, and swaps. In each case, interbank trading volume makes up the bulk of transactions. For each of the three types of contracts, the correlation across currencies between total and interbank volume exceeds .99.

On the New York Stock Exchange (NYSE), turnover in 1992 was 48 percent. While there is no convincing theoretical prediction for

[1] Tobin (1988, p. 250) makes this point: "the accounts of asset markets standard in both economics and finance . . . do not explain the volume of *transactions*." See also Tobin (1978).

[2] Higher trading volume by intermediaries is also consistent with the explanation that they have lower transactions costs.

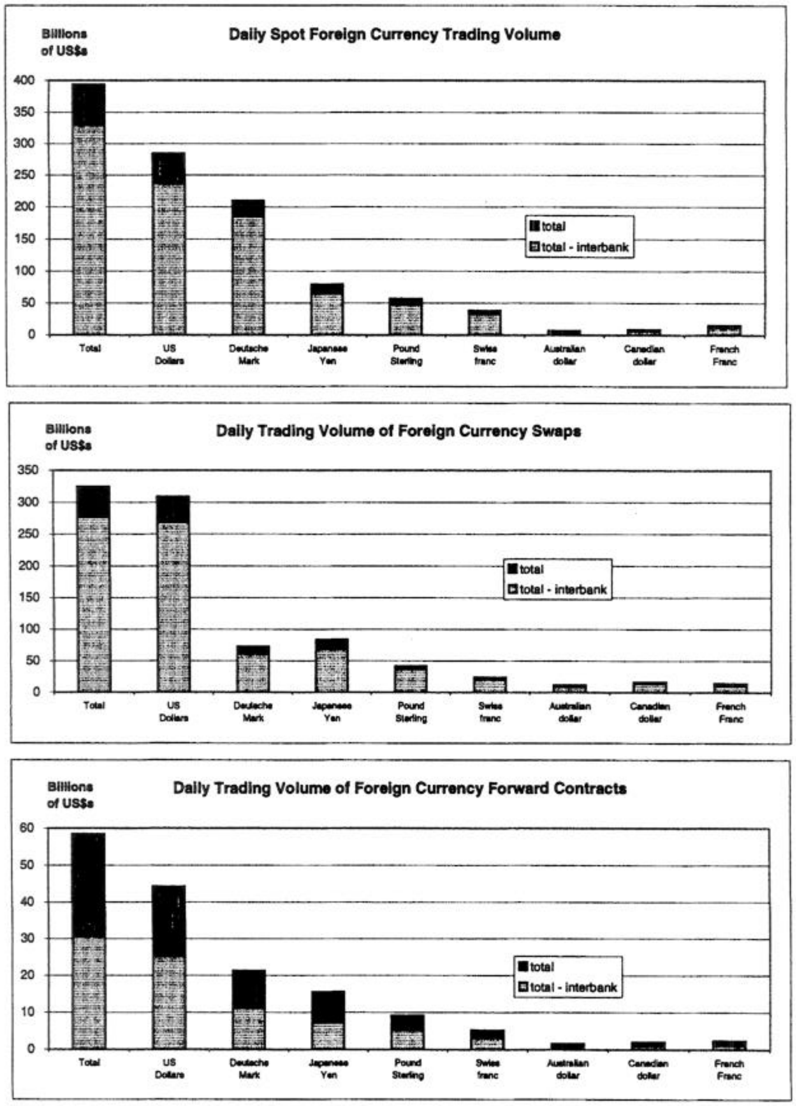


FIG. 1.—Foreign exchange trading volume. Source: Bank for International Settlements (1993).

assessing these numbers, many observers have the view that turnover is very high. For example, the Presidential Task Force on Market Mechanisms (Brady 1988) presents this viewpoint. As in the foreign exchange market, on the NYSE the increase in turnover has been accompanied by a rise in institutional ownership. This casual observation of a positive correlation between turnover and institutional ownership is confirmed when we take account of the decline in real trading costs over the post–World War II period. A regression of turnover on institutional ownership and real commissions per share shows that institutional ownership is still highly significant in explaining turnover.[3] As with foreign exchange, the available evidence is at least suggestive of a causal link between turnover and institutional control.

It seems difficult to explain the level of trading activity purely on the basis of "rational" motives for trade. Hedging and liquidity seem likely to explain only a small fraction of this trade, and it seems unreasonable to suppose that a small amount of such uninformed trade can support a large amount of informed trade. Hence the appeal of the "irrational" point of view. In contrast, we consider another motive for rational, uninformed agents to trade. We argue that it is capable of explaining a significant amount of trade. It is also consistent with the observed correlation between volume and institutional presence.

The motive stems from a contracting problem between professional traders and their clients or principals. We have in mind two settings in which the problem may arise. In one setting, an investor hires a fund management firm. The other setting is one in which a firm hires an employee to trade securities on its behalf. In both settings, there is likely to be a difficulty in writing incentive-compatible, efficient compensation contracts. In this context money managers may engage in ex ante unprofitable trades that have some chance of being profitable ex post. One goal of this paper is to investigate this possibility and to analyze the conditions under which such trade can be sizable.

In our setting portfolio managers who engage in producing information do not always uncover profitable trading opportunities. It can happen that inactivity (i.e., not trading) is the (first-best) optimal decision because the portfolio manager's effort at finding mispriced securities did not uncover any. The contracting problem that arises in our model is whether the delegated portfolio manager can

[3] Annual data on turnover and institutional ownership were taken from New York Stock Exchange (1993), and the data on real commission costs per share were kindly provided by Harold Mulherin (see Mulherin 1990). For the period 1955–88 the regression results were

$$\begin{aligned}\text{turnover} = \underset{(.16)}{-0.40} + \underset{(.48)}{2.58}\ &\text{institutional ownership \%}\\ + \underset{(9.04)}{20.00}\ &\text{real trading cost.}\end{aligned}$$

Standard errors are in parentheses, and $R^2 = .82$.

convince the client/principal that inactivity was his best strategy.[4] The difficulty is that the principal cannot distinguish "actively doing nothing" in this sense from "simply doing nothing." If the contract allows a reward for not trading, portfolio managers may simply do nothing; the contract may either attract incompetent managers or lead competent managers to shirk. If this makes it impossible to reward inactivity and limited liability prevents punishing ex post incorrect decisions, then the optimal contract may induce trading by the portfolio manager that is simply a gamble to produce a satisfactory outcome by chance. We call this noise trade or churning.[5] In the first part of the paper we show that noise trade will occur in equilibrium.

The contracting environment we study is a simple one in which the portfolio manager is unable to convince his principal that any inactivity is optimal. In our setting, inactivity is not rewarded because that would induce shirking by talented portfolio managers and it would attract incompetent portfolio managers. In many simple contracting environments, this problem could be solved by a two-part contract that specifies (i) a large bonus for taking correct trading positions and (ii) a smaller, lump-sum, payment for inactivity. Talented portfolio managers would be attracted by the chance of the bonus, but ex post, if they happened not to uncover trading opportunities, the lump sum would be chosen in preference to trading randomly as a gamble to earn the bonus. Incompetent portfolio managers would not sign the contract if the lump sum was not as large as their opportunity cost. Our environment is chosen with care so that this contract, and others like it, cannot eliminate the agency problem. Of course, if the contracting problem does arise in reality, as we suggest, it is presumably in the context of a more complex, repeated, environment that would not be as analytically tractable as the one specified here.

