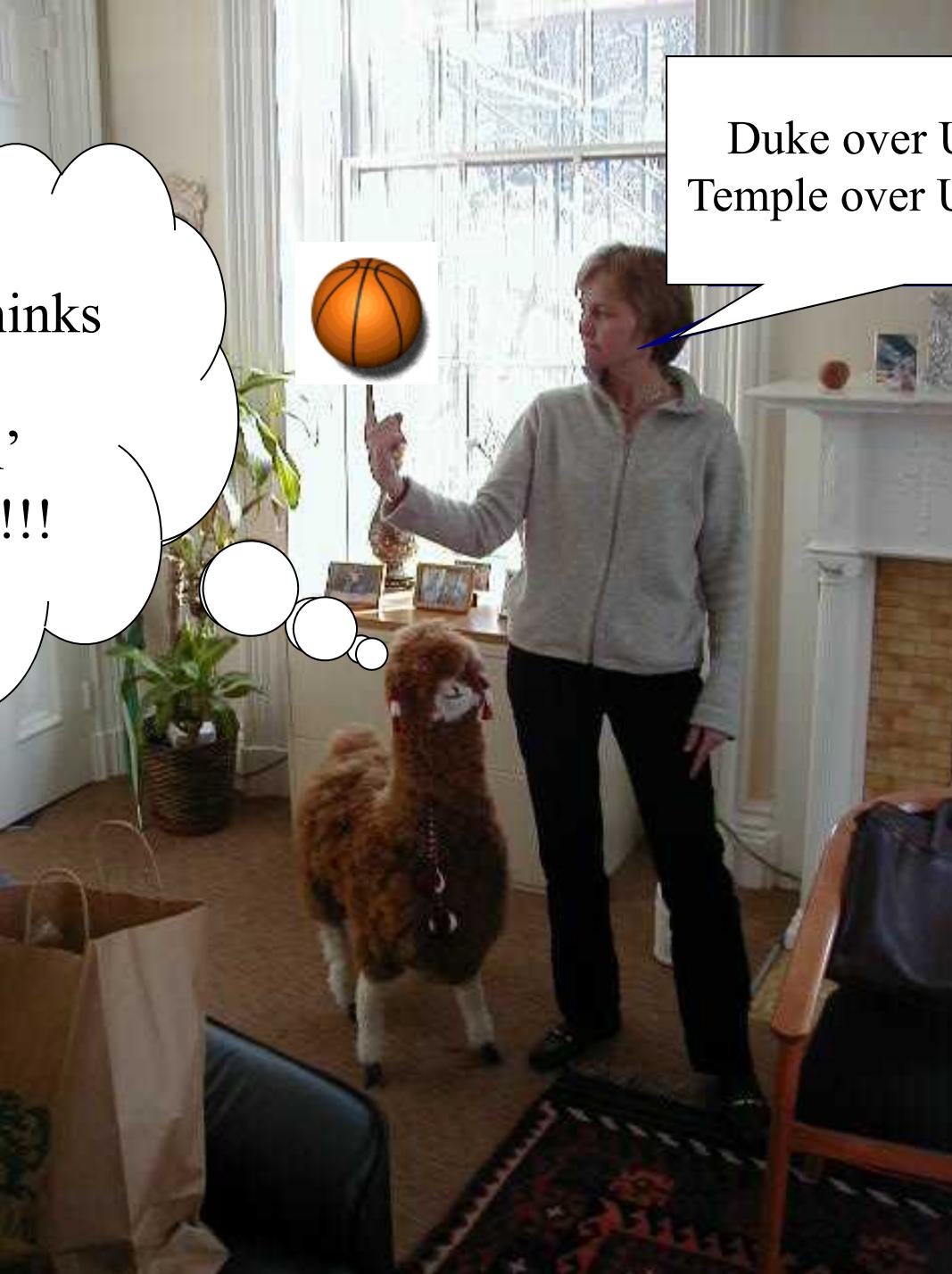




WWF....SMACKDOWN!!!!  
AWESOME BABY!!!!





