



Yale SCHOOL of MANAGEMENT

# The Number of Undocumented Immigrants in the United States

Mohammad M. Fazel-Zarandi, Jonathan S. Feinstein, Edward H. Kaplan

# Motivation



# Research Question and Contribution

- **Questions:**

1. How many undocumented immigrants are there in the United States?
2. Are the current methods of estimating the number of undocumented immigrants adequate?

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# Research Question and Contribution

- **Questions:**
  1. How many undocumented immigrants are there in the United States?
  2. Are the current methods of estimating the number of undocumented immigrants adequate?
- **Contribution:** Propose a new approach grounded on operational data and mathematical modeling that estimates annual population inflows and outflows from 1990 – 2016
- **Why:** Sets the scale of the issue

# Outline

- Motivation
- Current estimates
- Snapshot of our results
- The model and the simulation
- Results, Receptions, and Policy Implications

# Current Estimates

- Residual Method

- Passel (2016), Krogstad and Passel (2015), Baker and Rytina (2013), Warren and Warren (2013)

$$\begin{aligned} \text{Estimated Number of} \\ \text{Unauthorized Immigrants} \end{aligned} = \begin{aligned} \text{Estimated Total Foreign} \\ \text{Born Population} \\ (\text{Non-Citizen}) \end{aligned} - \begin{aligned} \text{Estimated Lawful} \\ \text{Immigrant Population} \end{aligned}$$

# Current Estimates

- Total Foreign Born Population
  - Based on *surveys* (American Community Survey or Current Population Survey)

13197082

**Person 1**

Please copy the name of Person 1 from page 2, then continue answering questions below.

Last Name \_\_\_\_\_

First Name \_\_\_\_\_ MI \_\_\_\_\_

7 Where was this person born?

In the United States – Print name of state.  
 Outside the United States – Print name of foreign country, or Puerto Rico, Guam, etc.

8 Is this person a citizen of the United States?

Yes, born in the United States → SKIP to question 10a  
 Yes, born in Puerto Rico, Guam, the U.S. Virgin Islands, or Northern Marianas  
 Yes, born abroad of U.S. citizen parent or parents  
 Yes, U.S. citizen by naturalization – Print year of naturalization  
 No, not a U.S. citizen

11 What is the highest degree or level of school this person has COMPLETED? Mark (X) ONE box. If currently enrolled, mark the previous grade or highest degree received.

**NO SCHOOLING COMPLETED**  
 No schooling completed

**NURSERY OR PRESCHOOL THROUGH GRADE 12**  
 Nursery school  
 Kindergarten  
 Grade 1 through 11 – Specify grade 1 – 11 →  \_\_\_\_\_

**HIGH SCHOOL GRADUATE**  
 12th grade – NO DIPLOMA  
 Regular high school diploma  
 GED or alternative credential

**COLLEGE OR SOME COLLEGE**  
 Some college credit, but less than 1 year of college credit  
 1 or more years of college credit, no degree  
 Associate's degree (for example: AA, AS)  
 Bachelor's degree (for example: BA, BS)

**AFTER BACHELOR'S DEGREE**  
 Master's degree (for example: MA, MS, MEng, MEd, MCW, MRes)

13 What is this person's ancestry or ethnic origin?

(For example: Italian, Jamaican, African Am., Cambodian, Cape Verdean, Norwegian, Dominican, French Canadian, Haitian, Korean, Lebanese, Polish, Nigerian, Mexican, Taiwanese, Ukrainian, and so on.)

14 a. Does this person speak a language other than English at home?

Yes  
 No → SKIP to question 15a

b. What is this language?

(For example: Korean, Italian, Spanish, Vietnamese)

c. How well does this person speak English?

Very well  
 Well  
 Not well  
 Not at all

15 a. Did this person live in this house or apartment 1 year ago?

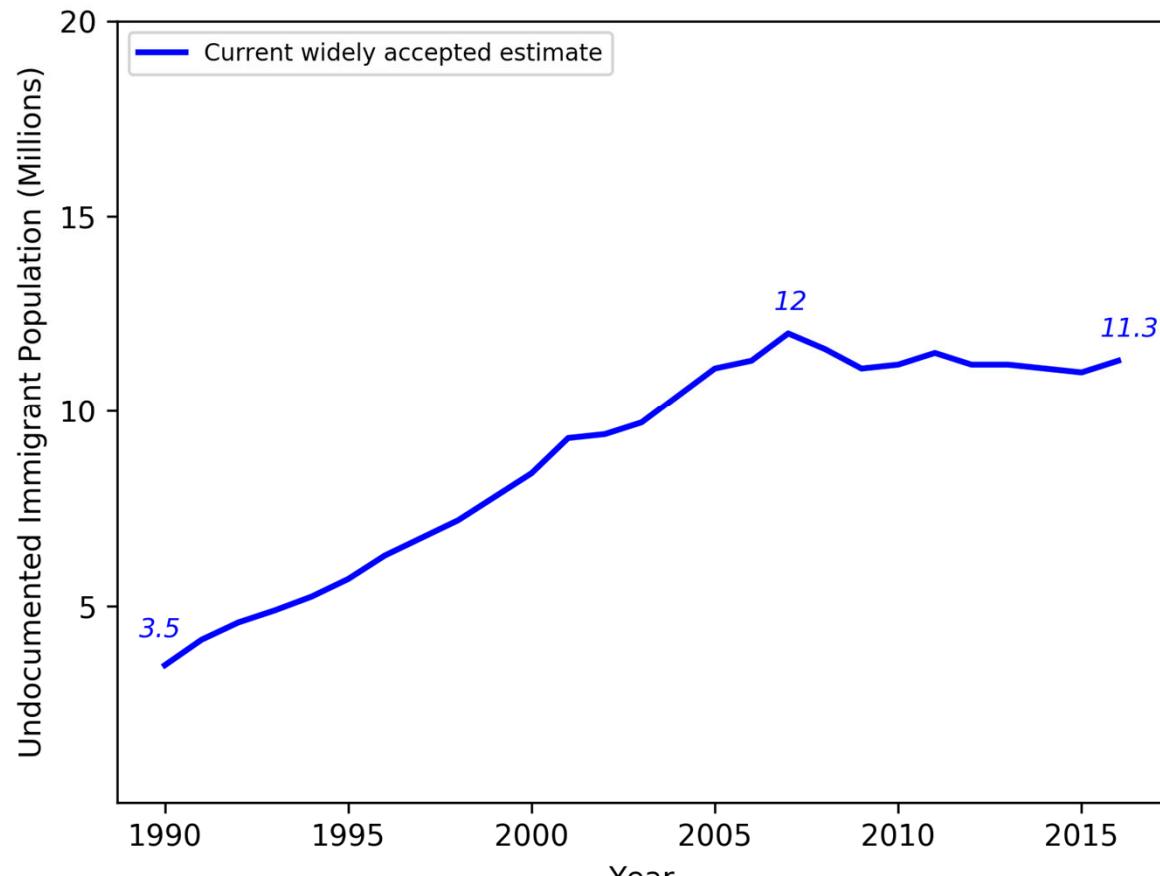
Person is under 1 year old → SKIP to question 16

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# Current Estimates

- Total Foreign Born Population
  - Based on *surveys* (American Community Survey or Current Population Survey)
- Estimate of Lawful Immigrant Population
  - Use Department of Homeland Security data on lawful arrivals

# Current Estimate



Source: Pew Research Center

# Current Estimates



# Hidden Population

- Difficult to locate members of the target population  
(Goel and Salganik (2010), Crawford et al.(2018))
  - Reaching a representative sample of all those born outside of the U.S.
    - Undocumented immigrants are more difficult to locate and survey
  - Accurate responses from survey respondents
    - Undocumented immigrants may misreport their country of origin, citizenship, and number of household residents

# Some Relevant Statistics

ACS  
response  
rates

Year	Response Rate	Refusal	Unable to Locate	No One Home	Temporarily Absent	Language Problem	Insufficient Data	Maximum Contact Attempts Reached	Other
2017	93.7	2.7	0.0	0.9	0.1	0.1	0.4	1.1	0.9
2016	94.7	2.1	0.0	0.9	0.1	0.1	0.3	1.1	0.7

Population: Origin and Language <sup>1</sup>		Percent Allocated				
Item		2017	2016	2015	2014	2013
Place of birth total population		9.3	9.1	9.2	8.8	8.6
Sex total population		0.1	0.1	0.1	0.1	0.1

ACS Question  
non-response  
rates among  
respondents

Source: U.S. Census Bureau

# Some Relevant Statistics

5% non-response

+  $0.95 \times 8\%$  question skippers

---

12.6% no clear answer  $\approx$  **40 million people**

- Ignoring deliberate misreporting
  - Place birth
  - Number of household residents

# Some Relevant Statistics

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12.6% no clear answer  $\approx$  40 million people

- Ignoring deliberate misreporting
  - Place birth
  - Number of household residents
- Non-response bias
  - Missing at random vs *missing on purpose*

**Census may be too blunt an instrument to reach a relatively small population with an incentive to remain undetected**

# Census Citizenship Question Debate

The New York Times

## *Court Blocks Trump Administration From Asking About Citizenship in Census*



Commerce Secretary Wilbur L. Ross Jr., center, ordered the Census Bureau to add a citizenship question to the 2020 census. Doug Mills/The New York Times

By Michael Wines

Jan. 15, 2019

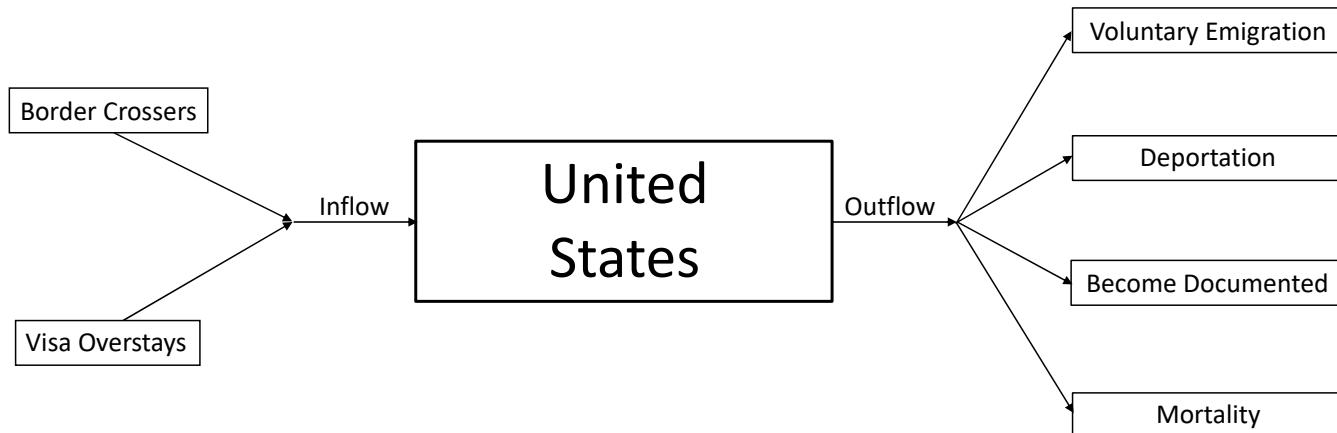


“The result will not only be a decrease in the quality of census data — something Defendants concede — but likely also a net differential undercount (that is, an **undercount of certain sectors of the population, including people who live in households containing noncitizens and Hispanics, relative to others**).”

