Yale SCHOOL *of* MANAGEMENT

YALE UNIVERSITY
School of Public Health

YALE UNIVERSITY
School of Engineering and
Applied Science

Football State Spaces and Decision Making

Edward H. Kaplan

William N and Marie A Beach Professor of Operations Research
Yale School of Management

Professor of Public Health
Yale School of Public Health

Professor of Engineering
Yale School of Engineering and Applied Science
New Haven, Connecticut

Football State Space

- What's needed for football state space?
  - Score differential
  - Down
  - Yards to go
  - Field position
  - Quarter
  - Time on clock
- Football data compared to baseball?
  - 16 game season/team vs 162/team for baseball

Simplification: Infinite Football

- Forget clock (!), quarter – imagine infinite game
- Keep track of current down and yards to go
- Idea – establish value for field position, down and yards to go
- Value is expected margin of victory in infinite game from current state on

Example: 3 State Football

- Suppose field is G---1---2---3---G
- Get ball at 1 and run a play
- With probability ½, get first down at 2; otherwise other team gets ball at 1
- If have ball at 2, wp ½ get first down at 3; otherwise other team gets ball at 2
- If have ball at 3, wp ½ score and get 1 point, otherwise other team gets ball at 3
- Same rules for other team in other direction

What Is Field Position Worth?

Let v_i be value of my having the ball at position i, $i = 1, 2, 3$

Note that if you have ball at position i, worth $-v_{4-i}$ to me (why?)

What Is Field Position Worth?

Let v_i be value of my having the ball at position i, $i = 1, 2, 3$

Note that if you have ball at position i, worth $-v_{4-i}$ to me (why?)

So, we have three equations:

$$v_1 = \frac{1}{2}v_2 + \frac{1}{2}(-v_3)$$

If I get first down, I get value of position 2, but if I turn it over, you get the ball at position 1, which is worth $-v_3$ to me (you are knocking on my goal!)

What Is Field Position Worth?

Let v_i be value of my having the ball at position i, $i = 1, 2, 3$

Note that if you have ball at position i, worth $-v_{4-i}$ to me (why?)

So, we have three equations:

$$v_1 = \frac{1}{2}v_2 + \frac{1}{2}(-v_3)$$

If I get first down, I get value of position 2, but if I turn it over, you get the ball at position 1, which is worth $-v_3$ to me (you are knocking on my goal!)

Second equation:

$$v_2 = \frac{1}{2}v_3 - \frac{1}{2}v_2$$

(same logic as above)

What Is Field Position Worth?

Let v_i be value of my having the ball at position i, $i = 1, 2, 3$

Note that if you have ball at position i, worth $-v_{4-i}$ to me (why?)

So, we have three equations:

$$v_1 = \frac{1}{2}v_2 + \frac{1}{2}(-v_3)$$

If I get first down, I get value of position 2, but if I turn it over, you get the ball at position 1, which is worth $-v_3$ to me (you are knocking on my goal!)

Second equation:

$$v_2 = \frac{1}{2}v_3 - \frac{1}{2}v_2$$

(same logic as above)

Third equation:

$$v_3 = \frac{1}{2}(1 - v_1) - \frac{1}{2}v_1$$

(if I score, I get a point and you get the ball at position 3, which is worth $-v_1$ to me; if I turn it over, you get the ball at position 3 but I don't score).

What Is Field Position Worth?

Let v_i be value of my having the ball at position i, $i = 1, 2, 3$

Note that if you have ball at position i, worth $-v_{4-i}$ to me (why?)

So, we have three equations:

$$v_1 = \frac{1}{2}v_2 + \frac{1}{2}(-v_3)$$

If I get first down, I get value of position 2, but if I turn it over, you get the ball at position 1, which is worth $-v_3$ to me (you are knocking on my goal!)

Second equation:

$$v_2 = \frac{1}{2}v_3 - \frac{1}{2}v_2$$

(same logic as above)

Third equation:

$$v_3 = \frac{1}{2}(1 - v_1) - \frac{1}{2}v_1$$

(if I score, I get a point and you get the ball at position 3, which is worth $-v_1$ to me; if I turn it over, you get the ball at position 3 but I don't score).