A woman in a light-colored zip-up hoodie and dark pants stands in a living room, facing a brown and white dog. A basketball is suspended in the air between them. The room has a fireplace, a window, and a potted plant. A large, white, cloud-shaped speech bubble originates from the dog's position.

She still thinks  
I'm a  
freakin'  
Husky!!!!

Duke over UCLA by 3!  
Temple over UCONN by 7!

# March Madness And The Office Pool

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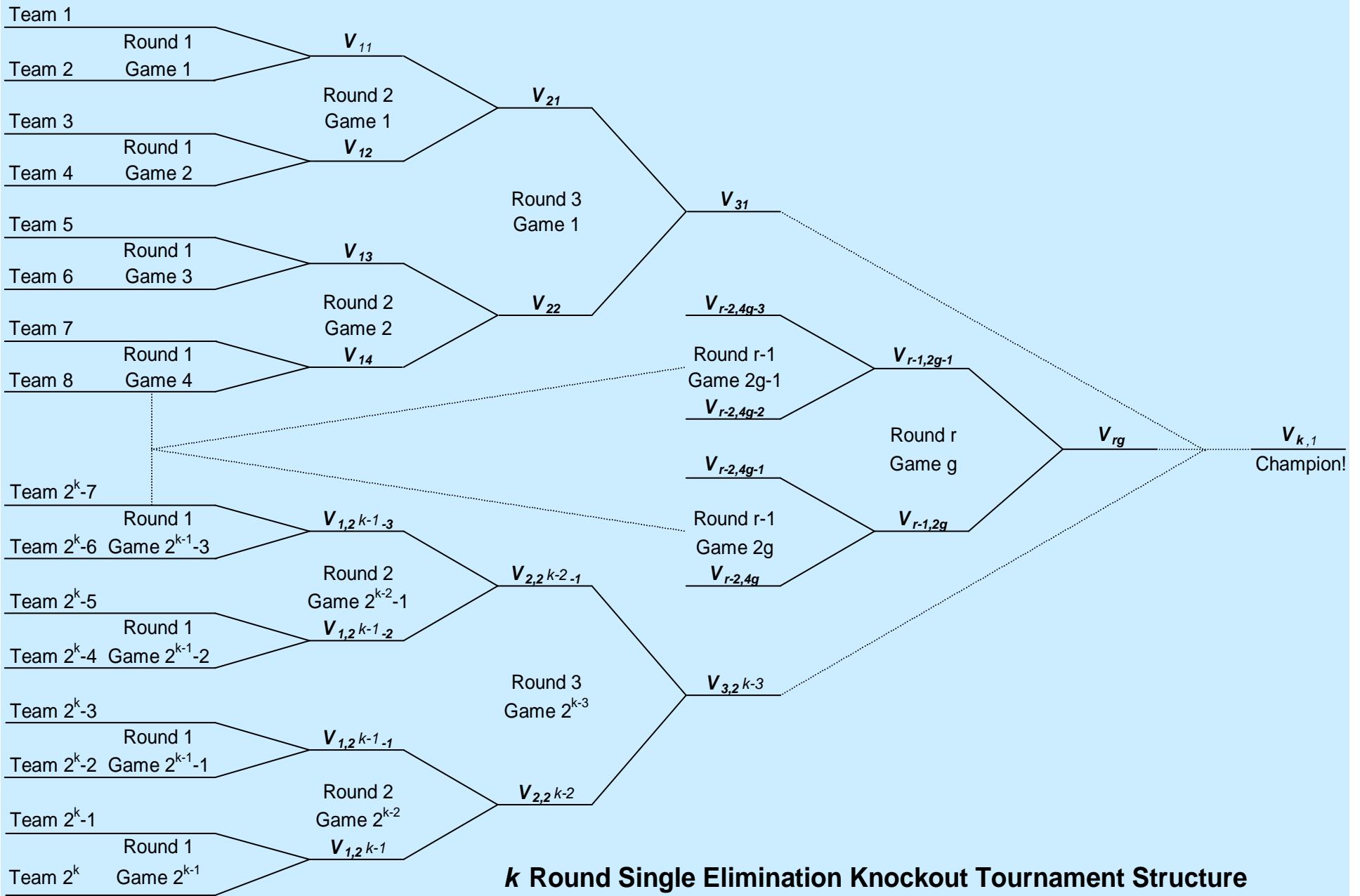
Edward H. Kaplan  
Stanley J. Garstka