Judge Jesse M. Furman (United States District Court in Manhattan)

# Our Estimate

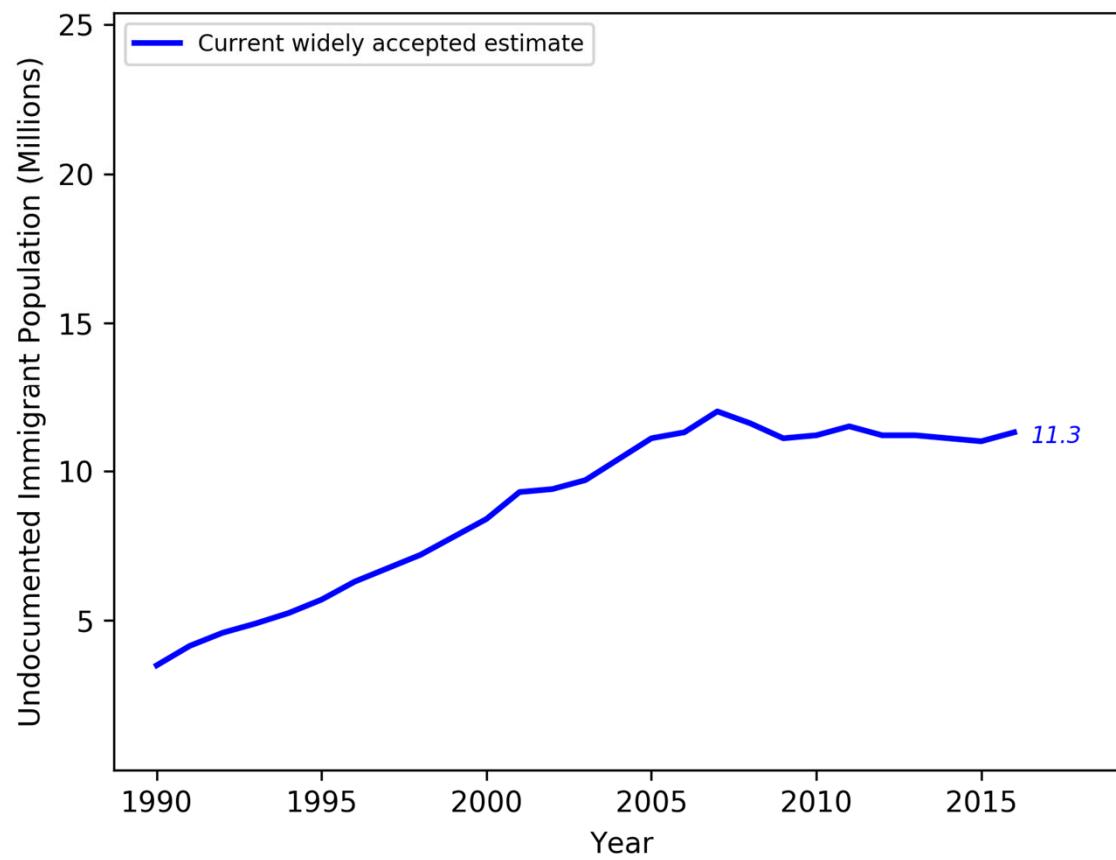
- Combine mathematical modeling with data analysis
- Our model tracks and estimates annual inflows and outflows from 1990 - 2016



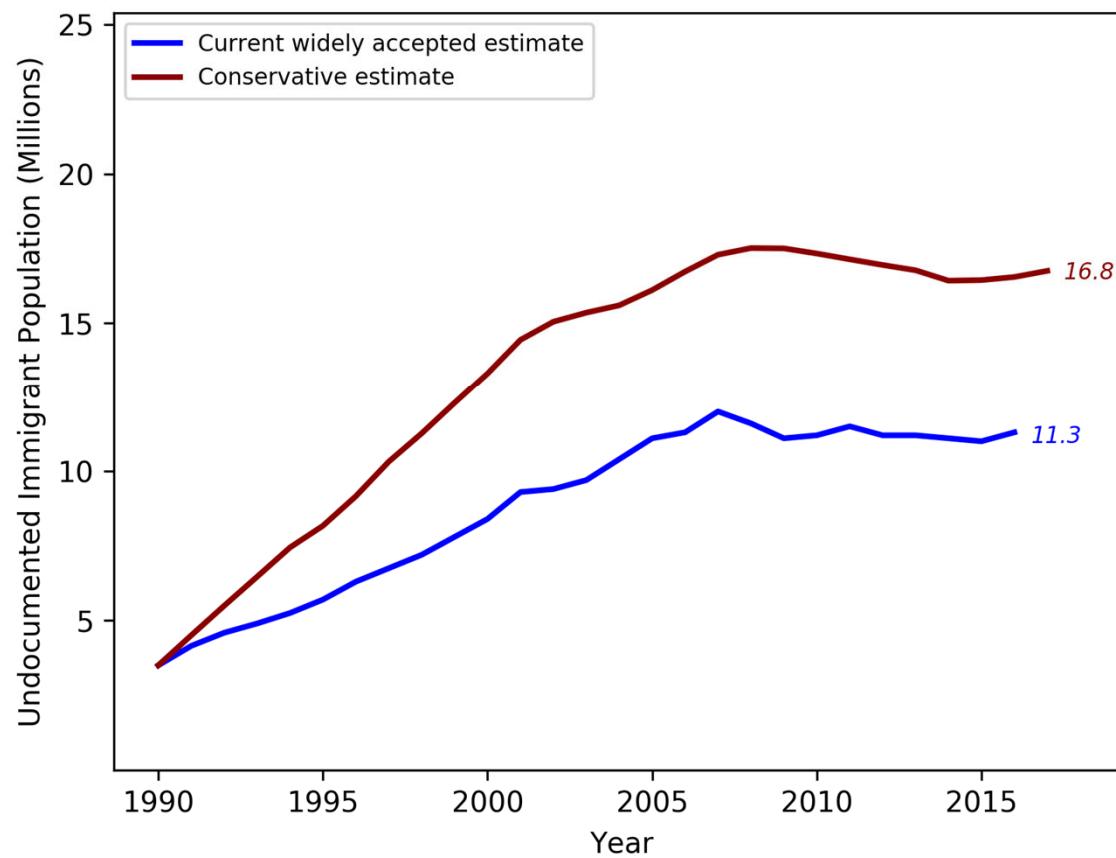
# Our Estimation Result

- I. Generate a **conservative estimate** (low-end) of the number of undocumented immigrants
- II. Generate probability distribution over the number of undocumented immigrants based on simulating our model over a wide range of assumptions

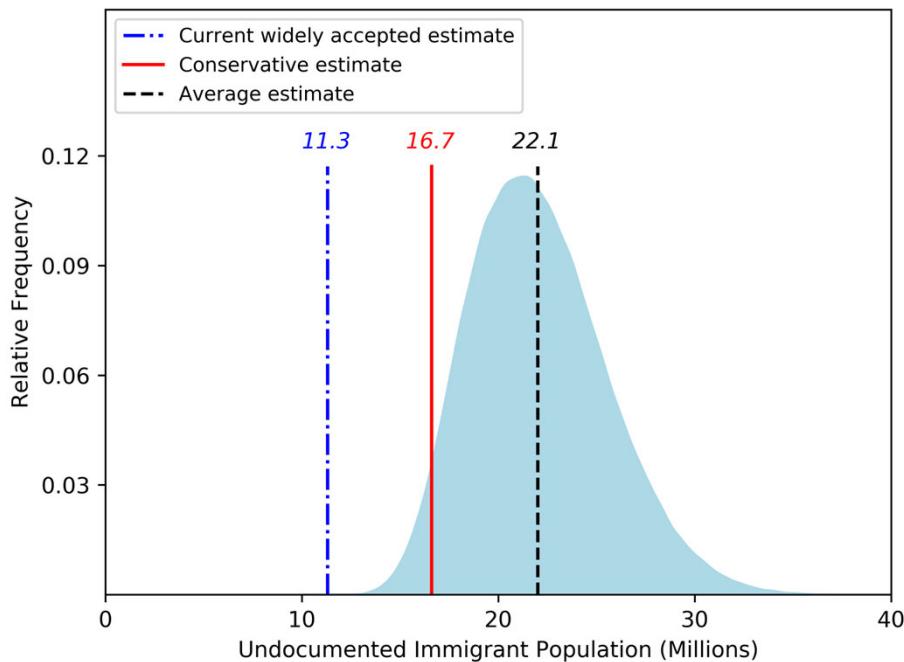
# Results



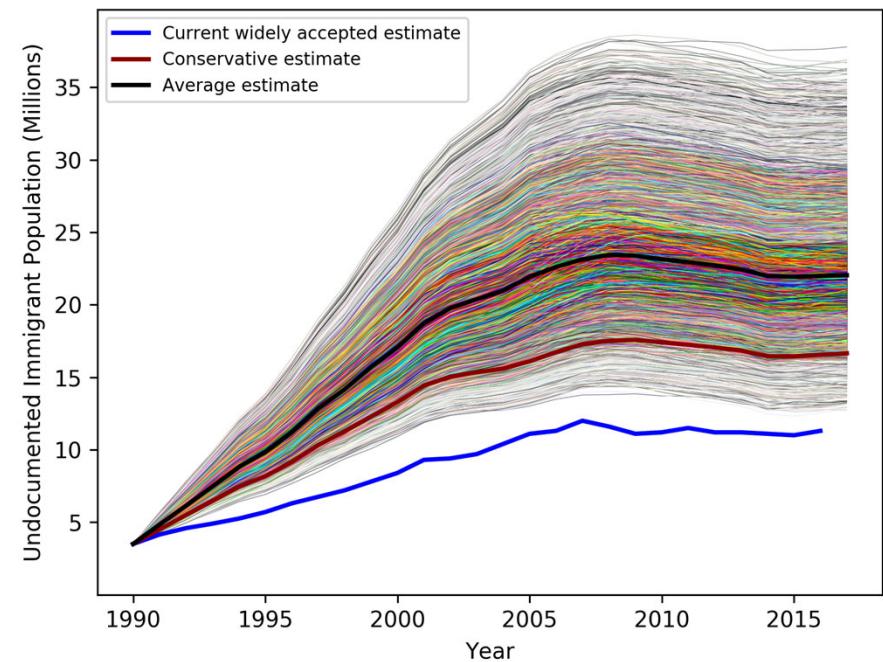
# Results – Conservative Estimate



# Results – 1,000,000 Simulation Runs



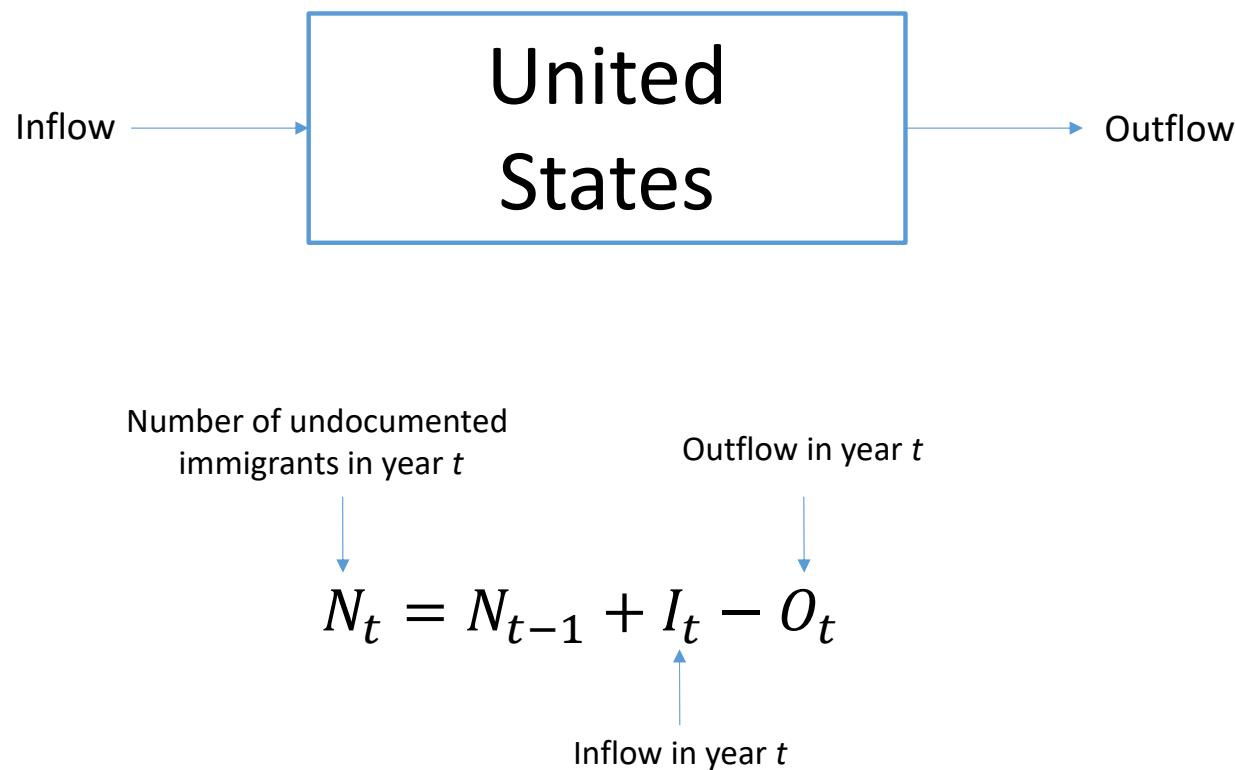
Probability Distribution (2016)