In the latter part of the paper, we consider the implications of noise trading for agents' welfare. Noise trading would appear to be costly for the principal since it lowers the expected rate of return on the portfolio. On the other hand, it will benefit hedgers: if managed portfolios earn lower rates of return, then uninformed hedgers earn higher returns. However, there is another effect. The higher return earned by the hedgers effectively reduces the cost of hedging; as a result, they will trade larger amounts. In turn, this increase in volume can support a larger amount of investment by an informed fund

[4] In practice, a similar effect would often arise when the manager identifies an asset as overvalued, but short-sale costs prevent holding a short position.

[5] The word "churning" is sometimes used in the different sense of trading to generate brokerage commissions.

manager. If the manager earns a smaller (percentage) return on a sufficiently increased investment, then he will be better off. We provide an example in which noise trading or churning by portfolio managers is Pareto-improving. In the example we provide, the security market would actually fail to exist without noise trade. This may not be as extreme as it may appear. It is often held that "illiquidity" can prevent the opening of a new market.

Our elementary welfare analysis is possible because we explicitly model the motives for trade of all the agents and, in particular, the hedging demand. Models that introduce "noise" or "liquidity" trade as an exogenous random variable cannot be used for this purpose. Endogenizing this component of trade, as we have done here, is neither original nor difficult. However, anything less than this has nothing to say about welfare. The most that can be done is to measure the transfer from uninformed agents to privately informed agents. Despite this problem, the literature has generally used this criterion for welfare analysis (e.g., Leland 1992). In our paper, modeling the hedgers' equilibrium response to informed traders is crucial.

We now briefly review related research. Two previous papers have derived churning as a result of the portfolio management contracting problem. Allen and Gorton (1993) study a model in which informed portfolio managers trade in one market and the uninformed managers (noise traders) trade in a market with a price bubble. The bubble is an exogenous price process whose only role is to allow uninformed managers to carry out negative-present-value risky trades. Trueman (1988) assumes that the objective function of the agent is to maximize the expectation of the principal's posterior belief that he will receive information. Churning occurs because uninformed managers attempt to imitate the informed. This problem could be overcome by the incentives of a suitably designed contract. This issue is one of the starting points of our paper. A number of other papers consider the contracting problem of buying information from an agent. This issue is tangentially related because these models treat information sale as isomorphic to portfolio management. See the discussion in Allen (1989) and the references therein (e.g., Bhattacharya and Pfleiderer 1985; Kihlstrom 1988; Allen 1990). These models do not imply churning (i.e., reporting false signals) because the agents can effectively be punished sufficiently hard to deter churning/lying. In our model, this possibility is precluded by the constraints of limited liability and limited personal resources of the agent. These models of information sale also do not consider the effect of the agency problem on security market prices or trading volume.

To summarize, our model is a general equilibrium model of portfolio management in a security market in which (i) there is a labor market for potential portfolio managers, (ii) these agents optimally contract with investors to manage their portfolios, (iii) the security is traded in a market in which prices are formed by a competitive market-making process, and (iv) the portfolio managers trade against hedgers who optimize when choosing their demands. In Section II, we present the model. The equilibrium is derived in Section III. We show that in equilibrium, the optimal contract induces noise trade. A small amount of uninformed hedging trade may support a large amount of noise trade. The welfare implications of this are considered in Section IV. An example shows that the noise trade caused by contracting problems can be Pareto-improving. Section V presents a conclusion.

## II. The Model

There are two dates, 0 and 1. A risky security is traded for cash at date 0, and at date 1 it pays a liquidating dividend of either $H$ or $L$ (with equal probability). We assume $H = 1$ and $L = 0$, but sometimes for clarity we maintain the $H$ and $L$ notation.

### A. *Delegated Portfolio Management*

A single principal has an opportunity to employ an agent to manage his portfolio. There are two types of agents who may potentially be employed as professional portfolio managers. The first type (talented managers) may receive private information about the value of the security, which would allow them to earn an above-average return for the principal. The second type (incompetent managers) have no chance of receiving private information. The population of agents consists almost entirely of incompetents, though there is a large number of talented managers as well.[6] An agent's type is private information. We assume that agents have no resources of their own, have the protection of limited liability, and are risk neutral.

Any agent has the choice of working for the principal (if offered a contract) or engaging in another activity (receiving remuneration $k$). However, this other activity is compatible with agreeing to manage the principal's portfolio as long as no trading actually occurs. In other words, $k$ is the payoff to the agent if he agrees to work for the principal but shirks, or if he chooses not to work for the princi-

[6] We omit the measure theoretic representation of this statement since it is obvious.

pal. The principal cannot observe whether the agent is engaging in the other activity. If a talented agent chooses to forgo this other activity and work for the principal, then with probability $\alpha$ he discovers the liquidation value of the security. In the discussion below we shall refer to an agent who forgoes the other activity as "actively working" for the principal. In contrast, an agent who accepts a contract to manage the portfolio but simultaneously engages in the alternative activity (so cannot trade for the principal) is "shirking." Agents might want to do this if the contract specifies a payment for doing nothing, that is, a payment in the event of no trade.

If a talented agent actively works for the principal, he may or may not receive private information. If he receives private information, he can manage the portfolio to generate superior returns. Even if he does not receive private information, he can still choose to trade and by chance may earn a superior return anyway. Of course, if he does trade without information, he is equally likely to earn an inferior return; but if this happens, the principal cannot penalize him because the agent has no private resources.

An incompetent agent faces the choice between shirking and actively working. If he shirks, he will earn $k$, together with any payment specified in the contract for not trading. Alternatively, if he actively works, he can trade at random in the hope of earning a superior return.

If the principal decides to hire an agent as his portfolio manager, he must design a contract to induce the agent to forgo the alternative activity and to maximize the return on the portfolio net of management expenses. The contract cannot condition directly on whether the agent undertakes the alternative activity, nor on whether the agent receives the private information since both of these situations are unobservable to the principal. However, the contract can condition on the realized value of the security and on the position the agent took.

The "alternative activity" can be interpreted in two ways. First, suppose that the portfolio manager is an employee and the principal is his employer. Since the agent is an employee, he can, presumably, be prevented from working at another firm. His alternative activity is to engage in leisure activities on the firm's time, for example, setting up his computer to display pornography from the World Wide Web rather than market prices, or taking his friends out to long lunches. It is not possible to engage in these activities to the same extent if the agent must actually trade.

A second situation is one in which the agent is a portfolio management firm that has been hired by an institutional investor. The portfolio management firm is able to choose how much effort to devote to this client's account. If the firm does not carry out any trades for the client, it will be able to spend more time recruiting other clients and working for them instead. Here the alternative activity is managing the portfolios of other clients.

Typically, an investment manager will have been hired to undertake a specific investment strategy, for example, choosing between different equities of a given class (growth stocks, high-technology stocks, etc.), choosing between bonds and stocks, choosing between different foreign currencies, or choosing between different national equity markets. "Doing nothing" in our model corresponds to keeping the portfolio allocation constant over time, whereas trading means that the portfolio weights are readjusted. Plausibly, a more active management strategy with frequent adjustment of the portfolio is time-consuming and restricts the manager's ability to engage in alternative activities.