Yale University  
School of Management

# What We're Going To Do:

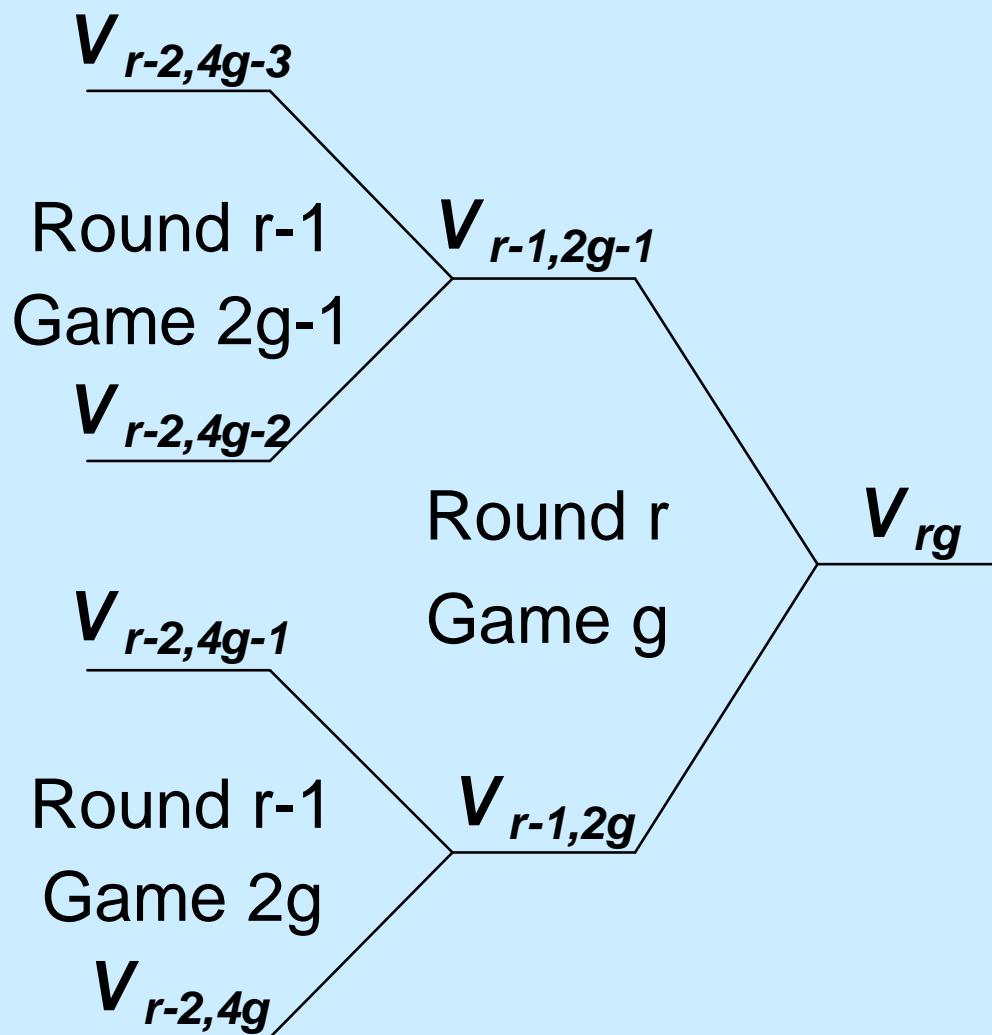
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- ◆ Describe tournament structure
- ◆ Embed office pools in tournament structure
- ◆ Illustrate with a random tournament (let Alli Baby pick the winners!)
- ◆ Illustrate with toy Markov tournaments
- ◆ Show how to maximize the expected number of points in a real pool
- ◆ Markov models for college tournaments
- ◆ Empirical results