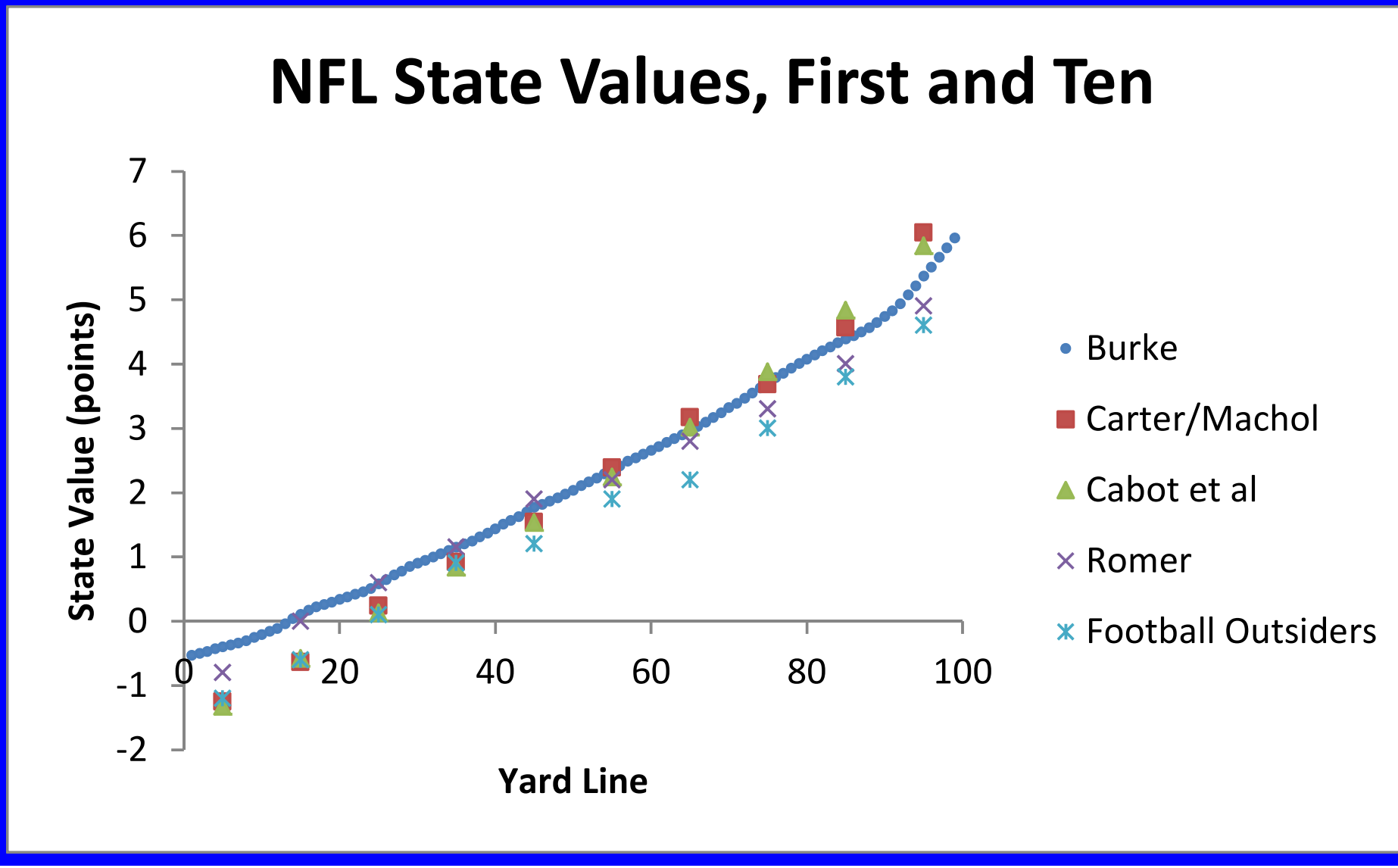
Solution to equations:

$$v_1 = -\frac{1}{4}; v_2 = \frac{1}{4}; v_3 = \frac{3}{4}$$

5 State Football

- See pp. 141-142 in the Mathletics text
- Only difference – now 5 states instead of 3
- Score 7 points instead of 1
- Example yields incremental value of 3.5 points for each advance in field position
- Note that 3 state model gave value of ½ for each advance in field position

How About 100 Yard, 4 Down Infinite Football?



Intermediate Football State Space

- Now the idea is to define a state space based on:
  - Down
  - Yards to go
  - Field Position
- Ignores the current score differential and the clock (quarter and time until it ends)
- Idea is to establish the value of field position
- Will bring time back later

Empirical Approach To Value Of Field Position

- Want a way to work with empirical data to save trouble of fancy model
- In baseball, got rid of dependence on time (inning) and point spread by focusing on runs to end of inning
- Football equivalent is to focus on *points to end of drive* (offensive possession) given *down, yards to go, and field position* (i.e. given state)
- Complete analysis courtesy of Keith Goldner at http://drivebyfootball.com

Expected Points To End Of Drive

- Ignore clock, quarter, score
- Focus on down, yards to go, field position
- Ask what happens at end of drive
- 8 ways drive ends (ignoring end half/game):