Note that any contract that attracts incompetents as well as talented managers will result in hiring an incompetent almost surely, since they predominate in the population. Clearly, such a contract will not arise in equilibrium since it entails a positive payment in return for nothing. By the same argument, talented managers under the optimal contract will choose to actively work rather than shirk (if they shirk, they are no better than the incompetents). Since there is a large supply of talented agents, a portfolio manager will be paid just enough to induce him to forgo the alternative activity. Since the contract cannot condition directly on the agent's type, shirking decision, or information arrival, the portfolio manager's incentives may be distorted. This may happen even though the contract is optimally designed and attracts only the talented managers. This agency cost may be reflected in the manager's trading when he has received no information. We describe this as "noise trade" or "churning."

### B. *The Security Market*

The security is traded in a centralized market in which a market maker sets prices and clears the market, and other agents trade for hedging motives. Hedgers should be viewed as a continuum of small traders. However, for convenience we shall simply refer to "a hedger" in what follows. The probability that an agent with a hedging need arrives is $\delta$. With probability $1 - \delta$ there is no hedger present.

Hedgers want to insure against an income shock. The income shock may be positively or negatively correlated with the security's liquidation value. With 50 percent probability, it will be positively correlated, and a hedger will be perfectly hedged by selling one share short. Specifically, the hedger's wealth is $W$ when the asset is worth $L$ and his wealth is $W + 1$ when the asset is worth $H$. With 50 percent probability, a hedger will be perfectly insured by buying one share; that is, his wealth is $W$ when the asset is worth $H$ and $W + 1$ when the asset is worth $L$. No other agents know whether there is a hedger present and, if so, whether his hedging need is positive or negative. In other words, in our market the amount of hedgeable risk is either $-1$, 0, or $+1$ with probabilities $\frac{1}{2}\delta$, $1 - \delta$, and $\frac{1}{2}\delta$, respectively.

Prices are set by a risk-neutral market maker who faces Bertrand competition as in Glosten and Milgrom (1985) or Kyle (1985). The portfolio manager and the hedger submit their orders to the market maker, who observes the order flow and sets the price equal to the expected value conditional on this order flow, meeting the net order from his inventory. The market maker observes the total buy orders and the total sell orders. For example, if the market maker observes total buy orders of $x$, he cannot distinguish whether this is the sum of small orders placed by a large number of hedgers or whether a single order of $x$ was placed by the portfolio manager.

The hedgers are an important component of the model. Since the portfolio managers (averaging over both informed and uninformed) earn excess returns, other agents must be losing money to them. The standard device in the literature (as discussed in the Introduction) is to model these money-losing agents as "noise" or "liquidity" traders who trade an exogenous random quantity. Rather than introduce such exogenous behavior, we explicitly model the utility functions of all agents, for two reasons.

First, one goal of our paper is to explain the existence of "noise" trade that is not motivated by informational advantage, risk aversion, or liquidity needs. Therefore, we cannot assume the result by introducing exogenous noise trade. Instead, we introduce agents who are willing to lose money as resulting from an explicit hedging need. They lose money on average but are better off because they are partially hedged.

Second, the amount the hedgers trade depends on the equilibrium prices, which in turn depend on the optimal portfolio management contract and hence on the amount of churning. Welfare analysis would be impossible without explicitly modeling the utility functions of all agents. Moreover, our example will depend crucially on the response of hedging demand to the equilibrium prices.

### C. *The Benchmark Model: Direct Investment*

To understand the economic effects of the agency problem, we compare the economy described above to another economy in which there is no agency problem. In this "benchmark" model, the principal manages his own portfolio and may himself, with probability $\alpha$, become informed about the value of the asset. Since he is trading with his own money, he will trade only if he receives information, and this will be understood by the market maker and the hedgers. We refer to this case as "direct investment." We solve this model in Appendix A.

We stress that our comparison is made between a world of agency problems and a world without such problems. We do not address the question of whether a portfolio owner with investment management ability would deliberately introduce an agency problem by hiring an agent as his portfolio manager.

## III. Equilibrium with Delegated Portfolio Management

An equilibrium of the model of delegated portfolio management (DPM) specifies (1) a decision for the principal on whether to employ an agent to manage his portfolio and, if so, a contract describing the agent's remuneration; (2) a decision for the portfolio manager on whether to shirk or actively work for the principal and, if the latter, a trading strategy (conditional on information arrival); and (3) a decision for the hedger on how large a position to take, such that everybody is maximizing utility, given the market maker's beliefs and the behavior of the others. In computing the equilibrium, we shall start by assuming that the hedgers will trade a quantity $\pm x$. The quantity $x$ is derived later. In addition, later we shall verify that a portfolio manager will also either buy or sell $x$ in order to pool with the hedgers (any other quantity would reveal his identity to the market maker).

### A. *The Optimal Portfolio Management Contract*

In this subsection we derive the optimal portfolio management contract. The analysis in this subsection is conditional on the assumption that the principal does decide to employ an agent. Later, we shall have to verify that the principal does not pay too much money to cancel the benefits of having the agent work. In other words, hiring the portfolio manager to produce superior returns should be more profitable than saving the management fee and picking stocks at random.

As explained above, the optimal contract will attract only talented agents. It will induce them not to shirk, and they may or may not become informed. It is clear that (under an optimal contract) a portfolio manager who receives information will buy $x$ on good news and sell $x$ on bad news. An uninformed manager may or may not trade $x$ depending on the incentives provided by the contract. We shall show that the optimal contract will indeed induce an uninformed manager to trade.

The possible outcomes observed by the principal at $t = 1$ are (1) the security value is $H$ and the portfolio manager bought $x$; (2) the security value is $H$ and the portfolio manager sold $x$; (3) the security value is $L$ and the portfolio manager bought $x$; (4) the security value is $L$ and the portfolio manager sold $x$; and (5) the portfolio manager did not trade. In case 5 the security value could be either $H$ or $L$. But there is no need to distinguish between these possibilities. In cases 1 and 4 the portfolio manager either was informed or was an uninformed noise trader who was lucky (i.e., had no information but traded $x$ anyway in the right direction). In cases 2, 3, and 5, it is clear that the portfolio manager was uninformed.

The payment in each of these cases could in principle be positive, and we denote these payments by $m_1$, $m_2$, $m_3$, $m_4$, and $m_5$, respectively.[7] Note, however, that because the agent has no private resources, he cannot be penalized with a negative payment, notably in cases 2 and 3.[8]

PROPOSITION 1. *Churning.*—Under an optimal contract, only talented agents will be employed, they will actively work for the principal, and they will churn in the event that they receive no information.

*Proof.* If a contract attracts incompetent agents, almost surely an incompetent will be employed. Principals will not be willing to pay for this. Among other things, this implies $m_5 = 0$, since any contract that offers a positive payment for not trading will attract a flood of incompetents. A talented agent who does not actively work is no better than an incompetent. Again, principals will not pay for this. Thus the principal offers a contract that attracts only talented agents and induces them to work. We now show that they will churn if they receive no information.

An agent who buys without information is equally likely to receive either $m_1$, if the security turns out to high-valued, or $m_3$, if he takes the incorrect position. Similarly, if he sells, he receives either $m_2$ or $m_4$. Thus the payoff from churning is

$$\max\{{}^1\!/\!_2 m_1 + {}^1\!/\!_2 m_3, {}^1\!/\!_2 m_2 + {}^1\!/\!_2 m_4\}. \quad (1)$$

[7] In principle, there could be infinitely many other outcomes; e.g., the agent bought 25,000,000 shares rather than $x$, etc. It is easy to show that a zero payment in all these outcomes is an optimal level (see subsection $E$ below).