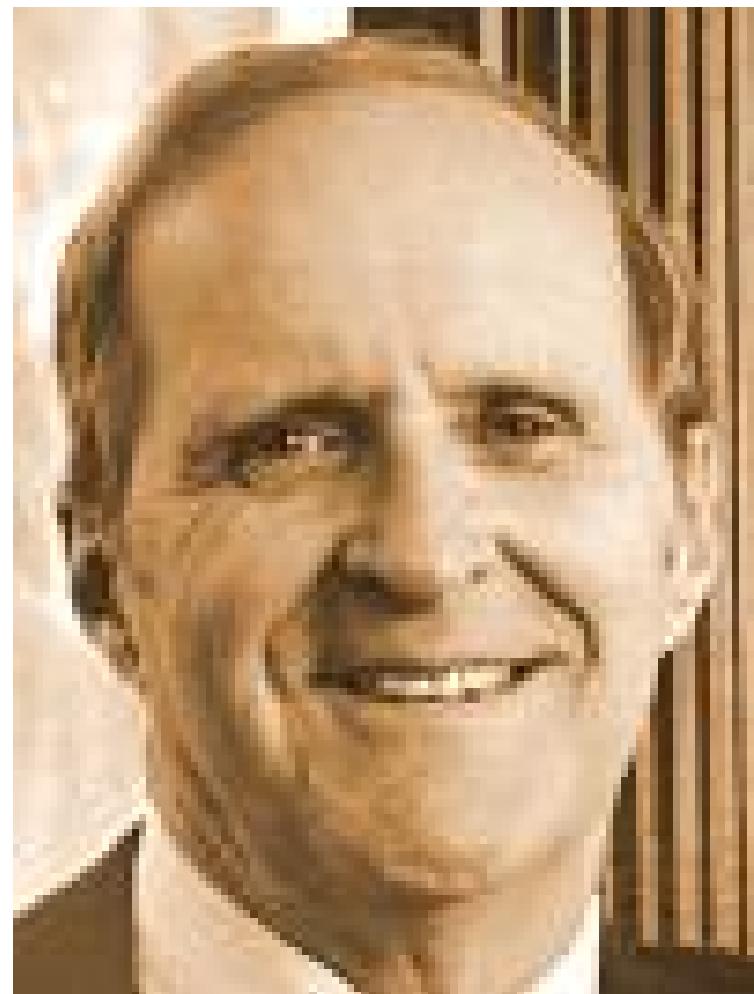
# Single Round Elimination Tournament Structure

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# The Victor!

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# Tournament Structure Facts

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- ◆ Tournaments with  $k$  rounds start with  $2^k$  teams, and require  $2^k - 1$  games
- ◆ e.g. NCAA tournament has 6 rounds, 64 teams, and 63 games
- ◆ The number of possible realizations equals  $2^{2^k - 1}$ , so for the NCAA tournament there are  $2^{63}$  or about  $9.22 \times 10^{18}$  (that is, more than 9 billion billion) possible realizations

# Office Pools

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- ◆ Points are awarded for correctly naming game winners *a priori*
- ◆ Score for the pool is the sum of the points awarded for each game
- ◆ (S)he who gets the most points wins
- ◆ Simplest case: pick as many games correctly as possible
- ◆ More complex case: points increase with tournament depth; upset points awarded

# Example: A Random Tournament



# Example: A Random Tournament

---

- ◆ For a team to win in round  $r$ , it must win  $r$  games in a row! This has probability  $1/2^r$
- ◆ In round  $r$  of a  $k$  round tournament, there are  $2^{k-r}$  games
- ◆ The expected number of games called correctly in round  $r$  is thus  $2^k/4^r$
- ◆ The expected fraction of correct calls goes to  $1/3$  (geometric series in  $1/4$ )!
- ◆ Variance (# correct) goes to mean / 1.05