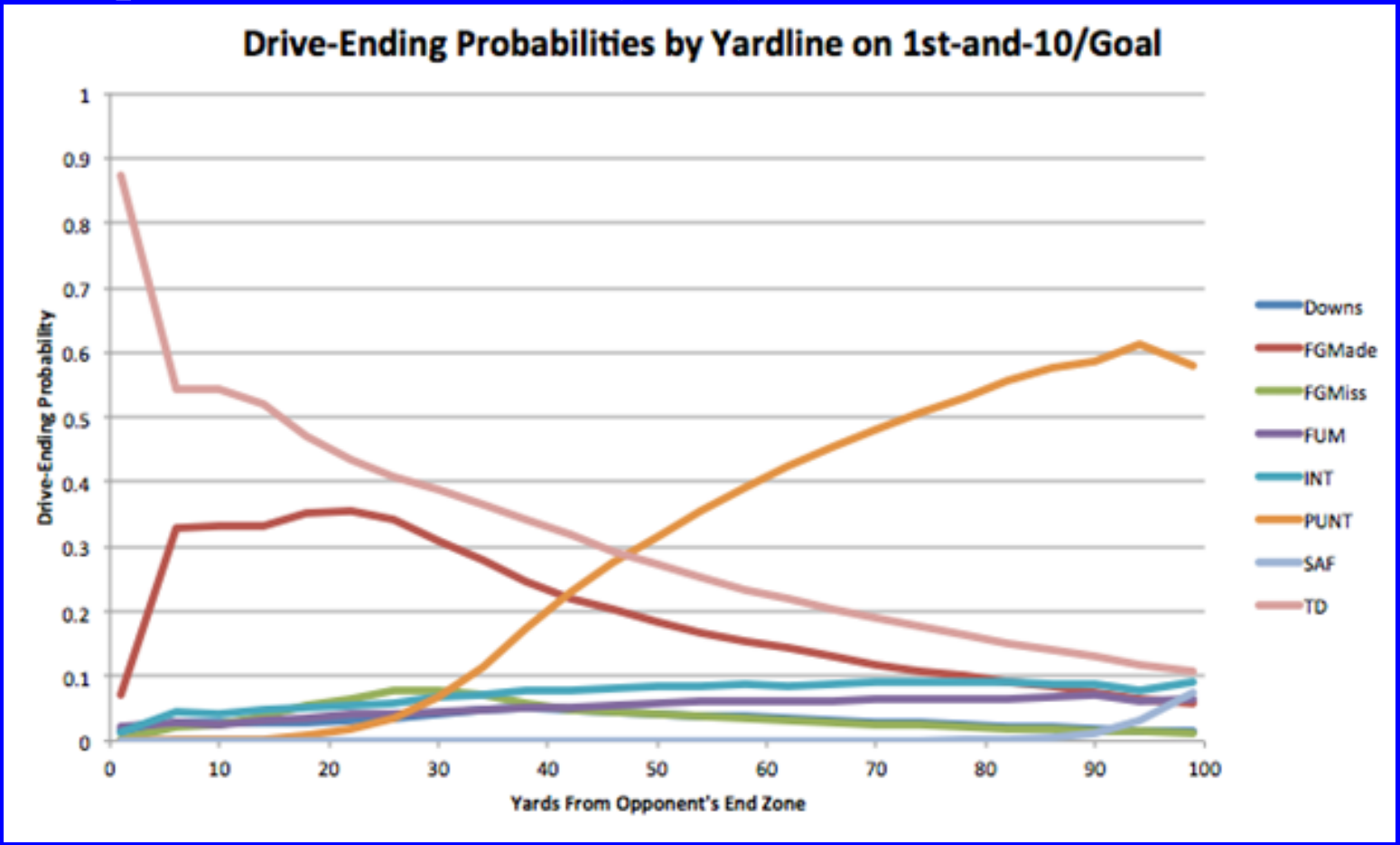
Turnover on downs	Punt
Made Field Goal	Missed Field Goal
Fumble	Interception
Touchdown	Safety

Drive By Football

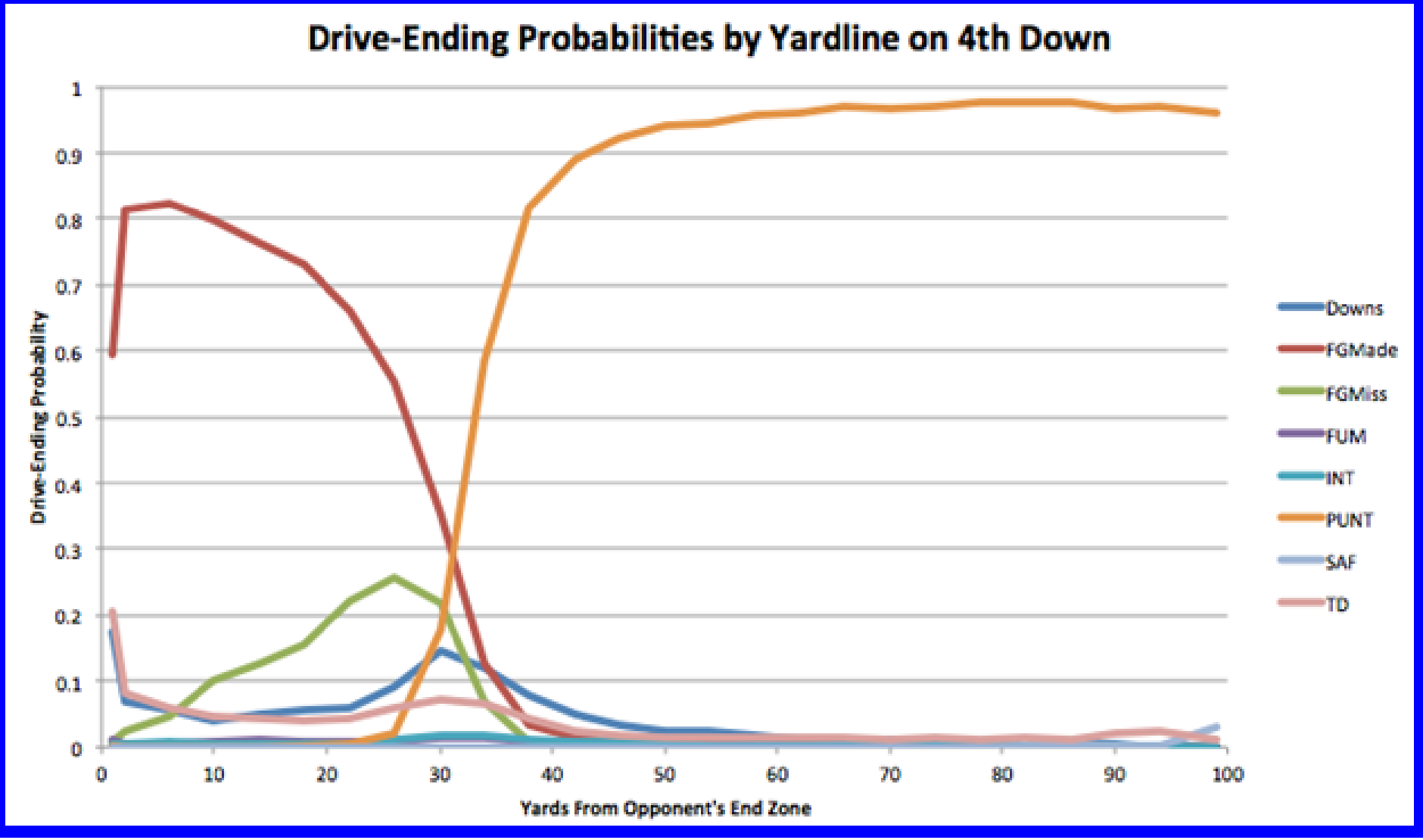
- So, by looking at 430,412 plays from 2000-11 (ignoring clock-ending drives), can see how many drives were in given *down*, *yards to go*, *field position* at some point (that is, how many observations per state)
- Count how many of these drives ended in turnover on downs, punt, field goal, touchdown, etc.
- Let's look at the data!!