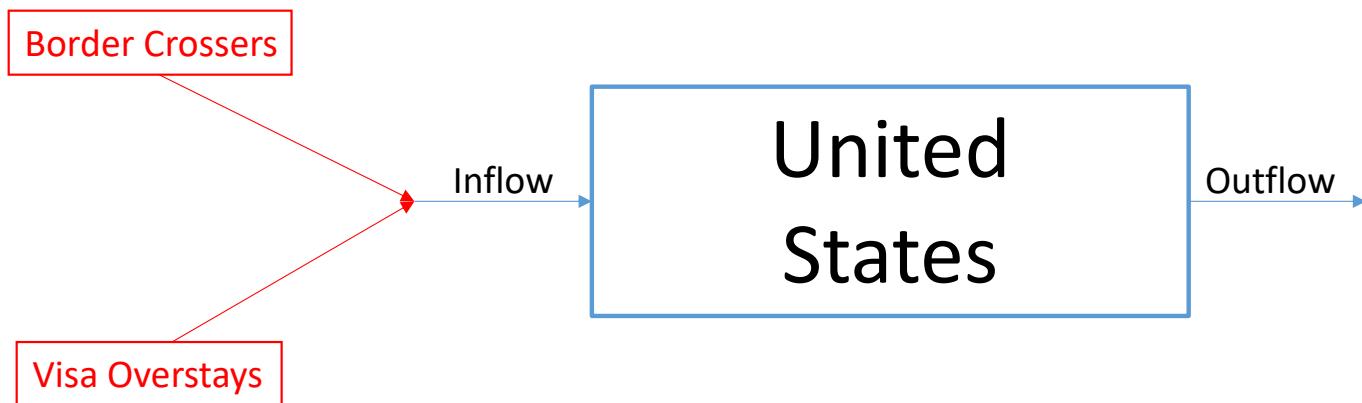
Trajectories

# The Approach

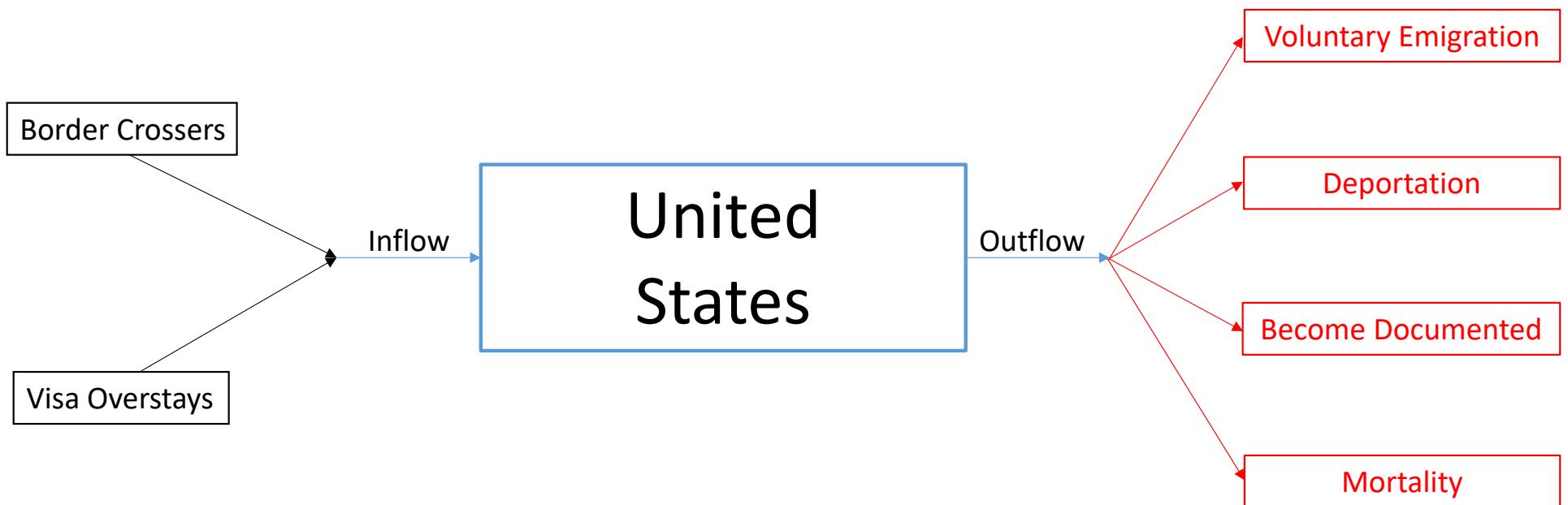
# The Model



# The Model



# The Model

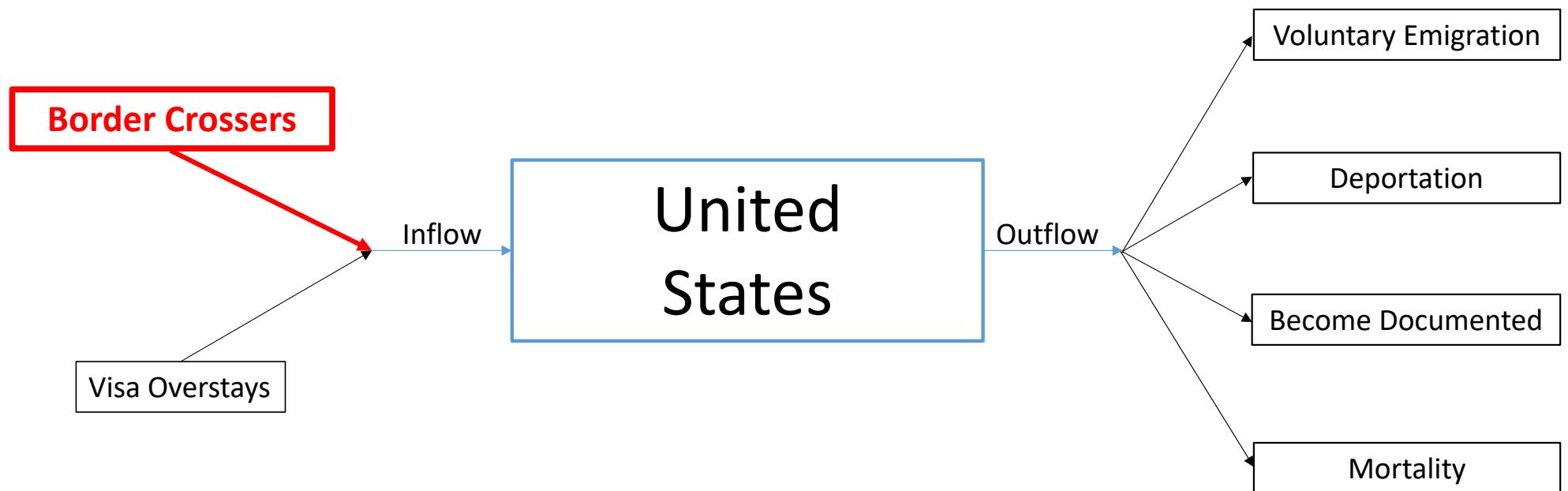


# Estimation Strategy

- Generate a *conservative estimate* (low-end estimate)
  - Underestimate Inflows
  - Overestimate Outflows

# Inflows

# The Model



# Illegal Border Crossers

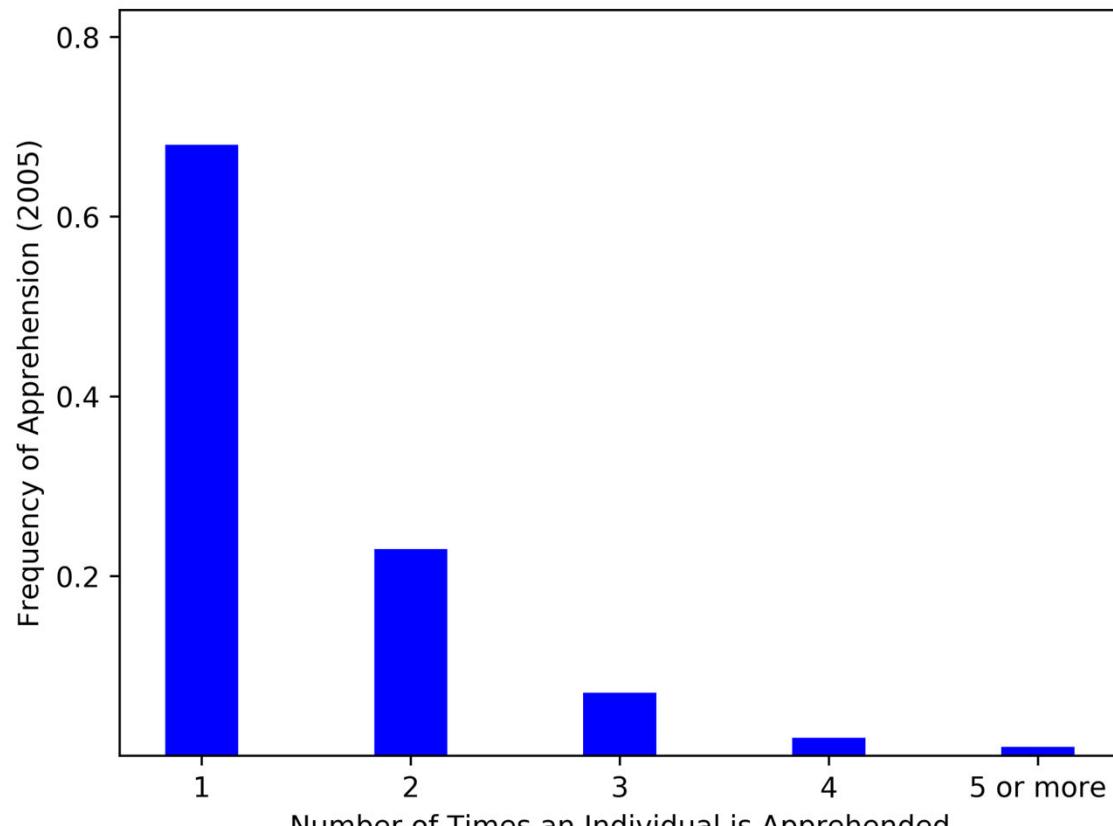
- Really hard component of the model
- To estimate this we use apprehension data on the Mexico - U.S. border
- Our model of the border crossers is based on standard repeated trials (Espenshade (1995)), i.e. a *Bernoulli process model*
- The model estimates the number of border crossings consistent with observed apprehensions that are happening in the data

# Number of Apprehensions



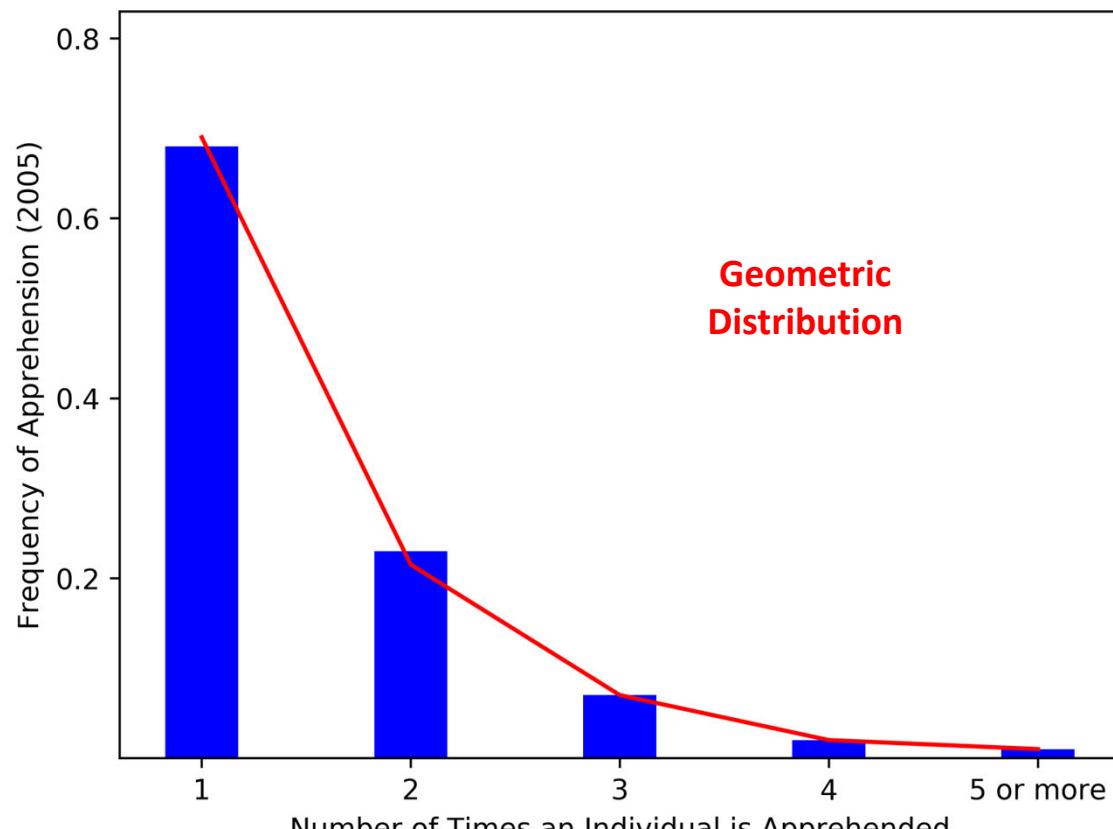
Data Source: U.S. Border Patrol

# Frequency of Apprehension (2005)



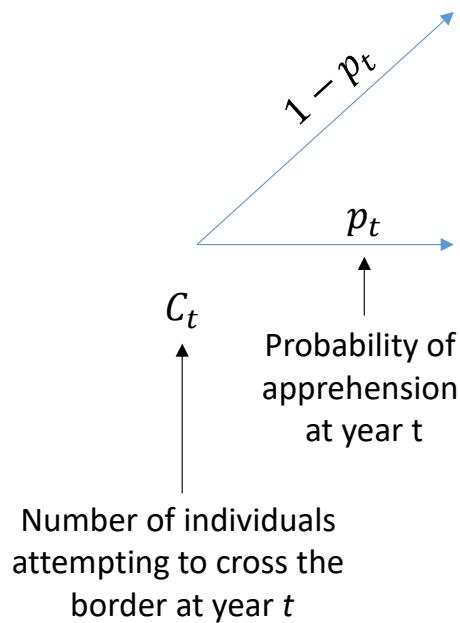
Data Source: U.S. Department of Homeland Security