[8] By convention, we assume that principals do not offer trivial contracts that pay zero in exchange for no work.

This must be strictly positive since at least one of the $m_i$ must be nonzero, all of them are nonnegative, and $m_5 = 0$. Therefore, if the talented agent receives no information, churning is a strictly dominant strategy. Q.E.D.

An optimal contract must satisfy the following two conditions:

$$\max\{\tfrac{1}{2}m_1 + \tfrac{1}{2}m_3, \tfrac{1}{2}m_2 + \tfrac{1}{2}m_4\} < k \tag{2}$$

and

$$\alpha(\tfrac{1}{2}m_1 + \tfrac{1}{2}m_4) + (1 - \alpha)\max\{\tfrac{1}{2}m_1 + \tfrac{1}{2}m_3, \tfrac{1}{2}m_2 + \tfrac{1}{2}m_4\} = k. \tag{3}$$

Condition (2) says that the expected payment must be small enough not to attract incompetent managers to actively work at managing portfolios. Since they never receive information, they would then churn. Note that when

$$\tfrac{1}{2}m_1 + \tfrac{1}{2}m_3 = \tfrac{1}{2}m_2 + \tfrac{1}{2}m_4, \tag{4}$$

the uninformed agent will be willing to randomize between buying and selling.

Condition (3) states that the expected payment to the talented manager must be just high enough to attract him away from alternative employment. With probability $\alpha$, he will get information and make the correct investment decision. With probability $1 - \alpha$, he will not become informed and will churn (recall that the optimal contract cannot specify a strictly positive payment for doing nothing). Since there are many talented managers (even though almost all potential managers are incompetent), the equilibrium expected reward to the manager must be just enough to attract him to actively work.

We can now compute the payments for an optimal contract. We consider symmetric contracts in which the payment is the same for both correct outcomes and also for both incorrect outcomes. Substituting $m = m_1 = m_4$ and $m_2 = m_3 = 0$ into equation (3) gives

$$m = \frac{2k}{1 + \alpha}. \tag{5}$$

This solution satisfies inequality (2) since $\tfrac{1}{2}m < k$.

It is clear that there are other contracts that are equivalent in terms of their incentives and their expected costs. Any contract in which $m_1 = m_4$, $m_2 = m_3$, $m_1 + m_3 = 2k/(1 + \alpha)$, and $m_1 > m_3$ is equivalent as long as $m_3$ is small enough to satisfy inequality (2). For our purposes this distinction is immaterial.[9]

[9] Note also that this contract is not renegotiation-proof since a (talented) portfolio manager who (actively works and) receives no information can offer not to churn (which is costly to the principal) in exchange for a payment of at least $\tfrac{1}{2}m$. However, principals who leave open the possibility of such renegotiation would be vulnerable

### B. *Order Flow in Equilibrium*

We now consider the order flow in equilibrium with portfolio management. The hedgers will trade $\pm x$. The quantity $x$ will be derived below. In order to pool with the hedgers, an informed portfolio manager will also either buy or sell $x$ since any other quantity would reveal his information to the market maker.

Consider the decision problem of an uninformed portfolio manager. If he does not trade, he will be revealed as uninformed. As we showed above, the contract will not reward him in this event, because otherwise it would attract a flood of incompetents. On the other hand, if he churns and trades $x$, there are two possible outcomes: First, by luck he may mimic the actions of an informed portfolio manager and be rewarded accordingly. Second, he may take the wrong action and reveal himself to be uninformed; in this case he cannot be penalized because of limited liability. Therefore, he will trade $\pm x$ at random. This was shown in proposition 1.

The market maker can therefore observe five possible different order flows: (1) sell $2x$: if there is a hedger who sells, together with either an informed portfolio manager with bad news or an uninformed portfolio manager who sold at random; (2) sell $x$: if there is no hedger and there is either an informed portfolio manager with bad news or an uninformed portfolio manager who sold at random; (3) zero net trade: if there is a hedger who sells, together with either an informed portfolio manager with good news or an uninformed portfolio manager who buys at random, or vice versa; (4) buy $x$ or (5) buy $2x$: these order flows are symmetric to selling.

### C. *Prices*

It is straightforward to compute the market maker's updated beliefs and prices in these five cases. Since the asset takes the value of either one or zero, the price is equal to the market maker's belief that the asset value is high. Suppose that the market maker observes an order flow of sell $2x$. The probability of this event is $\frac{1}{2}\delta[\frac{1}{2}\alpha + \frac{1}{2}(1 - \alpha)] = \frac{1}{4}\delta$, and the probability of the joint event of sell $2x$ and the asset is valuable is $\frac{1}{2}\delta\frac{1}{4}(1 - \alpha)$. Therefore, the price in this case is $\frac{1}{2}(1 - \alpha)$. Similarly, he could observe sell $x$. The probability of this event is $(1 - \delta)[\frac{1}{2}\alpha + \frac{1}{2}(1 - \alpha)] = \frac{1}{2}(1 - \delta)$, and the probability of the joint event of sell $x$ and the asset is valuable is $(1 - \delta)\frac{1}{4}(1 -$

to a flood of incompetent applicants for the management contract. We therefore assume that they are able to credibly precommit not to renegotiate. One could imagine a number of different ways to achieve this; e.g., the principal is a legal entity with a governing charter that precludes paying agents for doing nothing.

$\alpha$), so the price is again $\frac{1}{2}(1 - \alpha)$. Note that this is the same as for sell $2x$ because the only difference between the two cases is the presence of the hedger, which conveys no information. Zero net order flow conveys no information (by symmetry), so the price is the unconditional expectation, $\frac{1}{2}$. The probability of this case is $\frac{1}{2}\delta$. Finally, if the net order flow is buy $x$ or buy $2x$, the price is $\frac{1}{2}(1 + \alpha)$, by symmetry.

We shall denote the prices $p_S$ in the event of sell $x$ or $2x$, $p_0$ if there is zero net trade, and $p_B$ in the event of buy $x$ or $2x$.

### D. *The Hedger's Decision*

Consider the case of a hedger whose income shock is negatively correlated with the security value. When he buys $x$ he will pay one of two prices: $p_B$ or $p_0$. The possibilities are as follows: (1) There is another buyer and the security is worth $L$. This can occur only if the other buyer is an uninformed portfolio manager who randomly happens to buy, which occurs with probability $\frac{1}{4}(1 - \alpha)$. Since there is another buyer, the price is $p_B$. (2) There is another buyer and the security is worth $H$. This can occur if the other buyer, a portfolio manager, is informed, which occurs with probability $\frac{1}{2}\alpha$, or if the other buyer is uninformed and randomly happens to buy, which occurs with probability $\frac{1}{4}(1 - \alpha)$. The probability is therefore $\frac{1}{4}(1 + \alpha)$. Since there is another buyer, the price is $p_B$. (3) There is another order that is a sell order and the security is worth $L$. This can occur if the other order was submitted by an informed portfolio manager with bad news, which occurs with probability $\frac{1}{2}\alpha$, or the other order was randomly submitted by an uninformed portfolio manager, which occurs with probability $\frac{1}{4}(1 - \alpha)$. As before, the total probability is $\frac{1}{4}(1 + \alpha)$. In this case the price is $p_0$. (4) There is another order that is a sell order and the security is worth $H$. This occurs if an uninformed portfolio manager randomly submits a sell order, which occurs with probability $\frac{1}{4}(1 - \alpha)$. Again, the price is $p_0$.