Absorption Probabilities

- For each state, can calculate fraction of drives that end in each possible outcome (data)



Fourth Down Absorption Probabilities



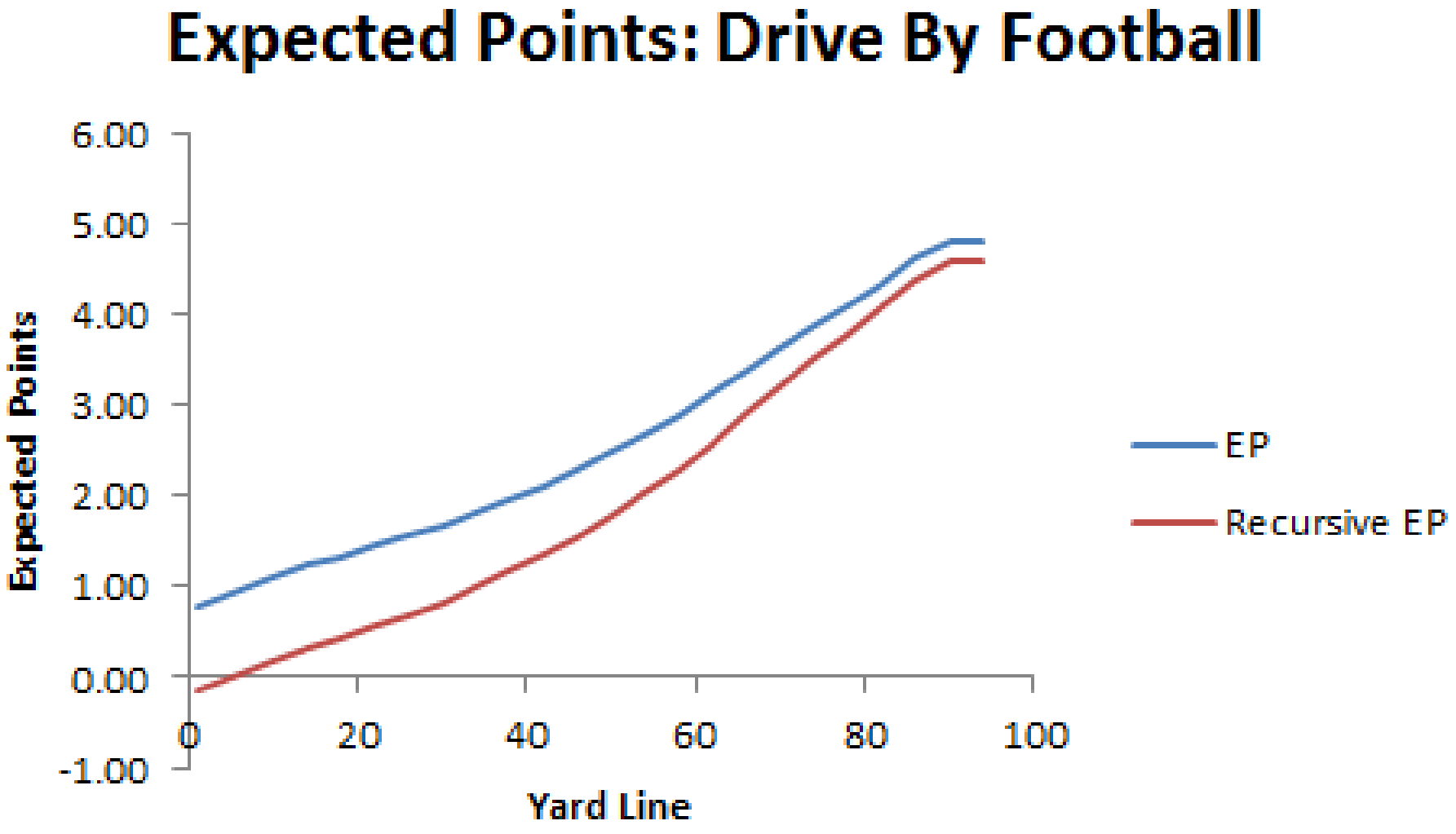
How About Expected Points?