# Frequency of Apprehension (2005)

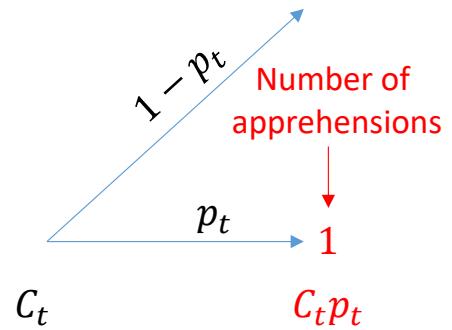


Data Source: U.S. Department of Homeland Security

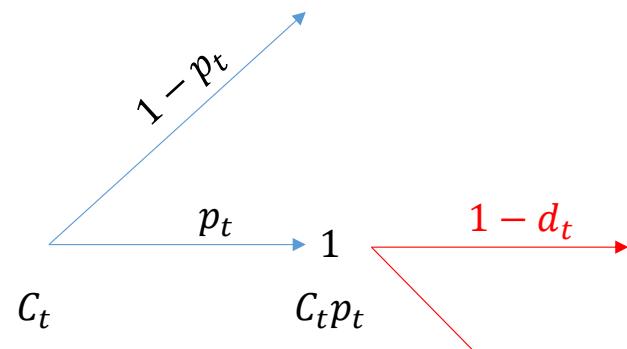
# The Border Crossing Model



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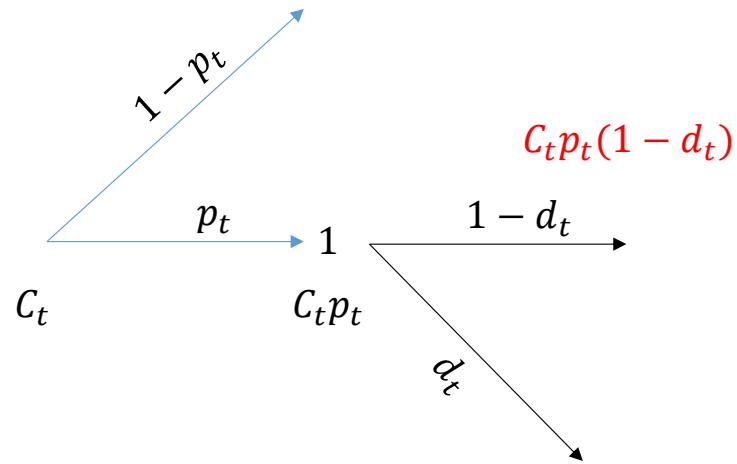


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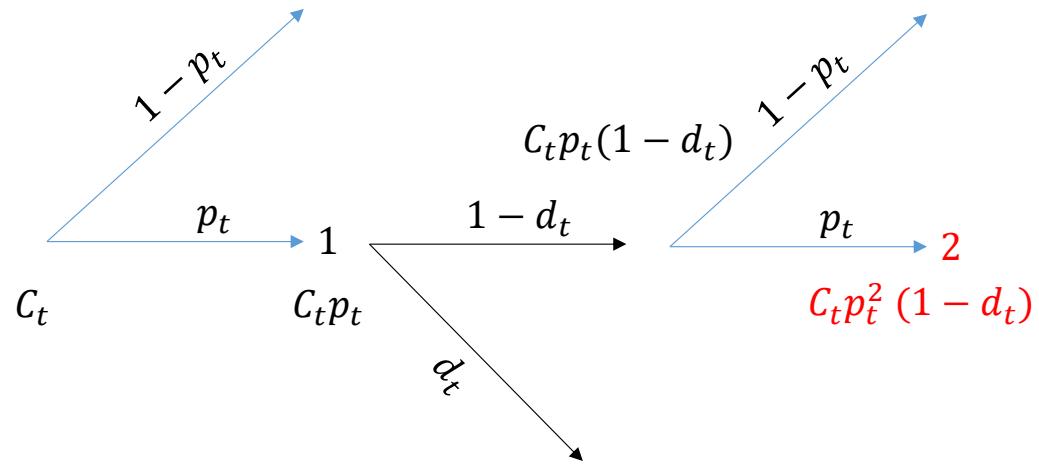


Deterrence  
due to apprehension  
(estimated from  
Encuesta sobre  
Migración en la  
Frontera Norte y Sur  
de México (EMIF))

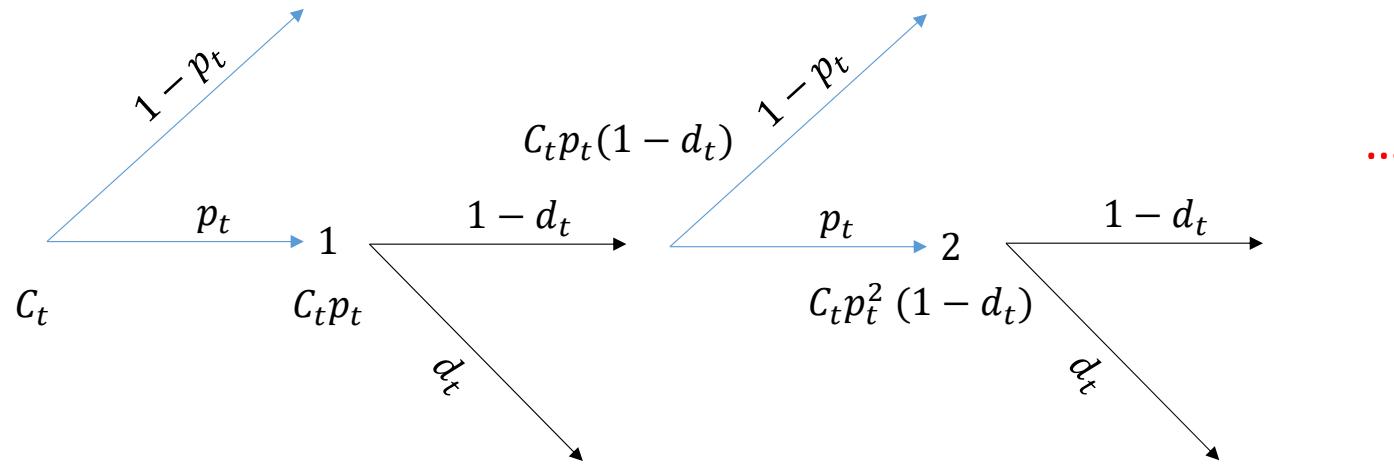
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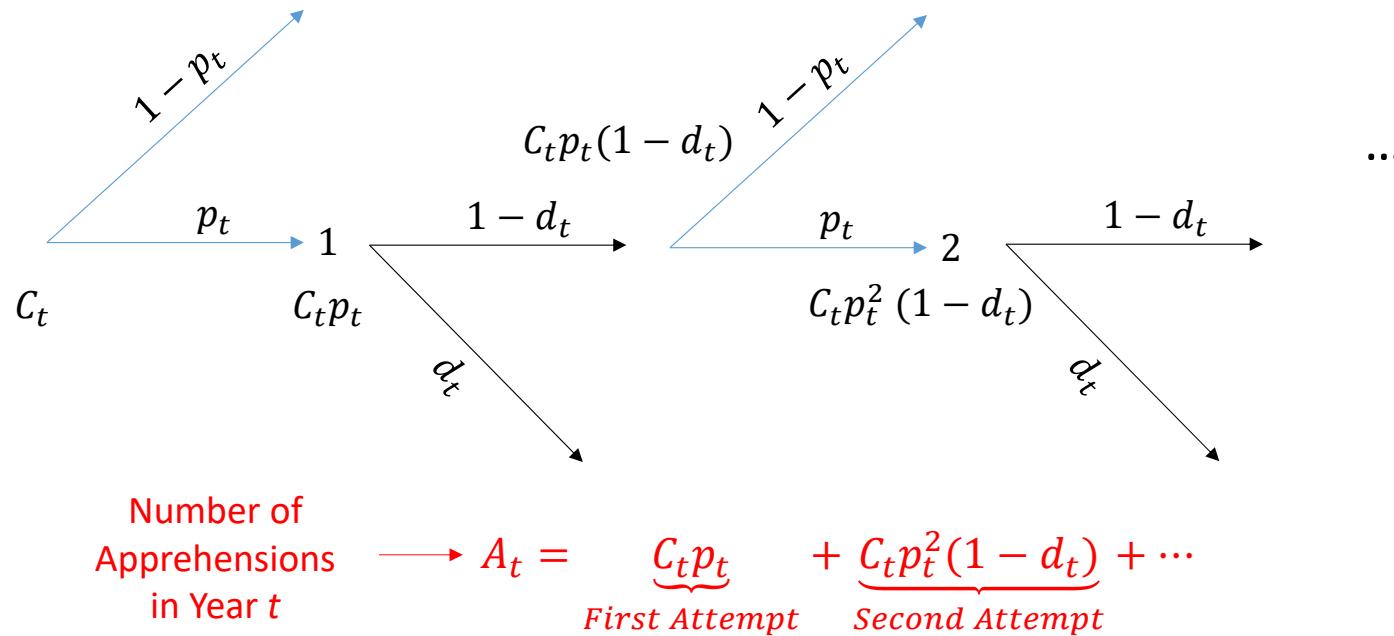
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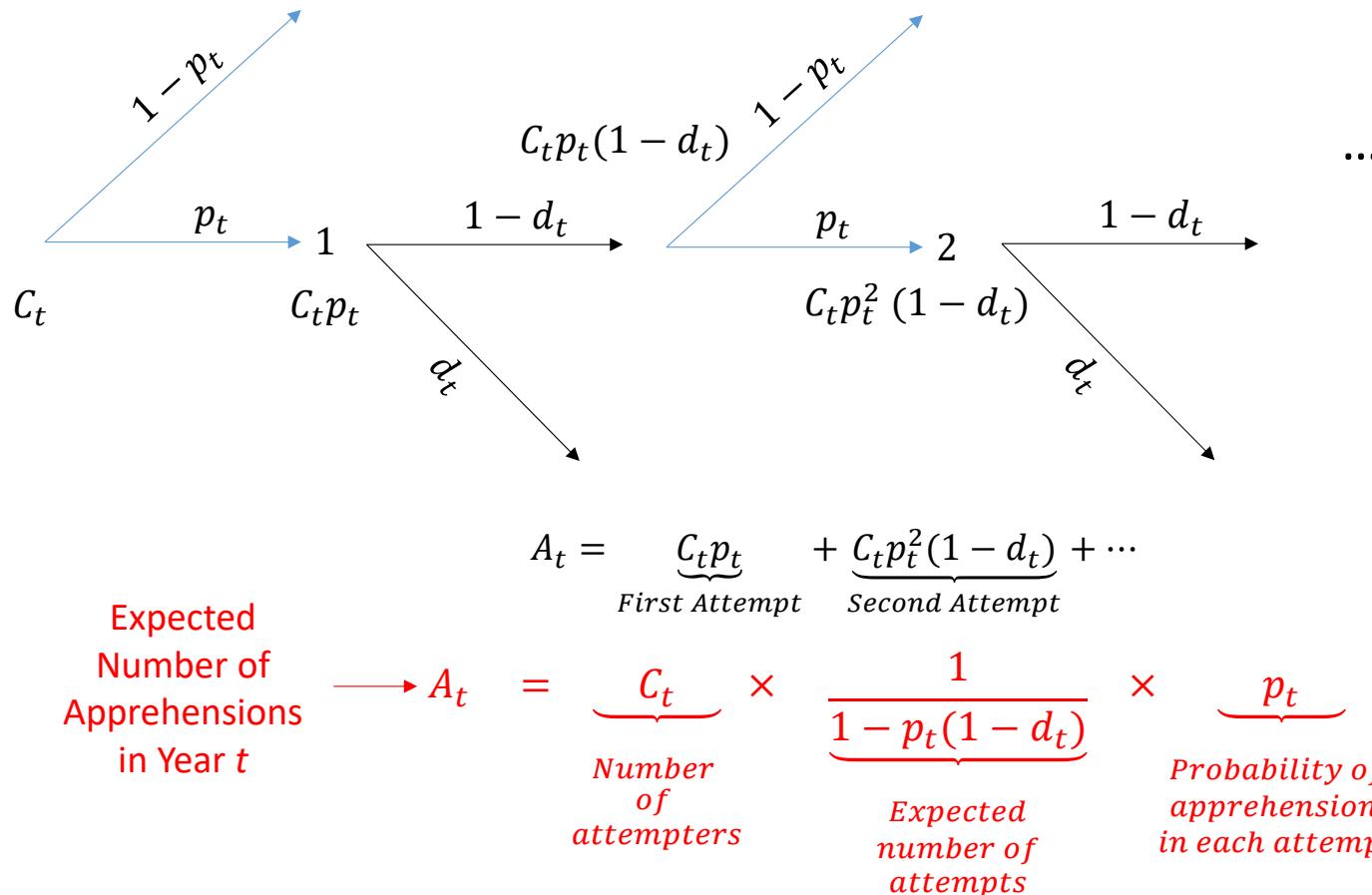
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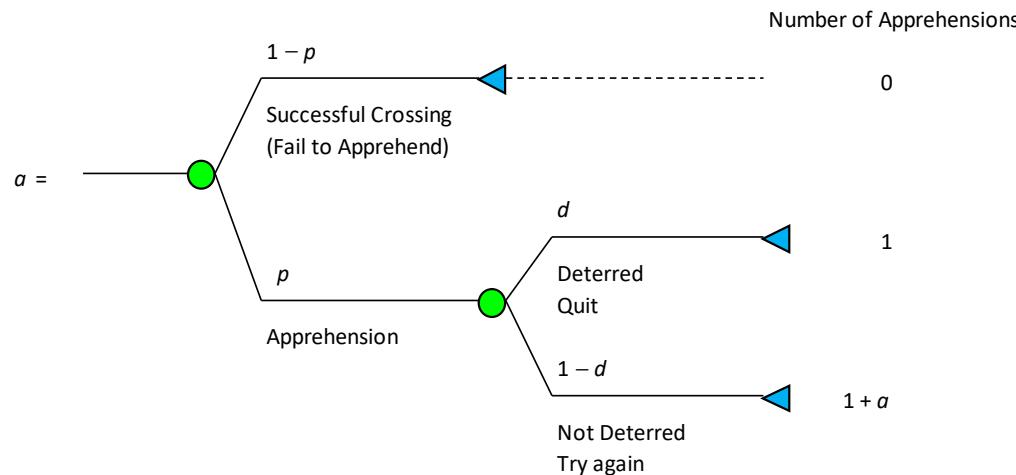