The hedger chooses to buy an amount $x$ of the security to maximize

$$\tfrac{1}{4}(1 - \alpha)U(W - p_B x + 1) + \tfrac{1}{4}(1 + \alpha)U(W - p_B x + x)$$
$$+ \tfrac{1}{4}(1 + \alpha)U(W - p_0 x + 1) + \tfrac{1}{4}(1 - \alpha)U(W - p_0 x + x).$$

The derivative of expected utility with respect to $x$ is

$$\tfrac{1}{4}(1 + \alpha)U'(W + x(1 - p_B))(1 - p_B)$$
$$+ \tfrac{1}{4}(1 - \alpha)U'(W + 1 - xp_B)(-p_B) + \tfrac{1}{4}(1 - \alpha)U'(W + \tfrac{1}{2}x)\tfrac{1}{2}$$
$$+ \tfrac{1}{4}(1 + \alpha)U'(W + 1 - \tfrac{1}{2}x)(-\tfrac{1}{2}).$$

Evaluating at $x = 0$ and requiring the derivative to be positive gives

$$\frac{U'(W)}{U'(W+1)} > \frac{(2-\alpha)(1+\alpha)}{(2+\alpha)(1-\alpha)}$$

as the condition that hedgers are sufficiently risk-averse for hedging to be nonzero.

Because of the symmetry in the hedger's decision problem, it is optimal to hedge equal but opposite amounts depending on whether the hedger has income shocks that are positively or negatively correlated with the asset value.

### E. *Out-of-Equilibrium Beliefs and Contract Payments*

The contents of this subsection are purely technical but are included for the sake of completeness. To this point we have considered only the possibility that all agents trade $\pm x$. To complete the construction of the equilibrium, it remains to verify that no agent has an incentive to deviate by trading other quantities. In particular, we must verify that the principal will not give the manager a contract rewarding him for trading any other quantity. The most reasonable specification of beliefs for the market maker at out-of-equilibrium quantities is that he believes that any deviation must come from a manager rather than a hedger, without changing his belief about the relative likelihood that the manager is informed or uninformed. Further, the market maker believes that any order for a quantity other than $x$ comes from a manager who would otherwise have traded $\pm x$ in the same direction.

Note that the principal makes money only because the manager's orders are sometimes indistinguishable from the hedger's equal and opposite orders, resulting in a price of $p_0$. At other times the informed manager makes money because the market maker confuses him with an uninformed manager and sets a price $p_B$ or $p_S$, but this exactly offsets losses made by the uninformed manager and ex ante does not benefit the principal.

We consider the case in which the manager would normally buy $x$ (the case of selling is symmetric). There are three possible deviations the manager can make. One possible deviation is to buy more than $x$. Then there is no possibility that the order will cross with a hedger selling $x$, which would have resulted in a price of $p_0$. The price will always be $p_B$. Clearly, the principal will not give the agent an incentive to deviate in this way. The second possible deviation is to buy less than $x$. Again, this will always lead to a price of $p_B$. The third possible deviation is to sell instead of buying. This causes the portfolio to lose money when the manager is informed and makes no difference when the manager is uninformed. Again, the principal will not design a contract to induce this deviation.

Finally, the hedgers have no incentive to deviate by definition of $x$. Since the hedgers are a continuum of infinitesimal agents, an individual cannot affect the aggregate trading volume and so cannot change the market maker's beliefs (see Sec. II*B*). The quantity $x$ is derived under precisely this assumption.

### *F. The Return on the Delegated Portfolio*

It remains to verify that the principal will indeed employ an agent as portfolio manager. If he does employ the agent as portfolio manager, he will earn a higher return than if he bought and held the market. On the other hand, he will have to pay a management fee on the portfolio. We now compute the expected return on the delegated portfolio.

When the portfolio manager buys, the price will be $p_0$ if the hedger arrives and sells or $p_B$ otherwise. The expected price is then $\frac{1}{2}\delta p_0 + (1 - \frac{1}{2}\delta)p_B$. When the portfolio manager sells, the expected price is $\frac{1}{2}\delta p_0 + (1 - \frac{1}{2}\delta)p_S$. If the portfolio manager is informed, he buys when the security is worth $H$ and sells when it is worth $L$, earning $\frac{1}{2}[H - \frac{1}{2}\delta p_0 - (1 - \frac{1}{2}\delta)p_B] + \frac{1}{2}[\frac{1}{2}\delta p_0 + (1 - \frac{1}{2}\delta)p_S - L]$ per share traded. If the portfolio manager is uninformed, he will pay the same expected prices but his orders will be uncorrelated with the value of the security, $[(\frac{1}{2}L + \frac{1}{2}H) - \frac{1}{2}\delta p_0 - (1 - \frac{1}{2}\delta)p_B] + [\frac{1}{2}\delta p_0 + (1 - \frac{1}{2}\delta)p_S - (\frac{1}{2}L + \frac{1}{2}H)]$, per share traded. Thus the expected earnings of the principal, net of management fees (expected value $k$), are

$$\begin{aligned} x(\alpha\{&\tfrac{1}{2}[H - \tfrac{1}{2}\delta p_0 - (1 - \tfrac{1}{2}\delta)p_B] + \tfrac{1}{2}[\tfrac{1}{2}\delta p_0 + (1 - \tfrac{1}{2}\delta)p_S - L]\} \\ &+ (1 - \alpha)\{\tfrac{1}{2}[(\tfrac{1}{2}L + \tfrac{1}{2}H) - \tfrac{1}{2}\delta p_0 - (1 - \tfrac{1}{2}\delta)p_B] \\ &+ \tfrac{1}{2}[\tfrac{1}{2}\delta p_0 + (1 - \tfrac{1}{2}\delta)p_S - (\tfrac{1}{2}L + \tfrac{1}{2}H)]\}) - k \\ &= x[\tfrac{1}{2}(1 - \tfrac{1}{2}\delta)(p_S - p_B) + \tfrac{1}{2}\alpha(H - L)] - k. \end{aligned}$$

Substituting for the prices, we obtain

$$\tfrac{1}{4}\delta\alpha(H - L)x - k. \tag{6}$$

If this expression is positive, the delegated portfolio management arises in equilibrium: it will be worthwhile for the principal to hire the agent. Clearly, this is more likely to hold if (1) hedging demand is large ($\delta x$ is large), (2) the chance that a portfolio manager will obtain information is high ($\alpha$ is large), (3) the variance of the secu-

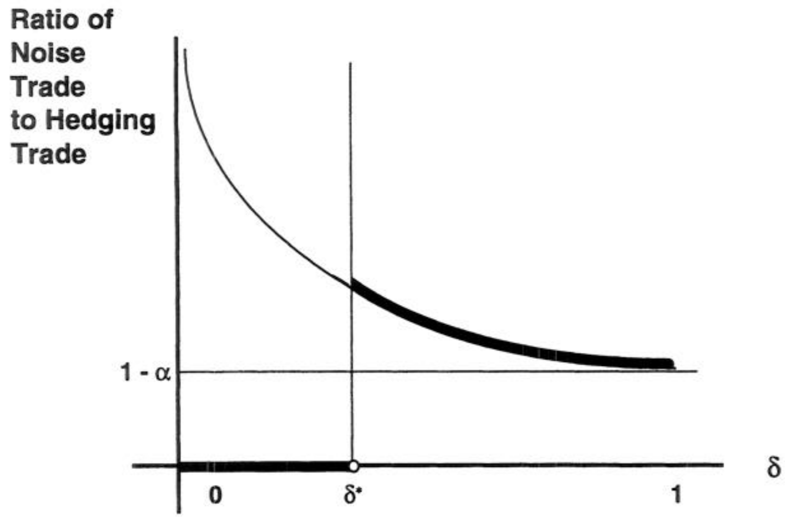


FIG. 2.—Ratio of noise trade to hedging trade

rity value is high ($H - L$ is large), and (4) the portfolio manager's opportunity cost is low ($k$ is small).