- Simplest idea – only count points on current drive; ignore turnovers, punts

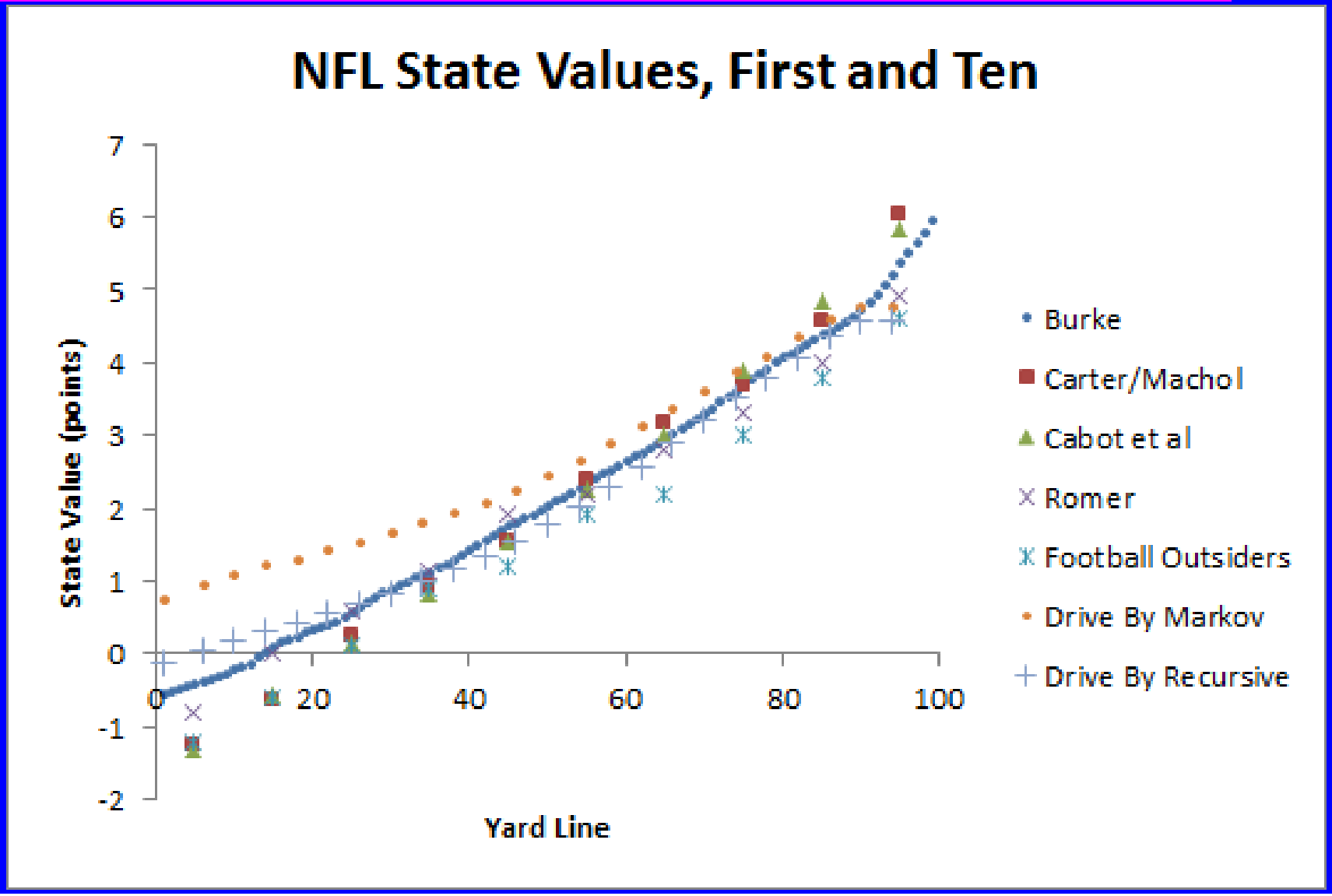
  Expected Points = 3 x Pr{Field Goal}
  +7 x Pr{Touchdown}
  – 2 x Pr{Safety}

- Account for turnovers/punts by looking at opposing field position afterwards and subtracting expected points for other team (*recursive* expected points)
- Same sort of equation we used for run value of baseball states (*v*'s)

Drive By Football Expected Points



How Do These Compare To The Other State Space Models?