# The Border Crossing Model



# Mean Apprehensions Over All Attempted Border Crossers

- Let  $p = \Pr\{\text{Apprehension per crossing attempt}\}$
- Let  $d = \Pr\{\text{Deterrence} \mid \text{Apprehension}\}$
- Let  $a = \text{mean number of apprehensions over all attempted border crossers}$



$$a = (1 - p) \times 0 + pd \times 1 + p(1 - d) \times (1 + a)$$

$$a = \frac{p}{1 - p(1 - d)}$$

If  $C$  persons are attempting to cross, then

$$E(\text{Apprehensions}) = Ca = C \frac{p}{1 - p(1 - d)}$$

# What Does the Model Imply So Far?

- Number of apprehensions at year  $t$ :

$$A_t = C_t \frac{p_t}{1-p_t(1-d_t)}$$

- Number of attempters at year  $t$ :

$$C_t = A_t \frac{1-p_t(1-d_t)}{p_t}$$

- Number of undocumented border crossers at year  $t$ :  $B_t = C_t - Q_t$  ← Individuals who give up

- Number of individuals deterred at year  $t$ :

$$Q_t = A_t d_t$$

- Number of undocumented border crossers at year  $t$ :  $B_t = A_t \frac{1-p_t}{p_t}$

# We Need The Apprehension Probability $p$

- Define  $\bar{A}$  = *recidivist* apprehensions (that is, # 2<sup>nd</sup> or higher apprehension)
- Recall  $A$  = total apprehensions
- All apprehensions = First time apprehensions + recidivist apprehensions
- $A = Cp + \bar{A}$
- Recall  $C = A \frac{1-p(1-d)}{p}$
- $\Rightarrow \bar{A} = A - Cp = A - A(1 - p(1 - d)) = Ap (1 - d)$
- Solve for  $p$ :  $p = \frac{\bar{A}}{A(1-d)}$
- Note:  $A(1 - d)$  is total undeterred apprehensions, which equals the number of crossing opportunities for *recidivist* apprehensions

# What Does the Model Imply So Far?

- Number of apprehensions at year  $t$ :

$$A_t = C_t \frac{p_t}{1-p_t(1-d_t)}$$

- Number of attempters at year  $t$ :

$$C_t = A_t \frac{1-p_t(1-d_t)}{p_t}$$

- Number of individuals deterred at year  $t$ :

$$Q_t = A_t d_t$$

- Number of undocumented border crossers at year  $t$ :

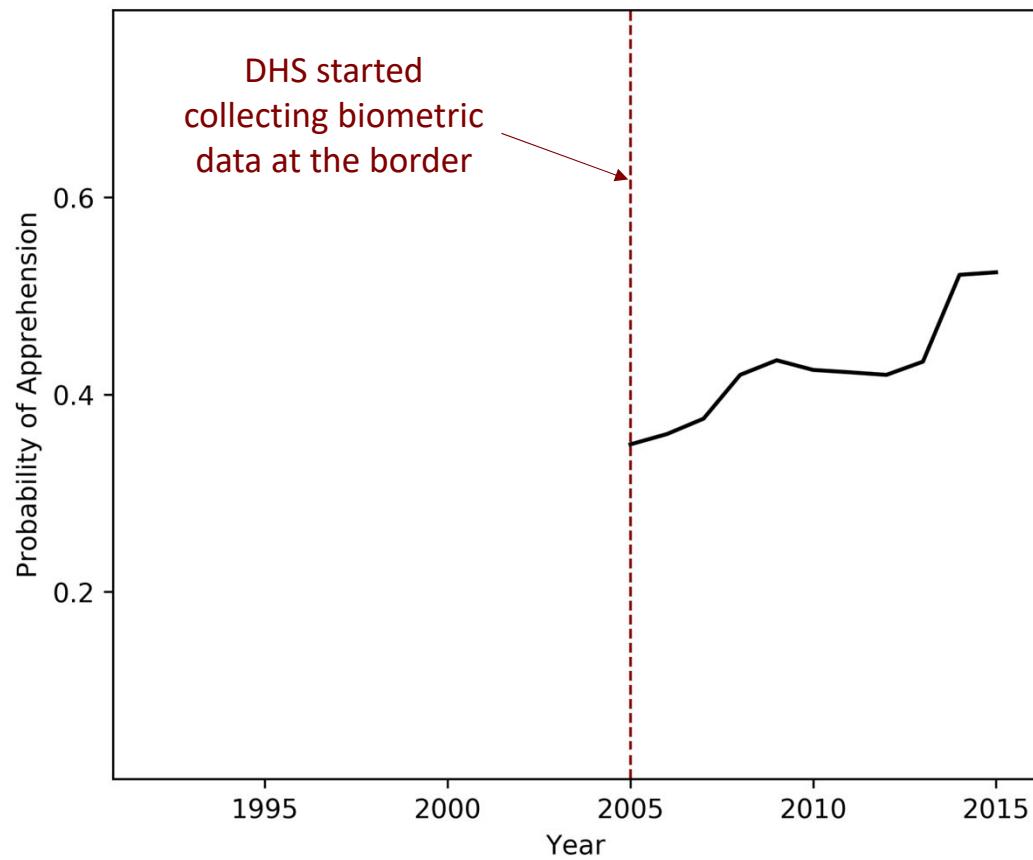
$$B_t = A_t \frac{1-p_t}{p_t}$$

- The probability of apprehension at year  $t$ :