If the hedging demand is too small, then it is impossible to cover the fixed cost $k$ of delegated portfolio management. The reason is that the superior return on a managed portfolio comes at the expense of the hedgers.

### G. *How Much Noise Trade Can the Market Support?*

Substituting for $H$ and $L$ in (6) and defining $\delta^* \equiv 4k/\alpha x$, we see that the market for portfolio money management exists as long as $\delta \geq \delta^*$. Since the expected amount of hedging trade is $\delta x$ and the amount of expected noise trade is $(1 - \alpha)x$, the ratio of expected noise trade to expected hedging trade is $(1 - \alpha)/\delta$. Figure 2 illustrates this. As the amount of hedging trade, $\delta$, falls, the ratio of noise trade to hedging trade increases. Furthermore, it does so at an increasing rate, $(1 - \alpha)/\delta^2$. In this sense, a "small" amount of hedging can support a "large" amount of noise trade.

## IV. Can Noise Trade Make Everybody Better Off?

Noise trading by portfolio managers reduces the profitability of an actively managed portfolio, relative to the benchmark case of direct investment. By the same token, hedgers are effectively able to insure their endowment risk at lower cost. Thus it might appear that the noise trade resulting from the agency problem inherent in delegated portfolio management would make hedgers better off and investors worse off. This conclusion would be too simplistic, however. Because noise trading lowers the effective cost of insurance, hedgers will respond by purchasing more. If the increase in hedging demand ($x$) is large enough, the investor-principal may actually earn a larger total amount (this is the standard result that, if the price elasticity exceeds one, a price fall will cause an increase in expenditure). The reason for the decline in the cost of insurance is the presence of noise trade. Of course, in a world without agency problems, a principal who manages his own portfolio is free to engage in noise trade if he wants to. What lowers the "price of insurance" in a world of delegated portfolio management, however, is the commitment to engage in noise trade by hiring an agent. A principal investing for himself would always prefer not to engage in noise trade, ex post, and thus the hedgers and the market maker would not anticipate any noise trade.

If the hedgers do respond to delegated portfolio management by increasing their demand sufficiently that the profits on a managed portfolio improve (relative to the benchmark case of direct investment), then the agency problem that generates noise trade creates a Pareto improvement: hedgers can hedge more cheaply, portfolio managers are indifferent (they are employed at a wage equal to their opportunity cost), and portfolio owners earn higher returns.

In this section, we formalize this argument. We compare an economy with delegated portfolio management to one in which there are no agency problems because principals may become informed with probability $\alpha$. We provide an example showing a Pareto improvement. Note that this conclusion could not be reached using the standard paradigm of inelastic liquidity demand.

Let $x'$ be hedging demand in the benchmark case of direct investment and $x$ hedging demand in the delegated portfolio management case. To motivate the construction of the utility functions in the example, consider the wealth levels of hedgers in DPM and direct investment. We consider buying hedgers only; selling is symmetric. A buying hedger in DPM will realize one of four possible wealth levels, in each of the four cases listed in Section III$C$: (1) with probability $^1/_4(1 + \alpha)$, wealth is $W + x(1 - p_B)$; (2) with probability $^1/_4(1 - \alpha)$, wealth is $W + 1 - xp_B$; (3) with probability $^1/_4(1 - \alpha)$, wealth is $W + x(1 - p_0)$; and (4) with probability $^1/_4(1 + \alpha)$, wealth is $W + 1 - xp_0$.

In direct investment, a buying hedger will also realize one of four wealth levels, as given in section $D$ of Appendix A: (1)

with probability $\frac{1}{2}\alpha$, wealth is $W + x'(1 - p_{+2})$; (2) with probability $\frac{1}{2}(1 - \alpha)$, wealth is $W + 1 - x'p_{+1}$; (3) with probability $\frac{1}{2}(1 - \alpha)$, wealth is $W + x'(1 - p_{+1})$; and (4) with probability $\frac{1}{2}\alpha$, wealth is $W + 1 - x'p_0$.

In order for the investor-principal to be better off under DPM, hedgers must hedge sufficiently more than in direct investment. We shall construct an example in which this property holds in the extreme, that is, $x = 1$ and $x' = 0$. In other words, in direct investment the cost of hedging is so high that hedgers choose not to hedge at all, whereas in DPM they choose to hedge fully. In that case, the possible wealth levels in DPM are as follows: cases 1 and 2 above have wealth $W + 1 - p_B = W + \frac{1}{2} - \frac{1}{2}\alpha$, and cases 3 and 4 reduce to $W + 1 - p_0 = W + \frac{1}{2}$. Note that, since they hedge fully under DPM, the hedgers care about the price they are paying, but not about the value of the asset. Under direct investment, with $x' = 0$, the situation is reversed: the hedgers are exposed to asset risk but not to price risk since they choose not to hedge: cases 1 and 3 have wealth $W$ and cases 2 and 4 have wealth $W + 1$.

For hedgers to make these choices, we need them to be very risk averse in the region from $W + \frac{1}{2} - \frac{1}{2}\alpha$ to $W + \frac{1}{2}$ and less risk averse on either side of this region. A likely candidate for such a preference is, therefore, a concave piecewise-linear utility function with kinks at $W + \frac{1}{2} - \frac{1}{2}\alpha$ and $W + \frac{1}{2}$. We shall show that with such a utility function, DPM is indeed Pareto-improving.

PROPOSITION 2. If hedgers have the following utility function,

$$u(\lambda) = \begin{cases} \lambda & \text{for } \lambda < W + \frac{1}{2}(1 - \alpha) \\ [W + \frac{1}{2}(1 - \alpha)](1 - a) + a\lambda & \text{for } W + \frac{1}{2}(1 - \alpha) < \lambda < W + \frac{1}{2} \\ (W + \frac{1}{2})(1 - b) - \frac{1}{2}\alpha(1 - a) + b\lambda & \text{for } \lambda > W + \frac{1}{2}, \end{cases}$$

where $0 < b < a < 1$, then delegated portfolio management Pareto-dominates direct investment.

*Proof.* See Appendix B.