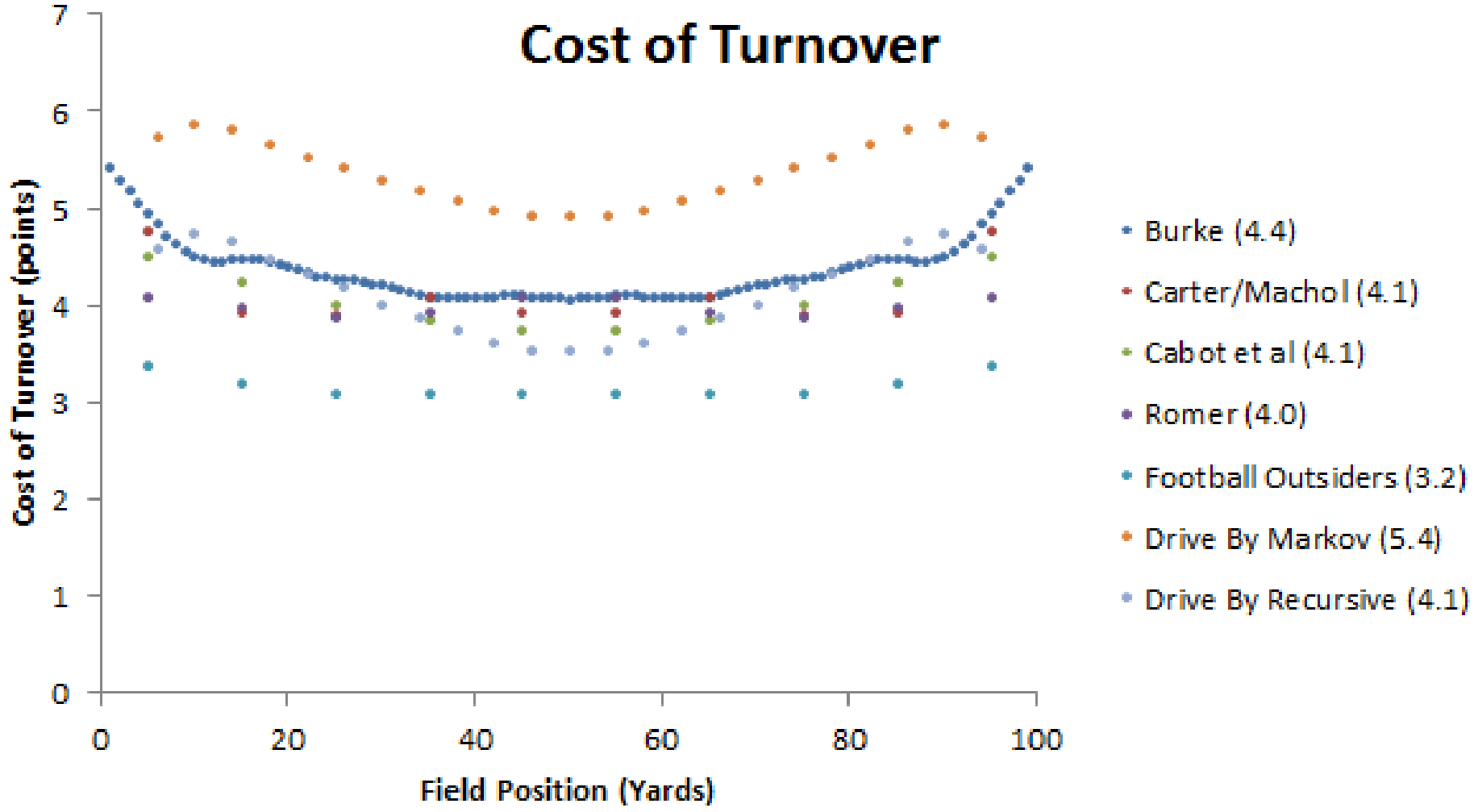
How To Value Turnovers

- Let $v(x)$ be the value of having the ball at field position x (0 is your goal line, 100 is opponent goal line)
- If turn the ball over at position x, you lose $v(x)$ while your opponent gains $v(100 - x)$
- Value added of a turnover to you is found from looking at your value at end of play ($-v(100 - x)$) and subtracting your value at the start ($v(x)$); Cost of Turnover $c(x)$ is negative of this
- $c(x) = v(100 - x) + v(x)$

The Cost of a Turnover is Symmetric in Field Position

- *Theorem*: The cost of a turnover $c(x)$ is a symmetric function of field position (x), that is, $c(x) = c(100 - x)$
- *Proof*: since $c(x) = v(100 - x) + v(x)$, then
$$c(100 - x) = v(100 - (100 - x)) + v(100 - x)$$
$$= v(x) + v(100 - x) = c(x)$$
- Let's check out some $c(x)$ functions for the state space models seen earlier

Estimated Cost of Turnovers



How To Use For Football Decision Making?

- Easy! Whenever you need to make a decision (e.g. Go for one point or two? Gamble on 4^{th} down or punt?), choose the option with the highest expected points!

Here's Another 4th and Belichick!

COMMENTARY

Debatable decision costs Patriots

They have been strong closers in the past

By **Mike Reiss** | ESPNBoston.com **Updated:** November 16, 2009, 6:59 PM ET

INDIANAPOLIS -- It will be the fourth-down call talked about across New England, a questionable decision by Patriots coach Bill Belichick that resulted in one of the team's most heartbreaking defeats in recent memory.

Holding a 34-28 lead with 2:08 remaining, why go for it on fourth-and-2 from their 28-yard line?

http://sports.espn.go.com/boston/columns/story?columnist=reiss_mike&id=4659027

Play failed, Colts scored and won game!

Salient Facts/Assumptions

- Pats have ball at own 28, 4th down, 2 to go
- Could punt the ball; assume Colts get ball at *their* 30 yard line
- Could go for first down
  - If successful, gain 1st and 10 at 30 yard line
  - If fail, Colts get 1st and 10 in same place
- Checking out Drive By Football results using EP:
  - If punt, Colts EP=1.67 (so –1.67 for Pats)
  - If gamble and successful, Pats EP = 1.67
  - If gamble and fail, Colts EP=3.63 (so –3.63 for Pats)

When Should Pats Gamble?

- Let f = probability of gaining 1st down
- Gamble if EP(gamble) > EP(punt), that is if

$$f \times 1.67 + (1-f) \times (-3.63) > -1.67$$

- Solves to $f > 0.37$
- Turns out the 4th and 2 is successful about 50% of the time, so suggests going for it was the right decision!
- BUT: EP ignores value loss from turnovers
- Should that make gambling better or worse?

When Should Pats Gamble? (Recursive Expected Points)