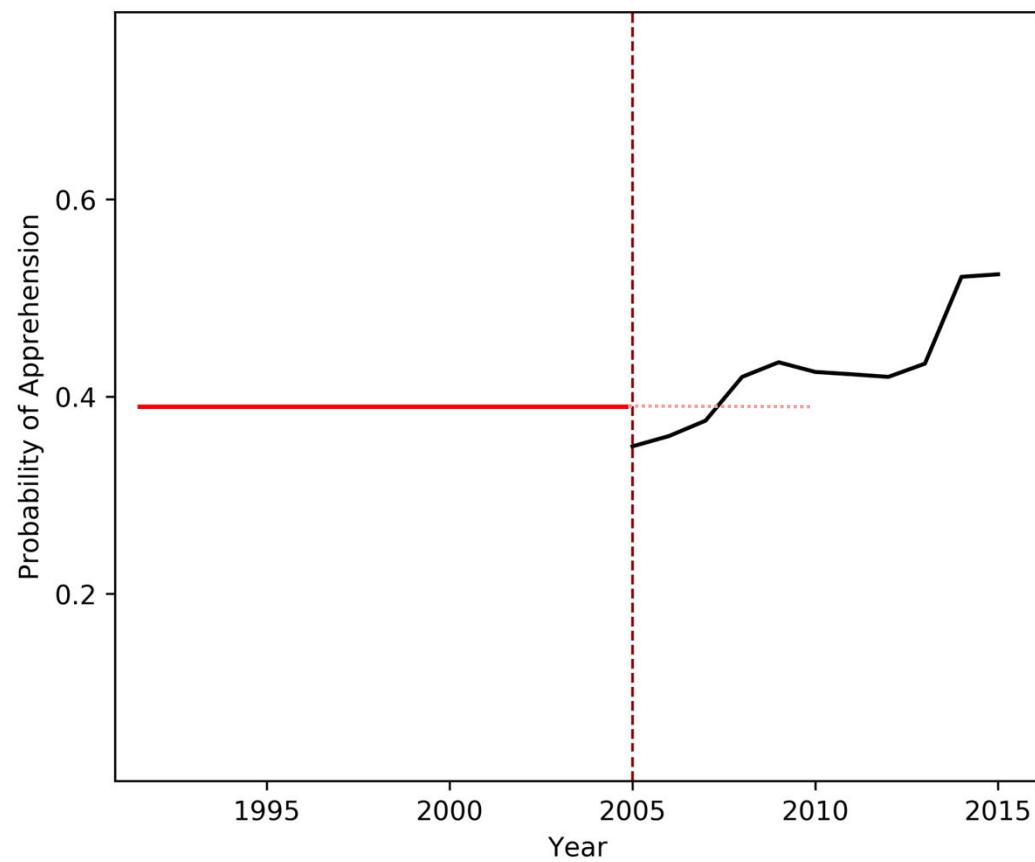
$$p_t = \frac{\bar{A}_t}{A_t(1-d_t)}$$

Number of recidivist apprehensions

# Probability of Apprehension

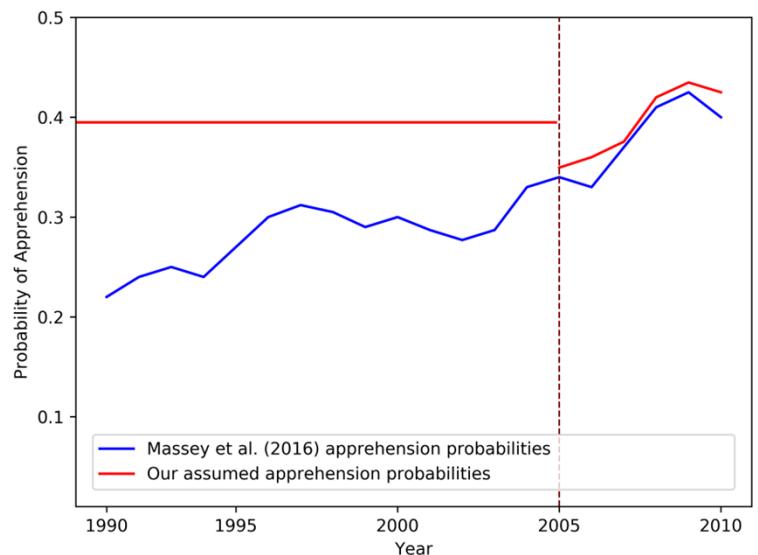


# Probability of Apprehension

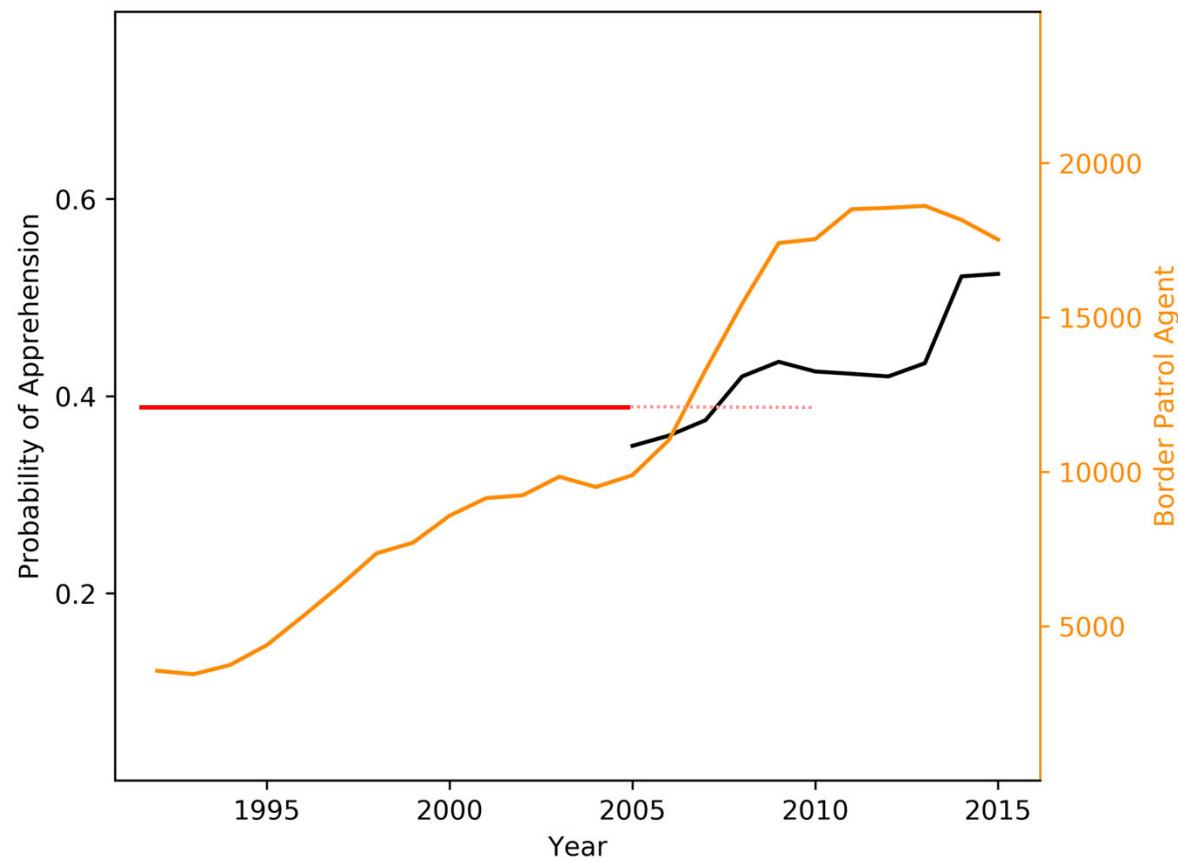


# Probability of Apprehension

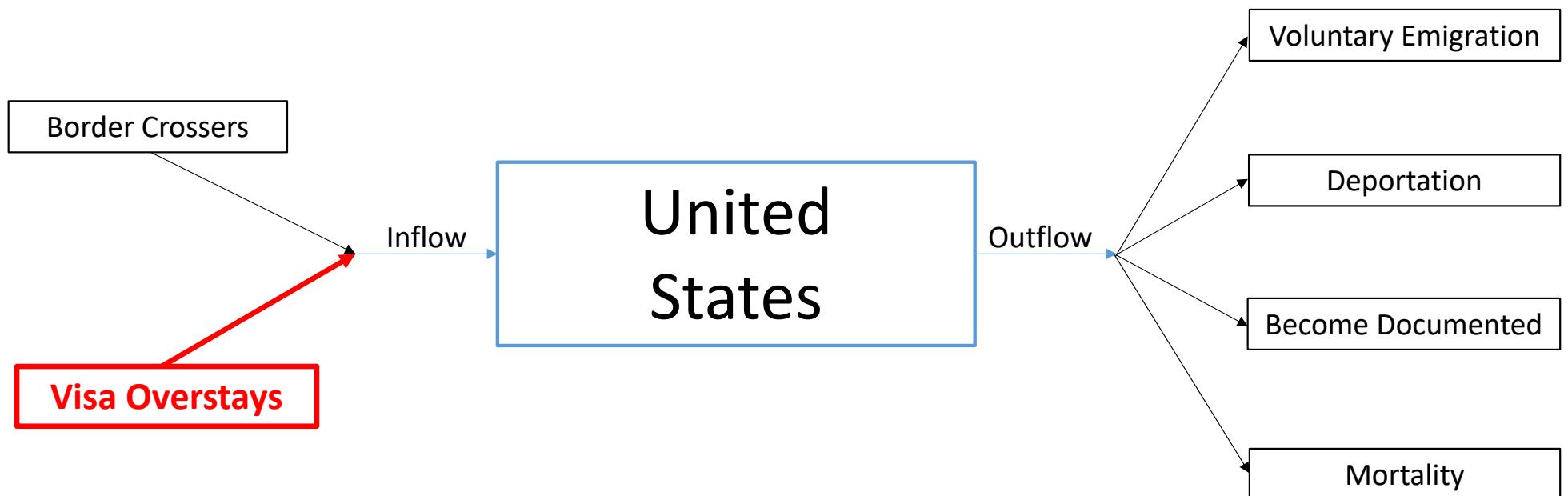
- What does the literature says?
  - Massey et al. (2016):
    - Start from the low 20% range in the 1990s ranging upward to approximately 30% in the earlier 2000s
  - Wein and Motskin (2009):
    - Estimate the 2003 rate at around 20%



# Probability of Apprehension



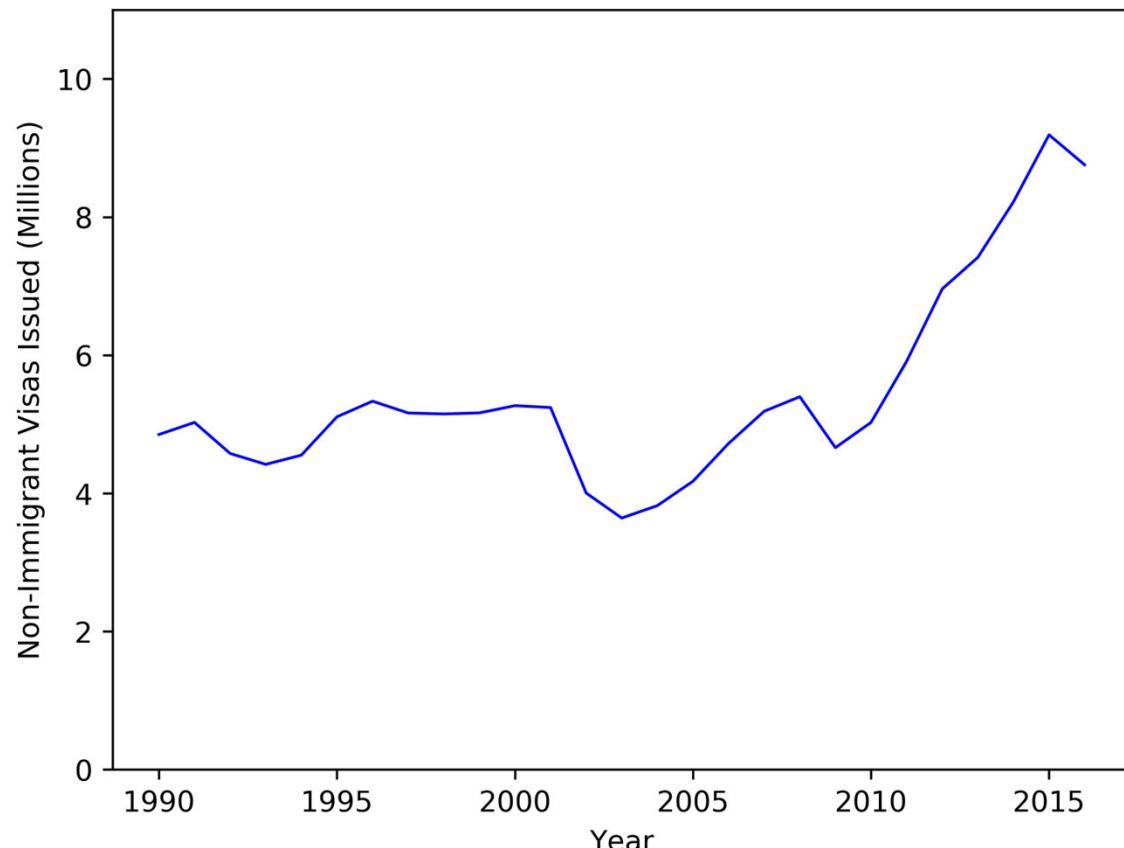
# The Model



# Visa Overstays

- Non-immigrants who are admitted to the U.S. lawfully, but do not leave after the period during which they have been allowed to remain in the U.S. legally ends
- Comprehensively measured by Department of Homeland Security starting in 2016

# Number of Non-Immigrant Visas Issued



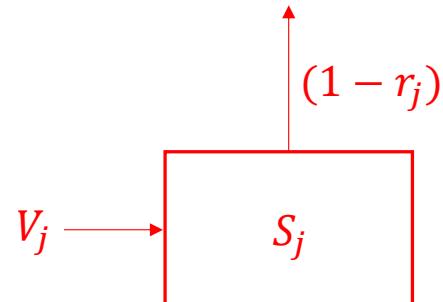
Data Source: U.S. Department of State

# Visa Overstays

$V_j$  : Number of visas issued in year  $j$

$S_j$  : Number of visa overstays in year  $j$

$r_j$ : Visa overstay rate in year  $j$



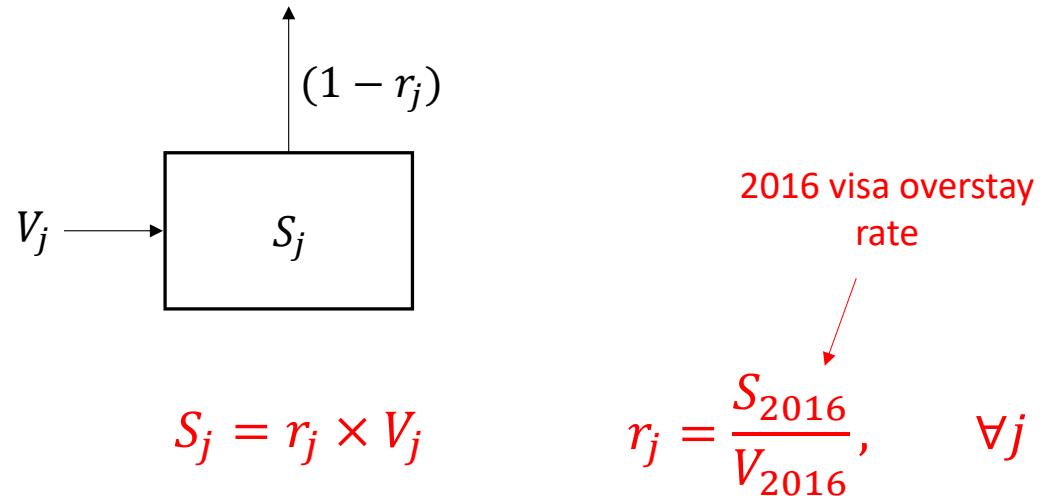
$$S_j = r_j \times V_j$$

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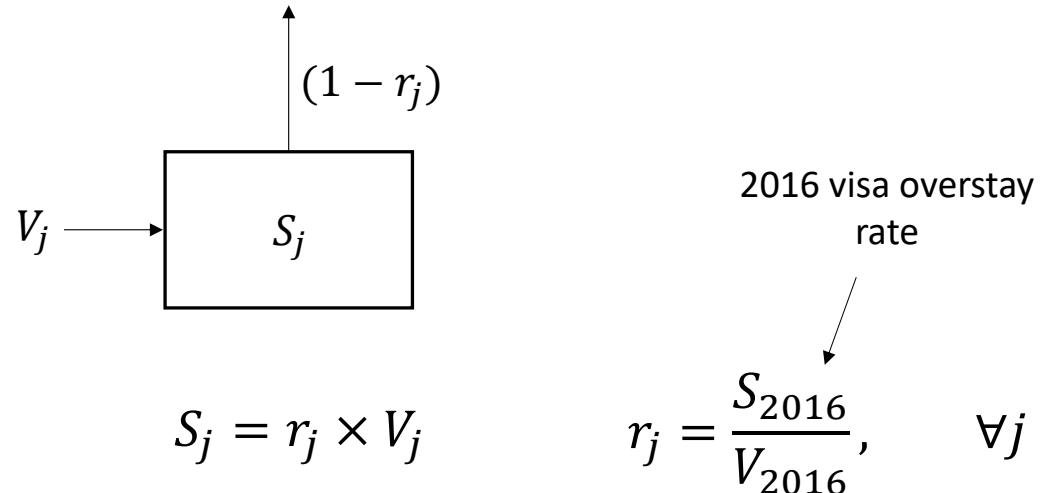


# Visa Overstays

$V_j$  : Number of visas issued in year  $j$

$S_j$  : Number of visa overstays in year  $j$

$r_j$ : Visa overstay rate in year  $j$



## Calibration:

Fraction of undocumented immigrants arriving in year  $j$  still in the US in 2015

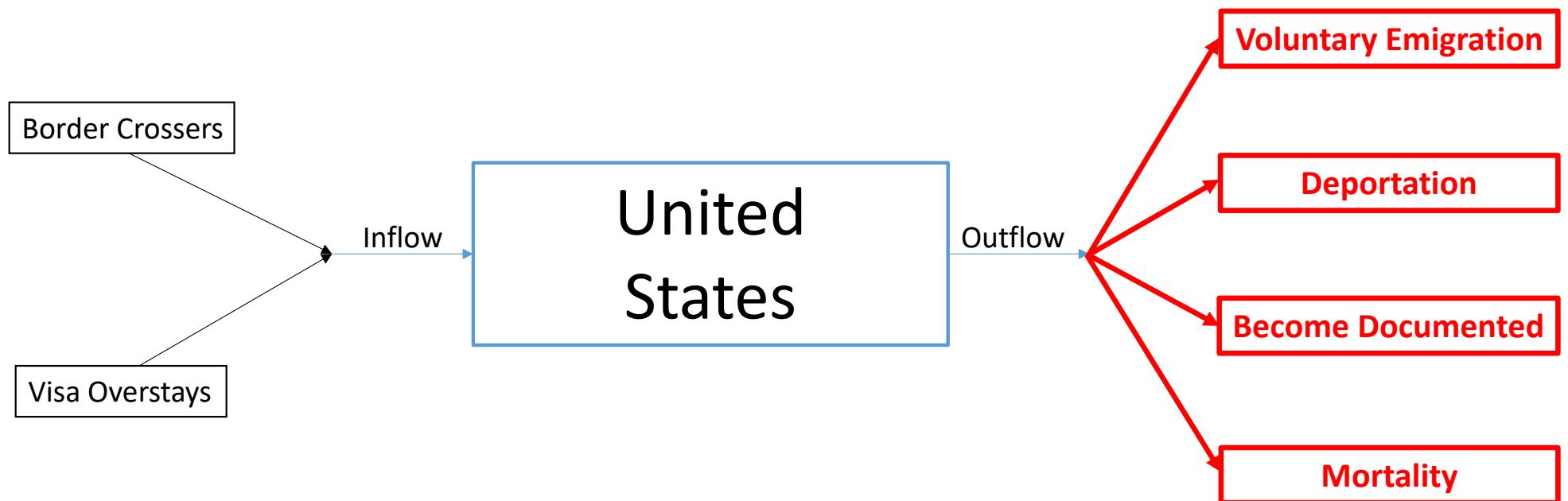
$$\sum_{j=1}^{t=26} S_j \times \Pr\{\tau_j \geq t-j\}$$

Model estimate of the population of overstayers in 2015

Number of overstayers in the current widely accepted estimate for 2015

# Outflows

# The Model



# Outflows

- Voluntary Emigration
- Deportations
- Become Documented
  - Including DACA in the outflows
- Mortality