# Possible Objectives

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- ◆ Maximize expected winnings (money is everything)
  - Need to worry about other players

# Possible Objectives

---

- ◆ Maximize expected winnings (money is everything)
  - Need to worry about other players
- ◆ Maximize expected number of points in the pool (we want to be right!)
  - Does not depend on the behavior of others
  - Does depend on probability assessments for game outcomes, and pool rules for points

# Example: 2001 Euro Final Four

Row Beats Column With Probability:	ПаваНайк	EFES Pilsen	ЦСКА	מכבי
ПаваНайк	--	0.6	0.6	1
EFES Pilsen	0.4	--	0.6	1
ЦСКА	0.4	0.4	--	1
מכבי	0	0	0	--

- ◆ Draw: Пава/Пilsen, מכבי/ЦСКА, winners
- ◆ Who should you pick to maximize the expected number of wins?

# Example: 2001 Euro Final Four

Row Beats Column With Probability:	Παναθηναϊκός 1908	ΕΦΕΣ Pilsen	Ε.Π.Ο. Ολυμπιακός	ЦСКА Москва
Παναθηναϊκός 1908	--	0.6	0.6	1
ΕΦΕΣ Pilsen	0.4	--	0.6	1
Ε.Π.Ο. Ολυμπιακός	0.4	0.4	--	1
ЦСКА Москва	0	0	0	--

- ◆ Clearly choose **μεταξιά** over ЦСКА
- ◆ Πανα or Pilsen?
- ◆ Who should you pick for the final?

# Example: 2001 Euro Final Four

---

- ◆ To maximize expected number of correct picks, pick **Пава** over Pilsen, **מכבי** over ЦСКА, and **מכבי** over **Пава** (expected wins:  $.6 + 1 + .4 = 2$ )
- ◆ Optimal to pick **מכבי** over **Пава**, *even though* **Пава** *beats* **מכבי** w.p. 0.6!
- ◆ Really picking **מכבי** to win final, *not* to beat **Пава**!
- ◆ Picking **Пава**, **מכבי**, **Пава** yields an expected  $.6 + 1 + .6 \times .6 = 1.96$  wins

# Example: Impact of Depth Points

---

Row Team Beats Column Team With Probability:

	A	B	C	D
A	--	0.5	0.6	0.2
B	0.5	--	0.7	0.2
C	0.4	0.3	--	0.6
D	0.8	0.8	0.4	--

- ◆ A plays B, C plays D, winners play
- ◆ What picks maximize  $E(\# \text{ wins})$ ?
- ◆ What picks maximize  $\Pr\{\text{Pick WINNER!}\}$ ?

# Example: Impact of Depth Points

Row Team Beats Column Team With Probability:

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C	0.4	0.3	--	0.6
D	0.8	0.8	0.4	--

- ◆ To maximize  $E(\# \text{ wins})$ , choose B-C-B (yields expectation of 1.35)
- ◆ Team D has highest chance (32%) of winning (pick A-D-D or B-D-D)

# Example: Impact of Depth Points

Row Team Beats Column Team With Probability:

	A	B	C	D
A	--	0.5	0.6	0.2
B	0.5	--	0.7	0.2
C	0.4	0.3	--	0.6
D	0.8	0.8	0.4	--

- ◆ Suppose you get 1 point for calling each of first two games,  $d$  points for championship
- ◆ If  $d < 20/7$ , pick B-C-B, otherwise pick A-D-D or B-D-D

# Optimizing in Real Time

- ◆ Let  $\mu_{rg}(i)$  denote the expected points through game  $g$  if team  $i$  to win that game
- ◆ Let  $\mu_{rg}^*$  denote the expected points optimally thru game  $g$
- ◆ Let  $\pi_{rg}(i)$  be the probability of correctly picking team  $i$  round  $r$
- ◆ Let  $\omega_{rg}(i) = \Pr\{i \text{ wins}$