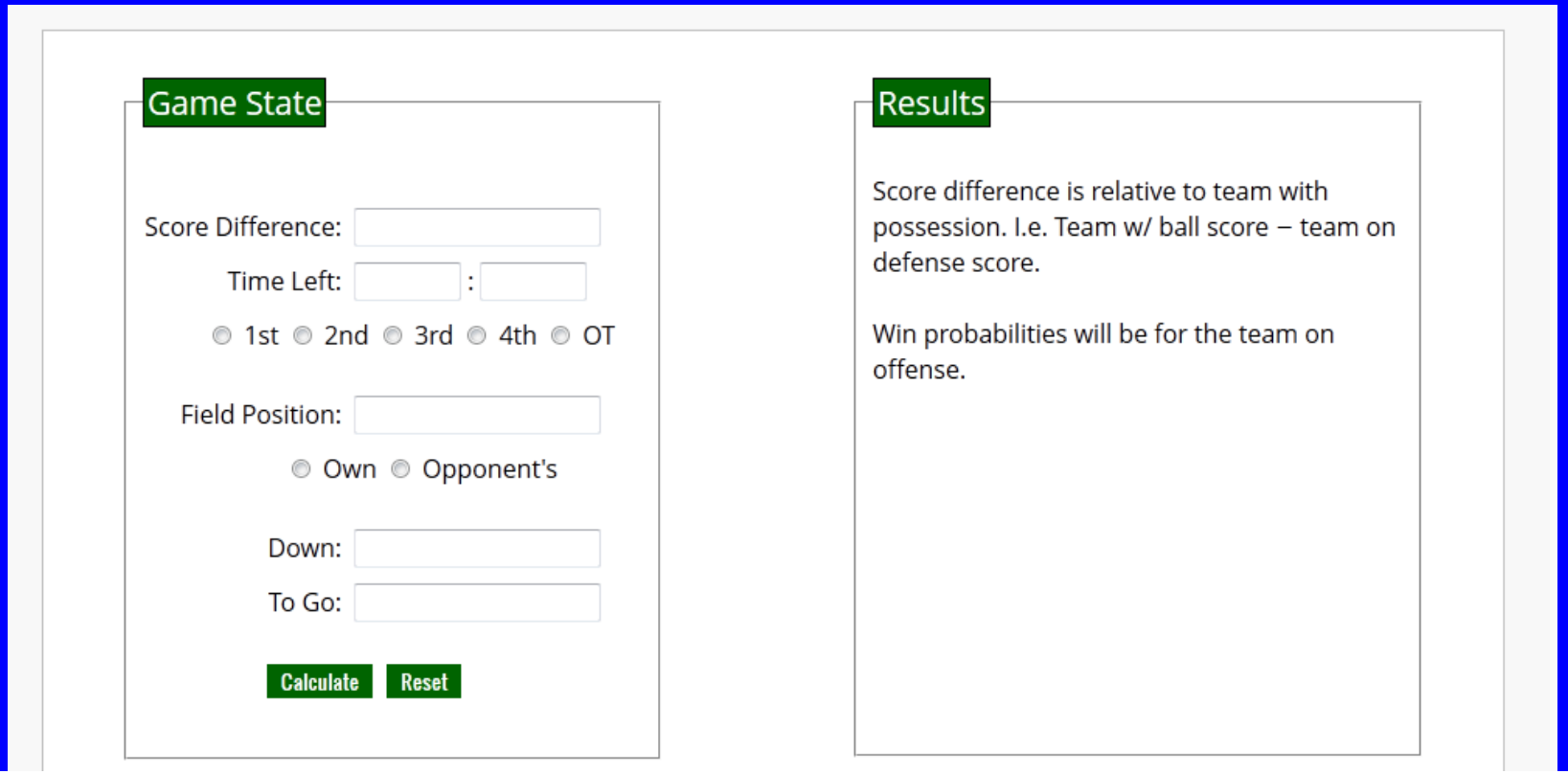
- Let f = probability of gaining 1st down
- Gamble if REP(gamble) > REP(punt), that is if

  $$f \times 0.82 + (1-f) \times (-3.2) > -0.82$$

- Solves to $f > 0.59$
- Turns out the 4th and 2 is successful about 50% of the time, so suggests going for it was the *wrong* decision!
- BUT: Pats were up 6, only 2:08 minutes left!
- Should that make gambling better or worse?

Bring Back The Clock!

- Advancednflstats.com
- Beyond infinite football – try to sew up model empirically by conditioning on how much time is left in a quarter



Back to Pats vs Colts

- Use advancednflstats to find:
- Pr{Colts win|down 6, own 30, 2 mins left, 1st and 10} = 0.26 – so Pats win wp .74
- Pr{Pats win|up 6, own 30, 2 mins left, 1st and 10} = 0.94
- Pr{Colts win |down 6, Pats 30, 2 mins left, 1st and 10} = 0.31 – so Pats win wp .69

So Using Win Probability…

- Gamble if Pr{Win|Gamble}>Pr{Win|Punt}
- This will be true when:

  f x 0.94 + (1–f) x 0.69 > 0.74
- Solves to f > 0.20 – so should be even *more* willing to gamble

4th Down Calculator

- Advancednflstats.com has automated 4th down decisions involving the choice of punting, going for 1st down, or kicking field goal with the object of maximizing win probability!
- Let's take a look…

One More Belichick…

- Super Bowl, Feb 3, 2008 Patriots vs Giants
- "Early in the third quarter, Belichick successfully challenged a non-call, and a review showed that, indeed, linebacker Chase Blackburn was the 12th man on the field for a punt. New England retained possession, but a Strahan sack eventually led Belichick to pass on a 49-yard field goal attempt." http://espn.go.com/nfl/recap?gameId=280203017

Salient Facts

- 3rd Quarter, 6:49 left, Pats lead 7-3, have the ball 4th and 13 at the Giants 31 yard line
- Try for field goal, or go for it?
- Pats went for 1st down and failed
- What does the 4th down calculator say?

Doesn’t Really Matter What They Did!

- Using the 4th down calculator:

Stat	Go4it	Punt	FG Att
Success Rate:	0.23	-	0.60
WP Success	0.80	0.70	0.76
WP Fail:	0.65	-	0.63
WP Total:	0.68	0.70	0.71
Break-Even:	0.38		

Touchdown! 1 Point or 2?