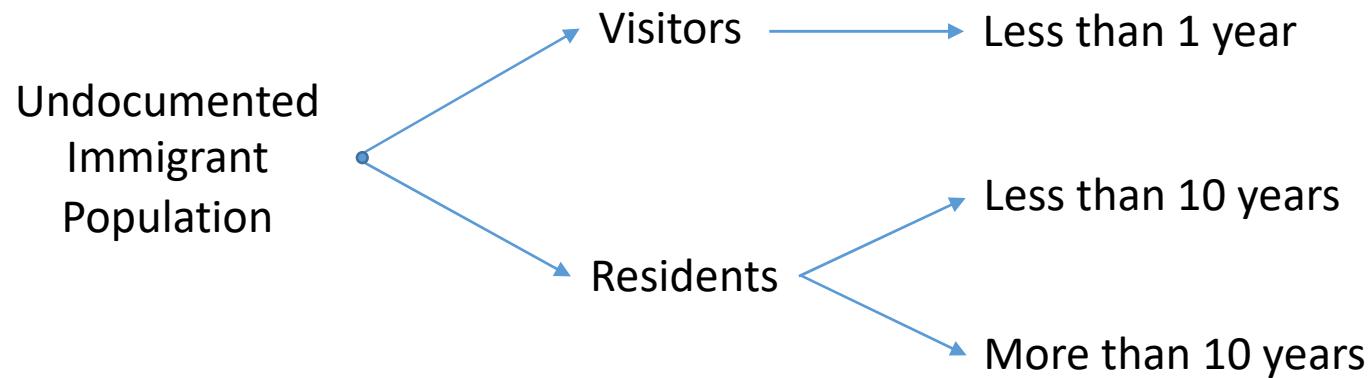
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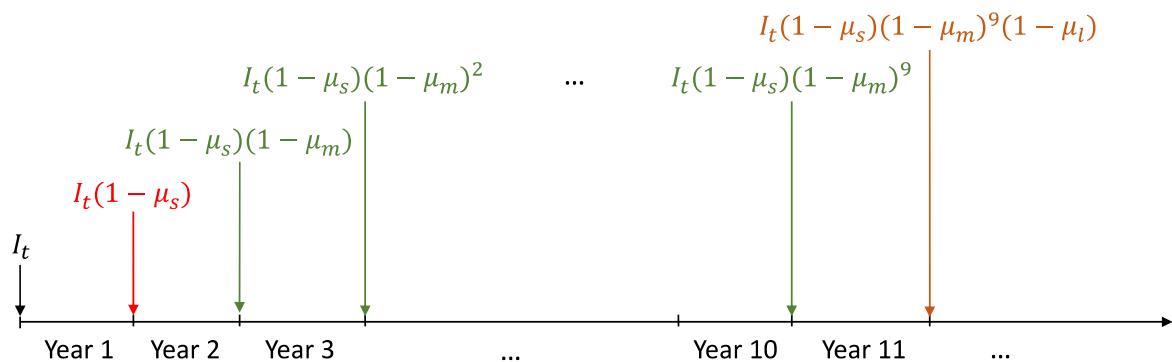
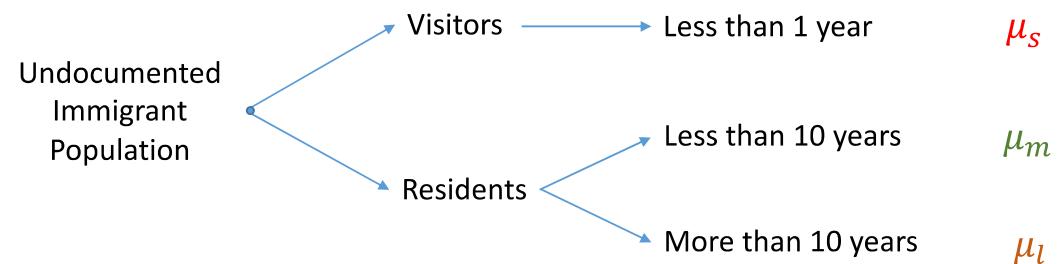
# Voluntary Emigration

- Emigration decreases with time spent in the country
  - Van Hook and Zhang (2011), Bhaskar et al. (2013), Warren and Warren (2013)
- For each undocumented immigrant we must keep track of duration in the country
  - Duration-dependent emigration rate

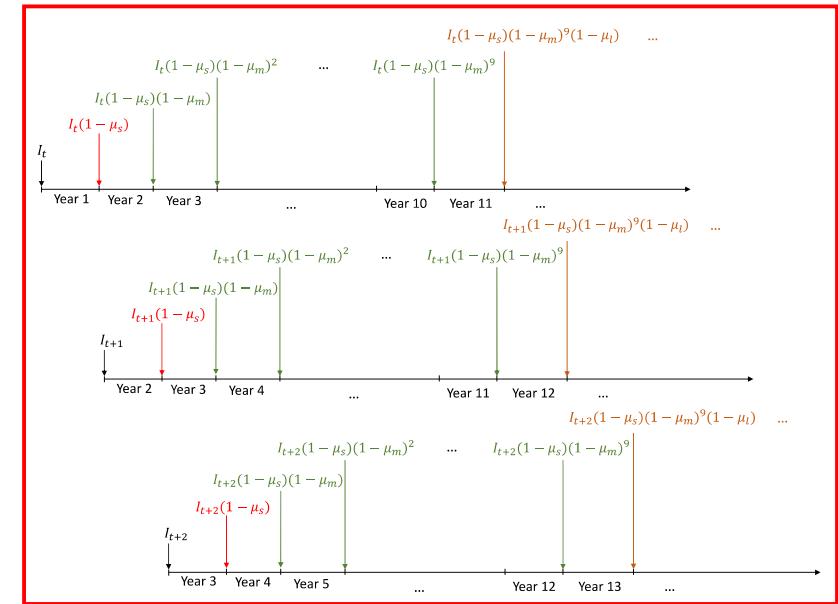
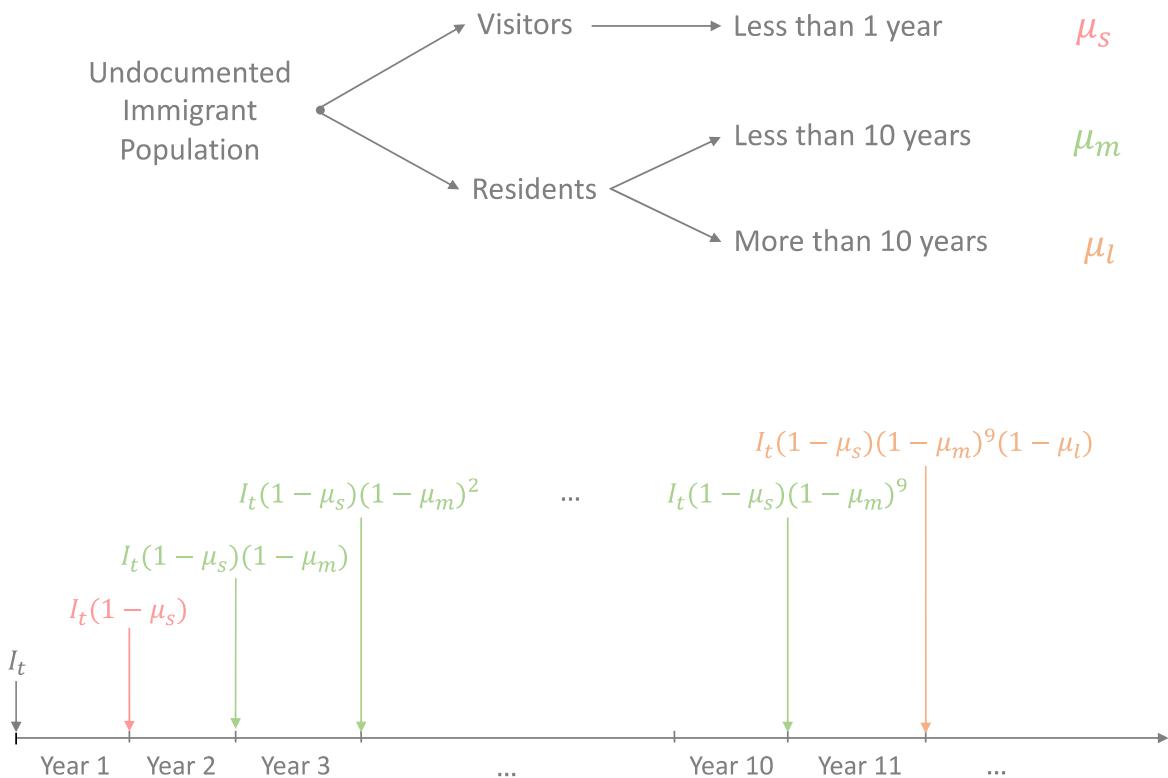
# Voluntary Emigration



# Voluntary Emigration – Simplified Example



# Voluntary Emigration – Simplified Example



# Voluntary Emigration - Data

- Residents: use ***largest*** values in published academic and government sources
  - Ahmed and Robinson (1994), Mulder (2003), Van Hook and Zhang (2011), Baker and Rytina (2013), Warren and Warren (2013), Bhaskar et al. (2013)
- Visitors: use data on first-year exit rate for visa overstayers
  - The assumed emigration rates are an upper bound for border crossers (Massey et al (2002), Massey et al. (2016))
- The emigration rates used significantly overestimate outflows

# The Final Formula

$$N_t = N_{t-1} + I_t - O_t$$

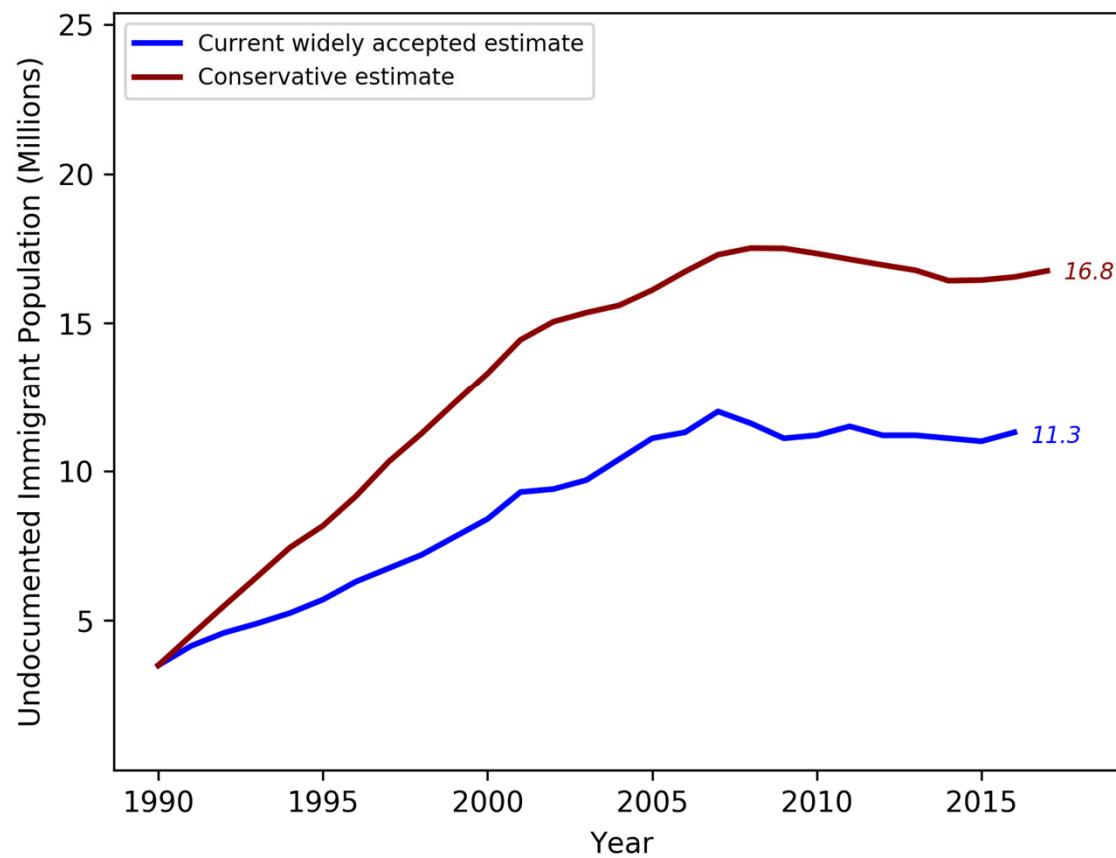


$$N_t = N_0 \times Pr\{\tau_0 > t\} + \sum_{j=1}^t \left( (I_j(1 - \mu_s) - D_j) \times Pr\{\tau_j > t\} \right)$$

$$I_t = S_t + B_t = r \times V_t + A_t \frac{1 - p_t}{p_t} \quad , \quad p_t = \frac{\bar{A}_t / A_t}{(1 - d_t)}$$

$$Pr\{\tau_j > t\} = \begin{cases} (1 - \mu_m - \delta)^{10} (1 - \mu_l - \delta)^{t-10}, & j = 0 \\ (1 - \mu_m - \delta)^9 (1 - \mu_l - \delta)^{t-j-9}, & 0 < j \leq t-10 \\ (1 - \mu_m - \delta)^{t-j}, & j > t-10 \end{cases}$$