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# Optimizing in Real Office Pools

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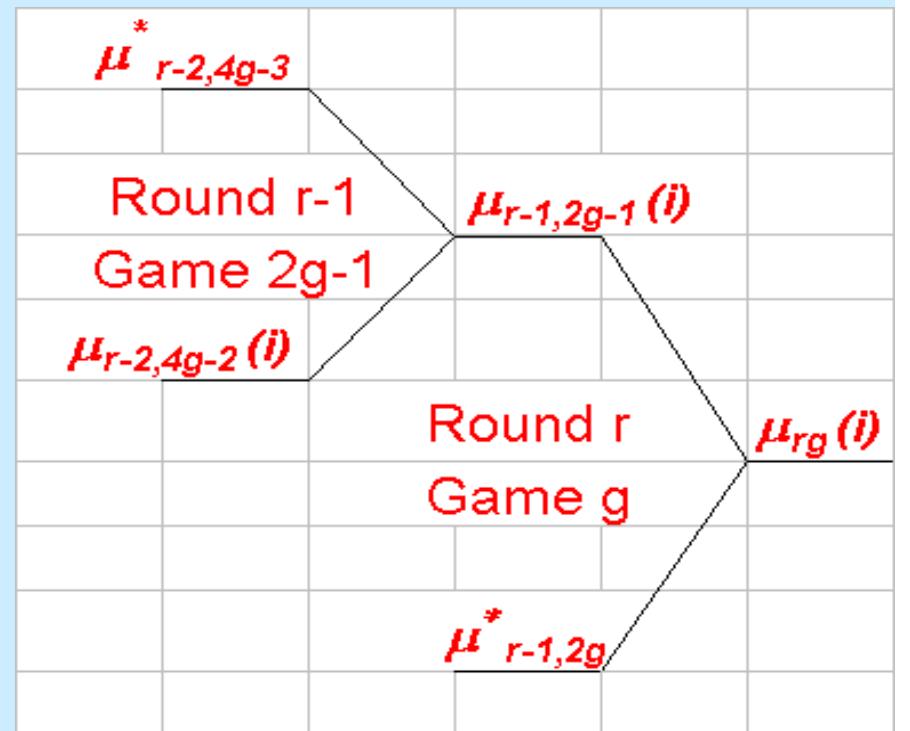
- ◆ Let  $\mu_{rg}(i)$  denote the expected number of points through game  $g$  in round  $r$  if choose team  $i$  to win that game and act optimally
- ◆ Let  $\mu_{rg}^*$  denote the expected points gained optimally thru game  $g$  in round  $r$
- ◆ Let  $\pi_{rg}(i)$  be the points awarded for correctly picking team  $i$  to win game  $g$ , round  $r$
- ◆ Let  $\omega_{rg}(i) = \Pr\{i \text{ wins game } g, \text{ round } r\}$

# Optimizing in Real Office Pools

$$\mu_{rg}(i) = \mu_{r-1,2g-1}(i) + \mu_{r-1,2g}^* + \pi_{rg}(i)\omega_{rg}(i) \quad \text{if } i \text{ plays in } r-1,2g-1$$

$$\mu_{rg}(i) = \mu_{r-1,2g-1}^* + \mu_{r-1,2g}(i) + \pi_{rg}(i)\omega_{rg}(i) \quad \text{if } i \text{ plays in } r-1,2g$$

$$\mu_{rg}^* = \max_{i \in \tau(r,g)} \mu_{rg}(i)$$



# Markov Models for College Basketball

---

- ◆ We need to compute the probabilities that any team  $i$  wins game  $g$  in round  $r$  ( $\omega_{rg}(i)$ )
- ◆ We do this via Markov models
- ◆ Markov assumption: in any game between two teams  $i$  and  $j$ , the probability that  $i$  beats  $j$  equals  $p_{ij}$  independently of other games

$$\omega_{rg}(i) = \omega_{r-1,2g-1}(i) \sum_{\ell \in \phi(i,r)} \omega_{r-1,2g}(\ell) p_{i\ell}$$