Our example illustrates the extreme case in which the lemons problems caused by the presence of informed traders cause a market to fail to exist. Churning, then, may cause the market to open (as in our example).[10] In our model, "noise trade" could be a public

[10] This is related to Pagano (1989). His model may have more than one equilibrium, each with a different amount of shares in existence. In a "thin" equilibrium (a small number of shares in existence), risk-averse agents are unwilling to trade because of the risk that there will be few buyers when they need to sell.

good because it increases the liquidity of the market. All agents benefit from this. In reality, firms, exchanges, and governments sometimes act to provide this good. Firms issuing new securities or underwriting securities are likely to be privately informed about the new securities they are issuing. To provide liquidity, they sometimes commit to trading these securities at prespecified prices for a period of time, creating liquidity by committing not to use their private information. Exchanges monitor the credit quality of counterparties when they require margin, thus reducing the possible information advantage of informed traders since credit quality becomes irrelevant. The government offers insurance on demand deposits and provides liquidity via the discount window. Our analysis highlights a more indirect way in which liquidity is created: It is an inadvertent by-product of professional money management.

## V. Conclusion

A portfolio manager will frequently find that the best investment policy is simply to hold the existing portfolio, in other words, to do nothing. The question is whether, in this situation, he will be able to credibly convince his client or principal that he is "actively" doing nothing. The client may instead believe that he is simply doing nothing. He may think that the portfolio manager has not spent any effort on producing information or he has no talent. Our paper describes a contractual relationship, and its economic consequences, in which actively doing nothing is indistinguishable from simply doing nothing. Ultimately it is an empirical question as to when they are indistinguishable. Designing a contractual relationship for portfolio management is to a large extent a matter of maximizing this distinction.

## Appendix A

### Equilibrium with Direct Investment

*A. Order Flow under Direct Investment*

Orders must be multiples of $x'$, the amount the hedger trades. The market maker will observe one of five possible order flows: (1) sell $2x'$: if there is both an informed agent who sells and a hedger who sells; (2) sell $x'$: if there is either an informed agent who sells (and no hedger) or a hedger who sells (and the agent does not receive information); (3) zero net trade: if no hedger arrives and the agent does not learn any information, or if a hedger arrives to sell and the informed agent learns that the security is of high value and buys, or vice versa; (4) buy $x'$ and (5) buy $2x'$ are symmetric to selling.

### B. *Prices and Beliefs*

The market maker's beliefs and the prices in these five cases are as follows. If he observes two sell orders, then the true value of the security is revealed; so the market maker's belief that the asset is worth one is zero and the price is also zero. We denote this price by $p_{-2}$. The probability of this event is $\frac{1}{4}\alpha\delta$. Next, he may observe a single sell order of $x'$. The probability of this event is $\frac{1}{2}\alpha(1 - \delta) + \frac{1}{2}(1 - \alpha)\delta$, and the probability of the joint event of sell $x'$ and the asset is valuable is $\frac{1}{4}(1 - \alpha)\delta$. Therefore, the market maker's belief about the value of the asset when he observes a single sell order is $\delta(1 - \alpha)/2(\alpha + \delta - 2\alpha\delta)$, which is the price. We denote this price by $p_{-1}$. There may be offsetting orders. This conveys no information (by symmetry), so the price, $p_0$, is the unconditional expectation, $\frac{1}{2}$. The probability of this event is $1 - \alpha - \delta + \frac{3}{2}\alpha\delta$. The market maker may observe a single buy order for $x'$. The probability of buying $x'$ is $\frac{1}{2}\alpha(1 - \delta) + \frac{1}{2}(1 - \alpha)\delta$. The probability of buying $x'$ and the asset is highly valued is $\frac{1}{2}\alpha(1 - \delta) + \frac{1}{4}(1 - \alpha)\delta$. Therefore, the updated belief is $(2\alpha + \delta - 3\alpha\delta)/2(\alpha + \delta - 2\alpha\delta)$, which is the price $p_{+1}$. Finally, he may observe two buy orders of $2x'$. This is symmetric with sell $2x'$. The price, $p_{+2}$, is one.

### C. *The Investor's Expected Returns*

From the point of view of an informed investor with good news, the probabilities of these possible events (and corresponding prices) are as follows: with probability $\frac{1}{2}\delta$, there is a hedger present who also buys and the market maker observes two buy orders. With probability $1 - \delta$, there is no hedger and the market maker observes a single buy order. Finally, with probability $\frac{1}{2}\delta$, there is a hedger who sells and the market maker observes two offsetting orders. If the informed investor has good news, it is impossible to have net asset sales. From the point of view of an informed investor with bad news, the probabilities of two sell orders, one sell order, and no net trade are, respectively, the same. The expected profits of the investor, therefore, are

$$\alpha x'\left(\tfrac{1}{2}\left\{1 - \left[\tfrac{1}{2}\delta \cdot 1 + \frac{(1-\delta)(2\alpha + \delta - 3\alpha\delta)}{2(\alpha + \delta - 2\alpha\delta)} + \tfrac{1}{2}\delta \cdot \tfrac{1}{2}\right]\right\}\right.$$
$$\left. + \tfrac{1}{2}\left\{\left[\tfrac{1}{2}\delta \cdot 0 + \frac{(1-\delta)\delta(1-\alpha)}{2(\alpha + \delta - 2\alpha\delta)} + \tfrac{1}{2}\delta \cdot \tfrac{1}{2}\right] - 0\right\}\right) - k$$
$$= \tfrac{1}{4}\alpha x' \frac{\delta(2 - \alpha - \delta)}{\alpha + \delta - 2\alpha\delta} - k.$$

This must be positive for the agent to forgo his alternative activity:

$$\frac{\frac{1}{4}\delta(2-\alpha-\delta)}{\alpha+\delta-2\alpha\delta} \geq \frac{k}{\alpha x'}.$$

*D. The Hedgers' Decisions*

Consider a hedger whose wealth is negatively correlated with the asset value. From the point of view of this hedger, there are four possible cases when he buys: (1) There is another buy order and the asset is worth one. This can happen only if the other trader is an informed trader who knows that the asset value is one. The price is $p_{+2} = 1$. The wealth of the hedger in this case is $W + x' - p_{+2}x'$, and the probability of this event is $\frac{1}{2}\alpha$. (2) There is no other order and the asset is worth zero. Again, there will be no other order only if there is no informed trader (in which case the asset is equally likely to be worth one or zero). The wealth of the hedger is $W - p_{+1}x' + 1$, and the probability of this event is $\frac{1}{2}(1-\alpha)$. (3) There is no other order and the asset is worth one. There will be no other order only if there is no informed trader, in which case the asset is equally likely to be worth one or zero. The wealth of the hedger is $W - p_{+1}x' + x'$, and the probability of this event is $\frac{1}{2}(1-\alpha)$. (4) There is a sell order and the asset is worth zero. This occurs if the informed trader sells the asset, in which case the price is $p_0$ and the wealth of the hedger is $W - p_0x' + 1$. This event occurs with probability $\frac{1}{2}\alpha$.

The expected utility of the hedger is

$$\begin{aligned}&\tfrac{1}{2}\alpha U(W) + \tfrac{1}{2}(1-\alpha)U(W+1-x'p_{+1})\\ &+ \tfrac{1}{2}(1-\alpha)U(W+x'(1-p_{+1})) + \tfrac{1}{2}\alpha U(W+1-\tfrac{1}{2}x').\end{aligned}$$

The utility of a hedger whose wealth is positively correlated with the asset value is exactly symmetric so that the choice of $x'$ for buying and selling hedgers is the same.