# Results – Conservative Estimate



# Incorporate Uncertainty Into The Model

- Build uncertainty into the model to take into account variability
  - Produce probability distribution over the number of undocumented immigrants
- Main source of uncertainty
  - Parameter uncertainty

# Incorporate Uncertainty Into The Model

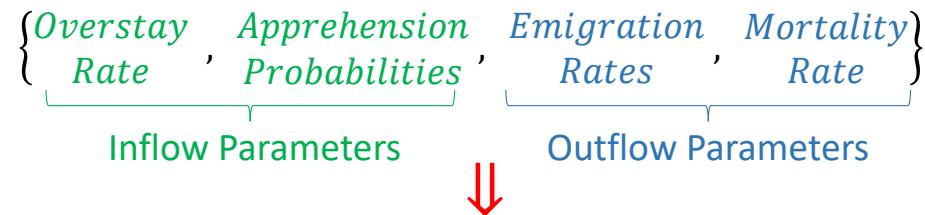
- Parameter Uncertainty:

$$\left\{ \begin{array}{c} \text{Overstay Rate}, \text{Apprehension Probabilities} \\ \text{Inflow Parameters} \end{array}, \begin{array}{c} \text{Emigration Rates}, \text{Mortality Rate} \\ \text{Outflow Parameters} \end{array} \right\}$$

- Taking into account:
  - Circular flows which link apprehension probability with emigration rates (Massey (2004, 2013), Massey and Pren (2012))
  - Cohort dependent emigration rates

# Incorporate Uncertainty Into The Model

- Parameter Uncertainty:



$$N_t = N_0 \times Pr\{\tau_0 > t\} + \sum_{j=1}^t \left( (I_j(1 - \mu_s) - D_j) \times Pr\{\tau_j > t\} \right)$$

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# Incorporate Uncertainty Into The Model

- Parameter Uncertainty:

The diagram illustrates the flow of parameters. At the top, a set of four items is shown: *Overstay Rate*, *Apprehension Probabilities*, *Emigration Rates*, and *Mortality Rate*. The first two items are grouped under the label "Inflow Parameters" with a green bracket, and the last two are grouped under the label "Outflow Parameters" with a blue bracket. A red double-headed vertical arrow points downwards from this set to a box containing the following equations:

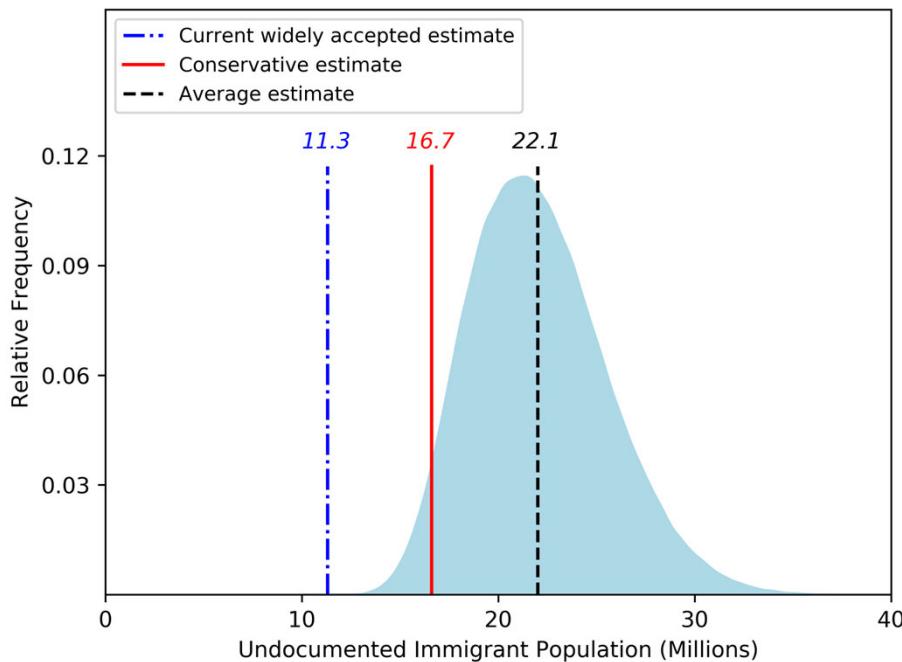
$$N_t = N_0 \times \Pr\{\tau_0 > t\} + \sum_{j=1}^t \left( (I_j(1 - \mu_s) - D_j) \times \Pr\{\tau_j > t\} \right)$$

$$I_t = S_t + B_t = r \times V_t + A_t \frac{1 - p_t}{p_t} \quad , \quad p_t = \frac{\bar{A}_t / A_t}{(1 - d_t)}$$

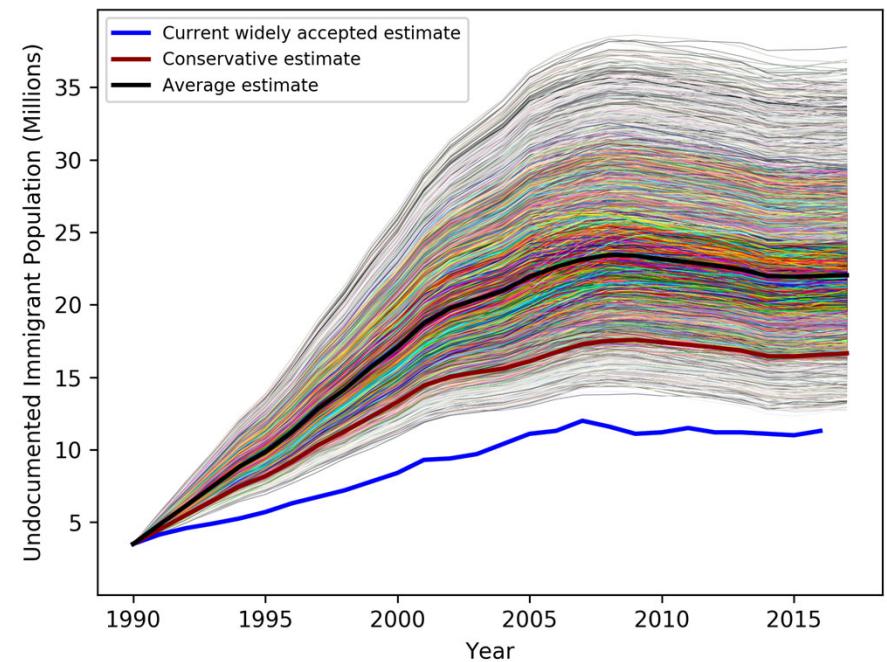
$$\Pr\{\tau_j > t\} = \begin{cases} (1 - \mu_m - \delta)^{10} (1 - \mu_l - \delta)^{j-10}, & j = 0 \\ (1 - \mu_m - \delta)^9 (1 - \mu_l - \delta)^{t-j-9}, & 0 < j \leq t-10 \\ (1 - \mu_m - \delta)^{t-j}, & j > t-10 \end{cases}$$

- Poisson structure with mean dependent upon the underlying parameter

# Results – 1,000,000 Simulation Runs



Probability Distribution (2016)



Trajectories

# Reception

The New York Times

LA  
Times

The Boston Globe

The Washington Post

THE  
HILL

Bloomberg

POLITICO

Mother Jones

Daily  
Mail

DAILY NEWS  
NEW YORK

NATIONAL  
REVIEW

npr



## Prior estimates of undocumented immigrants in the US were way off

BY MOHAMMAD FAZEL-ZARANDI, OPINION CONTRIBUTOR — 10/26/18 11:15 AM EDT  
THE VIEWS EXPRESSED BY CONTRIBUTORS ARE THEIR OWN AND NOT THE VIEW OF THE HILL

A screenshot of the The Hill website homepage. The header includes "TOPICS", "SEARCH", "DAILY NEWS", "3 FREE MONTHS", and "LOG IN". Below the header are several news thumbnails. A banner at the bottom features an advertisement for "CAFE ESCADRILLE" and a promotional offer for gift cards.

How many undocumented immigrants there really are, and why the number matters



By MOHAMMAD M. FAZEL-ZARANDI, JONATHAN S. FEINSTEIN and EDWARD H. KAPLAN

<https://video.foxnews.com/v/5839718007001#sp=show-clips>

# Reception

Some of the issues in dispute:

1. The undercount implied by the new model is too high.
2. The range of model-estimated populations is too large to be useful for policy purposes, while the residual method gives a much smaller range of uncertainty.
3. The voluntary emigration rates of undocumented immigrants employed in the model are too low.

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# Census/American Community Survey Undercount

- For around 40 million there is no clear answer to place of birth (ignored deliberate misreporting)
- Fill in the blanks using “hot deck” allocation
  - Missing at random vs *missing on purpose*
  - Due to *missing on purpose*, the number of “donors” to the hot deck will be disproportionately US born
  - Imputed value for the place of birth variable will disproportionately point to “born in the USA”
  - Undercounting number foreign born ⇒ undercount in number of undocumented immigrants

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2. **The range of model-estimated populations is too large to be useful for policy purposes, while the residual method gives a much smaller range of uncertainty.**
  - Precise estimate of wrong quantity: undocumented immigrants who are located and answering truthfully
  - Small variability stems from the sampling variation that accompanies large samples

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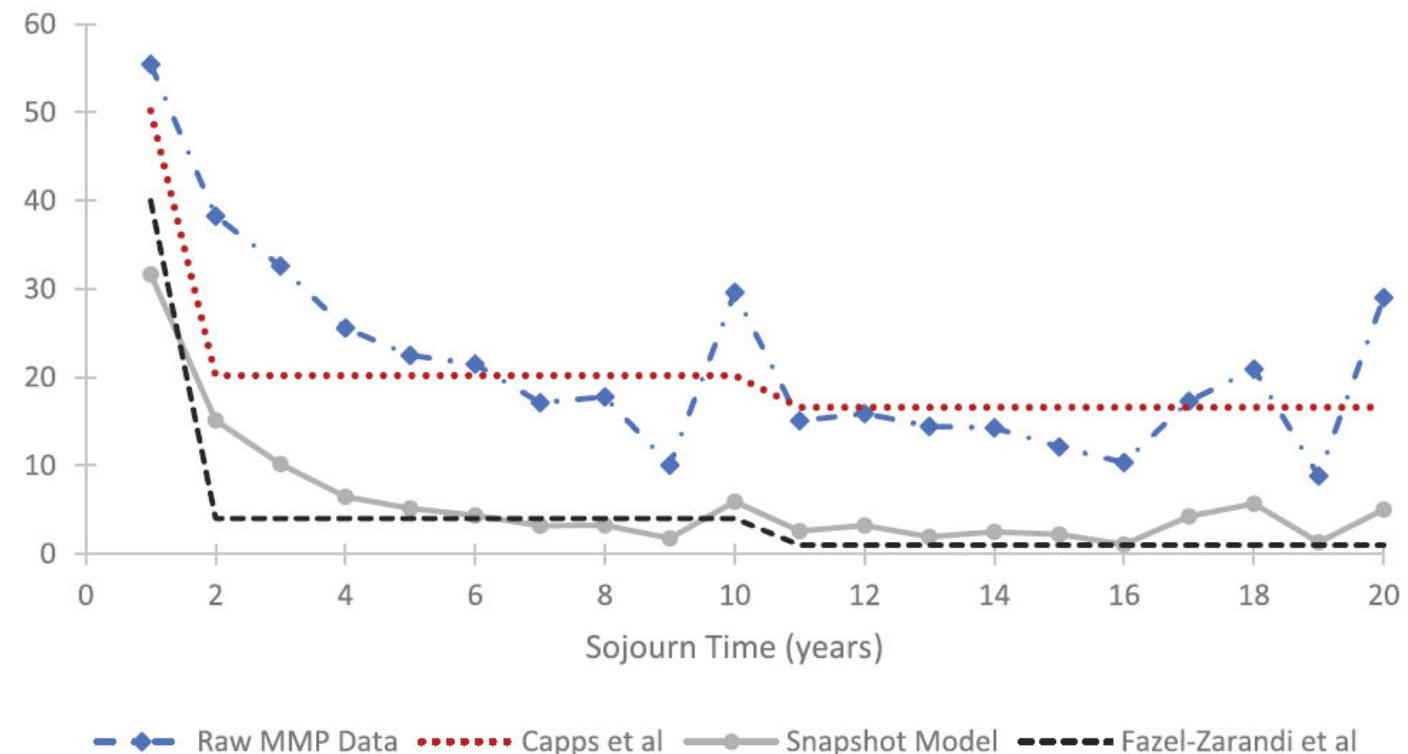
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  - Nearly everyone in the survey are in Mexico at the time of sampling (Massey et al (2016), Lessem (2018))
  - Misses those still in the United States at the time of the survey
- After accounting for the sampling bias, we find that the resulting emigration rates are consistent with our presumed rates

# Snapshot Models of Undocumented Immigration

Scott Rodilitz  <sup>1,\*</sup> and Edward H. Kaplan 

Annual Emigration Probability



**Fig 8.** Annual emigration probabilities from the United States.

# Policy Implications

- **Social services:**
  - agencies that have been working off of the previous estimate should recognize that the resources they have allocated for this population may be too low
- **Border control:**
  - In the last few years, the majority of new undocumented entrants are visa overstays
  - The number of illegal border crossers has substantially decreased
- **Crime:**
  - Further calls into question the claim of elevated risks of criminality surrounding undocumented immigrants.
  - The crime **rate** is much smaller