# Markov Models for College Basketball

---

- ◆ We consider three models:
  - regular season record
  - expert ratings (Sagarin ratings)
  - Las Vegas Odds

# Regular Season Model

---

- ◆  $p_{ij} = s_i / (s_i + s_j)$  (Bradley-Terry model, or model of *quasi-independence*)
- ◆ Estimate the parameters via (constrained) maximum likelihood based on regular season (and conference tournament) results
- ◆ Use 64 NCAA teams, 32 NIT teams, and all others lumped into *megateam* (so 96 free parameters)
- ◆ Provides connectivity among all teams

## 3 Team Example

---

- ◆  $a$  plays  $b$ ,  $c$  plays  $b$ , but  $a$  and  $c$  never play
- ◆  $f_{ab}$  and  $f_{cb}$  are respectively the observed fraction of time  $a$  and  $c$  beat  $b$
- ◆ Solve 3 equations:

$$\frac{s_a}{s_a + s_b} = f_{ab}, \frac{s_c}{s_b + s_c} = f_{cb}, s_a + s_b + s_c = 1$$

- ◆ Implied  $s_a$  depends on performance of  $c$ !

$$s_a = \frac{f_{ab}(1 - f_{cb})}{1 - f_{ab}f_{cb}}$$

# Sagarin Ratings

---

- ◆ Sagarin provides estimates of scoring rates  $\lambda_i$  for all Division I NCAA teams
- ◆ We assume *uncorrelated Poisson scoring!*
- ◆ Let  $X_{ij}$  be the point spread. Under the model,  $X_{ij}$  is approximately normal with mean  $\lambda_i - \lambda_j$  and variance  $\lambda_i + \lambda_j$
- ◆  $p_{ij} = \Pr\{X_{ij} > 0\}$

# Las Vegas Odds

---

- ◆ Las Vegas takes bets on NCAA games
- ◆ Two types of bets: point spreads and point totals (“over/under” bets)
- ◆ If market is correct, then can directly estimate scoring rates from quoted point spreads and point totals, and use uncorrelated Poisson model
- ◆ Implies that the actual point spread that will occur is normally distributed with mean given by Vegas point spread line, and variance given by Vegas over/under line!

# Tournament by Tournament

---

	NCAA 1999	NCAA 1998	NIT 1999	NIT 1998
	Actual/Expected/Standard Deviation of Wins			
Chance	-- / 21.3 / 4.5	-- / 21.3 / 4.5	-- / 10.7 / 3.2	-- / 10.7 / 3.2
Pick the Seeds	36 / -- / --	39 / -- / --	17 / -- / --	13 / -- / --
Regular Season	39 / 42.6 / 4.3	37 / 43.5 / 4.1	17 / 12.9 / 3.4	18 / 15.0 / 3.4
Sagarin	41 / 41.4 / 4.2	39 / 38.5 / 4.5	15 / 13.1 / 3.4	13 / 13.0 / 3.4
Las Vegas Odds	38 / 44.6 / 3.8	35 / 45.1 / 4.3	22 / 17.5 / 3.7	15 / 19.2 / 3.7

# Packard Pool

---

- ◆ Run by a math professor at Mesa State U.
- ◆ Points awarded if correctly call  $n^{th}$  seed to win in round  $r$  given by  $(1+n)\rho(r)/2$  where  $\rho(r)$  increases nonlinearly as  $r$  goes from 1 thru 6 ( $\rho(1) = 1,890$  while  $\rho(6) = 27,720$ )
- ◆ How would we have done?

# The Packard Pool!

---

	NCAA 1999	NCAA 1998
	Actual/Expected/Standard Deviation of Total Score	Actual/Expected/Standard Deviation of Total Score
Pick the Seeds	239,256 / -- / --	233,702 / -- / --
Regular Season	274,956 / 302,964 / 47,293	356,969 / 304,181 / 54,213
Sagarin	405,421 / 299,062 / 55,738	274,572 / 273,333 / 63,974
Las Vegas Odds	299,942 / 339,887 / 62,735	213,285 / 514,516 / 138,543

- ◆ Winner of 1999 pool scored 333,572 points
- ◆ Our Sagarin model would have won!
- ◆ Winner of 1998 pool scored 218,545 points
- ◆ Our Regular Season and Sagarin models would have won (as would picking the seeds)!