The derivative of expected utility with respect to $x'$ is

$$\begin{aligned}&\tfrac{1}{2}(1-\alpha)U'(W+1-x'p_{+1})(-p_{+1})\\ &+ \tfrac{1}{2}(1-\alpha)U'(W+x'(1-p_{+1}))(1-p_{+1})\\ &+ \tfrac{1}{2}\alpha U'(W+1-\tfrac{1}{2}x')(-\tfrac{1}{2}).\end{aligned}$$

Evaluating at $x' = 0$ and setting the derivative to be positive gives

$$\frac{U'(W)}{U'(W+1)} > \frac{\frac{1}{2}\alpha + (1-\alpha)p_{+1}}{(1-\alpha)(1-p_{+1})},$$

which is the condition for nonzero hedging in the case of direct investment. Note that $p_{+1} = (2\alpha + \delta - 3\alpha\delta)/2(\alpha + \delta - 2\alpha\delta)$; it may be verified that this condition is more stringent than the corresponding condition under delegated portfolio management.

## Appendix B

### Proof of Proposition 2

We begin by showing that in direct investment, $x' = 0$ is optimal. The hedger's expected utility in direct investment is

$$
\begin{aligned}
&\tfrac{1}{2}\alpha W + \tfrac{1}{2}(1-\alpha)[W + x'(1-p_{+1})] \\
&+ \tfrac{1}{2}(1-\alpha)[(W+\tfrac{1}{2})(1-b) - \tfrac{1}{2}\alpha(1-a) + b(W+1-p_{+1}x')] \\
&+ \tfrac{1}{2}\alpha[(W+\tfrac{1}{2})(1-b) - \tfrac{1}{2}\alpha(1-a) + b(W+\tfrac{1}{2})] \\
&= \tfrac{1}{2}W + \tfrac{1}{2}(1-\alpha)x'(1-p_{+1}) + \tfrac{1}{2}(W+\tfrac{1}{2})(1-b) \\
&- \tfrac{1}{4}\alpha(1-a) + \tfrac{1}{2}bW + \tfrac{1}{2}b(1-\alpha)(1-p_{+1}x') + \tfrac{1}{4}\alpha b.
\end{aligned}
$$

We require that the derivative with respect to $x'$ (from the right-hand side at $x' = 0$) be negative, so $\tfrac{1}{2}(1-\alpha)(1-p_{+1}) - \tfrac{1}{2}b(1-\alpha)p_{+1} < 0$, that is, $b > (1-p_{+1})/p_{+1}$. Next we compute the hedger's expected utility under DPM:

$$
\begin{aligned}
&\tfrac{1}{4}(1+\alpha)[W + x(1-p_B)] + \tfrac{1}{4}(1-\alpha) \\
&\times \{[W + \tfrac{1}{2}(1-\alpha)](1-a) + a(W+1-p_B x)\} \\
&+ \tfrac{1}{4}(1-\alpha)\{[W+\tfrac{1}{2}(1-\alpha)](1-a) + a(W+\tfrac{1}{2}x)\} \\
&+ \tfrac{1}{4}(1+\alpha)[(W+\tfrac{1}{2})(1-b) - \tfrac{1}{2}\alpha(1-a) + b(W+1-\tfrac{1}{2}x)] \\
&= \tfrac{1}{4}(1+\alpha)W + \tfrac{1}{2}(1-\alpha)[W + \tfrac{1}{2}(1-\alpha)(1-a)] \\
&+ \tfrac{1}{4}(1+\alpha)[(W+\tfrac{1}{2})(1-b) - \tfrac{1}{2}\alpha(1-a)] \\
&+ \tfrac{1}{4}(1-\alpha)[a(W+1) + aW] + \tfrac{1}{4}(1+\alpha)b(W+1) \\
&+ x[\tfrac{1}{4}(1+\alpha)(1-p_B) - \tfrac{1}{4}(1-\alpha)ap_B \\
&+ \tfrac{1}{4}(1-\alpha)\tfrac{1}{2}a - \tfrac{1}{4}(1+\alpha)\tfrac{1}{2}b].
\end{aligned}
$$

We require that the derivative with respect to $x$ (at $x = 1$ from the left-hand side) be positive, that is,

$$\tfrac{1}{4}(1-\alpha)a(\tfrac{1}{2}-p_B) + \tfrac{1}{4}(1+\alpha)(1-p_B-\tfrac{1}{2}b) > 0.$$

Now, when we substitute for $p_B = \tfrac{1}{2}(1+\alpha)$,

$$(1-\alpha)a(-\tfrac{1}{2}\alpha) + (1+\alpha)[\tfrac{1}{2}(1-\alpha) - \tfrac{1}{2}b] > 0,$$

or $b < (1-\alpha) - [a\alpha(1-\alpha)/(1+\alpha)]$.

It remains to verify that $a$ and $b$ can simultaneously satisfy this equation together with the previously derived condition under direct investment, as well as $0 < b < a < 1$ (see fig. B1). We now show that this is possible by taking as an example $a = b = (1-p_{+1})/p_{+1}$ (this point is circled in fig. B1). It remains to check algebraically that this point lies below the line $b = (1-\alpha) - [a\alpha(1-\alpha)/(1+\alpha)]$, that is, that

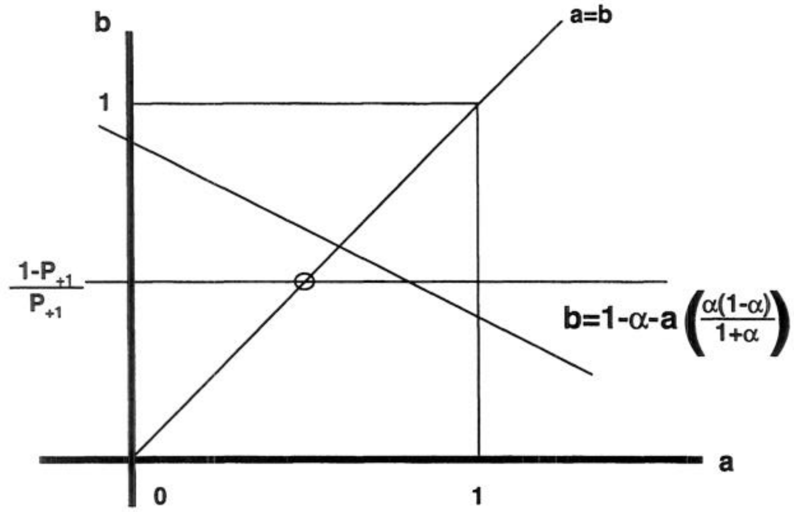


FIG. B1.—Parameter region in which delegated portfolio management dominates direct investment.

$$\frac{1-p_{+1}}{p_{+1}} < (1-\alpha) - \frac{\alpha(1-\alpha)}{1+\alpha}\frac{1-p_{+1}}{p_{+1}}.$$

Substituting $p_{+1} = (2\alpha + \delta - 3\alpha\delta)/2(\alpha + \delta - 2\alpha\delta)$ gives

$$\left[\frac{\delta(1-\alpha)}{2\alpha + \delta - 3\alpha\delta}\right]\left(\frac{1-\alpha}{1+\alpha}\right) < 1.$$

Since both of the terms in brackets on the left-hand side are less than one, the inequality is, indeed, satisfied. Notice that, by revealed preference, the hedgers are better off under delegated portfolio management, whereas the investor-principal is also better off since he makes zero profits under direct investment because there is no trade. This completes our example. Q.E.D.

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