- Sept. 14, 2008, Mile High, Denver, CO
- "With 0:24 left in regulation, Denver scored a touchdown to trail 38-37 pending the try. Denver coach Mike Shanahan then made the remarkable decision to attempt a two-point conversion rather than kick the extra point." http://www.footballcommentary.com/analysis2008week2.htm
- It worked! Denver won, 39-38.
- Was it a smart move?

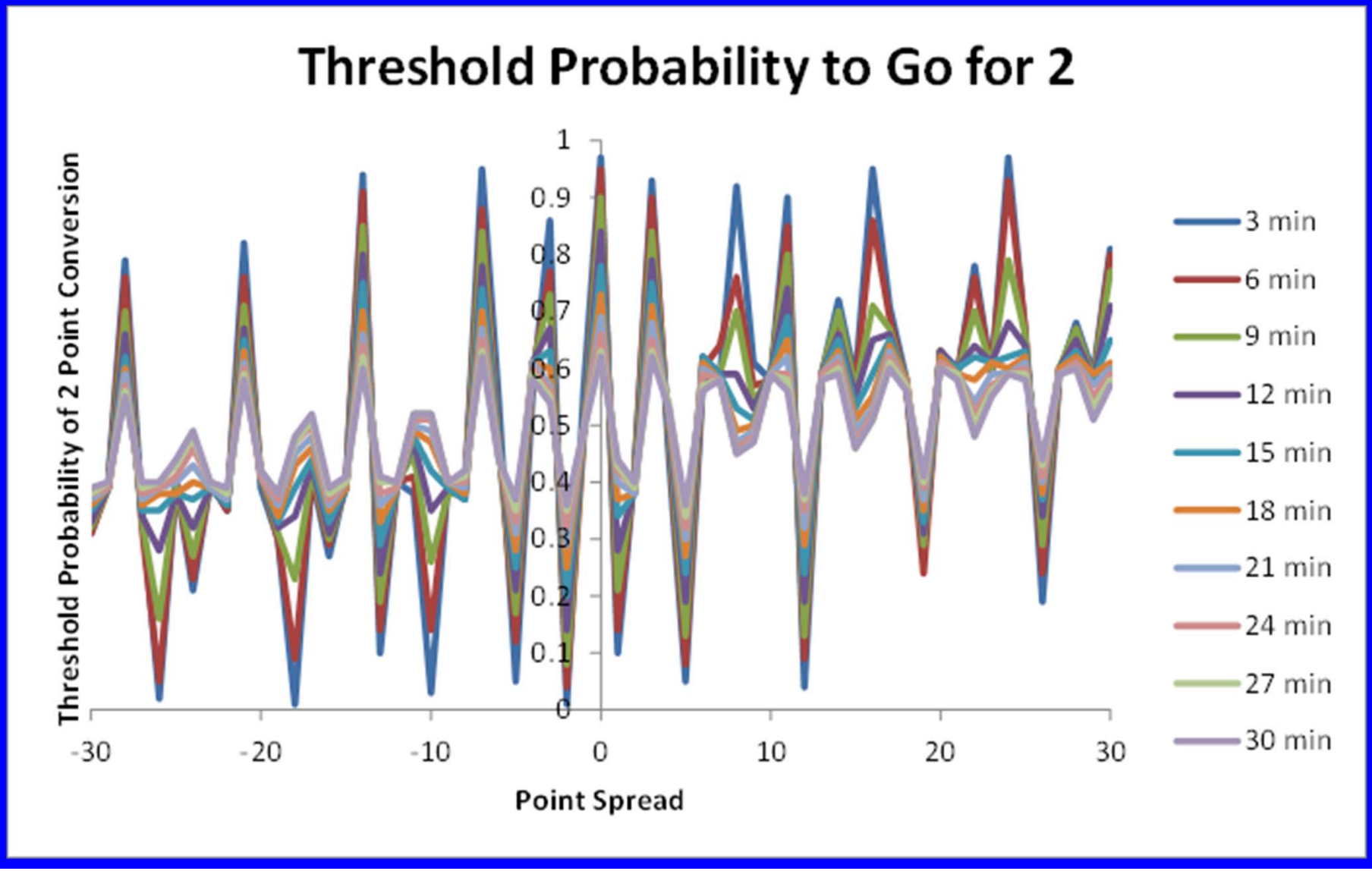
1 Point or 2?

- Fact: 2 point conversions have been successful about 48% of the time
  - (2000-2009 data, 708 “normal” 2 point conversions (ignore aborted kicks), 339 successes)
- If go to overtime, 50% chance of winning.
- Normal 1 point conversion successful 98.5% of time.
- Down 1, if go for tie, prob of winning =.985*.5 =.4925
- Prob of 2 point conversion (and win) = 0.48.
- Other factors: 24 seconds left, Chargers could still score off kickoff if Broncos were successful with 2 points; Broncos could get onside kick if 2 point not successful

Choice of 1 or 2 is Complicated!

- Complete analysis at footballcommentary.com (William Krasker)
- Taking complete account of score differential and time left in the game, Krasker found the threshold probability of success that would justify going for 2 points
- High threshold means bad idea to try; low threshold means good idea

Choice of 1 or 2 is Complicated!



Choice of 1 or 2 is Complicated!

- Ill-advised to go for 2 if, immediately after touchdown, trail by 28, 21, 14 or 7 points!
- Ill-advised to go for 2 if, immediately after touchdown, game is tied or lead by 3, 8, 11, 16, 24 or 30 points!
- Why???
- Hint – if decide to kick for single point, almost guaranteed that kick will be good
- Another hint – if behind, what must you do to catch up? If ahead, what must other team do?