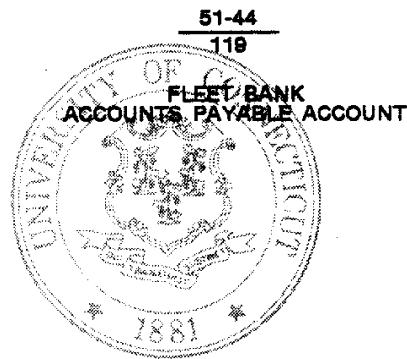
# 2000 NCAA Tournament Results

		CBS Sportsline Pool	Packard #2 Pool	ESPN Pool
<b># Participants</b>		95,000	40	580,000
<b>Upset Points?</b>		Yes	Yes	No
	Percentile	99.97	92.5	71.3
<b>Massey Ratings</b>	$x / \mu / \sigma$	426 / 239 / 76	341,807 / 275,271 / 59,876	780 / 799 / 203
	# Wins	39	38	37
	Percentile	83.1	90	61.9
<b>Sagarin Ratings</b>	$x / \mu / \sigma$	228 / 241 / 105	304,304 / 268,067 / 66,167	710 / 738 / 193
	# Wins	29	38	39
	Percentile	40.2	70	32.5
<b>Regular Season</b>	$x / \mu / \sigma$	173 / 248 / 93	262,872 / 277,201 / 76,480	530 / 713 / 185
	# Wins	37	31	
	Percentile	95.8	80	28.4
<b>Las Vegas Odds</b>	$x / \mu / \sigma$	263 / 258 / 54	281,388 / 315,024 / 51,254	510 / 1,027 / 196
	# Wins	40	40	39

# So, Did We Make Any Money?

UNIVERSITY OF  
**CONNECTICUT**  
STORRS, CONNECTICUT 06269

PAY *Two hundred fifty and 00/100 Dollars*  
TO THE  
ORDER  
OF EDWARD H. KAPLAN  
YALE SCHOOL OF MANAGEMENT  
PO BOX 208200  
NEW HAVEN CT 06520-8200



51-44  
119

FLEET BANK  
ACCOUNTS PAYABLE ACCOUNT

CHECK NO. 15-645073

CHECK DATE

03/26/00

CHECK AMOUNT

\*\*\*\*\*\$250.00

VOID AFTER 120 DAYS

SECOND SIGNATURE REQUIRED ABOVE \$9,999.99

A handwritten signature in black ink, appearing to read "D. Kaplan".

AUTHORIZED SIGNATURE

THE BACK OF THIS DOCUMENT CONTAINS AN ARTIFICIAL WATERMARK - HOLD AT AN ANGLE TO VIEW

15645073 10119004451 6772 8052

# How About 2001?

---

- ◆ Tried to predict *women's* tournament
- ◆ After first two rounds, the model correctly picked 42 of 48 games, beating 99.7% of 60,831 entries in *ESPN* pool
- ◆ But then...
- ◆ Of remaining 15 games, model got only 4 correct, ending up at the 48.3%-ile

# 2001 Men's Tournament

---

- ◆ Highlights: beat 71.3% in the 600,000+ *ESPN* pool
- ◆ Came in 3<sup>rd</sup> place!!!...
- ◆ ...in the Yale School of Management pool, winning \$17.50
- ◆ And, Stan and I won *four T-shirts* in the Fantasy Cup Tournament...
- ◆ ...for coming in 180<sup>th</sup> out of 1200+
- ◆ The winner got \$5,000

# Web Implementation

---

- ◆ Tom Adams’ “poologic” calculator at

<http://www.poologic.com/>

SO.....

---

Wanna dance??????

